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SURPLUS LABOUR, SYNCHRONISED LABOUR COSTS AND MARX'S LABOUR THEORY OF VALUE [1, 2]

SINCE the publication of Marx's systematic critique of Political Economy the basic analytic instrument of this critique, Marx's labour theory of value, has itself been the object of innumerable criticisms. However, while only a small group of critics has understood the formal definition of values,[3] the *purpose* of Marx's value analysis has quite generally been misunderstood. Among these misinterpretations there are typically three: the identification of the labour theory of value with the apparently mistaken statement that purposive human activity (labour) is the ultimate source of all material wealth (use value); next, the interpretation of it as a natural right theory of just price and finally its interpretation as a theory of competitive equilibrium prices. This last interpretation has been the basis of Böhm-Bawerk's fundamental criticism (1926) and still is representative of most of the modern criticism.[4] The list of criticisms of Marx's labour theory of value has however been enlarged quite recently by Samuelson and v. Weizsäcker (1971, a, b). Seemingly in contrast to the tradition of refutation, these economists claim to have *generalised* Marx's theory of value and exploitation by substituting for labour values their concept of synchronised labour costs.

The discussion of serious criticism has always been a useful way of clarifying one's understanding of the theories under attack. In the following

[1] I am grateful to Bob Rowthorn, Bertram Schefold, Gerhard Sessler and C. C. von Weizsäcker for valuable discussions.

[2] Some of the notation is not explained in the text. I: identity matrix; e: unit vector; $\mathbf{1}: = \Sigma e^j$; if the vector a is non-negative we write $a \geqq 0$, if a is semi-positive $a \geqslant 0$, if a is strictly positive $a \gg 0$.

[3] We refer here to those who maintain that the calculation of labour values, as of Böhm-Bawerk's average period of production, required an infinite regress in history. Another misunderstanding is confusion of prices in terms of the commodity labour power with values. This confusion has been common ever since the appearance of Böhm-Bawerk's interpretation. On these lines, even J. Robinson introduced Marx's values as being rather eccentric kinds of prices (in terms of the commodity labour power), based upon the assumption of equal ratios of profit to wages, rather than of equal rates of profit. The competitive set of prices, therefore, is considered as a correction of Marx Vol. I, *cf.*, J. Robinson (1966, ch. 2). It is perhaps interesting to note that the same misunderstanding can be found in J. Robinson's *Introduction to the Second Edition* (1966): " There is no reason to postulate any tendency for the rates of exploitation to be equalised so as to make prices proportional to values " (*op. cit.*, p. xi).

[4] Taking notice neither of the serious attack on Böhm–Bawerk's interpretation by Hilferding (1904) and Petry (1916), nor of v. Bortkiewicz's correction of the deficiencies in Marx's solution of the transformation problem (1906; 1907); even Böhm–Bawerk's polemic imputation of a " great contradiction " between Marx's own analysis in terms of values in *Das Kapital* Vol. I and the production price model in Vol. III has been commonly quoted in economic textbooks. So it is only since the rediscovery and refinement of v. Bortkiewicz's contribution by Sraffa (1960), Schwartz (1961, lecture 2) and Seton (1956), that the purpose of value analysis, and so of the transformation problem has been clearly visible.

tentative discussion of Marx's value analysis and the purpose behind it, we shall therefore explicitly refer to the recent criticism. Though, as it will be argued, Samuelson and v. Weizsäcker have missed the point by introducing the *dual* to Böhm-Bawerk's misinterpretation, their writings do suggest the need for some clarification, both of the concept of exploitation and of the law of value.

In the following paper we shall first of all introduce a careful distinction between physical relations of production and reproduction (*quantity system*), value analysis (*value system*) and exchange relationships (*price system*). These systems are, however, not considered as alternative realities, but as *different aspects* of the same capitalist system. Each of them is as real as the others, and each of them throws a light on a different aspect of this reality and, in this sense, is indispensable. On the basis of this distinction we shall try to reconstruct the purpose of Marx's value analysis. As a proof of its meaningfulness we shall consider the additional insights one can gain from its use. To simplify exposition and to be directly comparable with the work of Samuelson and v. Weizsäcker, we shall confine our analysis to situations of steady-state growth and, as far as a capitalist system is concerned, a competitive golden rule rate of profit. It has however to be emphasised that in capitalist economies the value system, like the two other systems, applies quite generally, and is by no means restricted to our peculiar equilibrium assumptions, nor to particular technological assumptions.

I. General Assumptions and Procedure

Consider an economy producing n goods in n different departments and using homogeneous labour everywhere and produced inputs. Every good is assumed to be a *basic* good, *i.e.*, to be required directly or indirectly as material input to the production of every good. The period of production is uniform and so taken as the time unit. There are no joint products. Means of production are therefore entirely used up during one production period. Techniques are given and constant over *time* and *scale*. For any good j $(=1, \ldots, n)$ they are defined by a semi-positive column vector $A^j \geqslant 0$ of material input requirements per unit of good j and the respective coefficient of direct labour requirements a_{0j} per unit of j. As regards the number of techniques available, for every good j we assume the existence of m_j efficient techniques. Let $S_j = \{1, 2, \ldots, m_j\}$ be the index-set of techniques available for the production of good j, then clearly the set of all available *production systems* is represented by the Cartesian product of the index-sets S_j, $S: = \prod_{j=1}^{n} S_j$. As a *production system*, represented by an element $\nu \in S$, we denote any technology matrix $A_\nu = (A^1, \ldots, A^n)$ and its respective vector of direct labour requirements $a_0(\nu)' = (a_{01}, \ldots, a_{0n})$, which allow the production of every commodity. The complete production system may be defined as

$$\hat{A} \partial= (A_1{}^1, \ldots, A_{m_1}{}^1; A_1{}^2, \ldots, A_{m_2}{}^2; \ldots; A_1{}^n, \ldots, A_{m_n}{}^n)$$
$$\hat{a}_0{}' \partial= (a_{01}(1), \ldots, a_{01}(m_1); \ldots; a_{0n}(1), \ldots, a_{0n}(m_n)),$$

and any *activity vector* as $x \partial= (x_j)$. Any technology matrix A_ν is therefore assumed to be semipositive and indecomposable.[1] For reasons that shall become clear later, it is called *viable* if one (and so all) of the statements in the last footnote hold. In what follows, consider $\nu \in S$ as viable and let physical units (of $\hat{A}$) be chosen such that the vector of row sums of A_ν is $A_\nu \mathbf{1} \ll \mathbf{1}$.[2]

As far as *commodity production* is concerned, we assume that commodities are exchanged at the end of every period of production and that markets are cleared at the competitive prices. Under *capitalist* commodity production labour power itself is a commodity and (in contrast to Marx) we assume that this particular commodity is paid after its use, at the end of the production period. Perfect competition also prevails in the labour market and therefore wages and the working day are equal for every worker. Only capitalists save.

Throughout the next three paragraphs we shall *introduce* the systems in order of decreasing generality. We therefore assume any production system $\nu \in S$ to be given. In this sense the analysis is *ex post* in character and $(A_\nu, a_0(\nu))$ can be considered as the production system *actually* used in the economy. After an explanation of Samuelson/v. Weizsäcker's *synchronised labour costs* on this level, we shall consider the *ex ante* choice of techniques in order to allow meaningful classification of the concept of synchronised labour costs as an instrument of optimal planning. This will enable us then to compare Marx's concept of exploitation with the supposed generalisation of Samuelson/v. Weizsäcker. Assuming globally labour-augmenting technical progress we shall finally discuss an extension of the model, the result of which the critics considered as inconsistent with a basic statement of Marx's

[1] For these matrices the following statements are equivalent:

(i) the dominant characteristic root of A, $\lambda^*(A)$, is positive and satisfies $\lambda^*(A) < 1$.
(ii) $(I - A)x = c$ has a non-negative solution $x \geqq 0$ for at least one given $c \gg 0$.
(iii) $(I - A)x = c$ has a non-negative solution for every $c \geqq 0$.
(iv) $(I - A)^{-1}$ exists and is strictly positive.
(v) $\sum_{k=0}^{\infty} (A)^k$ exists and is strictly positive.
(vi) a non-singular diagonal matrix $J > 0$ exists, such that $A^* \mathbf{1} \ll \mathbf{1}$ (where $A^* := J^{-1} A J$) is satisfied.

For these statements (and the proofs) *cf.* Lancaster (1968, p. 310), (Nikaido (1968) Ch. II) and (Schwartz (1961) lecture 2).

Also note, if any (and so every) of the above statements is true, then $\sum_{k=0}^{\infty} (A)^k = (I - A)^{-1}$ follows.

[2] Let A_ν be measured in any physical units. Then, if A_ν is viable, gross outputs λ_j exist, which, taken as the new physical units, guarantee the condition (vi) of the last footnote, if the λ_j's are taken as the Kronecker deltas of the diagonal matrix J.

own theory of exploitation. This statement, which we shall also consider as fundamental for Marx's theory, says, that a positive rate of exploitation is both necessary and sufficient for a positive rate of profit.

II. The Quantity System

Let us begin with the most general relationships of production and reproduction as they are specified by the production system in use,[1] (A, a_0), and the steady state equilibrium rate of growth $g \geqq 0$. This information gives us an impression of the division of labour within the society considered and the forces of production it has developed, but it cannot tell us anything about its specific social organisation of production and distribution. So, to sum up, production and reproduction are considered here as production and reproduction of use values (goods), or in the aspect of a *relationship between man and nature* (Marx).

1. *Production*

To produce any gross output vector x, sufficient means of production Ax have to be available and $L = a_0'x$ units of labour must be expended. This process of transforming goods through purposive human activity is called the *labour process* (Marx) and its existence is a general condition of human life. In order to be *viable* in the literal sense, the technology applied in this labour process has to allow a positive *net product* u

$$u = (I - A)\, x \qquad (1.1)$$

which generally will serve for current subsistence (whosoever consumes it) or as means of production for future growth. Production therefore always is production of net product u, and by this definition includes reproduction of the means of production used up.

Considered as an expenditure of human labour,[2] the production of every good j requires a definite amount of *socially necessary labour*, ω_j. Corresponding to a unit level of net product, this vector of socially necessary labour time per unit of the respective good is

$$\begin{aligned}\omega' &= a_0' + \omega' A \qquad (1.2)\\ &= a_0'(I - A)^{-1}\\ &= a_0' \sum_{k=0}^{\infty} (A)^k\end{aligned}$$

where a_0 is the vector of the respective direct labour inputs and $\omega' A$ the vector of labour inputs necessary to reproduce the respective means of production used up. *As technical change is excluded*, $\omega' A$ is also identical with the labour time historically performed for the production of these means of

[1] As long as the production system is given, we shall omit the index ν.

[2] As unproduced goods are excluded here, every material input can be reduced to the expenditure of labour. In the general case of several unproduced inputs, this procedure is obviously restricted to the produced inputs. This, however, does not make the calculation of values meaningless.

production used up. Therefore the labour time socially necessary to produce the *net* product u, as well as the labour time embodied in u, is equal to the labour time actually expended, L,

$$\begin{aligned} \omega' u &= \omega'(I - A)x \\ &= a_0' x = L \end{aligned} \qquad (1.3)$$

Evidently, the vector of socially necessary labour can be derived from the data of the production system (A, a_0) alone and is strictly positive, if and only if this production system is viable (see assumptions and footnote 1, p. 789).

2. *Reproduction*

Net product, it was said, always provides for current consumption, represented by the vector c, and/or for future growth. So, if reproduction is to be possible at an expanded scale, given by the overall growth rate $g > 0$, the vector gAx has to be taken out of u for means of production, so that $(1 + g)Ax$ means of production are available in the subsequent period

$$\begin{aligned} c &= u - gAx \\ &= (I - (1 + g)A)x \end{aligned} \qquad (1.4)$$

However, as the viability condition suggests, not every positive growth rate is feasible. Therefore, denoting the (positive) dominant eigenvalue of A by $\lambda^*(A)$ and the (strictly positive) associated eigenvector by x^*, the following relationships hold between non-negative c, g, x and L (for the equivalence of the viability assumption and $\lambda^*(A) < 1$, see footnote 1, p. 789:

if $c = (0)$: [1]

$$x^*\lambda^* = Ax^*, \quad x^* \gg 0; \quad \lambda^* = (1 + g^*)^{-1} \qquad (1.5)$$

$$g^* = \frac{1 - \lambda^*(A)}{\lambda^*(A)} > 0 \qquad (1.6)$$

if $c \geqslant 0$: [2]

$$g < g^* = \frac{1 - \lambda^*(A)}{\lambda^*(A)} \qquad (1.7)$$

$$(I - (1 + g)A)^{-1} = \sum_{k=0}^{\infty} (A)^k (1 + g)^k \gg 0 \qquad (1.8)$$

$$(I - (1 + g_0)A)^{-1} \gg (I - (1 + g_1)A)^{-1}, \text{ if } g_0 > g_1 \qquad (1.9)$$

$$x = (I - (1 + g)A)^{-1} c \gg 0 \qquad (1.10)$$

$$L = a'_0 x = a_0'(I - (1 + g)A)^{-1} c \qquad (1.11)$$

[1] *Cf.* Lancaster (1968), p. 310 and footnote 1, p. 789.

[2] From footnote 1, p. 789, we know that $(I - B)^{-1}$ exists, and is strictly positive, if and only if the dominant eigen-value of B is smaller than one, $\mu^*(B) < 1$. Now, define $B = (1 + g)A$ and apply the eigenvalue equation; $\det(\mu I - (1 + g)A) = 0$. Obviously, this equation is identical to $\det\left(\frac{\mu}{(1 + g)} I - A\right) = 0$. Therefore the following relationship holds: $\mu^*(B) = (1 + g)\lambda^*(A) < 1$, and the restriction on feasible growth rates, given in the text, is easily proved.

As (1.6) and (1.7) make clear, g^* is the maximal growth rate of the system: its attainment would require society to live on air and the original availability of an outfit Ax^* of means of production: it is therefore of no particular interest in itself. Given the appropriate amount of means of production, it does however help in formulating the restriction upon positive growth rates consistent with an arbitrary semi-positive vector of subsistence $c \geq 0$, (1.7), (1.11), and only the subsequent relationships will be of interest for us. So, if any given $c \geqslant 0$ is to be produced in the period considered, steady-state equilibrium growth at a feasible rate $g < g^*$ requires a definite composition and scale of gross output x and therefore a definite allocation and employment of labour. These requirements are indicated by (1.10) and (1.11). As a consequence, these equation systems will be labelled as physical equilibrium conditions of reproduction under steady-state growth. Finally (1.9), (1.10) and (1.11) together show how employment increases with g. This is what one would expect; in other words it confirms the existence of a trade-off between subsistence per labour unit and growth, if any $c \geq 0$ is treated as consumption-unit. Given the vector of subsistence per worker, therefore, the growth rate is determined. This leads to a social restriction upon feasible growth rates, which we shall not examine here.

3. *Surplus Product and Surplus Labour*

Let us now introduce the concepts of *surplus product* and *surplus labour*. In almost every society we can distinguish between one part of the net product which serves as subsistence for those who do the work and another part which serves for general social purposes. Accordingly we can divide the total labour time expended into two respective parts, *necessary labour* and *surplus labour*. Let $z \geqslant 0$ be the vector of means of subsistence for those who do the work ($c \geq z$) and gAx that part of the net product u which serves for future growth. Then the *surplus product* is defined as

$$\begin{aligned} u - z &= (I - A)x - z \qquad \ldots \qquad (1.12) \\ &= gAx + (c - z) \end{aligned}$$

and the amount of socially necessary labour to produce this surplus product will be called *surplus labour* m. Clearly surplus labour is the difference between total labour and the labour socially necessary to produce subsistence for those who work:

$$\begin{aligned} m &= \omega'(u - z) \qquad \ldots\ldots \qquad (1.13) \\ &= L - \omega' z = \omega' gAx + \omega'(c - z) \end{aligned}$$

So if $z = c$, surplus labour is that amount of labour which in the period considered has been spent to allow future growth. It is a surplus over what had been necessary to keep society in the same state of wealth.

4. *The Austrian Concept of Production*

We have so far defined production as production of u and therefore used a quite common concept of net product. This concept is however in contrast to that employed in the *Austrian* capital theory, which considers production as a process of *maturing* means of subsistence. Production in that theory is defined as production of means of subsistence and net product in this Austrian framework consists only of c and not of the whole of u. In other words, expansion of the means of production never counts as part of this net product.

Let us now analyse production from this point of view. To introduce some further Austrian categories, *stages of* production are defined as the time units which a bundle of goods still has to pass through on the way to maturity. Production structure is called *continuously staggered* (or, synchronised), if distribution of the working capital over the different stages guarantees steady-state equilibrium growth. So, if today the vector $c \geqslant 0$ of means of subsistence is to mature and if this vector is to grow in the future at a rate $g < g^*$, synchronisation requires in this period the employment of $a_0'c$ labour units in the stage of production *nearest* to maturity and the employment of $a_0'A(1+g)c$ labour units in the preceding stage, etc. The structure and scale of the commodity bundles actually in work in the different stages of production therefore is given by the sequence

$$(c, A(1+g)c, (A)^2(1+g)^2c, \ldots, (A)^n(1+g)^nc, \ldots),$$

and the respective structural allocation of labour is given by

$$(a_0'c, a_0'A(1+g)c, a_0'(A)^2(1+g)^2c, \ldots, a_0'(A)^n(1+g)^nc, \ldots).$$

As a consequence of this Austrian definition of production, the concept also of socially necessary labour is different. Accordingly, the socially necessary labour to produce one unit of good j not only includes the labour time required to reproduce the means of production used up, but also the labour time required to expand future production. Let us denote the vector of (Austrian) socially necessary labour, which corresponds to unit net product level, as Π, then

$$\begin{aligned} \Pi' &= a_0' + \Pi'A(1+g) \qquad (1.14) \\ &= a_0'(I-(1+g)A)^{-1} \\ &= a_0' \sum_{k=0}^{\infty} A^k(1+g)^k \end{aligned}$$

and our interpretation is confirmed. It is this concept of socially necessary labour which Samuelson and von Weizsäcker introduced as their *synchronised labour costs* or *rational values* (1971a, b), and its Austrian origin is apparent. In the case of $g = 0$, Π and ω clearly are identical. But to call Π a generalisation of ω because of this would be misleading.

Looked at from the point of view of our distinction between production and reproduction, the synchronised labour costs provide a particular economic interpretation of our physical equilibrium condition of expanded reproduction. This is obvious from the identity between (1.11) and the synchronised labour costs of the (Austrian) net product

$$\begin{aligned} \Pi' c &= a_0'(I - (1+g)A)^{-1}(I - (1+g)A)x \qquad . \qquad (1.15) \\ &= a_0' x = L = \omega' u \end{aligned}$$

As a last point we may emphasise that the concepts of necessary and surplus labour have also different meanings in the Austrian theory. If z is again the vector of means of subsistence for those who work, then necessary labour in the Austrian definition means $\Pi' z$, and surplus labour is equal to $\Pi'(c - z)$. Irrespective of the particular growth rate $g < g^*$, surplus labour must therefore be positive, if and only if $(c - z) \geqslant 0$. To compare this with the traditional definition, note that Austrian necessary labour therefore contains as a first part (traditional) necessary labour and as a second part (traditional) surplus labour used for providing for steady-state growth of z over time. So, if $z = c$, (Austrian) necessary labour $\Pi' z$ is equal to (traditional) necessary labour $\omega' c$ *plus* (traditional) surplus labour $\omega' g A x$ and Austrian surplus labour is zero.

$$\Pi' c = \omega' c + \omega' g A x \qquad . \quad . \quad . \qquad (1.16)$$

As we made clear, the differences between Π and ω rely on different concepts of production. Both concepts may be useful in their way, but there is nothing which could not be discovered without the use of Π. If however the traditional definition is rejected in favour of the Austrian one, basic insights into the *social* relationships of production are made impossible. This point will be one of the results of part III, where capitalist commodity production is to be discussed.

III. The Value System

So far we have analysed social production without even mentioning what type of society we refer to: let us now turn to a particular mode of social production, capitalist production. As two of its outstanding characteristics, use values (goods) are produced *for* exchange (commodity production) and labour power is itself a commodity. However, those who work are owners of themselves and what they sell is only their labour *power*.[1] They sell it to those who monopolise the means of production as private property (capitalists) and, as labour power is their only property, they are *forced* to sell it. To complete the stylised picture, production is organised by independent firms and therefore presumes the purchase of labour as well as of means of

[1] In Marx (1969), an unknown work up to 1933, instead of the term *labour power* Marx used the clearer term *capacity for work* (Arbeitsvermögen).

production. Owning means of production therefore implies both the complete control of the labour process and the ownership of the outputs of this labour process. Under capitalism this mode of production is *dominant* and we shall not take into account here the other modes of production coexisting in every actual capitalist society.

Marx defines the value of a commodity as the labour time socially necessary to produce it. The value of commodity j is therefore given by the labour time directly expended, a_{0j}, plus, so far tautologically, the value of the means of production used up. In our definition of socially necessary labour (1.2), the latter component was defined as the labour time *actually* expended for the reproduction (or future replacement) of these means of production. Due to our assumptions, these magnitudes are identical with the labour time (historically) *embodied* in these means of production. To avoid misunderstanding, let us, however, generally define the value of means of production as the labour-time currently performed for providing future replacement of means of production at a sufficient rate to allow a stationary consumption level to be maintained. This definition is consistent with (1.2) and therefore ω will be considered as the vector of values of commodities. It should however be clear, that the expression " value of a commodity " is only meaningful if there are individual firms producing *for* exchange. Value, as a theoretical category, presumes exchange, and in this sense is sociologically specified.[1]

1. *The Purpose of the Value Analysis*

At the beginning we argued, that the purpose of Marx's value analysis is not to be seen as an explanation of equilibrium prices. This can be documented by many passages from *Capital*, and to support this view it is sufficient to point out that it was Marx who criticised Ricardo for his confusion of equilibrium prices and values.[2] But what is the purpose of analysing production as a value creating process and of measuring the flow of commodities between individual contractors in terms of values? As an introduction to the following paragraphs, let us summarise what we consider to be the purpose of Marx's value analysis.

As a general statement, value analysis is an instrument to lay bare the social relationships between men which are hidden behind the sphere of exchange (circulation). It directs our attention to a set of relationships between men which can be discovered neither on the basis of the quantity

[1] This marks a difference in the applicability of the terms *socially necessary labour* and *value*. However, one might even argue that socially necessary labour time itself is only meaningful if the expenditure of labour can be taken in abstraction from its concreteness (abstract labour), which certainly presumes its malleability.

[2] As a consequence, Marx was conscious of the transformation problem as a part of his whole theory long before *Capital* was written. It is perhaps interesting to note that the (imperfect) solution of this problem we find in *Capital* Vol. III originates from a letter to Engels from 2.8.1862, *cf.* (Marx, (1964b), pp. 263–68).

system nor on the basis of the price system. These are the social relationships within the *labour process*. As it exposes the roots of social inequality under capitalism it provides an explanation of the *origin* of profit; and as it is based on an invariable standard unit of commodities it provides a unique measure of Marx's concept of exploitation. Finally, value analysis helps to reveal the laws of motion of the capitalist mode of production.

2. *Commodity Fetishism*

Analysing the social relationships of any society whatever, one always has to distinguish between the set of relationships itself, the transparency of it for the members of that society and finally, the legalisation more or less accepted which establishes the stability of these relationships. To take a simple example, under a feudal mode of production, where " the direct producer . . . was under obligation based on law or customary right to devote a certain quota of his labour or his produce, to the benefit of his feudal superior " (Dobb, 1967, p. 2), the social relationships within production were quite obvious. The *legal* relationships between men were a direct index of their social relationships and everybody perceived whether he was forced to devote *surplus labour* or had the rights to dispose of it. The justification of these relationships as a divine order was effective for a long period.

In contrast to pre-capitalist modes of production, there is under capitalism no direct control of the complicated network of production and reproduction, and there is instead private property and voluntary exchange of goods and labour power. Social relationships therefore appear as a set of market relationships. They are seen in the guise of relationships between contractors equipped with common legal rights, and equality and freedom (the slogans of bourgeois revolution) seem to have historically replaced inequality, coercion and exploitation. This picture of capitalism, typical for political economy, is however false. In comparing the new society, capitalism, and the prototype of a power-regulated society, feudalism, on the basis of the legal position of men, it fails to reveal the survival of inequality, coercion and exploitation as structural elements of capitalism. In Marx's opinion this picture is therefore *ideological*: but it is not an ideology in the sense of a consciously dispensed " opium for the people." As this picture of capitalism is suggested by the sphere of exchange itself, it is an ideology in the sense of a self-deception (commodity fetishism). Under capitalism, therefore, power relationships are not transparent. In order to uncover them, the veil of " commodity fetishism " has first to be lifted.

3. *Meaning and Measurement of Exploitation*

Perhaps the easiest step is to show that capitalism, as a mode of production, presupposes the existence of a class of people devoid of any means of production. Free and equal as *citizens*, these people are yet *forced* to offer the " voluntary " sale of their labour power. As under feudalism, therefore, they are forced to supply a part of their working day as surplus labour. As a further important consequence of their position as wage workers, the sale places the worker under the absolute control of *capital* regulating the labour process. It therefore enforces *alienated labour.*

Let us begin with the first proposition. What we have to lay bare is a social power-relationship—enforced surplus labour—as hidden behind profit. Thus surplus labour has to be shown to be part of every worker's working day and consistent with the voluntary sale of labour power. There is, however, one difficulty in exposition: in order to speak of profits, the price system has to be introduced, and in order to speak of surplus value the value analysis has to be performed. So, to avoid this difficulty, let us anticipate a result of part IV based on a comparison of the value and the price system. This is the statement, that positive surplus value is both necessary and sufficient for positive profits. This enables us to consider surplus value, in a yet unspecified sense, as the abstract form of profit. As an *alternative* way out of this difficulty of exposition, one may, for the moment, assume commodities to be exchanged at their values. This is the same procedure as that used in Marx's *Capital* Vol. I, which has provoked so much misunderstanding.[1]

Consider now the labour process as a value creating process under the control of capital. Again, z is the vector of means of subsistence consumed by all those who work: in this context it therefore is the vector of aggregate wage-goods bought by the workers employed out of their wages. For the sake of simplicity, we take as our time unit the working day, fixed by the labour contract.[2] Then, $v = \frac{1}{L}(z)$ is the vector of the daily means of subsistence per worker. Due to our normalisation and a competitive labour market, v also equals overall real wages per labour unit.[3] Any capitalist engaged in the production of commodity j employs per unit activity, a_{0j} units of labour and the vector A^j of means of production. As we know from (1.2), the value of commodity j is

$$\omega_j = a_{0j} + \omega' A^j$$

[1] Marx, however, gives further arguments for his procedure. In order to rule out individual profits due to losses of other firms, he assumes for the beginning that commodities are exchanged at their values. Obviously this assumption is sufficient to rule out this possibility. On its basis, Marx was able to refute the dogma of early socialists like Proudhon, that profit accrues to capitalists as a result of the exchange mechanism (*cf.* Marx (1964c)).

[2] It is perhaps worth emphasising, that the real wage is defined here irrespective of the consumption habits of the individual worker. The logic of the argument is as follows: Find out what workers in the aggregate have consumed out of the wages they had received. This gives our vector z. Then define the average real wage per labour unit as a multiple of z, where $1/L$ fits as multiplier. If the (money) wage-rate is equal for every worker, then set $(1/L)z$ as the real wage per labour unit. Clearly, we do not want to explain the individual worker's consumption pattern. On the contrary, we want to lay bare the social relationships behind the economic events which actually took place. In this sense the value analysis is an *ex post* analysis.

[3] For an explicit introduction of the variable "working day," *cf.* Okisio (1963) and Morishima (1973, Ch. II).

and what the labour process had added to $\omega' A^j$, is the labour time directly expended within the time unit. The value equivalent of the real wage rate, *i.e.*, the socially necessary labour to produce v, $\omega' v$, is what the capitalist has to pay in value terms for the use of one labour unit (value of the commodity labour power). Consequently, part of the "value added" (per unit activity) a_{0j}, goes to the workers, and what remains in the hands of the capitalist is *surplus value* S_j.

$$\begin{aligned} S_j &= a_{0j} - a_{0j}(\omega' v) \\ &= a_{0j}(1 - \omega' v) \end{aligned} \qquad (2.1)$$

The ratio between surplus value S_j and the wages paid to those who create this surplus value, is Marx's rate of exploitation e_j

$$\begin{aligned} e_j &= \frac{S_j}{a_{0j}(\omega' v)} \\ &= \frac{1 - \omega' v}{\omega' v} \end{aligned} \qquad (2.2)$$

Clearly, $\omega' v$ is that part of the working day (of 1 time unit) which the individual worker has to spend in order to produce the value equivalent of his daily means of subsistence, and $(1 - \omega' v)$ is the *surplus labour* the individual worker has to perform per working day as a consequence of the labour contract. As a result, enforced individual surplus labour is consistent with freedom and parity in the eyes of the law. This follows directly from the position of labour as wage-labour and of means of production as capital. The rate of exploitation thus invariably measures the division of every worker's working day into one part, necessary for providing for his own real wages and another part, surplus labour. Obviously, the rate of exploitation is independent of the scale of output, and if real wage rates are equal in all departments, the rate of exploitation also is the same everywhere

$$e_j = e, j = 1, \ldots, n \qquad (2.3)$$

In the framework of the quantity system we introduced the concepts of socially necessary labour and of surplus labour. We will now show that the ratio of aggregate surplus value to the value of total labour power employed is identical to the ratio of surplus labour to necessary labour

$$\begin{aligned} e &= \frac{L - L(\omega' v)}{L(\omega' v)} \\ &= \frac{a_0' x - \omega' z}{\omega' z} \qquad (2.4) \\ &= \frac{\omega' u - \omega' z}{\omega' z} \end{aligned}$$

An interesting identity concerning this ratio e, is that

$$e = \frac{\omega' u - \omega' z}{\omega' z} \qquad . \quad . \quad . \quad . \quad . \quad (2.5)$$

$$= \frac{\omega' g A x + \omega'(c - z)}{\omega' z}$$

where $(gAx + (c - z))$ is that part of the net product which is in the hands (and at the disposal) of capitalists.

So far, however, only the aggregate value analysis is included in our quantity system. There are material differences between our concepts from the quantity system (socially necessary labour and surplus labour) and the respective concepts from the value system (value and surplus value). While *surplus labour* represents the *aggregate* surplus product, individual *surplus value* represents an individual share of the aggregate surplus product and therefore reflects a particular mechanism of distribution of surplus product. This mechanism is specific for capitalism, and the concepts of value and surplus value, therefore, presume production as commodity production and labour as wage-labour. To avoid misunderstandings, in our (so far unproved) interpretation of surplus value as the abstract form of profit, we did not, however, assume individual surplus value and profit to be identical. In fact, the price mechanism implies, in general, a redistribution of the surplus value created in the *different* sectors.

To complete this section let us now examine why Marx considers e as an appropriate index of a social power-relationship, *exploitation.* This will enable us finally to compare the social meaning of Marx's rate of exploitation with the one based on the concept of synchronised labour costs (or rational values). In order to contrast the social meaning of concepts like surplus labour and surplus value, consider first a peasant community which decides collectively on the three basic questions of *what, how* and *for whom* the community shall produce. Clearly, this community may decide to produce some surplus product for future growth or again as subsistence for those who are unable to work, and so would decide to perform surplus labour. But as this results from an egalitarian decision process, nobody could consider this surplus labour as an index of exploitation. Positive surplus labour, therefore, is not a sufficient condition for establishing that there is exploitation. Now consider surplus value, which in its character of an abstract form of profit represents a capitalist mode of production, *i.e.*, labour as wage-labour and means of production as capital. In speaking, therefore, of surplus value, we imply that individual workers are forced to perform surplus labour and that this, as a further consequence of the labour contract, is appropriated by those who control the labour process. In contrast to our stylised peasant community wage-workers, therefore, do not here decide *how* to produce; they do not decide whether any surplus product shall be produced at all, nor what composition this surplus vector should have; and finally, they do not decide *for whose* disposal this surplus shall be produced. This is why surplus value can be considered as an index of a social power-relationship between men; in fact as an index of exploitation.

Now compare Marx's definition of exploitation with that of Samuelson/v. Weizsäcker. As we already know, the Austrian concepts of net product and surplus product underlie the definition of synchronised labour costs. The corresponding rate of exploitation, therefore, is defined as the ratio of Austrian surplus labour to Austrian necessary labour (see (1.5))

$$\epsilon = \frac{\Pi'(c - z)}{\Pi' z} \quad . \quad . \quad . \quad . \quad (2.6)$$

$$= \frac{1 - \Pi' v}{\Pi' v}$$

$$\text{where } v = \frac{1}{L}(z)$$

Clearly, ϵ represents the division of the working day between labour socially necessary to produce the daily means of subsistence *plus* the respective additional means of production necessary for steady-state growth and its residual. Irrespective of the growth rate g, therefore, the Austrian-type ϵ is positive if and only if capitalists consume out of their profits. On the other hand, in the case $g > 0$, the Marxian e is positive even if capitalists save all their profits. It is therefore fair to conclude that ϵ evaluates the capitalist system according to the extent that capitalists perform their social function to accumulate. It cannot, however, serve as an index of the social power-relationships we revealed above, and therefore is by no means a generalisation of Marx's concept of exploitation.

IV. The Price System

Commodities do not, in general, exchange at their values. In order to complete the analysis we shall, therefore, turn to another aspect of the capitalist mode of production, the exchange relationships (prices). In contrast to the other aspects we have analysed, the exchange relationships are directly observable. Decisions are based on them and they, therefore, have a practical meaning. The actual form of production as commodity production, however, tends to obscure both the physical relationships of production and reproduction *and* the social relationships between men. It is for these reasons, that the quantity and the value systems are needed as theoretical tools of analysis.

1. *Equilibrium Prices*

We begin with the statement of the exchange relationships which are consistent with our equilibrium assumptions. These are the competitive prices, or briefly, prices. We require that the price vector of the produced commodities, $\bar{p}$, allows both the payment of an overall competitive (money) wage-rate, $w \geqq 0$, and an overall competitive rate of profit, $r \geqq 0$, on the invested capital [1]

$$\begin{aligned} \bar{p}' &= (1+r)\bar{p}'A + w\,a_0' \quad . \quad . \quad . \quad (3.1) \\ &= w\,a_0'(I-(1+r)A)^{-1} \\ &= w\,a_0' \sum_{k=0}^{\infty} (A)^k(1+r)^k \end{aligned}$$

As one would expect, not every positive rate of profit is feasible. The maximal rate of profit, r^*, consistent with $\bar{p} \gg 0$, is equal to the maximal growth rate, $r^* = g^*$. This rate r^* would imply $w = 0$. Therefore the restriction upon feasible, positive rates of profit consistent with $w > 0$ is identical to (1.7). Substituting g for r, (1.8) and (1.9) also hold, and this is all we need. Since only the cases $r < r^*$ are of interest, we can choose the commodity labour-power as the numeraire, $p = \frac{1}{w}(\bar{p})$. According to (1.8), (1.9), therefore, the vector $p(r) \gg 0$ increases with r, and as in the quantity system, we can verify the existence of a trade-off between the real wage-rate and the rate of profit, if the subsistence-unit has been chosen. To the real wage-rate $\frac{1}{p'v}$ (where $vL \leqq c$) there then corresponds a unique rate of profit and a strictly positive price vector. Finally, the following relationships hold between prices in terms of the commodity labour-power and the synchronised labour costs (see (1.9), (1.14), (3.1))

$$\begin{aligned} p' &= a_0'(I-(1+r)A)^{-1} \quad . \quad . \quad . \quad (3.2) \\ &\gtreqless \Pi', \text{ if } r \gtreqless g \end{aligned}$$

[1] We did not take into account here the possibility of different prices at t (end of the production period) and at $t-1$ (beginning of the production period). If we do so, then competitive equilibrium requires

$$\bar{p}(t)' = (1+r)\bar{p}(t-1)'A + w(t)a_0'$$

However, if we assume $w(t) = w$, then equilibrium prices of produced commodities are independent of t

$$\begin{aligned} \bar{p}(t)' &= w\,a_0' \sum_{k=0}^{\infty} (A)^k(1+r)^k \\ &= w\,a_0'(I-(1+r)A)^{-1} \end{aligned}$$

On the other hand, if we do not specify the time path of $w(t)$ and allow in general $\bar{p}(t) \neq \bar{p}(t-1)$, then different " own rates of interest " are possible. This, however, would contradict our competitive assumptions.

2. *Exploitation and Profit*

We are now in a position to link the competitive prices with our value analysis. As a basis, we assert the result that a positive rate of exploitation e is both necessary and sufficient for a positive rate of profit r.[1] The proof is given in Appendix A.

This result is sufficient to establish the consistency of our value analysis. In particular, it underpins our interpretation of profits as the phenomenal form of surplus value. Marx's theory of value, therefore, is consistent and an efficient tool wherewith to extract the social content of economic relationships and so, in this sense, to provide an explanation of the origin of profits. This does not, the reader is warned, imply any identity or proportionality between individual surplus value and profit, and in the single establishment surplus value therefore does not exactly represent the individual capitalist's slice of aggregate surplus value. It only represents the part *created* by his workers, not, in general, the slice he gets from the cake.[2]

V. Choice of Techniques and the Law of Value

As a general assumption we considered the production system applied as given. Both the concept of socially necessary labour and that of the price equations were therefore unambiguous. In order to classify the notion of synchronised labour costs as a central planner's tool and to evaluate Marx's *Law of Value*, we now turn to the choice of techniques. As a decision criterion of planners' choice, let us stipulate the minimisation of current labour requirements. Under capitalism, however, decisions are made on the basis of exchange relationships: competition, therefore, prescribes the choice of the unit-cost minimising production system. Given the rate of profit, competitive equilibrium leads to the choice of that production system which minimises all prices simultaneously in terms of the wage unit.

In the following paragraphs we shall show that, given the steady state rate of growth $g \geq 0$, the planners' optimal production system is that one

[1] *Cf.* Morishima (1973), Ch. II and Okisio (1963). Clearly, this statement is not generally true for the relationship between Samuelson/v. Weizsäcker's rate of exploitation and profits. In the Golden Rule case, positive profits are consistent with $\epsilon = 0$, and $r > 0$ is necessary and sufficient for $\epsilon > 0$ only if $r > g$. To prove the necessity part, begin with

$$\begin{aligned} \bar{p}' &= w\, a_0'(I - (1 + r)A)^{-1} \\ &\geqslant w\, a_0'(I - (1 + g)A)^{-1} \quad \text{if } g < r \\ &= w\, \Pi' \\ &= (\bar{p}'v)\, \Pi' \end{aligned}$$

and continue as in Appendix A. As $\epsilon > 0$ implies capitalists' consumption, $\epsilon > 0$ is equivalent to $g < r$. For the proof of the sufficiency part, remember that $\epsilon > 0$ implies $e > 0$.

[2] The determination of what the individual capitalist gets would not be difficult. It would, however, depend on arbitrary assumptions about the part of surplus *product* which represents the respective individual profit. It would therefore be meaningless.

which simultaneously minimises the synchronised labour costs of every good. As a corollary, in the Golden-Rule case, the planners' choice coincides with what would result from competitive equilibrium (see (3.2)). On the basis of these results we are finally in a position to discuss another widely misunderstood part of Marx's analysis, the Law of Value.

1. *Choice of Techniques*

In order to prove the above propositions consider $g \geqq 0$ as given and assume the subset $S^* = S$ of feasible production systems consistent with g not to be empty, *i.e.*, $S^* := \{\sigma \in S | (1+g)\lambda(A_\sigma) < 1\} \neq \phi$. Define $\hat{A}$ and $\hat{a}_0$ as the respective complete production system (see assumptions) and $\hat{I}$ as [1]

$$\hat{I} = (e^1, \ldots, e^1;\ e^2, \ldots, e^2;\ \ldots;\ e^n, \ldots, e^n).$$

Then $(\hat{I} - (1+g)\hat{A})x = c$ has non-negative solutions for arbitrary $c \geqq 0$. Consider now any $c \gg 0$ as given, then in order to determine the optimal techniques, the planners have to minimise current labour requirements

$$\hat{a}_0{}'x = \min$$

subject to the constraints

$$(\hat{I} - (1+g)\hat{A})x = c$$
$$x \geqq 0$$

This linear-programming problem has optimal solutions and from Lancaster's proof of the *dynamic non-substitution theorem* (Lancaster; Sect. 6. 7) we know two important results: [2] this minimum problem has an optimal basic-solution and the respective optimal basis is also optimal for every non-negative $c \geqq 0$. As the *basis* represents a production system from S^*, the optimal production system, say $\nu \in S^*$, is therefore independent of the position of c and satisfies [3]

$$a_0(\nu)'x_\nu \leqq a_0(\mu)'x_\mu \text{ for every } \mu \in S^* \quad . \quad . \quad (4.1)$$

On the basis of these results it is proved in Appendix B, that the vector of synchronised labour costs of production system ν satisfies

$$\Pi(\nu) \leqq \Pi(\mu) \text{ for every } \mu \in S^* \quad . \quad . \quad . \quad (4.2)$$

The optimal production system, therefore, is that one which simultaneously minimises the synchronised labour costs. This makes it meaningful to describe synchronised labour costs as a tool for use in central planning.

[1] e^j represents the unit vector with 1 at the j-th place. As in the definition of $\hat{A}$, the number of vectors e^j is m_j $(j = 1, \ldots, n)$.

[2] To be precise, Lancaster does not prove the *dynamic* non-substitution theorem but assumes the stationary case $g = 0$. For given $g > 0$, there is, however, no difference in the proof, and we shall just refer to Lancaster therefore.

[3] x_ν and x_μ are the truncated activity vectors. The zero-elements are omitted and the vectors therefore are the respective output (gross) vectors.

As we had shown earlier (1.14), in the Golden-Rule case Π equals the respective price vector in terms of the commodity labour power. It therefore follows immediately that the same choice as above is made on the basis of capitalist competition, in the special case $r = g$.

2. *The Law of Value*

Both results show that the choice of techniques cannot be described as a simultaneous minimisation of values. This is true for competitive capitalism as well as for a rationally-planned system. Consequently, the notion that commodities tend to be produced by the value-minimising techniques cannot be accepted. This notion, however, has usually been regarded as a central part of Marx's analysis, *i.e.*, as Marx's "Law of Value." We now argue that this interpretation is mistaken and betrays a basic misunderstanding of Marx's value analysis.

As we remarked earlier, Marx builds his whole analysis in *Capital* Vol. I on the *preliminary* assumption that under competitive equilibrium commodities are exchanged at their values. We shall not discuss now the reasons for this procedure (for one reason *cf.* footnote 1, p. 797), but emphasise that on this basis the "Law of Value" is indeed correct. It describes the operation of a social mechanism, competition, which forces individual capitalists to apply the unit-cost-minimising techniques. In the general case, however, when exchange rates are discussed explicitly and systematic deviations between prices and values are allowed for, Marx clearly points out that decisions (like technical choice) are made on the basis of prices and not of values. In Marx's opinion, technical choice is therefore not generally governed by the "Law of Value" (*cf.* Marx (1964), Ch. 10).

This, however, does not at all make the value analysis meaningless. That analysis is an instrument for laying bare the social power-relationships hidden beneath the exchange relationships and, in this role, presumes the relationships to be analysed as given. It analyses the relationships *post factum* and, therefore, applies after techniques have been chosen and after wages have been spent on subsistence (*cf.* our definition of real wages and footnote 2, p. 797). This is implied in the very purpose of the value system.

VI. Technical Change [1]

In this paper we have not done justice to the "Darwinist" element in Marx and, in particular, to Marx's "Laws of Motion" of capitalism. In order to discuss finally the objections against the generality of our relationships between profits and exploitation, implicitly raised by Samuelson (1971) and v. Weizsäcker (1971, a and b, p. 27–9), consider now one of

[1] In writing this section I have benefited from reading an exchange of letters between Prof. v. Weizsäcker and B. Schefold.

Marx's Laws of Motion. This is the statement that capitalist competition enforces the "permanent revolutionization of the labour process," *i.e.*, the accelerated development of new techniques which both allow improved control of the labour process itself *and* the reduction of individual values.[1] We shall not go into the details of Marx's exposition but turn to Samuelson v. Weizsäcker's simplified model of technical progress. It has, however, to be emphasised that the existence of technical change is by no means foreign to Marx's analysis and that Marx's comprehensive account of this aspect of the dynamics of capitalism has been much helped by his application of value analysis.[2]

As a simplified model of technical progress, assume (direct) labour requirements to be diminishing at the given and constant proportional rate $\gamma \geqq 0$. Let material input requirements be constant over time and assume the optimal production system to be given. Time units are defined as before and we indicate the (minimal) vector of (direct) labour requirements available at the beginning of the period between $t - 1$ and t by the symbol t. Prices, values and synchronised labour costs of the outputs produced in this period are also indicated by t. Then, given the labour requirements of some initial period, $a_0(0)$, labour requirements in the period between t and $t - 1$, $a_0(t)$, are given by

$$a_0(t) = a_0(0)(1 + \gamma)^{-t} \quad . \quad . \quad . \quad (5.1)$$

Prices, therefore, are given by (3.1) or (3.2) [3] and values, defined as the labour time socially necessary to produce one unit of j *and* to reproduce the respective means of production used up

$$\omega_j(t) = a_{0j}(t) + \omega(t)'A^j$$

are given by (1.2). As a result, the rate of technical progress only effects a reduction of values over time

$$\omega(t + 1) - \omega(t) = -\gamma\omega(t + 1) \quad . \quad . \quad (5.2)$$

[1] Generally speaking, the reduction of at least one component of the previous vector of input requirements (including labour). This, however, implies a reduction of individual values.

[2] *Cf.* Marx (1962, Ch. 10–13) and (1969).

[3] In order to be consistent with the assumed competitive equilibrium, let us assume prices and the rate of profit to remain constant over time and, therefore, money wages to grow at the rate of technical progress. As the following argument suggests, this procedure is necessary in order to guarantee the operationality of equilibrium prices. Let w and r be constant over time, then

$$\begin{aligned} p(t)' &= a_0(t)' + p(t-1)'A(1+r) \\ &= a_0(t)' \sum_{k=0}^{\infty} (A)^k(1+r)^k(1+\gamma)^k \\ &= a_0(t)'(I - (1+r)(1+\gamma)A)^{-1} \end{aligned}$$

Therefore, if prices are positive, then they decrease over time. Moreover, the condition for positive prices

$$(1 + r)(1 + \gamma)\lambda^*(A) < 1$$

imposes a further restriction upon feasible rates of technical progress, consistent with $r \geqq 0$.

and does not affect the specified vector itself. Every previous statement can therefore be applied to this case and, in particular, positive surplus value is still necessary and sufficient for positive profits (*cf.* Section IV. 2). It is only the identity of socially necessary labour and the labour historically expended which is restricted to cases without technical change.

Samuelson and v. Weizsäcker, however, identify Marx's values with the following definition

$$\omega(t)' = a_0(t)' + \omega(t-1)'A \quad . \quad . \quad (5.3)$$

They therefore come to the conclusion that values depend on γ and are identical to Π (*cf.* (1.14)) if employment is constant (*i.e.*, $g = \gamma$)

$$\begin{aligned} \omega(t)' &= a_0(t)' + \omega(t-1)'A \quad . \quad . \quad . \quad (5.4) \\ &= a_0(t)' \sum_{k=0}^{\infty} (A)^k (1+\gamma)^k \\ &= a_0(t)'(I - (1+\gamma)A)^{-1} \\ &\gtreqless \Pi(t)', \text{ if } \gamma \gtreqless g, \end{aligned}$$

and also identical to prices, if $r = \gamma$

$$\omega(t)' \gtreqless p(t)', \text{ if } \gamma \gtreqless r \quad . \quad . \quad . \quad (5.5)$$

Consequently, Marxian e and Austrian ϵ are identical if $g = \gamma$, and if capitalists save all their profits in this case, then $e = 0$ would be consistent with positive profits.

It should, however, be clear, that Samuelson and v. Weizsäcker did not find any counter-example to the generality of Marx's fundamental theorem of exploitation. They simply identified values with historical labour costs and so did not apply Marx's definition (*cf.* parts II, III). There is also no reason to deviate from Marx's definition in order to avoid the implied " losses " due to the falling values of the means of production. Even though there are falls in the values of the means of production bought at the beginning of the period (*cf.* equation (5.2)), it would be misleading to call these falls " losses." This would be meaningful only if prices were *assumed* to equal values. And even then one could only say that this assumption about prices implied losses. Moreover, synchronised labour costs also fall with the rate γ and " losses " would accrue if commodities were exchanged at their synchronised labour costs. But nobody would therefore sacrifice the meaningful definition of Π (cf. (1.14)) in order to exclude these " losses." As in the case of definition (5.3), such a procedure would also call in question the operationality of these modified synchronised labour costs

$$\begin{aligned} \Pi(t)' &= a_0(t)' + \Pi(t-1)'A(1+g) \quad . \quad . \quad (5.6) \\ &= a_0(t)' \sum_{k=0}^{\infty} (A)^k (1+g)^k (1+\gamma)^k \\ &= a_0(t)'(I - (1+g)(1+\gamma)A)^{-1} \end{aligned}$$

as $\Pi(t) \gg 0$ would require $(1+g)(1+\gamma)\lambda^*(A) < 1$, and $\omega(t) \gg 0$, $(1+\gamma)\lambda^*(A) < 1$.

Obviously these conditions are without any economic meaning. Both the vector of socially necessary labour and the vector of synchronised labour costs have to be considered as explications of some aspect of the quantity system, in which the maximal growth rate is unaffected by the rate of technical progress.

From the above discussion we may conclude that it is both consistent and meaningful to define socially necessary labour in terms of labour currently performed. Moreover, there are strong arguments against Samuelson's and v. Weizäcker's approach, which is based on labour historically, rather than currently, performed. These objections render pointless their supposed counter-example against Marx's fundamental theorem of exploitation. Again, synchronised labour costs are in no sense a generalisation of Marx's values, and the corresponding concepts of exploitation reveal material differences in social content.

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Appendix A

To prove necessity, assume $r > 0$ (where $r < r^*$). By definition of the real wage per labour unit, the (money) wage-rate is $w = \bar{p}'v$. The assumption, therefore, implies (see (1.2), (1.9), (3.1))

$$\begin{aligned} \bar{p}' &= w\, a_0'(I - (1+r)A)^{-1} \\ &\gg w\, a_0'(I - A)^{-1} \\ &= w\, \omega' \end{aligned}$$

i.e.,

$$\bar{p}' \gg (\bar{p}'v)\omega'$$

As $v \geqslant 0$, multiplication by v preserves the inequality and the statement follows immediately from the definition of e, (2.2), (2.3),

$$\bar{p}'v > (\bar{p}'v)(\omega'v)$$

i.e.,

$$(1 - \omega'v) > 0$$

$$e > 0$$

In order to prove sufficiency, note the equivalence of the statements "$e > 0$ implies $r > 0$" and "$r \leqq 0$ implies $e \leqq 0$" (where $r \geqq -1$). Assume now $-1 \leqq r \leqq 0$. This implies

$$\begin{aligned} \bar{p}' &= w\, a_0'(I - (1+r)A)^{-1} \\ &\leqq w\, \omega' \gg 0 \end{aligned}$$

and after multiplication with $v \geqslant 0$,

$$\bar{p}'v \leqq (\bar{p}'v)(\omega'v)$$

i.e.,

$$(1 - \omega'v) \leqq 0$$

$$e \leqq 0$$

$e > 0$, therefore, is also sufficient for $r > 0$.

Appendix B

As a first step we shall exclude the existence of any $\mu \in S^*$ which allows

$$\Pi(\mu) \leqslant \Pi(\nu) \quad . \quad . \quad . \quad . \qquad \text{(i)}$$

In order to do so, assume the validity of (i). By definition of Π (see (1.14)) this implies

$$\Pi(\mu)'c < \Pi(\nu)'c \qquad (c \geq 0),$$

$$a_0'(\mu)x_\mu < a_0(\nu)'x_\nu$$

This, however, contradicts (4.1).

As a last step we have to prove that no $\mu \in S^*$ exists such that

$$\Pi(\mu)_k < \Pi(\nu)_k \text{ for every } k \in Z \subset \{1, \ldots, n\} \quad . \qquad \text{(ii)}$$

$$\Pi(\mu)_r \geqq \Pi(\nu)_r \text{ for every } r \in \{1, \ldots, n\}/Z \quad . \quad . \qquad \text{(iii)}$$

For sake of the proof assume the contrary. From the non-substitution theorem we know that (4.1) hold irrespective of $c \geqq 0$. Therefore, choose $c^* \geqq 0$ such that

$$c_r^* = 0, \text{ for every } r,$$

and this implies

$$\Pi(\mu)'c^* < \Pi(\nu)'c^*$$

$$a_0(\mu)'x_\mu < a_0(\nu)'x_\nu$$

Again, this contradicts (4.1) and therefore the validity of (4.2) is proved.

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