15 The Empirical Strength of the Labour Theory of Value

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INTRODUCTION[1]

The purpose of this chapter is to explore the theoretical and empirical properties of what Ricardo and Smith called natural prices, and what Marx called prices of production. Classical and Marxian theories of competition argue two things about such prices. First, that the mobility of capital between sectors will ensure that they will act as centres of gravity of actual market prices, over some time period that may be specific to each sector (Marx, 1972, pp. 174–5; Shaikh, 1984, pp. 48-9). Second, that these regulating prices are themselves dominated by the underlying structure of production, as summarized in the quantities of total (direct and indirect) labour time involved in the production of the corresponding commodities. It is this double relation, in which prices of production act as the mediating link between market prices and labour values, that we will analyze here.

At a theoretical level, it has long been argued that the behaviour of individual prices in the face of a changing wage share (and hence changing profit rate) can be quite complex (Sraffa, 1963, p. 15; Schefold, 1976, p. 26; Pasinetti, 1977, pp. 84, 88-89; Parys, 1982, pp. 1208-9; Bienenfeld, 1988, pp. 247-8). Yet, as well shall see, at an empirical level their behaviour is quite regular. Moreover these empirical regularities can be strongly linked to the underlying structure of labour values through a linear 'transformation' that is strikingly reminiscent of Marx's own procedure.

In what follows we will first formalize a Marxian model of prices of production with a corresponding Marxian 'standard commodity' to serve as the clarifying numeraire. We will show that this price system is theoretically capable of 'Marx-reswitching' (that is, of reversals in the direction of deviations between prices and labour values). We will then develop a powerful natural approximation to the full price system, and show that this approximation is the 'vertically integrated> version of Marx's own solution to the transformation problem. Lastly, using US input-output data developed by Ochoa (1984), we will compare actual market prices to labour values, prices of production and the linear approximation mentioned above. It will be shown that various well-known propositions in both Ricarclo and Marx, concerning the underlying regulators of market prices, turn out to have strong empirical backing. In particular, measured in terms of their average absolute percentage deviations, prices of production are within 8.2 per cent of market prices, labour values are within 9.2 per cent of market prices and 4.4 per cent of prices of production, and the linear approximation is within 2 per cent of full prices of production and 8.7 per cent of market prices.[2] Lastly, we find that Marx-reswitching is quite rare (occurring only 1.7 per cent of the time), and moreover is confined to cases where the price-value deviations are small enough to be empirically unimportant. All these results point to the dominance of relative prices by the structure of production, and hence to the great importance of technical change in explaining movements of relative prices over time (Pasinetti, 1981, p. 140).

MARXIAN PRICES OF PRODUCTION AND A MARXIAN STANDARD SYSTEM

Lower-case variables are vectors and scalars, and upper-case ones are matrices. Dimensionally, all row vectors are (I x n), column vectors (n x I), and matrices (n x n).

- a_0 = row vector of labour coefficients (hours per dollar of output).
- A = input-output coefficients matrix (dollars per dollar of output).
- D = depreciation coefficients matrix (dollars per dollar of output).
- K = capital coefficients matrix (dollars per dollar of output).
- T = diagonal matrix of turnover times.
- U = diagonal matrix of industry capacity utilization rates.
- w = wage rate.
- r = rate of profit.
- p = vector of prices of production.
- v = vector of labour values.
- m = vector of market prices.

Both flows and stocks, per unit output flow, enter into the definition of unit prices of production. But whereas flow-flow coefficients such as labour or material flows per unit of output may be taken to be relatively insensitive to changes in capacity utilization (which is the *premise,* for instance, of input-output analysis), the same cannot be said of stock-flow coefficients such as capital requirements per unit of output. In this case, any presumed stability of coefficients for a given technology must refer to the ratio of stocks to normal capacity output, or equivalently to the ratio of *utilized* stocks to actual output (Shaikh, 1987, pp. 118–19, 125–26; Duménil and Lévy, pp. 250–2). With this in mind, the total stock of capital advanced consists of the money value of utilized fixed capital per unit of output (pKU) and the utilized stocks of circulating capital per unit of output $(pA + wa_0)TU$, where the turnover times matrix T translates the flow of circulating capital into the corresponding stock (Ochoa, 1984, p. 79). Then Marxian prices *of* production will be defined by:

$$p = wa_0 + p(A + D) + r([pA + wa_0]T + pK)U \tag{15.1}$$

Let $A_1 = A + D$, $B = (I - A_1)^{-1}$, $H = (K + A)UB$, $a_1 = a_0.T.B$, and $v = a_0.B$. Then from equation 15.1 we can write $p = wv + rpH + r.w.a_1$. But since the row vector a_1 can be written as $a_1 = a_0TB = a_0B(B^{-1}TB) = v(B^{-1}TB) = vT_1$, where $T_1 = (B^{-1}T.B) = (I - A_1).T(I - A_1)^{-1}$,

$$p = wv + rwvT_1 + rpH \tag{15.2}$$

which yields

$$p = wv(I + rT_1)(I - r \cdot H)^{-1} \tag{15.3}$$

We know that the wage rate and profit rate are inversely related, so that $p = p(r)$ (Sraffa, 1963, ch. 3). At one limit we have $w = 0$, $r = R$ = the maximum rate of profit, so from equation 15.2.

$$(1/R) \cdot p(R) = p(R) \cdot H \tag{15.4}$$

which implies that $1/R$ is the dominant eigenvalue of H.

At the other limit, $w = W$ the maximum wage, and $r = 0$. Then from equation 15.2, $p(0) = Wv$ – that is, prices are proportional to labour values when $r = 0$. The Marxian standard system will be defined by a column vector Xs, such that

$$(1/R) \cdot Xs = H \cdot Xs \tag{15.5}$$

so that Xs is the dominant eigenvector of $\boldsymbol{H}$.

Letting X = the gross output vector in the actual system, we scale the output vector of the standard system in such a way that the standard sum of values = the actual sum of values.

$$v \cdot Xs = v \cdot X \tag{15.6}$$

We scale the price system such that (for all r) the standard sum of prices equals the standard (and actual) sum of values.

$$p(r) \; Xs = v \cdot Xs \tag{15.7}$$

This price normalization is equivalent to expressing all money values in the ***standard labour value of money,*** $v{\cdot}Xs/p{\cdot}Xs$. Alternatively, since at $r = 0$, equation 15.2 yields $p(0) = W{\cdot}v$, where W = the maximum money wage, the normalization $p(r){\cdot}Xs = v{\cdot}Xs$ (for all r) implies $W = 1$ – that is, that the maximum money wage is the numeraire. To define the wage-profit curve implicit in the general price system, from equations 15.2, 15.5 and 15.7 we write

$$pXs = wv(I + r \cdot T_1)Xs + r \cdot p \cdot H \cdot Xs$$

By construction, $H \cdot Xs = (1/R)Xs$, and $pXs = vXs$. Define $ts = (v \cdot T_1 \cdot Xs)/(v \cdot Xs)$ = the average turnover time in the standard system. Then we get $1 = w(1 + r{\cdot}ts) + (r/R)$, ***so*** the ***Marxian standard wage-profit curve*** is given by

$$w = (1 - [r/R])/(1 + r \cdot ts) \tag{15.8}$$

Once the standard commodity is selected as the numeraire (equations 15.6–7), then what was previously the money wage, w, is now the wage defined in terms of the standard labour value of money, or equivalently as a fraction of the maximum money wage, W.

Note that the Marxian standard wage-profit curve is not linear. If we had constructed our price system as a Sraffian one with wages paid at the end, so that wages advanced, ***w.a*** did not appear as part of total capital advanced in equation 15.1, then equations 15.2 and 15.8 would reduce to the Sraffian expressions shown below, and the wage relation would be linear.

$$p = wv + rpH \tag{15.2a}$$

$$w = 1 - (r/R) \tag{15.8a}$$

Even so the standard commodity, X_S, we have defined here is ***not*** generally the same as a Sraffian one. It can be shown that even when the wage–profit curve is linear, there are in fact ***two*** standard commodities that will do the trick (see Appendix 15.1).

MARX-RESWITCHING

In Marxian analysis the direction of individual price-value deviations is quite important, since it determines transfers of surplus value between sectors and regions, and between nations on a world scale (Shaikh and Tonak, 1994, pp. 34–7). Yet one of the properties of a general price of production system is that relative prices can switch direction as the rate of profit varies (Sraffa, 1963, pp. 37-8). I will refer to this phenomenon as 'Marx-reswitching'.

Consider the simple case of a pure circulating capital model, in which we abstract from fixed capital so that $K = 0$ and $D = 0$, and from turnover time so that $t_i = 1$ for all i and hence $T = 1$. Then the Marxian price system and wage curve in Equations 15.1, 15.3 and 15.8 reduce to

$$p = w(1 + r)v + rpH \tag{15.2b}$$

where now $H = A(I - A)^{-1}$

$$w(1 + r) = 1 - (r/R) \tag{15.8b}$$

Then for $a_0 = (0.193\ \ 3.562\ \ 0.616)$ and

$$A = \begin{bmatrix} 0.05 & 0.768 & 0.02 \\ 0 & 0 & 0.169 \\ 408 & 0 & 0.10 \end{bmatrix}$$

we get $R = 1.294$ and $v = (0.845\ 4.211\ 1.494)$. Figure 15.1 shows that the standard price-value ratio, $pv_3(r)$, initially rises above 1 and then falls below it, signalling a Marx-switch at roughly $r = 1.1$.

The preceding numerical example demonstrates that Marx-reswitching is possible. But it neither establishes the conditions under

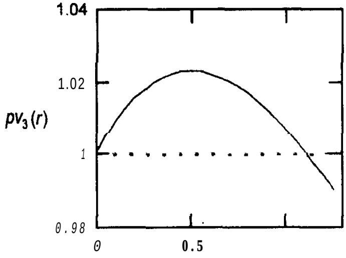


Figure 15.1 Standard price-value ratio, Sector 3

which it occurs, nor its likelihood. Although we cannot pursue the point here, further analysis suggests that when such instances occur, they do so only when an individual commodity's capital composition is 'close' to the standard one, so that its price of production is close enough to its labour value for 'Wicksell' effects (the effects of general price-value deviations on the money value of capital advanced) to have a significant influence. This is evidently the case in the preceding numerical example. More importantly, we shall see that it is also the case in every one of the (rare) empirically observed instances of reswitching (only six cases out of 355 over all years) in the US data. If true, it implies that Marx-reswitching is unimportant at an empirical level: first, because it is rare; and second, because even when it does occur, it does so only when the transfer of value involved is negligible because the pricevalue deviation is small.

APPROXIMATING PRICES OF PRODUCTION

A price system of the form in equations 15.2 and 15.8 (or indeed of the Sraffian equivalent in equations 15.2a and 15.8b) is in principle capable of very complex behaviour as far as individual prices are concerned. But there is an underlying core which is quite simple. To see this, wc begin by expressing cquation 15.2 in terms of a single price, p_i of the *ith* sector.

$$p_i = wv_i + r \cdot k_i(r) \tag{15.9}$$

where $k_1(r) = W(T_1^i + p(r) \cdot H^i$. T_1^i and H^i are the *ith* columns of the turnover matrix T_1 and the vertically integrated capital coefficients matrix H, respectively, so the term $K_i(r)$ represents the money value of the vertically integrated capital advanced per unit output of the *ith* sector.

We know from Sraffa (1963) that as $r \to R$, in *every industry i* the (money value of the) output-capital ratio, q_i approaches the standard output-capital ratio, $q_s = R$. This can be derived directly from equation 15.4. Note that this standard ratio R, which is the vertically integrated output-capital ratio of every industry at $r = R$, is also the *labour value of vertically integrated output-capital of the standard system.* To see this, multiply equation 15.5 on both sides by the labour value vector, v, to get $v \cdot Xs / v \cdot H \cdot Xs = R = q_s$

At the other limit, when $r = 0$ and the standard wage $w = 1$, we get $p = v$ (standard prices equal labour values) and the *ith* sector's output-capital ratio becomes $q_{oi} = v_i / (H^i + T_i^i)$, which is reciprocal of the labour value of the sector's vertically integrated technical composition of capital (that is, the ratio of the total labour time required for the production of commodity i to the total labour time materialized in the total capital inputs for this same commodity).'

We see therefore that for $0 < r < R$ the output-capital ratio $q_1(r)$ of every industry must lie between its *own labour value* output-capital ratio, q_{oi} and the *common standard labour value* output-capital ratio q_s. With this in mind, we turn to a simple approximation of the price system. The general system of equation 15.2 can be expressed as

$$p = wv + r \cdot wvT_1 + rpH = (wv[I \; I\text{-} \; r \cdot T_1] + r \cdot vH) + r(p - v)H \tag{15.10}$$

In this expression, the first term on the right-hand side $(w[I + r \cdot T_1] + r \cdot vH)$ represents the component of prices of production that arises when constant capital (fixed capital and inventories) *is valued at its labour* value, while the remaining term represents the further effects of price-value deviations on the value of capital stocks. The first term is therefore the vertically integrated equivalent of Marx's transformation procedure, as presented in volume III of *Capital.* We may call it the Marx component of prices of production. The second term, on the other hand, may called the Wicksell-Sraffa component (Schefold, 1976, p. 23). On the assumption that this second term is small (which we will test shortly), we may approximate price of production via the Marx component alone:

$$p'(r) = wv + r(wT_1 + H) = (w[I + r \cdot T_1] + r \cdot H)v \qquad (15.11)$$

Equation 15.11 implies a corresponding approximation for the output-capital ratio. Here the approximate unit capital advanced is $k_i'(r) = wv(T_1)^i + vH^i$, so that the output-capital ratio is

$$q_i'(r) = p_i'/k_i' = (wv_i + r \cdot k_i')/k_i' = (wv_1/[wT_1^i + H^i]) + r \qquad (15.12)$$

This latter approximation[4] yields the sectoral labour value ratio $q_{0i} = v_i/(H^i + T_i^i)$ when $r = 0$ and $w = 1$, and yields the standard labour value ratio (standard output–capital ratio) $q_s = R$ when $r = R$ and $w = 0$. In other words, the simple approximation to prices of production in equation 15.11 is equivalent to approximating each sector's output-capital ratio in terms of components that *depend only on labour values,* and in such a way that each sectoral output-capital ratio approximation is *exact* at the two endpoints $r = 0$ and $r = R$.[5]

The linear price approximation in equation 15.11 is a vertically integrated version of Marx's own transformation procedure. It is both analytically simple and, as we shall see, empirically powerful. However, before we proceed to the empirical analysis, it is worth noting that quadratic and higher approximations of the general price system of equation 15.2 can be easily developed. In effect, the the linear approximation *p'(r)* was created by sustituting the value vector v for the price vector p(r) on the right-hand side of equation 15.2, which amounts to ignoring the (Wicksell) effects of price-value deviations on the vertically integrated capital stock. A quadratic approximation can in turn be created by substituting *p'(r)* for *p(r),* which amounts to ignoring the effects of the errors in the linear approximation on the vertically integrated capital stock, and so on.[6] Although the quadratic approximation has little improvement to offer for US data, it will turn out to be useful in our discussion below of empirical applications of the pure circulating capital model Marzi and Varri, 1977.

EMPIRICAL RESULTS: MARKET PRICES, LABOUR VALUES AND PRICES OF PRODUCTION

The empirical calculations presented here are based on the data developed by Whoa (1984), covering the input-output years 1947, 1958, 1963, 1967 and 1972. Work is underway to extend the results to the years 1977, 1982 and 1987 (the last available input-output year). Further details are in Appendix 15.2.

Since most data patterns are similar across all the input-output years, we will generally use the 1972 data to illustrate them. Any exceptional patterns will then be separately identified. It is useful to note at this juncture that because input-output tables are cast in terms of aggregated industries, there is no natural measure of 'output' for a given sector. One must pick a level such as (say) $100 worth of output in each sector, which means that the market price for this output is $100 for each sector. Such a procedure poses no real problems for the calculation of unit labour values or prices of production, but when comparing vectors it does require one to distinguish between 'closeness of tit' in the sense of the deviation (distance) between them from the correlation between them (Ochoa, 1984, pp. 121-33; Petrovic, 1987, pp. 207-8). General measures of the proportional deviation between two vectors, such as the mean square error (MSE), root mean square error (RMSE), mean absolute deviation (MAD) and mean absolute weighted deviation (MAWD) are all line, and give essentially similar results for this data. But the correlation coefficient R, or the R^2 of a simple linear regression, are *not* meaningful in this case because (by construction) market prices show no variation, and hence will show no covariation with the other vectors. In what follows we will therefore select the mean absolute weighted (proportional) deviation (MAWD), each sector's weight being equal to its share in the labour or money value of total gross output. For two vectors with components x_i, y_i, and with weights z_i, mean absolute weighted deviation $(MAWD) = \Sigma(|y_i - x_i|Z_i)/\Sigma x_i z_i$

Market Prices, Labour Values and Prices of Production at the observed Rate of Profit

For each input-output year, total labour times' $v = a_0(I - A_1)^{-1}$ are calculated directly. Using the actual (uniform) rate of profit in each input-output year (Ochoa, 1984; p. 214), we calculate standard prices of production (prices of production in terms of the standard commodity) from equations 15.2 and 15.8 Since we have only average annual rates of capacity utilization u for the economy as a whole (Shaikh, 1987), we do not use them when calculating individual prices of production. We do use them, however, when subsequently comparing the time trend of the observed actual and maximum profit rate r and R, respectively, to those of the normal-capacity rates r_u = r/u and $R_u = R/u$.[8]

Standard prices of production are defined by the scaling $p(r) \cdot Xs = v \cdot X$ for all r (since this defines the standard commodity as the numeraire), so they are implicitly in the same units as labour values (which they equal at $r = 0$). They can therefore be directly compared to labour values. To make market prices comparable to both, we rescale market prices to units of labour time by multiplying the market price vector, m by the standard value of money $= m \cdot Xs/v \cdot Xs$. This makes all three vectors have the same sum of prices, and hence the same average level, which facilitates direct comparisons of their levels. It does not, of course, change relative market prices in any way.

In all years, both total labour times and prices of production are quite close to market prices. Table 15.1 summarizes the mean average percentage deviation (MAWD) between various pairs of vectors.

Table 15.1 establishes that both labour values and prices of production are quite close to market prices, with average percentage deviations of 9 per cent for the former and 8 per cent for the latter. It also establishes that labour values and prices of production are closer to each other than to market prices, with an average deviation of only 4.4 per cent between the two.

Table 15.1 Average percentage deviations (MAWD), (rescaled) market prices, labour values and prices of production at observed rates of profit

	1947	1958	1963	1967	1972	Average
Labour value vs market price	0.105	0.090	0.092	0.102	0.071	0.092
Price of production vs market price	0.114	0.075	0.076	0.084	0.063	0.082
Labour value vs price of production	0.056	0.038	0.038	0.048	0.038	0.044

Figure 15.2 illustrates the strong empirical connection between labour values and market prices for 1972, with the horizontal axis representing the total market value of standard sectoral outputs (ms_iXs_i, where ms_i = observed market prices m_i rescaled in the manner discussed above) and the vertical axis representing the corresponding total labour values. A 45” line is also shown for purposes of visual reference.

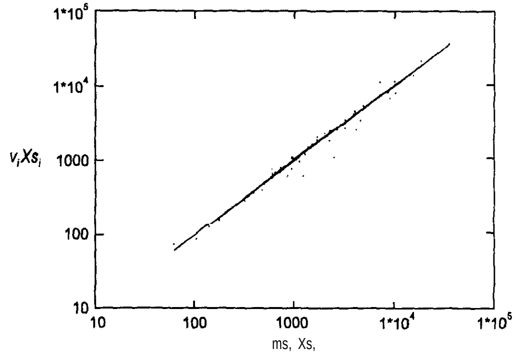


Figure 15.2 Total labour values vs total (rescaled) market prices, 1972 (log scales)

Figure 15.3 plots total sectoral prices of production p_iXs_i (sectoral standard outputs valued at prices of production) versus corresponding (rescaled) market prices ms_iXs_i.

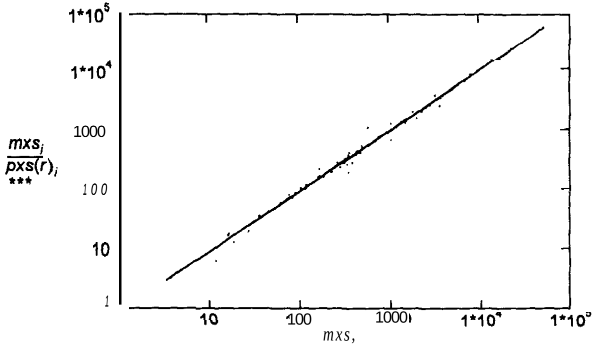


Figure 15.3 Total prices of production (at observed r=0.188) vs total (rescaled) market prices, 1972, (log scales)

Calculating Marxian Standard Prices of Production as Functions of the Rate of Profit

The next set of results pertain to the behaviour of standard prices of production as the rate of profit varies between r = 0 and r = R. Four things immediately stand out. First, in all years the relationship between the rate of profit and individual prices of production is ***almost invariably linear.*** Second, instances of Marx-reswitching are very rare (six cases out of 355 total prices in all the years). And third, the previously developed linear approximation to prices of production, which represents a vertically integrated version of Marx's own 'transformation procedure', performs exceedingly well: ***the average deviation over all years between the approximation and full prices of production is on the order of* 2 *per cent!*** And fourth, *in* relation to market prices, the linear approximation performs slightly better than full prices of production in one year and slightly worse in the others, with an average deviation of only 8.7 per cent (compared with 8.2 per cent for full prices of production in relation to market prices).

Figure 15.4 displays the movements **of** standard price of production–labour value ratios $PvT(r)_i$ as the ratio $x(r) = r/R$ varies between 0 and 1 (that is, as r varies between 0 and R) for 1972. The striking linearity of these patterns holds in all other years. In reading the various graphs, it is important to note that their vertical scales vary. Also of interest are the two instances of Marx-reswitching that occur in sectors 56 (aircrafts and parts) and sector 60 (miscellaneous manufacturing). Figure 15.5 and 15.6 present a close-up of this phenomenon. Over all years, there are only six cases of reswitching out **of** 355 prices series, and as hypothesized, in each case the switches in the direction of standard price-value deviations occur ***only when*** the price is itself very close to value throughout the range of the rate of profit.

Since labour values and market prices are given in any input-output year, the essentially linear structure of standard prices of production with respect to the rare of profit implies that the average deviation between prices of production and labour values (and market prices) increases more or less monotically with the rate of profit r. It is of interest, however, to note that the ***range*** of these deviations is quite small: ***even at the maximum rate of profit, price-value deviations average only 12.8*** per ***cent over all years.*** Table 15.2 reports these upper limits in each year.

Table 15.2 Average deviations of standard prices of production from labour values, at r = *R*

	1947	19.58	1963	1967	1972	Overall average
Average deviation at $r = R$	0.193	0.119	0.111	0.115	0.102	0.128

Testing the Linear Approximation to Full Prices of Production

We turn next to the relation between full standard prices of production and the linear approximation developed in equation 15.11. As noted earlier, this approximation, which represents a vertically integrated version of Marx's own transformation procedure, performs extremely well as a predictor of full prices of production (with an overall average deviation of only 2 per cent) and as a predictor of market prices (with an average deviation of 8.7 per cent). Figure 15.7 illustrates for 1972 a (typical) scatter between the two sets of prices, which are so close that the scatter looks like a straight line even though there is no reference line on this graph. Figure 15.8 plots the path of the corresponding average deviation as x(r) = r/R varies. Note that the largest deviation is only 2.5 per cent, and that the endpoint at $r = R$ is only 1.5 per cent. This too is typical.

Table 15.3 Actual and normal-capacity rates of profit

	1947	1958	1963	1967	1972
Actual profit rate, r	0.247	0.179	0.212	0.233	0.188
Maximum profit rate, R	0.806	0.700	0.739	0.748	0.670
Capacity utilization, u	0.876	0.819	0.995	1.129	1.088
Adjusted actual profit rate, r_u	0.281	0.219	0.213	0.207	0.173
Adjusted maximum profit rate R_u	*0.921*	*0.842*	*0.743*	*0.663*	*0.616*

Finally, as noted earlier, Marx's analysis of the trends of actual and maximum rates of profit abstracts from the fluctuations produced by cyclical and conjunctural phenomena. As such, the relevant empirical measures are normal (capacity adjusted) rates, not observed ones. In this regard it is interesting to see what a difference it makes to the perceived trends of r and R when one adjusts for capacity utilization. Table 15.3 presents the observed rates of profit r (Ochoa, 1984, p. 214).

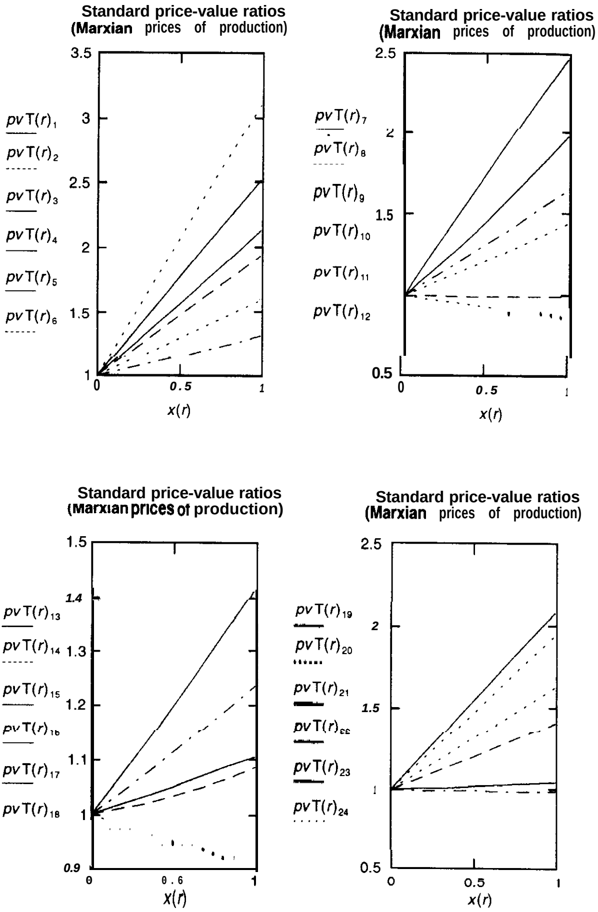
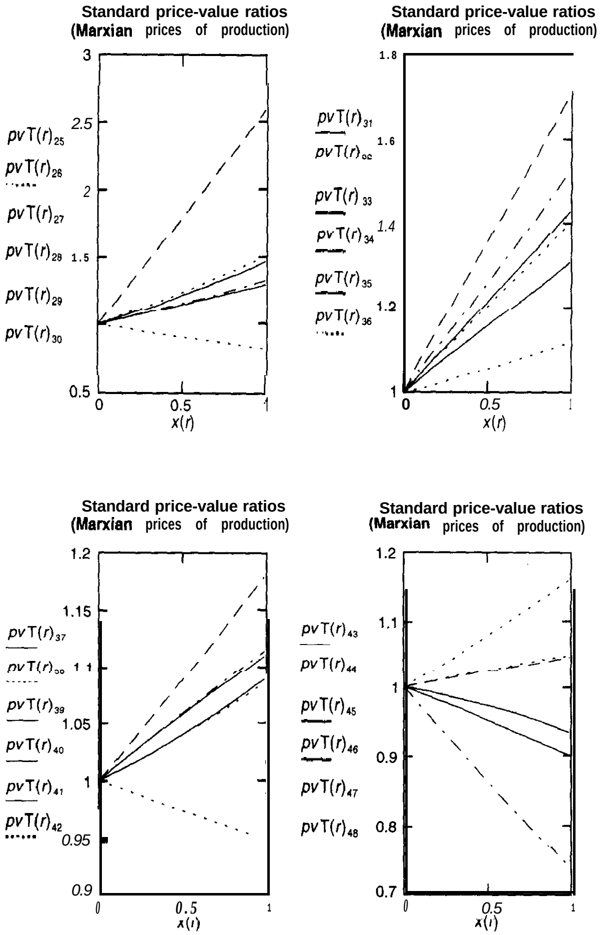


Figure 15.4 The behaviour of standard price-value ratios as x(r) = r/R varies, 1972

Figure 15.4 continued



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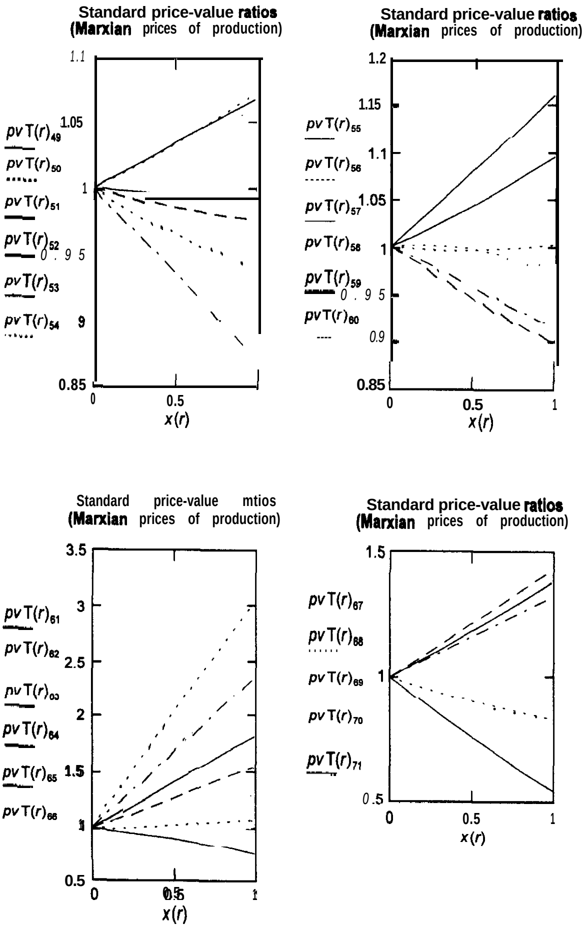

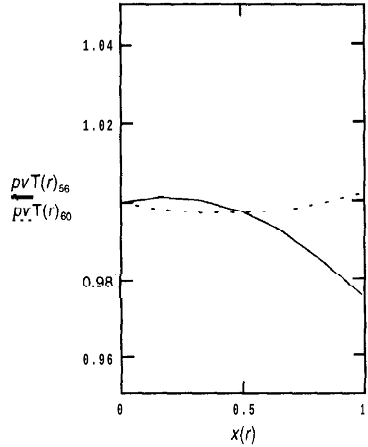

Figure 15.5 Price-value reswitching, Sectors 56 and 60, 1972

$pvT(r)_{56}$	$pvT(r)_{60}$
1	1
1.0013	0.998
1.0003	0.9969
0.9972	0.9967
0.992	0.9974
0.9847	0.999
0.9753	1.0016

Figure 15.6 Price-value reswitching, Sectors 56 and 60, 1972

our own calculations for the maximum rate of profit *R* and data on capacity utilization rates (Shaikh, 1987, Appendix B), which is then used to calculate normal capacity rates of profit, and $r_u = r/u$, as discussed previously. Note the adjusted rates exhibit a falling trend,

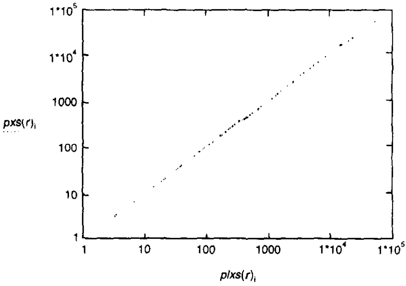

Figure 15.7 Price approximation vs full prices of production (at obserrved r=0. 188), 1972 (log scale)

while the unadjusted ones have no clear pattern. This highlights the potential importance of such adjustments.

SUMMARY AND CONCLUSIONS

This chapter has explored the theoretical and empirical links between market prices, prices of production and labour values. Prices of production are important because in a competitive system they directly regulate market prices; and labour values are important because they serve both as the foundation of prices of production and as their

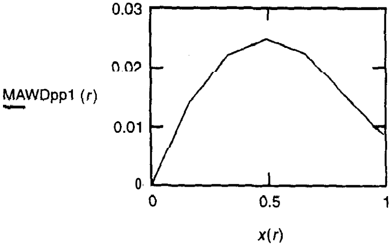

Figure 15.8 Average diviations, price approximation vs full prices of production, 1972 dominant components over time. This last aspect is particularly important, because over time technical change alters relative labour values and hence relative prices of production.

To address the above links, we first developed a model of prices of production that accounts for stocks, flows, turnover times and capacity utilization rates. These prices were in turn normalized by means of a Marxian standard commodity, which is generally different from the familiar Sraffian one. It is known that as the rate of profit r varies from zero to the maximum rate of profit R, prices of production can change in complex ways. We have shown that they are capable of reversing direction with respect to labour values, a phenomenon that we call Marx-reswitching. But on both theoretical and empirical grounds, this is not likely to be of any practical importance. On the other hand, a linear approximation to standard (that is, normalized) prices of production, one that can be viewed as a vertically integrated equivalent to Marx's own 'transformation' procedure, turns out to be of great significance. All of its structural parameters depend only on labour value magnitudes. And at an empirical level, it turns out to be an extremely good approximator of full prices of production (within 2 per cent), and hence an equally good explanator of market prices (within 8.7 per cent).

In our empirical analysis we compared market prices, labour values and standard prices of production calculated from US input-output tables for 1947, 1958, 1963 and 1972 using data initially developed by Ochoa (1984) and subsequently refined and extended by others (Appendix 15.2). Across input-output years we found that on average labour values deviate from market prices by only 9.2 per cent, and that prices of production (calculated at observed rates of profit) deviate from market prices by only 8.2 per cent (Table 15.1 and Figures 15.2-3).

Prices of production can of course be calculated at all possible rates of profit, r, from zero to the maximum rate of profit, R. The theoretical literature has tended to emphasize the potential complexity of individual price movements as r varies. Such literature is generally cast in terms of pure circulating capital models with an arbitrary numeraire. But our empirical results, based on a general fixed capital model of prices of production with the standard commodity as the numeraire, uniformly show that standard prices of prices of production are virtually linear as the rate of profit changes (Figure 15.4). Since standard prices of production equal labour values when $r = 0$, this implies that price-value deviations are themselves essentially linear functions of the rate of profit. For this reason, the linear price approximation developed in this chapter performs extremely well over all ranges of r and over all input-output years, deviating on average from full prices of production by only 2 per cent (Figures 15.6-7) and from market prices by only 8.7 per cent (as opposed to 8.2 per cent for full prices of production relative to market prices).

What explains the linearity of prices of production over all rates of profit? It is certainly not because prices of production are close to labour values, as Figure 15.4 makes clear: in 1972 the coefficient of variation (standard deviation over the mean) of direct capital-labour ratios expressed in labour value terms is 0.080, and that of vertically integrated capital-output ratios is 0.04. Nor is it due to the particular size of the maximum rate of profit, R, since multiplying the matrix H (whose dominant eigenvalue is $1/R$) by different scalars has virtually no effect on the linearity of individual prices.

A large disparity between first and second eigenvalues is another possible source of linearity.' But here, although the ratio of the absolute values of the first to second eigenvalues varies across input-output years from 2.76 to 232.20, near linearity holds in all years. This at least raises the question of how 'big' such a ratio must be to produce near linearity.

There are some clues, however. The choice of a standard commodity as numeraire is evidently important, as Sraffa so elegantly demonstrates. Obviously, if individual prices of production expressed in terms of the Marxian standard commodity are linear in r, choosing any arbitrary commodity as numeraire is equivalent to creating ratios of linear functions of r, and these can display (simple) curvature. So choosing the appropriate numeraire 'straightens out' individual price curves to some extent. But this is only part of the story. If one abstracts from fixed capital (so the matrices $K = 0$, $D = 0$), and from turnover time (so $T = I$) then the resulting 'pure circulating capital' model *does* show substantial curvature in the movements of individual prices of production even when prices are expressed in terms of the (new) standard commodity. This suggests that the *structure* of stock/flow relations represented by K (rather than their size, since varying R makes virtually no difference) also plays an important role. Circulating capital models are quite popular in the theoretical literature, which may explain the theoretical presumption that prices of production are curvilinear with respect to the rate of profit. But of course the discrepancies between the full model and the circulating capital model only point to the unreliability of this presumption. Moreover, even in this case any curvature of individual prices of production remains fairly simple (being convex or concave throughout), Marx-reswitching is just as rare, the linear price approximation captures about 80 per cent of the structure of prices of production, and the simple quadratic approximation discussed at the end of the section on 'Approximating Prices of Production' captures 92 per cent.

The puzzle of the linearity of standard prices of production with respect to the rate of profit is certainly not resolved. But its existence emphasizes the powerful inner connection between observed relative prices and the structure of production. Even without any mediation, labour values capture about 91 per cent of the structure of *observed* market prices. This alone makes it clear that it is technical change that drives the movements of relative prices over time, as Ricardo so cogently argued (Pasinetti, 1977, pp. 138-43). Moving to the vertically integrated version of Marx's approximation of prices of production allows us to retain this critical insight, while at the same time accounting for the price-of-production-induced transfers of value that he emphasized. On the whole these results seem to provide powerful support for the classical and Marxian emphasis on the structural determinants of relative prices in the modern world.

APPENDIX 15.1 MARXIAN AND SRAFFIAN STANDARD COMMODITIES

The Marxian standard commodity Xs can be different from a Sraffian one, even though both yield the same wage-profit curve. Consider the simple case of a Sraffian model with circulating capital that turns over in one period in each industry (so that $T = I$), infinitely lived fixed capital (so that D = [0]) and wages paid at the end of the period (so that wages do not appear as part of the capital advanced). Then

$$p = wa_0 + pA + rpK$$

At w = 0 we get $p(R) = p(R)A + RpK$. Sraffa's standard system is the quantity dual $Xs' = A \cdot Xs' + RK \cdot Xs$, so that the standard net product $Ys' = (I - A)Xs' = RK \cdot Xs$. This implies that $(1/R)Xs' = (I - A)^{-1}K\ Xs'$, so that Xs' is the right-hand dominant eigenvector of the matrix $(I - A)^{-1}K$. Sraffa also normalizes prices by setting the sum of prices of the standard *net* output Ys' equal to the sum of labour values of this net output. This latter quantity is the amount of living labour in the standard system, which is in turn scaled to be the same as that in the actual system: $p \cdot Ys' = v \cdot Ys' = v \cdot Y$, where $Y =$ net output in the actual system (Sraffa, 1963, p. 20).

For the very same price system, we derive the Marxian standard by noting that at w = 0 the price system can be written as $(1/R) \cdot p(R) = p(R) \cdot (K \cdot [I - A]^{-1})$ and we define the Marxian standard commodity' by $(1/R) \cdot Xs = (K[I - A]^{-1}) \cdot Xs$, so that Xs is the dominant right-hand eigenvector of the matrix $K(I - A)^{-1}$. Recall that we normalize quantities by setting the sum of labour values of *total* output = the actual sum of values ($v \cdot Xs = v \cdot X$) and normalize prices by setting the standard sum of prices of total output = the standard sum of values of total output ($p \cdot Xs = v \cdot Xs$).

It is known that the matrices $K(I - A)^{-1}$ and $(I - A)^{-1}$ have the same eigenvalues. But they do not, in general, have the same eigenvectors (Schneider, 1964, p. 131). Therefore, in general the two standard commodities, Sraffian and Marxian, will be different. Only in the case of pure circulating capital (K = A), uniform turnover rates = 1, and wages paid at the end of the production cycle (as in this illustrative model), will the two matrices, and hence the two standard commodities, be the same.

In spite of their differences, the two different standard commodities will nonetheless both yield linear wage profit curves, albeit with the wage expressed in terms of a different numeraire.

To see this for the Sraffian standard, write the illustrative price equation as $p(I - A) = wa_0 + rpK$. The Sraffian standard commodity is defined by $Ys' = R \cdot K \cdot Xs'$, where $Ys' = (I - A) \cdot Xs'$, and the price normalization is $pYs' = vYs'$, where $v = a_0 \cdot (I - A)^{-1}$, so we can write $p(I - A)Xs' = pYs' = w(a_0 \cdot Xs') + r \cdot p \cdot K\, Xs' = w(v \cdot Ys') + (r/R) \cdot p \cdot Ys'$. Thus $w' = 1 - r/R$. Note that here the wage w' is the wage share in the Sraffian standard system net product per worker, because the price normalization implies that $pYs'/a_0Xs' = 1$.

For the Marxian standard, we express the same price system in the form $p = wv + r \cdot p \cdot K \cdot (I - A)^{-1}$. The Marxian standard commodity is defined by $(1/R) \cdot Xs = (K[I - A]^{-1})$, and with prices normalized by $pXs = vXs$, we get $pXs = wv \cdot Xs + (r/R)pXs$, so that $w = 1 - r/R$. In this case w represents a share of the maximum wage W, because when $r = 0$, $p(0) = W \cdot v$, so that the normalization $pXs = vXs$ (for all r) implies that $W = 1$ – that is, that W is the numeraire.

APPENDIX 15.2 DATA SOURCES AND METHODS OF CALCULATION

All input-output data is from Ochoa (1984) at the 71-order level: the labour coefficients vector a_0, and matrices of input-output coefficient A, capital stock coefficients K, depreciation coefficients D, and turnover times T. Sectoral output units are defined as $100 worth of output, so all market prices equal $100 by construction. The current data set spans the input-output years 1967, 1958, 1963, 1967 and 1972, but work is underway on a revised and more comprehensive data set spanning both earlier and later input-output tables, based on the work of Michel Julliard. Ara Khanjian, Paul Cooney, Greg Bongen and Ed Chilcote. Since sectoral capacity utilization rates are unavailable at present, we set $U = I$ in the calculations of labour values and prices of production, although we do use the aggregate capacity utilization rate (Shaikh, 1987, Appendix B) to adjust actual and maximum rates of profit (see Table 15.3).

***Table* 15A.1 Sector list**

Industry no.	*Industry name*	*BEA Z-O no.*	*Industry no.*	*Industry name*	*BEA I-O No.*
1	Agriculture	1	37	Screw machine products	41
2	Iron & ferroalloy ores mining	5	38	Other fab. metal prods.	42
3	Nonferrous metal ores mining	6	39	Engines & turbines	43
4	Coal mining	7	40	Farm machinery & equipment	44
5	Crude petrol. & natural gas	8	41	Construction mach. & equip.	45
6	Stone, clay mining quarrying	9	42	Materials handling equipment	46
7	Chem. & fertilizer mineral mining	10	43	Metalworking mach. & equip.	47
8	New & repair construction	11	44	Spec. indust. machs. equip.	48
9	Ordance & accessories	13	45	Gen. indust. machs. & equip.	49
10	Food & kindred products	14	46	Machine shop products	50
11	Tobacco manufacturer	15	47	Office & computing machines	51

Table 15A.1 Sector list (Contd)

Industry no.	*Industry name*	*BEA Z-O no.*	*Industry no.*	*Industry name*	*BEA Z-O No.*
12	Fabrics, yarn & thread mills	16	48	Service industry machines	52
13	Misc. textile goods & floor cov.	17	49	Electric trans. equip.	53
14	Apparel	18	50	Household appliances	54
15	Misc. fabricated textile prod.	19	51	Electric wiring & lighting	55
16	Lumber wood prod. exc. containers	20	52	Radio, TV & comm. equip.	56
17	Wooden containers	21	53	Elec. components & access	57
18	Household furniture	22	54	Misc. electrical machinery	58
19	Other furniture & fixtures	23	55	Motor vehicles	59
20	Paper & allied products	24	56	Aircraft & parts	60
21	Paperboard containers & boxes	25	57	Other transportation equip.	61
22	Printing & publishing	26	58	Professional & scientific inst.	62
23	Chemicals & allied products	27	59	Photographic & optical gds.	63
24	Plastics & synthetic materials	28	60	Misc. manufacturing	64
25	Drugs, cleaning & toilet prep.	29	61	Transportation	65
26	Paints & allied products	30	62	Communications exc. brdcst	66
27	Petroleum refining	31	63	Radio & TV broadcasting	67
28	Rubber & misc. plastic products	32	64	Public utilities	68
29	Leather tanning	33	65	Wholesale & retail	69
30	Footwear & other leather products	34	66	Finance & insurance	70
31	Glass&glass products	35	67	Htels & repr. places exc. auto	72
32	Stone & clay products	36	68	Business serv.; R&D	73

Table 15A.1 Sector list (Contd)

Industry no.	*Industry name*	*BEA I-O no.*	*Industry no.*	*Industry name*	*BEA I-O NO.*
33	Primary iron & steel mfg.	37	69	Auto repair & services	75
34	Primary nonferrous metals mfg.	38	70	Amusements	76
35	Metal containers	39	71	Medleduc serv. nonprof. org.	77
36	Heating & fabricated metal prod.	40			

Notes

1. I wish to thank Gerard Duménil, Dominique Levy and Alan Freeman for helpful comments, Edward Ochoa for making available his input–output data, and Greg Bowgen and Ed Chilcote for their help with this data.
2. My results are similar to Ochoa's as far as inter-industry comparisons of labour values, prices of production and market prices are concerned (Ochoa, 1984). But whereas he uses actual gross output as the numeraire, I use the standard commodity. Also, like Bienenfeld my focus is on the determinants and behaviour of individual pricevalue deviations (Bienenfeld, 1988).
3. The term (v . $[K + AT]' + a_0T^i)/(a_0)_i$ is the ratio of the labour value of the direct capital advanced to the direct labour time required in production (see equation 15.1). If one calls this the *ith* 'materialized composition of capital', then the ratio of the labour value of total capital advanced to total labour required = $v \cdot (H + T_1^i$ /vi) = l/q, is the i^{th} vertically integrated materialized composition of capital.
4. The approximation is linear in w and r, but non-linear in r alone as long as turnover times differ across industries. Suppose all turnover times were alike, so that $T = t \cdot I$. Then $T_1 = B \cdot T \cdot B^{-1} = T = t \cdot I$, and the standard turnover time $ts = (v \,.\, T_1 \,.\, X_s)/(v \cdot X_s) = t$, and the wage rate $w = (1 - r/R)/(1 + r \cdot ts) = (1 - r/R)/(1 + r \cdot t)$. Substituting these into equation 15.11 yields $p'(r) = ([I - r/R] + r \; H)v$, which is linear in r.
5. Needless to say, we could have instead approximated output-capital ratios directly, and then used this to derive an approximation to the price system. But then the analytical simplicity of the price approximation is generally lost. Since the simple price approximation is also empirically very powerful, there seems to be no gain in an alternate procedure.

6. Bienenfeld (1988) chose to extend my (previously developed) linear approximation by creating a quadratic approximation that is exact at both $r = 0$ and $r = R$. But the economic interpretation of the terms involved is obscure.
7. We do not distinguish between production and non-production labour in these particular estimates, but then it is not clear that such a distinction is appropriate when modellmg individual prices, since the cost of activities such as wholesale retail trade will show up in the total costs of a commodity (Shaikh and Tonak, 1994, pp. 45-51).
8. The maximum profit rate R is the output-capital ratio of the standard system PX_s/PK_s, where both Xs and Ks are evaluated in any common price system (prices of production, market prices or labour values). To adjust for capacity utilization, we can either compare actual output flow Xs to utilized K u, or normal capacity output X_s/u to actual capital stock K. In either case the normal capacity maximum rate of profit $R_u = R/u$.
9. In recent private correspondence, Gerard Duménil and Donminique Lévy have shown that this could be a sufficient condition for ncar linearity. I had come to the same conclusion on the basis of my iterative procedure for linking Marx's 'transformed values' to full prices of production, since the speed of convergence depends on this ratio (Shaikh, 1977, mathematical appendix, unpublished).
10. The Marxian standard commodity can be shown to be related to the von Neumann ray (Shaikh, 1984, pp. 60–1).

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