THE SHIBATA–OKISHIO CONNECTION: LABOR THEORY OF VALUE AND RATE OF PROFIT

BY

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I. INTRODUCTION

This paper attempts to clarify a relatively unknown connection between two mathematical Marxian economists in Japan. The postwar Japanese contribution to mathematical economics is notable, and is perhaps greatest in the field of Marxian economics. Michio Morishima (1923-2004) introduced these contributions, including his own ideas, to the English-speaking world. However, as Morishima himself noted in his seminal work (Morishima 1973, p. viii), much of his analysis had already been anticipated by Nobuo Okishio (1927-2003).

Most Marxian economists, especially those in Western literature, have ignored the subtle differences between Okishio and Morishima. For example, they noted that Okishio claimed that his theorem on the falling rate of profit demonstrated Marx's inconsistency (Fine and Saad-Filho 2004). This led some Marxian economists to attempt to refute the theory rather than develop it as a potential contribution.

Aside from mathematical formalism, however, Okishio and Morishima also had different attitudes towards the implications of Marxian value theory. Morishima regarded his work on Marxian economics as a part of his trilogy on economic growth (Morishima 1973, p. vii). Despite a sophisticated analysis very similar to Okishio's, Morishima ultimately suggested that ''Marxian economists . . . ought to change their attitude towards the labor theory of value'' (*ibid*., p. 193). In other words, Morishima advocated the abandonment of the labor theory of value. Okishio, however, argued for the theoretical validity of Marxian value theory until the end of his life, and pursued a ''new society'' that would supplant capitalism, in which all members of the society would participate equally in making decisions about production.[1]

[1]These theoretical and political positions are succinctly presented in his posthumous book (Okishio 2004). Okishio's short book review on Morishima's work (1973) is also helpful for understanding their differences at a purely analytical level (Okishio 1977, pp. 270–5).

In order to elucidate the differences between Okishio and Morishima, this paper focuses primarily on a Kyoto University economist, Kei Shibata (1902-1986), who was a precursor of future Japanese contributions to analytical Marxian economics and general equilibrium theory. In particular, the Shibata–Okishio connection is important in the history of Marxian economics because almost all of the controversial issues in the 1970s and 1980s had already been raised by Shibata in the 1930s. This point has been previously noted in related literature (Groll and Orzech 1989; Howard and King 1992). Shibata's work, however, was considered only as an attack on Marxian value theory, similar to the treatment of Okishio's work. Shibata's particular position in the recon-struction of Marxian economics using the Walrasian method has not been thoroughly examined, particularly in Western literature. Although both Morishima and Okishio followed in Shibata's footsteps, their directions were quite divergent. This paper will argue that the seeds of both of these directions were already germinating in Shibata's theoretical works. In particular, this paper will pursue the theoretical connection between Shibata and Okishio, especially regarding the labor theory of value and rate of profit.

These connections are laid out in the following structure. Section II provides biographical notes on Shibata and Okishio. Section III examines a prototype of the Okishio theorem presented by Shibata. In particular, the focus will be on the Okishio theorem's character as a counterfactual explanation. It will be argued that Okishio faithfully extended Shibata's insight and generalized it. Section IV discusses Shibata's solution to the so-called transformation problem, which precedes the ''iterative solu-tion'' provided by Okishio and Anwar Shaikh in the 1970s. Okishio further developed Shibata's intuition by providing a general proof. However, in trying to maintain the function of value for explaining equilibrium prices, he abandoned an aggregate equa-lity between total profit and total surplus value. The fundamental Marxian theorem was thus required to fill the gap. In Section V, Shibata's view on the role of value theory will be examined. This was the position of Shibata against which Okishio was most strongly opposed. It will be noted that Okishio provided the fundamental Marxian theorem in order to criticize Shibata's ambivalent attitude towards the labor theory of value. Section VI concludes.

II. BIOGRAPHICAL NOTES

Beginning in 1926, Shibata studied Marx's *Capital* as a student of a famous Japanese Marxist, Hajime Kawakami (1879-1946) at Kyoto Imperial University (now Kyoto University).[2] Although Kawakami resigned for political reasons in 1928, Shibata became a lecturer in 1929, the same year that Yasuma Takada (1883-1972) was appointed as a professor. After studying modern economics under the influence of Takada, Shibata thought that Karl Marx's theory of capital accumulation could be integrated into the economics of Léon Walras.

Shibata's article entitled ''Marx's analysis of capitalism and the general equilib-rium theory of the Lausanne school'' was published in the *Kyoto University Economic*

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[2]For a detailed chronology of Shibata's life and works, see Sugihara, Kumon, and Okamura (1996).

Review (1933), the first Western-language journal published by a Japanese university.[3] In this article, he was essentially supportive of Marxian economics:

> What is it, then, that makes Marxian economics so powerful and the general equilibrium theory of the mathematical school so inert? It is simply this, that whereas in the Marxian economics the organization of capitalistic production and the laws of its development are analyzed in a direct way, in the general equilibrium theory the main attention is directed to the analysis of the mental structure of the individuals who take part in the organization of capitalistic production (Shibata 1933a, p. 108).

However, Shibata argues that "by settling [some] points can the general equilibrium theory be made to attain great potency in analyzing both the organization of capitalistic production and the laws of its development" (*ibid.*, p. 109).[4]

Although Shibata published several important papers on Marxian value theory in the *Kyoto University Economic Review*, he was never a Marxist. His political positions nonetheless seemed to be influenced by Marxism. After studying in the United States from 1936 to 1938, Shibata returned to Japan and proposed a "new order economic mode" in order to introduce a controlled economy, rather than allowing free capitalism. For example, he proposed that capitalists should be deprived of the power to appoint (and dismiss) "firm leaders," and that profits must remain under state control.[5] He organized a Study Group on the New Order (Shin Taisei Kenkyukai) in 1940 and argued for "communal totality," implying "the maximization of the produced real income of the society as a whole," which authorities suspected was a disguised communist movement (Yagi 1999, p. 188). Ironically, after the Second World War, he was banished from the university by Allied Forces General Headquarters on suspicion that he was "a supporter of militarism and ultra-nationalism."

Shibata returned to the academic world as a professor at Yamguchi University in 1952, and continued research to argue for "the theory of human emancipation." In another largely unnoticed book (Shibata 1959) published in English by a Japanese publisher, Shibata tried to synthesize all major economic theories, including Marx, Keynes, and the general equilibrium theory. However, his evaluation of Marxian value theory, at least from a purely analytical perspective, did not change in this book.

That Shibata became a "forgotten economist" (Tsuru 1985 [2006], p. 237) is partly due to his expulsion from Kyoto University. As a result, he did not have enough students to form a "school" in the manner of other economists (e.g., Uno School at

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[3]This paper was originally written in Japanese and published in January, 1933. It was noted by Oscar Lange and motivated him to write the paper, "Marxian economics and modern economic theory," which was published in the *Review of Economic Studies* in 1935 (Lange 1935 [1968], p. 68).

[4]Shibata published a two-volume work of about 2,000 pages, *Riron Keizaigaku* (*Theoretical Economics*) in 1935, which was not regarded as particularly successful in the academic world (Tsuru 1985 [2006], p. 247). In this book, he also stated that "The equation system of the general equilibrium of the Lausanne school will become an effective means to the research on the economic theory when it is simplified in an appropriate way" (quoted from Negishi 2004, p. 4).

[5]At that time in Japan, many leftists joined with Fascists and even ultra-Nationalists and tried to protect workers' interest under the wartime control (Morris-Suzuki 1989, Chapter 3). Particularly during the war, the concepts of socialism and fascism were intertwined with patriotism, under the umbrella of a state-controlled economy. Incidentally, this was typical of military-led development state model in East Asia, such as South Korea (Kim 2004).

Tokyo University). Furthermore, strong antipathy still existed towards the mathematical methods of Marxian economists (Akama 2000, p. 130).

On the other hand, Okishio studied Marxian economics from 1945, initially as a student at Kobe University of Commerce (now Kobe University). Although he studied mathematical economics starting with John R. Hicks and John Maynard Keynes, he was soon attracted to Marxian economics. This was mainly due to his experiences during the occupation after the Second World War, including food shortage, unemployment, severe inflation, and repression of the labor movement.[6] According to his interview with a Korean journal (Okishio 1995, p. 228), Okishio was impressed by Marx's conception of class struggle at the time.

It is worth noting that Okishio tried to bridge Marxian economics and mainstream economics in Japan (Akama 2000, p. 131). For example, he was the president of Riron Keizai Gakkai (Theoretical Economic Association) of the mainstream economists from 1979 to 1980 and an active member of Keizai Riron Gakkai (Economic Theoretical Association), an association of Marxian economists. This was very exceptional considering that most Japanese economists belong to only one of the two associations.[7]

Interestingly, Shibata, Okishio, and Morishima were all from Kansai (Kyoto-Osaka-Kobe), an area characterized by cultural homogeneity.[8] As economists in the Kyoto-Osaka-Kobe area organized study groups and seminars very frequently, it is obvious that Okishio and Morishima had interacted with each other from earlier days (Nakatani 1990, pp.130–1). Morishima explicitly admitted to Okishio's influence in his book:

> I must acknowledge here that Dr. Seton's contribution to the 1961 paper was particularly significant, because I was not very familiar with Marxian economics when we wrote it in 1957. With much help from Professor Okishio's books (unfortunately, all in Japanese), I had gathered almost all the material for this book by September 1968 (Morishima 1973, pp. vii–viii).

As an undergraduate student at Kyoto University, Morishima took an economic philosophy course from Yasuma Takada, a colleague of Shibata (Morishima 1994). Although Morishima did not note this explicitly, his work on Walras' economics emphasizing that "Walras' theory of exchange and production is . . . but an overture to his general equilibrium theory of capital formation and circulation." (Morishima 1977, pp. vii–viii) was very similar to Shibata's project:

> What is it, then, which the structure of the general equilibrium theory contains and which makes the analysis of the organization of capitalistic production and consequently the grasp of the laws of its development logically possible? And how can it be

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[6]Interestingly, a severe food problem in big cities such as Tokyo, Kyoto, and Osaka was one of the social situations driving Japanese economists' interest in the stability condition of the market introducing mathematical analysis (Ikeo 1994, p. 584). Not surprisingly, Morishima was one of the representatives.
[7]One can glimpse the conflict between the two associations with "no fruitful conversation" from Morishima's description (1973, p. 1).
[8]Okishio was born in Kobe. Born in Osaka, Morishima was brought up in Kobe. Shibata was born and brought up in Fukuoka, but went to Kyoto Imperial University.

eliminated? The object of the present article is to study these points (Shibata 1933a, p. 108).

In contrast, Okishio explicitly noted his indebtedness to Shibata in the acknowledgements of his seminal paper proving the so-called Okishio theorem (Okishio 1961). This is why Negishi (1989) coined the term the ''Shibata-Okishio theorem.'' In particular, with respect to the three topics discussed in the following sections of this paper, Okishio always regarded Shibata as both a precursor to his work and his main antagonist. Unlike Morishima, Okishio wrote his articles primarily in Japanese and published several books as collections of these articles. Specifically, his two-volume work, *Marukusu Keizaigaku* (*Marxian Economics*), published in 1977, clearly shows his theoretical relationship to Shibata's works.

III. THE TENDENCY OF THE RATE OF PROFIT TO FALL

The Okishio theorem shows that, if the real wage rate in terms of consumption goods is constant, any cost-reducing technique in terms of the present price-wage configuration will increase (or maintain), but not decrease, the equilibrium rate of profit. Here the equilibrium is static in the sense that ''profit rates are equalized in every sector'' (Okishio 2000, p. 493).

Two points are worth noting here: First (Point A), this theorem raises the issue of the criterion for technical choice on the side of capitalists. Capitalists, if rational, will not introduce any new technology that is expected to increase the unit cost of production. In particular, the expected cost must be calculated in price terms, not in value terms. What the theorem tells us is that it is misleading to assume a specific form of technical progress without considering capitalists' micro-foundation. Second (Point B), this theorem refers to a counterfactual case in which the real wage is kept constant. The real wage will undoubtedly and necessarily change as a result of introducing new technology. Okishio compared the typical state of a capitalist economy, in which there is a change in the real wage due to labor market dynamics, to a hypothetical state in which the real wage does not change.[9] The implication of the Okishio theorem is that a falling rate of profit may occur, simply due to an increase in the real wage.

Shibata's contribution is important because it shows the two lines of thinking discussed above explicitly, even though he does not extrapolate to more general scenarios.

A good place to begin is the summary of Shibata's model (1934). Using a three-sector model composed of money (gold, sector 1), means of production (sector 2), and means of consumption (sector 3), Shibata compared the initial situation (''Instance 1'') with four ''Instances'' introducing technology that raised the organic composition of capital. Numerical examples are set so that the capital-labor ratios in sectors 2 and 3 are equal, but not equal to sector 1.

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[9]This gives us an explanation for the reason Okishio, in his last paper (Okishio 2000), seemingly denied his theorem. Given the counterfactual character of the theorem, however, to note its restrictive premises does not necessarily indicate denial of it.

His examples can be easily represented using general notations such as the notion used in Table 1, where a_{ij}, l_j, p_j, b_j and r denote, respectively, the amount of commodity i needed to produce one unit of commodity j, the amount of direct labor needed to produce one unit of commodity j, the price of commodity j, the real wage in the jth sector, and the equalized rate of profit. Cases 2, 3, and 4, respectively, correspond to the situation in which prices of commodities are increasing, constant, and decreasing. While case 4 refers to the Okishio theorem, case 5 involves the contra-positive to the theorem in the sense that if the equilibrium rate of profit is known to have fallen ($\Delta r < 0$), the real wage must have increased ($\Delta b_2 = \Delta b_3 > 0$).

Of interest here are some peculiar points in Shibata's illustration (1934), which have also been noted previously by Howard and King (1992). First, as commodity 1 represents both money and luxury goods, it does not enter into the production of any other commodity. This means that a_{1j} is zero for all j. Second, under the new technology, the physical ratios of the commodity and labor inputs change with the same magnitude in all the sectors.

The physical capital-labor ratios are the same in both sectors, which was interpreted as an indication that Shibata's analysis did not go beyond previous discussions on the same issue by Natalie Moszkowska. In this sense, "Shibata's argument . . . could be reduced to a single sector" (Howard and King 1992, p. 137).

It is true that Shibata's three-sector example does not differ from the one-sector model with respect to the determination of the equilibrium rate of profit.[10] In Shibata's examples, however, the coefficients of sector 1 (gold) are assumed to be constant. Therefore, the value of the ith commodity relative to the value of money commodity (λ_i / λ_1) becomes different from the price of the ith commodity ($p_i / p_1 = p_i$).[11] This implies that value-price deviations are permitted. In this sense, Shibata's exposition allows for the explanation of the criterion for technical choice in terms of price, rather than in terms of value.

Related to this point is the following comment:

> For Shibata, on the other hand, the introduction of a new technique raises both q [organic composition of capital] and the level of productivity. The rise in productivity becomes an essential feature of the increase in the organic composition of capital (Groll and Orzech 1989, p. 257).

While it is true that Shibata's new technique accompanies an increase in the level of labor productivity, it is easily confirmed that his examples also consider the cost

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[10]As is well known, according to the Perron-Frobenius theorem, the equilibrium rate of profit has a specific relation to the maximum eigenvalue of the input coefficient matrix. However, $a_{1j} = 0$ (for all j) implies that the first row of the matrix does not have any nonzero element, and therefore, the maximum eigenvalue can be obtained from a 2×2 matrix in which the column and the row relating to the first sector are removed.

[11]As commodity 1 is money, its price (p_1) is equal to 1.

Table 1. Shibata's "Instances" (1934)

Case	Changes	Results
2	$\Delta a_{22}=\Delta a_{23}>0, \Delta l_2=\Delta l_3<0$	$\Delta r < 0$
		$\Delta p_2 = \Delta p_3 > 0$
3	$\Delta a_{22}=\Delta a_{23}>0, \Delta l_2=\Delta l_3<0, \ \Delta p_2=\Delta p_3=0$	$\Delta r = 0$
4	$\Delta a_{22}=\Delta a_{23}>0, \Delta l_2=\Delta l_3<0, \ \Delta p_2=\Delta p_3<0$	$\Delta r > 0$
5	$\Delta a_{22}=\Delta a_{23}>0, \Delta l_2=\Delta l_3<0, \ \Delta p_2=\Delta p_3<0, \ \Delta b_2=\Delta b_3>0$	$\Delta r < 0$

criterion for technical choice in price terms. Using general notation, it is known that all four of his instances satisfy the following condition:[12]

$$(a_{2j} + \Delta a_{2j})p_2 + (l_j + \Delta l_j)b_j p_3 < a_{2j}p_2 + l_j b_j p_3 \quad (j = 2, 3)$$
$$\therefore \ \Delta a_{2j}p_2 + \Delta l_j b_j p_3 < 0 \tag{1}$$

On the other hand, Shibata explicitly noted the Point (B) discussed above:

> . . . I hope, have made it clear that the elevation of the organic composition of capital in a capitalistic society does not, of itself, cause a reduction of the average rate of profit, but that rather it tends to bring about a rise of the average rate of profit. *However, this by no means proves that the fall of the organic composition of capital brings about a fall in the rate of profit. Nor does it imply that a fall of the average rate of profit may not be simultaneous with the elevation of the organic composition of capital. It simply shows that if the elevation of the organic composition of capital and the fall of the average rate of profit really happen at the same time, this phenomenon requires to be explained in a different way from the one which Marx adopted* (Shibata 1934, p. 71, emphasis added).

According to Shibata, the falling rate of profit can result either from "the increase of real wages or the shortening of the working day" (*ibid.*, p. 71). Case 5 in Table 1 indicates the increase in the real wage. A shortening of the working day, *ceteris paribus*, may be interpreted as a decrease in the rate of exploitation. Namely, it is admitted that some change in value production can cause a falling rate of profit.

However, Shibata was more interested in the trend of "the rate of income" than the rate of profit. Using Marx's notations, the rate of income is denoted as $(V + S)/(C + V)$, where C, V and S denote constant capital, variable capital, and surplus value. According to Shibata (1934, p. 34), technical progress is compatible with a decreasing rate of income, which means conflict between the economic

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[12]This is also confirmed by the following question, posed by Shibata, which has a negative answer: "Can we admit the assertion that '[sic] the capitalist producers will willingly adopt any new method of production no matter how much it may raise the cost (in price) of production, and consequently the price of the conditions concerned, if it only effects a fall in *value* of the commodities concerned?' " (Shibata 1939, p. 60, italics original). This quote was from Shibata's refutation against Shigeto Tsuru's criticism on Shibata's 1934 paper. Paul Sweezy footnoted Shibata's papers (Shibata 1934, 1939) as an example of an "unconvincing" argument (Sweezy 1942, p. 104). Considering Tsuru's cooperation with Sweezy at that time, this evaluation might be affected by Tsuru's reading of Shibata's papers.

welfare of the entire society and the objective of capitalists. Table 2 shows the situation Shibata had in mind.

In Situation 2, despite the rising rate of profit, the rate of income decreases. For Shibata, this implies that capitalism cannot achieve ''communal totality,'' defined as the maximization of the rate of income. The ''rate of income'' (Syotokuritsu) was one of the specific concepts that Okishio used to measure the degree of unequal exchange. Okishio, however, defined it as income per unit of direct labor input (Okishio 1956, p. 54). This is but one example of the Shibata–Okishio connection, which has terminological continuity with modifications in substance.

What, then, is Okishio's contribution?

Okishio (1961) proved the theorem in a general multi-sector context. Furthermore, while Shibata's examples did not clearly distinguish technology that improved labor productivity from technology that reduced costs as an unintended by-product, Okishio explicitly formulated his model in a general context. Okishio (1961) distinguished ''cost criterion'' represented by (1) from ''productivity criterion'' as

$$(a_{2j} + \Delta a_{2j})\lambda_2 + (l_j + \Delta l_j) < a_{2j}\lambda_2 + l_j \quad (j = 2, 3)$$
$$\therefore \Delta a_{2j}\lambda_2 + \Delta l_j < 0 \quad (2)$$

As $b_j p_3$ is equal to the money wage (w_j), two criteria are equal only if $p_2/w_j = \lambda_2$. However, this is inconsistent with the condition that ''there must be positive profit in every industry,'' namely $p_j/w_j > \lambda_j$ (Okishio 1961 [1993], p. 86).

Of course, there is no reason to believe that every industry must have positive profit in a real capitalist economy. Okishio, however, thought that this was necessary for a normal reproduction of a capitalist society, and used it as one pillar of his fundamental Marxian theorem. According to the interview noted above (Okishio 1995), Okishio said that he had trouble understanding why Marx's *Capital* starts with the assumption of equal exchange, which was inconsistent with Japanese reality in the late 1940s. One of the main goals of his fundamental Marxian theorem was to ''prove the theory of surplus value without assuming equal exchange'' (Okishio 1995, p. 232). In this sense, the distinction between ''productivity criterion'' and ''cost criterion'' is more closely related to the whole reformulation of Marxian value theory, rather than to the clarification of the technical choice criterion.[13] This is a critical difference between Okishio and Shibata.

Furthermore, Okishio (1977, p. 251) specifically criticized Shibata for assuming the direction of price change at the start. However, if, one considers that Shibata's exposition is based upon tedious numerical calculations and not upon a generalized model, these cases can be interpreted as representing all of the possible comparative static cases. As Shibata's emphasis is on the last two cases in Table 1, this criticism is not so decisive.

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[13]Unlike Howard and King 's notes (1992, p. 286), the fundamental Marxian theorem must be traced back to the year 1955, not Seton (1957) or Okishio's English paper (Okishio 1963). Okishio already provided the outline of the fundamental Marxian theorem in his Japanese paper, ''Kachi to Kakaku'' (Value and Price) published in *Kobedaigaku Keizaigaku Kenkyu* (*Kobe University Economic Studies*) in 1955, which was later included in Okishio (1977).

Table 2. Falling rate of income with increasing rate of profit

Situation	C	V	S	Rate of profit	Rate of income
1	100	100	100	50%	100%
2	200	100	180	60%	93.3%

In a nutshell, with regard to the falling rate of profit, Okishio's theories are in the same vein as Shibata's insight. Between Okishio and Shibata, there is not much dispute.

IV. A SOLUTION TO THE TRANSFORMATION PROBLEM: THE MARKOV PROCESS

Shibata (1933b) provided a Markov process solution to the so-called problem of transforming Marxian values into prices of production. He started with the recognition of the problem raised by Ladislaus von Bortkiewicz:

> The failure of Marx to make clear these circumstances was due solely to the fact that he did not consider thoroughly the result of the average rate of profit (the fact that the cost value itself is caused to deviate from value by the average of the rates of profit) (Shibata 1933b, p. 65).[14]

As is well known in Marxian literature, prices of production deviate from values unless very restrictive assumptions are made. In Shibata's presentation, this is represented by the fact that deviation ratios between value and price of production in two departments are not equalized to 1. Therefore, the cost price of each department must be recalculated based on prices of production, and not values. However, as just one more calculation will not solve the problem completely, it is necessary to iterate "the process of averaging the rates of profit a second and a third time" (Shibata 1933b, p. 61). In Japan, Yasuma Takada had already examined this procedure in his book in 1931 (Takada 1931), concluding that the deviation ratio diverges to infinity assuming that the average rate of profit remains constant in every step of the iteration procedure (Okishio 1977, p. 207). Shibata, on the other hand, argued that the transformation problem could be solved if Marx's method were pursued thoroughly (Shibata 1933b, p. 66). Using Marx's framework of the simple reproduction scheme in *Capital* volume 2, Shibata illustrated how this is possible.

Georg von Charasoff, Natalie Moszkowska, and others had already tried to provide this Markov process solution (Howard and King 1992, pp. 230–1). Shibata knew about these predecessors, so he could not take full credit for the original theorization. Furthermore, he did not give a general proof, but only gave a numerical example. His equilibrium condition for the simple reproduction was unnecessary for the procedure.

Okishio (1972) was the first to give a general proof of this iterative procedure.[15] Okishio's algorithm for the iterative procedure may be represented as (Nakatani and Rieu 2003, pp. 53–4):

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[14]Here "these circumstances" mean the fact that the general rate of profit in price terms is not equal to that in value terms. And "cost value" is obviously a mistranslation of "cost price."

[15]Shaikh (1977), which was mimeographed in 1973, was the first in Western literature (Morishima and Catephores 1978, p. 166). On the other hand, Okishio (1974) is an English translation of Okishio (1972).

$$p_0 = \lambda \tag{3}$$

$$p_{s+1} = (1 + r_s)p_s(A + bl) \tag{4}$$

$$r_s = \frac{p_s x - p_s(A + bl)x}{p_s(A + bl)x} \tag{5}$$

Here λ, p, A, and l denote, respectively, the value vector, price of production vector, input coefficient matrix, and labor input vector, while b and x are real wage basket and gross product, respectively, defined as column vectors. The subscript indicates the stage in the procedure.

However, as Morishima and Catephores (1978, pp. 167–8) proved, this transformation procedure converges to another system of prices of production if one changes equation (5) to the following equation, where S denotes total surplus value:

$$r_s = \frac{S}{p_s(A + bl)x} \tag{6}$$

While the algorithm with (6) instead of (5) can guarantee equality between total profit and total surplus value, which was not the case in Okishio's procedure, it can not guarantee equality between total price and total value, which was originally an aim of Okishio's procedure. In these two algorithms, prices of production as final solutions diverge, implying that this is simply a matter of choosing the constraint, or, in other words, a matter of choosing a numéraire.[16]

Again, Okishio's contribution was to provide a general proof for the Markov process. However, he gave privileged status to the iterative procedure (3), (4), and (5), implying an abandonment of the equality between total profit and total surplus value. For him, this would not create any problems because his fundamental Marxian theorem could validate the causal connection between total profit and total surplus value, even though equality was not guaranteed. Therefore, Okishio's theory eventually comes down to the establishment of the fundamental Marxian theorem. Okishio's proof of the fundamental Marxian theorem precedes the proof of the Markov process (see note 13). Thus, it must be that the logical reasoning in his mind ran from the fundamental Marxian theorem to the transformation problem, as was the case in the Okishio theorem.

V. THE ROLE OF VALUE THEORY

Although Shibata (1933b) proved that it is possible to consistently formulate Marx's transformation procedure, this does not necessarily indicate that he agreed with the

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[16]For example, Glick and Ehrbar (1987) used an iterative procedure to find a solution based on the so-called ''New Interpretation'' of Marxian value theory. They used another numéraire of a constant money wage, which clearly shows that any numéraire can be chosen to constitute the iterative procedure.

role of value theory as conceived by traditional Marxian economists. Instead, he presented an argument very similar to the so-called redundancy thesis that the Marxian value concept is simply a ''complicating detour'' (Samuelson 1971).

Shibata emphasized at the outset that ''the aim . . . is to make clear whether or no [sic] it is impossible to explain productive prices and the average rate of profit, unless we first know values . . . '' (Shibata 1933b, p. 51). He asked ''if price cannot be explained by any other means, cognition of value will remain an indispensable premise for cognition of price. Is it, however, impossible to explain price except from the premise of value?'' (*ibid*., p. 66). He then concluded the paper by stating

> Thus, we shall be able to conclude that the determination of the ratios of exchange of various commodities and the various phenomena based on it can be adequately explained theoretically without the knowledge of value and also that the cognition of value is a matter of coloring them from a specific point of view of the world (*ibid*., p. 68).

The problem is more fundamental in his 1939 paper:

> . . . it is questionable 1) whether we are not allowed with equal justification to assert that in so far as different products are socially treated as equal the individual concrete utilities of different goods must be considered as being reduced to some common entity, viz. a social abstract utility and 2) whether it is necessary to refer to the social abstract entity or value, at all, whether considered as a social abstract labor or as a social abstract utility. . . . It is the second problem, viz. the problem concerning the controversy between the value theory of exchange value, that demands our attention (Shibata 1939, pp. 41–2).

In other words, he was no longer interested in the choice of value entity, but in the necessity of the value concept itself. This is precisely the problematic of general equilibrium theory.

Unlike his 1933 paper, Shibata (1939) provided a general argument. Although he uses a five-sector model with a specific input-output structure, this can be easily generalized.

Using general notations, we can represent Shibata's ''system A'' (prices of production) and ''system B'' (value-price[17]) as follows, where e denotes the rate of exploitation:

$$p = (pA + pbl)(1 + r) \qquad (7)$$

$$\lambda = \lambda A + (1 + e)\lambda bl \qquad (8)$$

According to Shibata (1939), (7) and (8) can be connected using the price-value ratios $(p_i/\lambda_i's)$. If we denote D is a matrix whose ith main diagonal elements are $d_i \equiv p_i/\lambda_i$'s, and all the other elements are zero, the following can be obtained:

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[17]Value–price is defined as ''value divided by money's value'' or ''the value expressed in its relation to the value of money'' (Shibata 1939, p. 46). This term was also used by Okishio to emphasize that the dimensions of value and price are different (Okishio 1974 [1993], p. 43).

$$\lambda D = (\lambda DA + \lambda Dbl)(1 + r) \qquad (9)$$

This can be reduced to system (7) using the definition of d_i's, which implies that it is possible to obtain an identical result even if starting from system (7) instead of system (8). As Shibata noted:

> This by itself proves that the results concerning prices and the general profit rate which are obtained by the equation system B are precisely the same as those arrived at by means of the equation system A. And this means again that reference to value does not affect the results concerning prices and general profit rate (Shibata 1939, p. 47).

This conclusion suggests that the value system and the prices of production system are mutually independent. In particular, both systems are determined by physical and technical conditions of production. As Howard and King (1992, p. 138) noted, this proposition was a milestone in the debate on value theory because it anticipated what is known as the redundancy thesis of value theory.

However, there is still room for the role of value theory in Shibata's thinking. This was explicitly stated in Shibata's *Riron Keizaigaku*:

> Marx's theory . . . may be criticized for using value concept itself (setting aside the problem of whether regarding value as labor or not, and how to consider the structure of value determination) because one can explain exchange value without referring to value concept. This, however, does not imply that value concept is generally redundant, but that value concept is redundant from the viewpoint of specifying exchange value determination (Shibata 1935, p. 35).

In his postwar work, Shibata became clearer about the necessity of value theory in economics:

> This, however, is meant for asserting neither that the discussion on social value is generally unnecessary nor that it is generally erring. It is really necessary in economics, because economics is not a mere purpose-less positive science but is one of those sciences which are dedicated to the cause of some purpose or other just as the medical science is, which is dedicated to the cause of maintaining and improving human physiological health. . . . Economics is dedicated to the cause of emancipation of the laboring masses. . . . The impropriety of the Ricardian theory of value—as developed further by Marx—consists in the fact that it overlooks the role played by the entrepreneurs (Shibata 1959, p. 28).

However, Shibata's theory does not precisely clarify the usefulness of value theory. Two theoretical tendencies coexist in Shibata's economics. One is that the value concept is redundant in the general equilibrium framework, and the other is that value may be necessary from the viewpoint of ''social economics,'' which emphasizes historical and structural changes in the social system.[18]

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[18]''Social economics'' is part of the history of economic thought in Japan. Recently, ''social economics'' has been used instead of the conventional term ''political economy'' to indicate Marxian economics or, more broadly, radical non-mainstream economics (e.g., Yagi 2006).

Not surprisingly, this position was what Okishio most strongly opposed in Shibata's work. Criticizing Shibata's ''redundancy thesis,'' Okishio believed that the fundamental Marxian theorem could bridge the value system and price system:

> . . . it is difficult to agree with Shibata's argument that it is sufficient to start with price equations with an abrupt assumption of equal-rate-of-profit, without a labor theory of value, because the concept of embodied labor is indispensable in order to elucidate why equal-rate-of-profit is a positive constant (Okishio 1977, p. 207).

In this paper's notations, Okishio's task was to show that $r > 0$ in (7) and (9) necessarily requires $e > 0$ in (8). This is the necessity aspect of the fundamental Marxian theorem. It is related to the reason why Okishio (1961) focused on proving the theorem's necessity, instead of its sufficiency (Morishima 1973, p. 53).

Morishima's attitude towards the fundamental Marxian theorem should be noted here. Although Morishima gave the theorem its name (''fundamental''), he abandoned the labor theory of value or at least tried to recast it in the context of ''the von Neumann revolution'' (Morishima 1973, p. 196). Okishio notes this point:

> I can never agree with [Morishima's] conclusion that labor theory of value should be abandoned. Does he abdicate or support what he calls ''Fundamental Marxian theorem''? If he wants to support it, value concept is indispensable (Okishio 1977, pp. 274–5)[19]

Interestingly, Morishima, in his reply to Paul Samuelson, indirectly answers Okishio's question by saying

> Anyway, we may conceive of Marx without the labor theory of value, as long as we agree that the Fundamental Marxian Theorem is the core of his economic theory (Morishima 1974, p. 73).

Conclusively, while Morishima thought that the fundamental Marxian theorem could be maintained without value theory, for Okishio, it forms a basis for all the related issues in value theory.

VI. CONCLUDING REMARKS

Shibata's contribution to value theory issues has been evaluated by focusing on the Shibata–Okishio connection. Shibata's economic work contained two lines of reasoning running from general equilibrium theory to labor theory of value, combining the redundancy thesis with the viewpoint of social economics. Okishio tried to revive the analytical aspect of labor theory of value, no less than to generalize Shibata's analysis. The fundamental Marxian theorem has a central place in Okishio's theory in that it complements his Markov process solution to the transformation problem and the presentation of the Okishio theorem. In Okishio's view, the fundamental Marxian theorem is a logical refutation of the redundancy critique.

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[19]This note was originally published in *Keizai Kenkyu* (*Economic Studies*) in 1974.

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