THE TEMPORAL SINGLE-SYSTEM INTERPRETATION OF MARX’S ECONOMICS: A CRITICAL EVALUATION

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ABSTRACT

The temporal single-system (TSS) quantitative approach to Marx's economics is analysed. It is shown that TSS models lack a clear equilibrium concept and a coherent (dis)equilibrium methodology, and that Marx’s propositions on value and exploitation are tautologically obtained (i) by constructing a money costs theory of value, where by assumption values are equal to market prices, apart possibly from short-run deviations; and (ii) by arbitrarily assuming that the undefined monetary expression of labour time is positive. In general, the shortcomings of the analytical framework make TSS claims, including the proofs of the law of the tendential fall in the profit rate, unwarranted.

1. INTRODUCTION

In the last 15 years, a growing body of literature on Marxian economics has appeared, based on a temporal single-system (TSS) approach to Marx’s theory of value: i temporal, due to the emphasis on disequilibrium and dynamics and on a sequential determination of values and prices based on historical costs, as opposed to simultaneous valuations based on current values and prices; and single system because, unlike ‘dualistic’ approaches in which prices and values derive from separate accounts, TSS values and prices are determined interdependently.

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1 See, for example, Kliman and McGlone (1988), Giussani (1991), the contributions in Freeman and Carchedi (1996), and Kliman and McGlone (1999).

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The TSS approach is intended to be a rigorous and ‘general’ formalisation of Marx’s theory of value—that is a different paradigm for economics in Marx’s framework’ (Freeman and Carchedi (1996, p. xiii)). According to TSS proponents, TSS models vindicate the internal consistency of Marx’s theory, and prove ‘in completely general form the propositions which Marx has been accused of getting wrong’ (ibid., p. xviii). This feature ‘clearly recommends [TSS] as a superior interpretation’ (Kliman and McGlone (1999, p. 35)), as opposed both to the standard approach, which leads to the ‘transformation problem’ (see Desai (1991) for a survey), and to the ‘new interpretation’ (Duménil (1980), Foley (1982)), which provides a coherent model of Marx’s value theory by choosing only a ‘subset of propositions to maintain... as the essential core of the theory’ (Foley (1982, p. 40)). Under the TSS approach, the literal truth of all Marx’s propositions is claimed: ‘(a) all of Marx’s aggregate value–price equalities hold; (b) values cannot be negative; (c) profit cannot be positive unless surplus-value is positive; (d) value production is no longer irrelevant to price and profit determination; (e) the profit rate is invariant to the distribution of profit; (f) productivity in luxury industries affects the general rate of profit; and (g) labor-saving technical change can cause the profit rate to fall’ (Kliman and McGlone (1999, p. 55)).

Given the scope and relevance of these claims, the TSS approach has stimulated an intense debate on value theory and Marx’s economics, and several critiques have been expounded on the TSS approach both from a general philosophical perspective (e.g. Foley (2000), Laibman (2000), Skillman (2001), Mongiovi (2002)) and on specific issues, such as the TSS approach to Marx’s theory of the falling profit rate (e.g. Foley (1999), Laibman (1999, 2001)). In this paper, general philosophical and exegetical issues are left aside, while a thorough analysis of the TSS quantitative interpretation of Marx’s theory is presented in order to evaluate TSS methodological claims on dynamics and disequilibrium in value theory, the robustness of substantive claims (a)–(g), and the general implications of TSS models, compared to alternative interpretations of Marx’s theory.

The formal approach adopted has two advantages. First, while abstracting from doctrinal disputes, the analysis of TSS models shows that, if a TSS approach is adopted, no new insights are gained on either methodological or substantive issues in Marx’s theory, while much is lost in terms of analytical rigour and clarity with respect to alternative approaches. From a methodological viewpoint, TSS models lack a clear definition of the equilibrium concept and a coherent (dis)equilibrium methodology. If anything, the analysis of TSS models confirms that ‘the labour theory of value does not provide the framework to account for disequilibrium and dynamics in capitalism’ (Duménil and Lévy (2000, p. 142)).
From a substantive viewpoint, TSS claims (a)–(f) on value theory and Marx’s theory of exploitation are tautologically obtained by constructing a ‘money costs theory of value’, where values are a priori assumed to be equal to observed market prices, apart possibly from short-run deviations, and by arbitrarily assuming that the undefined monetary expression of labour time (MELT)—the variable that expresses ‘the social equivalence of money and labor time which is inherent in commodity production’ (Foley (1982, p. 39)) and thus converts value magnitudes into money magnitudes—is positive. Similarly, the TSS ‘proofs’ of a tendential fall in the profit rate are either tautologically true but theoretically irrelevant, or based on arbitrary and often inconsistent assumptions.

Second, by showing that all Marx’s propositions are assumed to be correct, the analysis of TSS models suggests that, as an interpretation of Marx’s theory, the TSS approach is not particularly enlightening, even though, unsurprisingly, it ‘corresponds to the original in a way that others do not’ (Kliman and McGlone (1999, p. 43)). Former debates on value theory are not ‘superseded’ (Freeman (1996, p. 225)) by TSS; instead, in this paper it is confirmed that the adoption of a coherent methodology and a clear distinction between values and prices would imply that not all Marx’s results hold, as is well known in the literature on Marxian economics. Contrary to TSS claims, this leads one to question the TSS literal interpretation of Marx’s theory and the idea that the latter is a ‘package deal’ (Kliman (2001, p. 110)), rather than to the conclusion that ‘not much is left’ of Marx’s economics.

The rest of the paper is structured as follows. Section 2 analyses the TSS discrete-time model of Marx’s theory of value, focusing on claims (a) and (b). In section 3, the analysis is extended to the TSS interpretation of Marx’s theory of exploitation and claims (c)–(f). Section 4 evaluates the TSS approach to the Marxian theory of the falling profit rate: the analysis strengthens Foley’s (1999) and Laibman’s (1999, 2001) critiques by directly questioning the analytical soundness of TSS results. Section 5 contains the conclusions.

2. VALUE AND PRICE

In this paper we adopt the following notation: at \( t \), \( p_t \) is the \([1 \times n]\) vector of sales prices; \( A \) is the \([n \times n]\) input–output matrix; \( \lambda_t \) is the \([1 \times n]\) vector of TSS values; \( l \) is the \([1 \times n]\) vector of labour coefficients; \( b_w \) is the \([n \times 1]\) workers’ consumption vector; \( x \), is the \([n \times 1]\) vector of productive activities; the scalar \( e_t \) is the MELT. In this section, the TSS quantitative interpretation of Marx’s value theory is analysed focusing mainly on Kliman and McGlone
(1999), which contains the most complete discrete-time formalization of the TSS approach.²

First, in a temporalist perspective, since prices and values of inputs which enter production at \( t \) will generally differ from prices and values of outputs sold in the market at \( t + 1 \), price and value determination should be based on historical costs. Second, a first single-system interdependence, the dependence of values on sales prices, is obtained by assuming that ‘the values of constant and variable capital depend on the prices, not the values, of means of production and subsistence’ (Kliman and McGlone (1999, p. 38)). Thus, the dynamic equations describing the motion of TSS sales prices and values can be written as

\[
\frac{p_{t+1}}{\varepsilon_{t+1}} = \frac{p_A}{\varepsilon_t} + I + g_t
\]

(1)

\[
\lambda_{t+1} = \frac{p_A}{\varepsilon_t} + I
\]

(2)

where \( g_t \) is the transfer vector, which captures the idea that a good’s sales price \( p_j \) will ‘differ from its own value, due to a gain (or loss) of some amount of value, \( g_{jt} \), in exchange’ (ibid., p. 36). Since ‘exchange cannot cause value to be gained or lost in the aggregate’ (ibid., p. 38), it is assumed that

\[ g_t x_t = 0 \]

(3)

In the general case with a variable MELT, analysed in Kliman (2001), the TSS inflation rate is defined as \( i = (\varepsilon_{t+1} - \varepsilon_t)/\varepsilon_t \), i.e. ‘inflation occurs if the same amount of value, as measured in labour time, is expressed as a greater monetary sum’ (ibid., p. 107). Thus, since labour costs are \( p b_{wl} \), aggregate nominal profits are \( \Pi^N = (p_{t+1} - p_A - p b_{wl}) x_t \), while aggregate real profits, \( \Pi^R \), are defined as

\[
\Pi^R_t = \left[ \frac{p_{t+1}}{1+i} - p_A - p b_{wl} \right] x_t
\]

(4)

and aggregate surplus value, \( S_t = s_t x_t \), is defined as

² Section 2 complements the formal sections of Duménil and Lévy (2000), Skillman (2001) and Mongiovi (2002). As shown in Veneziani (2002), all the conclusions can be extended to the TSS continuous-time model set up by Freeman (1996).
Finally, TSS production prices $p^*$ are given by (Kliman and McGlone (1999, p. 51))

$$p^*_{t+1} = (1 + r_t) p_t (A + b, l)$$

(6)

where $r_t$ is the TSS interpretation of Marx’s general profit rate:

$$r_t = \frac{s_t x_t}{(p_t A x_t + p_t b, l x_t)/\epsilon_t}$$

(7)

Thus, (6) and (7) establish the second single-system interdependence, as production prices depend on value magnitudes via the profit rate $r_t$.

A first feature of the TSS model to note is that, contrary to what Kliman and McGlone (1999, p. 38) suggest, claims (a)–(f) are not proved: they tautologically follow from the definitions, thanks to arbitrary assumptions. Claims (a) and (c) are obtained by assuming (3), a condition that is simply imposed, not derived: post-multiplying (1) and (2) by $x, s$ and using (4) and (5), it follows that $p_{t+1} x, s = \epsilon_{t+1} \lambda_{t+1} x_t$ and $\Pi^R_t = \epsilon_t S_t$, all $t$; claim (b) trivially follows from (2) by assuming $p_t \geq 0$, where $0 = (0, \ldots, 0)'$; claims (d)–(f) follow from (7) because the price rate of profit is defined as the average value rate of profit; and all claims crucially depend on the arbitrary assumption that the undefined MELT, $\epsilon_t$, is positive, for all $t$.

Second, both the equilibrium concept and the distinction between dynamics and disequilibrium are unclear, and the model lacks a coherent equilibrium, or disequilibrium, methodology. Unless equilibrium is very narrowly interpreted only as a steady state, disequilibrium plays no essential role in (1)–(7), which are compatible with an economy on a dynamic equilibrium path, with markets clearing in every period. Moreover, the vector $p^*_{t+1}$ is determined based on a uniform profit rate, a typical long-run condition which, according to TSS rhetoric, is just ‘a very particular case’ (Kliman (2001, p. 99)), or a rather restrictive postulate (Freeman (1996, p. 249)), but which holds in the TSS model even outside a steady state by arbitrarily assuming that the profit rate is equal to the average. However, ‘if market prices do not coincide with prices of production, there is no reason to think

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3 It might even be argued that ‘the equations of sequential values assume market clearing’ (Duménil and Lévy (2000, p. 126, italics added)).
that the profit rate will be uniform across sectors. To assume a uniform profit rate in such circumstances amounts to imposing an arbitrary condition on the sectoral mark-ups’ (Mongiovi (2002, p. 408)).

These methodological problems undermine the theoretical and analytical significance of TSS claims, and in particular the solution to the transformation problem. Kliman and McGlone deny that the TSS ‘interpretation eliminates the inconsistency in Marx’s value theory by supplying extra unknowns, in effect by modelling a perpetual disequilibrium in which “anything goes”’ (Kliman and McGlone (1999, p. 50)), because $p_t$ and $r_t$ are determined prior to $p_{t+1}^{*}$, and thus in (6) there are $n$ equations and $n$ unknowns. This claim is not entirely convincing, because the TSS assumption that $e_t = 1$, all $t$ (ibid., p. 36), is completely arbitrary outside a steady state and, as shown in section 3, the TSS MELT is undefined, so that, given (7), the production price equations (6) are indeed under-determined.

However, even assuming, for the sake of the argument, Kliman and McGlone’s (1999) claim to be convincing, TSS production prices have no obvious analytical interest, and no explanation of their theoretical status is provided. They are hypothetical prices obtained by applying a constant mark-up to unit money costs of production evaluated at last period’s market prices. They are neither equilibrium prices around which market prices gravitate, nor the outcome of a clear disequilibrium process. Actually, the introduction of an apparently redundant set of prices seems justified only by the need to establish, as noted above, ‘the other single-system interdependence—the dependence of output prices on value magnitudes’ (ibid., p. 37). Instead, since claims (a) and (b) and the TSS solution to the ‘transformation problem’ are based on (1)–(3), it seems natural to analyse primarily the relation between values and market prices. Therefore, first, consider a steady state, where $g_x = 0$ and (1) and (2) become

\[
\begin{align*}
\frac{p}{\varepsilon} &= \frac{pA}{\varepsilon} + I + g \\
\lambda &= \frac{pA}{\varepsilon} + I
\end{align*}
\]  

(1’)

Thus, the TSS system has $n$ degrees of freedom, unless it is assumed that in a steady state $g = 0$—or, equivalently, that in a steady state $\lambda = p/\varepsilon$ and goods exchange at embodied labour values—and a formal definition of MELT is provided.\(^4\) Actually, if, given the constancy of MELT, the \textit{ad hoc}

\[^4\] Given (3), $g = 0$ provides only $n - 1$ independent conditions.
TSS normalization $\varepsilon = 1$ is assumed, which, according to TSS authors, implies no loss of generality from a TSS viewpoint (e.g. Kliman and McGlone, 1999, p. 36), it might be simply said that the model is under-determined unless one assumes that in a steady state $\lambda = p$.

Alternatively, since $g_t$ is determined after market prices are realized, it would be necessary to assume that the steady state is never reached and prices determine values ‘historically’. However, this option would rest on the dubious methodological procedure of assuming either ‘equilibrium’ (e.g. by assuming a uniform $r_t$ in (6)) or ‘disequilibrium’ according to the need, in order to avoid indeterminacy. Moreover, such an ad hoc assumption would imply a significant loss of generality by leaving equilibrium unexplained. Indeed, in general the model would have little, if any, explanatory power, since all variables in (1)–(7) would be determined by observed, unexplained market prices. Finally, since it is arbitrary to assume $\varepsilon_t = 1$, all $t$, outside the steady state and, as shown in section 3, the TSS MELT is undefined, the system would remain under-determined.

Thus, assuming $\varepsilon = 1$, the equilibrium condition $\lambda = p$ should be imposed, as a matter of logical and methodological consistency, adding another ‘result’ to claims (a)–(g): (h) in a steady-state equilibrium, values are equal to observed market prices, and goods exchange at embodied labour values. In other words, the TSS approach solves the transformation problem by constructing a ‘money costs theory of value’, where by assumption $\lambda = p$, apart possibly from short-run deviations, and $g_t$ captures the difference between some kind of normal profits, due to production, and extra profits, determined by short-run market conditions.

If, as suggested by Desai (1979), the transformation problem is interpreted as an econometric identification problem of linking the three circuits of capital, in particular commodity and money capital, the TSS approach assumes the problem away by deleting one circuit altogether. What is defined as a ‘justification of the circular, Hegelian type, [where] what was initially taken as a premise has... been substantiated as a result’ (Kliman and McGlone (1999, p. 44)), simply amounts to assuming a priori what needs to be proved. Thus, (1)–(7) also contradict the (dubious) TSS claim that alternative approaches to value theory ‘should be tested by the normal method of science, [i.e.] which best explains the observed facts’ (Freeman (1996, p. 249)): (1)–(7) do not explain facts, and prove very little; indeed, they tautologically escape evaluation and falsification.

Thus, contrary to Foley’s (2000) suggestion, TSS and the ‘new interpretation’ seem quite distant, both theoretically and methodologically. In the latter approach, prices and values are conceptually distinct and MELT is used ‘to move back and forth between money and labour accounts’ (Foley (2000,
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Moreover, it is the specific definitions of MELT as \( e^N = p(I - A)x/lx \), and of the value of labour power as the money wage, that make it possible to retain ‘the central ideas of the labor theory of value, . . . [although] they cannot and do not retain all of the results that hold when prices are proportional to labor values’ (Foley (1982, p. 42)). In contrast, in the TSS approach the literal truth of all Marx’s propositions is claimed, even without a definition of MELT being provided. Indeed, TSS authors ‘fail to put forward a single, consistent definition of [MELT]’ (Foley (2000, p. 33)) because, contrary to Foley’s suggestion, it is not ‘necessary to carry out their purpose’ (ibid.): in the TSS approach there exist no distinct money and value accounts, and the single-system qualification reduces to the assumption that, apart from out-of-steady-state deviations, values are proportional to observed market prices. Thus, as shown in (1’) and (2’), the TSS MELT is just the undefined factor of proportionality between values and prices, which can be arbitrarily—and, from the TSS standpoint, without loss of generality—assumed equal to one.

The extra unknowns, \( g \), temporalism and ‘disequilibrium’ are necessary not to solve the transformation problem but to be able to define two sets of variables instead of one; i.e. to have some sort of ‘transformation problem’ to solve. However, even out of a steady state, (2) ‘is puzzling . . . and a considerable deviation from Marx’s labor theory of value. Sequential values are consubstantial with prices, within a labor–market price theory of value’ (Duménil and Lévy (2000, p. 127)). More strongly, (1) and (2) show the temporal and logical primacy, if not the exclusive relevance, of observed market prices: the sequence \{\( p_t \)\} \( t = 0,1, \ldots \) unidirectionally determines the time paths of all other variables \{\( \lambda_t, g_t, e_t, p^*_t \)\} \( t = 0,1, \ldots \). Given that everything happens within the money circuit, and in the sphere of circulation, it is legitimate to wonder why, even outside a steady state, one should be interested in values as distinct from prices in the first place.

As a final remark, consider the relation between values and production prices. Let \( s_t^* = r_p(A + b \omega) \). From (1)–(7), after some algebra,

\[
p_{t+1}^* = \frac{p_{t+1} + \epsilon_t g_t - (\epsilon_t s_t - s_t^*)}{1 + i} = \epsilon_t \lambda_{t+1} - (\epsilon_t s_t - s_t^*)
\]

Thus, under the TSS assumption \( \epsilon = 1 \), if a uniform profit rate prevails in a steady state, so that \( s = s^* \) and \( p^* = p \), as seems necessary to assume for consistency, then the last of a long series of TSS ‘results’ follows: (i) in a steady state, \( p^* = \lambda \). That is, the transformation between values and production

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5 For instance, given these definitions, aggregate profits are proportional to aggregate surplus value, and aggregate value added is proportional to aggregate direct labour (see section 3 below).
prices is also trivially solved in the TSS framework by assuming that they are proportional or, without loss of generality, equal, apart from short-run deviations.

3. EXPLOITATION, PROFITS AND TIME

In this section, the TSS theory of exploitation is examined, focusing on Kliman’s (2001) formal analysis. According to TSS authors, even setting aside the well-known problems related to joint production (Steedman (1977)), all simultaneist interpretations of Marx’s value theory, and not only the standard one, are incompatible with Marx’s theory of exploitation, because ‘in those systems in which the prices and values of inputs are determined simultaneously with the prices and values of outputs, the extraction of surplus labour is insufficient and, generally, unnecessary for the existence of positive profit’ (Kliman (2001, p. 97)). Instead, claims (c)–(f) prove that the TSS interpretation ‘implies that surplus labor is both necessary and sufficient for real profit to exist’ (ibid., p. 106), and thus only under TSS does Marx’s exploitation theory of profit hold.

In the standard interpretation, the link between surplus labour and profits is given by the fundamental Marxian theorem (FMT; Okishio (1963)), which in the generalization proved by Roemer (1981)—and used by Kliman (2001) as a benchmark for discussion—might be stated as follows: let \( \lambda^e = (I - A)^{-1} \) be the vector of embodied labour values, and let \( \phi_i = (I - A - b_w) x_t \) be the vector of net outputs. Let a reproducible solution (Roemer (1981, p. 19)) be a steady-state vector \( p_t \), such that, in every \( t \), capitalists maximize profits, consumed goods are replaced, workers receive a subsistence wage—which implies \( \phi_{jt} \geq 0 \), all \( j \) and \( t \)—and endowments are sufficient for production plans. Since in the standard interpretation \( \Pi_i = p_i \phi_i \) and \( S_i = (I - \lambda^e b_w) x_t = \lambda^e \phi_i \), then the FMT (Roemer (1981, p. 48, Theorem 2.11)) proves that under stationary expectations, in a reproducible solution, given the requirement of a non-negative \( \phi_i \), all \( t \), \( \Pi_i > 0 \) if and only if \( S_i > 0 \), all \( t \).

By means of numerical examples, Kliman (2001) claims that if a different definition of reproducibility is adopted which requires, for example, all net outputs to be positive over a sufficiently long time span, but allows for \( \phi_{jt} < 0 \), some \( j, t \), then there exist reproducible economies in which \( \lambda^e \) and \( p_t \) are such that \( \Pi_i > 0 \) while \( S_i < 0 \), some \( t \), and vice versa. Although this does not refute the FMT, according to Kliman, it shows that the FMT is theoretically

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6 For a survey see, for example, Desai (1991).
unsatisfactory because it holds only under Roemer’s restrictive and unrealistic definition of reproducibility.

Similarly, in the ‘new interpretation’, \( \Pi_t = p_t(I - A - b_s) x_t \), and \( S_t = l x_t - (p_t b_s lx_t / e_t^{NI}) \), where \( e_t^{NI} = p_t(I - A)x_t/lx_t \), and therefore \( \Pi_t = e_t^{NI} S_t \). However, unless \((I - A)x_t\) is a non-negative vector, \( e_t^{NI} \) can be negative, depending on \( p_t \): according to Kliman, this feature discloses ‘a serious conceptual flaw in the claim that the monetary expression of the value added by living labor can be measured by the price of the net product’ (Kliman (2001, p. 102)), and it proves that even in the ‘new interpretation’ \( S_t > 0 \) is not sufficient to have \( \Pi_t > 0 \).

There are several reasons why these arguments seem rather unconvincing. Consider, for instance, the standard interpretation: although the generalized FMT allows for different production sets available to capitalists, and thus for non-uniform profit rates (Roemer (1981, pp. 47–50)), it is untrue that it ‘examines the relation between profit and surplus labor under all possible market prices’ (Kliman (2001, p. 100)).\(^7\) Even if one questions the requirement that \( f_{jt} \geq 0 \), all \( j \) and \( t \), the FMT should be ‘conceived of as applying in a general expectations framework at a stationary state’ (Roemer (1981, p. 40)). Instead, Kliman’s (2001) examples are arbitrary and his economies, in which \( f_{jt} < 0 \), some \( j \), \( t \), are clearly not in a reproducible solution as defined above, but no alternative definition is provided: their dynamic structure and capitalists’ behaviour are simply not discussed. Although they can ‘reproduce’ themselves in a merely physical sense, no argument is provided to show that they are in a ‘reproducible’ (dis)equilibrium solution.

Even in a non-stationary path, the price and value vectors in Kliman’s examples are unlikely to be the outcome of an economy with profit-maximizing capitalists. For instance, Kliman (2001, pp. 101–2) claims that in the standard approach it is possible to have \( \Pi_t < 0 \) even if \( S_t > 0 \). However, given the technology with non-depreciating circulating capital, capitalists would never operate activities with negative profits, i.e. \( x_{jt} = 0 \), for all goods \( j \) such that \( p_{jt} < 0 \), and thus \( \Pi_t \geq 0 \).\(^8\)

In general, due to the lack of a proper dynamic framework with a definition of reproducibility and equilibrium (or, given TSS methodological claims, a model of disequilibrium dynamics), Kliman’s critiques of both the standard and the ‘new interpretation’ reduce to the trivially true, and rather uninteresting, algebraic statement that there are arbitrary combinations of the...

\(^7\) In Kliman (2001) there is no analysis of capitalists’ choices: if they can operate all the activities of the linear technology, the only possible equilibrium \( p \) is the equal-profit-rate vector (see Roemer (1981, p. 20, Theorem 1.2)), and all Kliman’s (2001) “results” are unwarranted.

\(^8\) If \( A \) is indecomposable, as in Roemer (1981, p. 48), Kliman’s economy is simply not viable.
variables such that $\Pi_t > 0$ while $S_t < 0$, and vice versa; i.e. to the claim that in a disequilibrium, conceived as a state where ‘anything goes’, the variables can take any arbitrary values so that the FMT may not hold, and the ‘new interpretation’s’ MELT may be negative. Appealing to the ‘real world’, claiming that the postulate of positive net outputs ‘is violated in every actual economy, [and] the theorems do not apply to the real world’ (Kliman (2001, p. 103)), does not make the argument more compelling.

Most important, even assuming, for the sake of the argument, Kliman’s (2001) critiques to be convincing, it is difficult to see how the TSS approach might provide a superior interpretation of Marx’s theory of exploitation, as claimed by TSS authors based on claims (c)–(f). Consider the TSS equation describing the dynamics of the temporalist MELT (Kliman (2001, p. 107)), which, in the linear setting adopted in this paper, can be derived by post-multiplying (1) by $x_t$:

$$\frac{p_{i+1} x_t}{\varepsilon_{t+1}} - \frac{p_i A x_t}{\varepsilon_t} = l x_t$$

(8)

According to Kliman, ‘examination of [(8)] shows that if $[p A x], [l x], [p x]$, and the initial condition $[\varepsilon_0]$ are positive and finite, then all subsequent terms of the $[\varepsilon]$ series must also be positive and finite’ (ibid., p. 108). Hence, claims (c)–(f) hold, proving that, of the existing interpretations of Marx’s value theory, it is only under the TSS approach that ‘the exploitation theory of profit holds’ (ibid., p. 106): in particular, since, by (4) and (5), $\Pi^R_t = \varepsilon_t S_t$, then $\Pi^R_t > 0$ if and only if $S_t > 0$.

This algebraically correct conclusion begs the question: why should $\varepsilon_0$ be positive in the first place? At most, (8) describes the motion of MELT, but it does not define it, and thus it says nothing about the sign of $\varepsilon_0$, while the TSS model (1)–(7) is inherently under-determined. (Moreover, (8) forcefully shows that the TSS assumption $\varepsilon_t = 1$, all $t$, is totally arbitrary out of a steady state.)

In a dynamic perspective one might argue that, for $t$ large, $\varepsilon_t$ converges to some positive finite value, regardless of $\varepsilon_0$. But then a steady-state argument must be adopted, in contradiction with the TSS ‘disequilibrium’ rhetoric, and in any case nothing would guarantee that $\varepsilon_t \geq 0$ far from the steady state. Again, the desired result can only be obtained by arbitrarily assuming $\varepsilon_0 \geq 0$, i.e. by assuming $\varepsilon_t \geq 0$, all $t$, which is equivalent to assuming a priori that claims (c)–(f)—and, indeed, claims (a) and (b) and the solution of the transformation problem—hold. Therefore, the emphasis on historical versus simulta-

neous valuation seems misplaced and the TSS approach does not offer a ‘superior’ interpretation of Marx’s theory of exploitation: no new insights are gained with respect to alternative approaches, while much is lost in terms of analytical rigour and conceptual clarity.

4. THE FALLING RATE OF PROFIT

The *law of the tendential fall in the profit rate* (henceforth, FPR) is among the most debated issues in Marxian economics, together with its refutation by Okishio’s theorem (OT; Okishio (1961)), which in the generalization proved by Roemer (1981) may be stated as: ‘If technical change is introduced by capitalists only when it is cost reducing at current prices, then the equilibrium [price] rate of profit will rise’ (ibid., p. 97, Theorem 4.6). In the received view, OT settles ‘in a fundamental way, the Marxian conjecture of [FPR] due to competitive innovations by price-taking capitalists’ (Roemer (1981, p. 98)). Given the restrictive assumptions on technical progress, the unrealistic description of the economy with identical firms, and the possibility of diverging value and price profit rates (e.g. ibid., pp. 95–6), TSS authors might be right to stress that such an interpretation stretches the result too far. However, TSS models do not refute OT ‘in the strict logico-mathematical sense’ (Kliman and McGlone (1999, p. 53)), because all TSS ‘refutations’ are based on patent violations of the formal assumptions of OT. Furthermore, in this section it is shown that TSS models and examples do not convincingly prove that the profit rate *might* fall, due to technical change and accumulation, and thus they do not even ‘refute’ OT in a more general, loose sense. Actually, in general, the TSS approach does not seem to provide valuable tools to evaluate the Marxian conjecture of FPR and OT.

The most structured attempt at proving that the profit rate might fall due to accumulation is in Kliman (1996), where a one-good model with technical progress is set up. Let $X$, $A$, $F$ and $L$ denote, respectively, the aggregate values of output, circulating constant capital, non-depreciating fixed capital and labour. Kliman (1996) assumes $X_t = X_0(d)^t$, $F_t = F_0(d)^t$, $A_t = A_0(d)^t$, $L_t = L_0(c)^t$, where $d > 1$ and $c < d$, due to continuous mechanization. Let $l_t = L_t/X_t$, $a = A_0/X_0$ and $n = L_0/X_0$. In a rather puzzling, unexplained deviation from (1)

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10 For a survey see, for example, Groll and Orzech (1989) and the contributions in Caravale (1991).

11 As shown in Veneziani (2002), even less convincing is the TSS attempt, based on the TSS continuous-time model set up by Freeman (1996), to prove the stronger claim that FPR is ‘not merely valid, but scientifically and rigorously exact’ (Freeman (1996, p. 272)).
and (2), in Kliman (1996) the causality between prices and values is reversed, and market prices are assumed to be proportional to values by definition, even outside a steady state. More precisely, TSS values are assumed to move according to 
\[ \lambda_{t+1}X_t = \lambda_t A_t + L_t, \]
or
\[ \lambda_{t+1} = \lambda_t a + l_t \]
while price at \( t \) is defined as
\[ p_t = e_t\lambda_t \]
and Kliman (1996) assumes \( e_t = e \), all \( t \). Hence, multiplying both sides of (9) by \( e \) and noting that \( l_t = n(c/d)' \), it follows that \( p_{t+1} = p_t a + en(c/d)', \) or
\[ p_t = (p_0 - \bar{p})(a)' + \bar{p}(c/d)' \]
where \( \bar{p} = enl(c/d - a) \). Next, the pre-mechanization profit rate is defined as
\[ r_0 = \frac{p_0(X_0 - A_0 - b_wL_0)}{p_0(A_0 + b_wL_0 + F_0)} = \frac{1 - a - b_w n}{a + b_w n + f} \]
where \( f = F_0/X_0 \). The material rate of profit at \( t \), \( r_t^m \), is defined as
\[ r_t^m = \frac{X_t - A_t - b_wL_t}{A_t + b_wL_t + F_t} = \frac{1 - a - b_w n(c/d)'}{a + b_w n(c/d)' + f} \]
That is, \( r_t^m \) is equivalent to a simultaneist profit rate, with both inputs and outputs evaluated at prices \( p_t \). Instead, the temporalist value/price profit rate at \( t \), \( r_t \), ‘calculated on the basis of historical cost’ (Kliman (1996, p. 217)) is defined as
\[ r_t = \frac{p_{t+1}X_t - p_tA_t - pb_wL_t}{(p_tA_t + pb_wL_t + K_t)} \]
where \( K_t = p_0F_0 + \sum_{i=1}^t p_i(F_i - F_{i-1}) \) is the historical value of fixed capital at \( t \). Based on (9)–(14), Kliman (1996, pp. 217–19) claims that \( r_t^m \) rises to \( r_\infty^m = (1 - a)(a + f) \); that \( r_t < r_\infty^m \), all \( t \); and finally that, if \( c \leq 1 \), \( r_t \) tends to zero, while if \( c > 1 \), \( r_t \) tends to a finite \( r_\infty \), with \( r_\infty < r_\infty^m \) and, possibly, \( r_\infty < r_0 \).
According to TSS authors, although these results do not prove that the profit rate must fall, they ‘explode a century of dogmas’ (Freeman and Kliman (2000, p. 244)), because OT ‘asserts that no viable technical change lowers the profit rate. Even one counterexample is sufficient to refute the theorem’ (ibid., p. 247). Thus, from a TSS viewpoint, Laibman’s (1999, 2001) and Foley’s (1999) critiques miss the point, because they ‘do not even attempt to prove that [TSS] results are wrong, and indeed, their own results confirm ours. Instead they have chosen to construct particular examples of profit rates that, although computed temporally, move in tandem with the material rate (Laibman (1999, pp. 212–14)), fall below it forever but by a finite amount (Foley (1999, pp. 231–2)), and so forth’ (Freeman and Kliman (2000, pp. 247–8)). Therefore, in this section both the robustness of TSS results and the TSS refutation methodology in general are directly questioned.

1. Despite a lengthy discussion on the ‘micro-enforcement’ of FPR (Kliman (1996, pp. 219–21)), capitalists’ behaviour is not modelled: technical progress is completely exogenous, and capitalists are assumed to be somewhat mysteriously compelled to invest according to a fixed rule, regardless of what happens to the price of output and to the profitability of investment. Thus, one might argue *a priori* that the model does not really take up OT’s challenge.

2. Even assuming, for the sake of the argument, that the model is convincing, it is hard to see how Kliman’s (1996) results might refute OT. By (11), it follows that

\[ p_{t+1} - p_t = (a - 1)p_0 \bar{p} \alpha + \bar{p}(c/d - 1)(c/d)' \]

so that TSS historical and replacement costs do not ‘increasingly diverge’ (ibid., p. 215): they converge to \( p = 0 \).\(^{12}\) Hence, a comparative statics analysis is possible, where the mechanization process is interpreted as a single technical change driving \( l_t \) to zero. Thus, first, \( p = 0 \) is not an equilibrium in the analytical setting of OT (e.g. Roemer (1981)): in the standard production price equations setting the new technique yields a higher profit rate. Second, even adopting the TSS price equation (11), Kliman’s (1996) conclusions may have some analytical support only in the implausible, singular case of technical change leading \( l_t \) and \( p_t \) to zero. If \( p_t \) converges to any value different from zero, not only might it be argued *a priori* that the static equilibrium pre-mechanization profit rate (12) should be compared to the higher post-mechanization static

\[^{12}\text{According to Kliman (1996, p. 215), they diverge because } \lim_{t \to \infty} \frac{p_{t+1}}{p_t} = \max(a, c/d) \neq 1. \text{ Based on this argument, one would have to conclude that if } y_{t+1} = ky_t, k < 1, \text{ then } y_t \text{ does not converge to zero. In Table 10.1 (ibid., p. 216)), } p_t \text{ displays an exponential pattern of decay.} \]
equilibrium (material) profit rate; it can also be proved that the TSS profit rate \( r_t \) increases.

Consider the general one-parameter family of mechanization processes \( \mathcal{F} = \{ l_t \in \mathbb{R} : l_t = l_1 + (cld)' l_2 , l_2 > 0 , n = l_1 + l_2 \} \), which includes \( l_1 = 0 \) as a special case. The material rate rises to \( r_m^\infty = (1 - a - b_n l_1)(a + b_n l_1 + f) \), and, apart from the subset of measure zero with \( l_1 = 0 \), the TSS profit rate rises to the same value.

**Proposition:** For all \( l_1 > 0 \), if \( l_t = l_1 + (cld)' l_2 \), then \( \lim_{t \to \infty} r_t = r_m^\infty \).

**Proof:** Let \( k_1 = l_1 e(l - 1) \) and \( k_2 = l_2 e(cld - a) \). From (7) and (8), \( p_t = (p_0 - k_1 - k_2)(a)' + k_1 + k_2 (cld)' , p_{t+1} - p_t = (1 - a)(p_0 - k_1 - k_2)(a)^{t-1} + k_2 (1 - cld)(cld)^{-1} \) and \( K/X_t = \left[ \sum_{i=1}^{\infty} (p_0 - k_1 - k_2)(1 - a) = (1 - a)f(1 - cld) \right] (cld)f(c)^{-1} \right](a)' + p.f. \) Since \( a < 1 \) and \( c < d \), the first term vanishes as \( t \to \infty \), and \( \lim_{t \to \infty} K/X_t = k_2 f. \) Since \( \lim_{t \to \infty} p_t L_t/x_t = k_1 l_1 \), the result follows from (14).

3. Even setting aside the doubts about the TSS theory of value and price (as modelled in (1)–(3) or, even more puzzlingly, in (8)–(10)), there are several features of the model that seem objectionable. For instance, although MELT is undefined, Kliman (1996, p. 215) refers to Foley’s (1986) definition, according to which MELT ‘will change over time because of changes in the productivity of labour’ (ibid., p. 15). But then, if technical change is the focus of the analysis, it seems extremely restrictive and arbitrary to assume \( \varepsilon \) constant, in order to ignore ‘purely nominal deviations of price from value’ (Kliman (1996, p. 215)). However, if the assumption is dropped, Kliman’s conclusions seem unwarranted.

As a mere illustration of the latter point, assume that \( l_t = (cld)' n \), as in Kliman (1996), but \( \varepsilon_t = (h)' \varepsilon_0 \), where \( h > 1 \) is the exogenous growth factor of \( \varepsilon \) due to labour productivity, as suggested by Foley’s (1986) definition. Assume, as a first approximation, that \( \varepsilon \) grows at the same rate as productivity, i.e. \( h = d/c. \) Let \( \bar{p}' = \varepsilon_0 n/(1/h - a) \). From (9) and (10), it follows that \( p_t = (p_0 - \bar{p}') (ah)' + \bar{p}' \), and therefore \( K/X_t = \left[ \sum_{i=1}^{\infty} f(p_0 - \bar{p}')(1 - ah)(ah)^t \right] / (a)' + p.f. \) Since \( 1/h > a \) and \( d > 1 \), it follows that \( \lim_{t \to \infty} K/X_t = \bar{p}' f \), and \( \lim_{t \to \infty} r_t = r_m^\infty = (1 - a)/(a + f) ). \) \[ \]

4. The arbitrary assumption of a constant undefined MELT is also *prima facie* inconsistent with (9) and (10). If either \( \varepsilon_{t+1} = \varepsilon_{t+1}^{NI} = p_{t+1}(X_t - A_t)/L_t \), or the ‘temporalist’ approach suggested by Foley (1999, p. 230) is adopted, where \( \varepsilon_{t+1} = [p_{t+1} X_t - p_t A_t + (p_{t+1} - p_t) F_t]/L_t \), then \( \varepsilon \) cannot be constant for any finite

\[ \]

\[ \] 13 It is necessary to assume \( 1/h > a \) for a positive equilibrium price to be possible.
t. Even more perplexing is that in (10) \(\varepsilon_t\) enters the definition of \(\rho_t\), so that any definition of \(\varepsilon_t\) based on \(\rho_t\) is circular, as is evident if one adopts the one-good version of (8), the ‘dynamic definition’ of the TSS MELT.\(^{14}\) But then it is unclear what the temporalist MELT might be, Kliman’s claims, which depend on the constancy of \(\varepsilon_t\), seem unwarranted, and the whole model seems extremely ad hoc.

Although these shortcomings suggest that Kliman’s (1996) model does not provide a convincing ‘refutation’ of OT, TSS authors claim to ‘have provided not one, but many . . . counterexamples’ (Freeman and Kliman (2000, p. 247)), in which a suitably defined profit rate falls and thus OT is ‘refuted’: in all these cases too—mostly numerical examples, rather than formal models—it is not difficult to show that mere algebraic results do not correspond to theoretically relevant propositions.\(^{15}\) However, in the rest of this section, only one other particularly informative example of the TSS ‘refutation methodology’ is analysed, based on Freeman and Kliman’s (2000) discussion of Foley (1999); the arguments developed here can be easily extended to the other TSS results.

In Foley (1999), a special case of Kliman’s (1996) model is analysed, where \(A_t = 0\) and \(L_t = 1\), all \(t\), so that \(r^m_t = (X_t - b_w)/F_t\), and a monetary rate of profit is defined as \(r^\text{mon}_t = [p_{t+1}(X_t - b_w) + (p_{t+1} - p_t)F_t]/p_tF_t\), or

\[
r^\text{mon}_t = \left(\frac{p_{t+1}}{p_t}\right)r^m_t + \left(\frac{p_{t+1} - p_t}{p_t}\right).
\]

(15)

In this setting, \(\varepsilon^\text{NI}_t = p_{t+1}X_t\), so that \(\varepsilon^\text{NI}_t = \varepsilon^\text{NI}\), all \(t\), if and only if \(p_{t+1} = \varepsilon^\text{NI}/(d'X_0\), all \(t\). Hence, if \(d = 1 + \delta\), \(\delta \geq 0\), on the price path that keeps \(\varepsilon^\text{NI}_t\) constant, (15) simplifies to \(r^\text{mon}_t = (r^m_t - \delta)/(1 + \delta)\). Foley concludes that TSS results do not hold because \(r^\text{mon}_t\) increases and ‘continuing technical change can depress \[r^\text{mon}_t\] below \[r^m_t\], but the two do not diverge asymptotically’ (ibid., p. 232).

According to Freeman and Kliman, ‘Foley is simply mistaken’ (2000, p. 266). They note that if the economy is initially in a static equilibrium with \(r^\text{mon}_0 = r^m_0\) and \(p_1 = p_0 = \varepsilon^\text{NI}\), then by (15) there is always a rate of disinflation \(\delta\) large enough so that (i) when technical progress starts, \(r^\text{mon}_t\) falls below \(r^m_0\); and (ii) since \(r^m_t\) is bounded above by 1/\(f\), although \(r^\text{mon}_t\) rises with \(r^m_t\), \(t \geq 2\), \(r^\text{mon}_t\) remains below \(r^\text{mon}_0\). This, they claim, ‘refutes [OT].’ Even when Kliman’s example is “corrected” in a way that keeps the New Interpretation’s

\(^{14}\) This observation motivates the assumption of exogenous growth of MELT in point 3.

\(^{15}\) See, for example, Kliman (1997), Kliman and McGlone (1999) and Freeman and Kliman (2000).
MELT constant, the resulting price and profit rate paths do contradict [OT]' (ibid.).

It is difficult to see how this (algebraically correct) result might refute OT. Why should the price path be such that MELT is constant? How is the price path determined? TSS authors seem to believe that in order to refute OT it is sufficient to find an arbitrary price path such that a profit rate (e.g. Foley’s (1999) monetary profit rate) falls. In this case, the emphasis on temporalism versus simultaneism seems misplaced and TSS value theory is not necessary to ‘refute’ OT; it suffices to note that in (15) a sufficiently high deflation rate leads to a fall in the monetary profit rate. But does a result based on permanent deflation describe a credible tendency of a capitalist economy? Many arbitrary ‘refutations’ of this sort can be obtained, but it is doubtful that these algebraic examples describe a plausible pattern of the profit rate, and thus that they are suitable to discuss the Marxian conjecture of a falling rate of profit, let alone refute OT.

5. CONCLUDING REMARKS

In this paper, the TSS quantitative interpretation of Marx’s value theory is critically evaluated. It is argued that if a TSS approach is adopted, no new insights are gained on either methodological or substantive issues in Marx’s theory, while much is lost in terms of analytical rigour and clarity. From a methodological viewpoint, TSS models lack a clear equilibrium concept and a coherent (dis)equilibrium strategy. From a substantive viewpoint, it is argued that all TSS claims are tautologically obtained by assumption—in particular, as concerns the ‘transformation problem’, by assuming that values are proportional to observed market prices, apart from short-run deviations, and by arbitrarily assuming that the undefined proportionality factor, the monetary expression of labour time, is positive.

Much remains to be done in Marxian economics, both at the theoretical and at the empirical level, but, in the light of the results presented in this paper, the TSS approach does not seem to provide a promising line for further research.

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