The Normative and Positivistic Inferiority of Marx's *Values* Paradigm*

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I. Introduction

Professor Nguyen [5] honors my 1971 postmortem on Marx's "transformation problem," Samuelson [10], by auditing its logic and results. It is gratifying that Nguyen's procedures (1) "will in general confirm Samuelson's result;" (2) give, for *values*, the result "found to be exactly that given in Samuelson's paper;" (3) give, for *prices*, "precisely that given in Samuelson's Table 3B;" and (4) show that "The conclusion reached by Professor Samuelson is . . . demonstrated in a fairly straight-forward fashion . . ." by a matrix characteristic-value formulation.

But, Professor Nguyen points out, Baumol [1] and Morishima [3;4] do not agree that my exposition was optimal. And Nguyen states that his result is not in complete agreement with my "erase-and-replace" description of how one goes from Marx's 1867 values to pre-1867 and post-1867 competitive prices "to the extent that the two 'alternative and discordant systems' that Professor Samuelson contemplated are shown [by Nguyen's analysis] to be not entirely arbitrary as was indicated by him [Samuelson]..." in his 1970 and 1971 papers.

I can add many names to the list of writers who found my "erase-and-replace" characterization as somehow overdramatic. One of the latest is a distinguished Soviet economist of Novosibirsk, K. Valtukh [20, 179] who dubs my 1971 analysis a "polemic outburst."

What I can most usefully do here is to relate the Nguyen analysis to previous discussions of the transformation program, and show that it precisely fits in with my intended contrast between alternative and inconsistent paradigms. Then, constructively, I can use the occasion to show why the *values* paradigm fails as a normative guide to policy and as a positivistic description of how a competitive system will display its statical and historical laws of motion and development.

The reader will be able to verify that no cogent derivation in Morishima [3;4], Baumol [1], Valtukh [20], or Nguyen [5] in any sense negates the truth of propositions proved by me or hypothesized by my pen.

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II. Analysis

To produce 1 of good *j* a period from now, you must have as inputs now a_{0j} of direct labor and $(a_{1j}, a_{2j}, ..., a_{nj})$ of goods 1,2,...,n. (Note that I have restored Nguyen's transposition of a_{ij} notation to its customary form.) Each unit of labor needs, if it is to be available in the steady state, the stipulated vector of subsistence wage-goods $(m_1, ..., m_n)$.

In 1971, I derived the implied rate of profit, R^* , and the implied vector of *real* prices, $(P_j/W)^*$, as roots of the relations

$$[P_j/W] = \mathbf{a}_0(1+R) [\mathbf{I} - \mathbf{a}(1+R)]^{-1}$$
(1a)

$$W/(\sum_{j=1}^{n} P_j m_j) = 1.$$
 (1b)

But I could indifferently have used the eigenvector formulation of the basic 1957 Seton [18] paper that I cited. It introduces as "feeding coefficients," which summarize the amounts of the goods needed as subsistence inputs to maintain the needed labor, the matrix which Nguyen calls **B** in his treatment of my 3-good example. In traditional notation **B** is $[m_i a_{0j}]$.

In place of (1) above, I could have written the Seton-Nguyen equivalent form

$$[P_j/W][(a_{ij}+m_ia_{0j})(1+R)] = [P_j/W]$$
(2a)

$$[P_j/W][m_i] = 1, \text{ normalization.}$$
(2b)

Substitution confirms that (1) and (2) are completely equivalent, and it is no coincidence that Nguyen verifies the correctness of my 1971 numbers for real prices and profit rate. Neither formulation is better than the other, substantively or for throwing light on what Marx did or should have done.

Now for the paradigm of marked-up Marxian values. Denoting the common rate of surplus value, used to mark up wage payments in all industries by r^* , I derived the resulting real marked-up values, (p_j/w) and r^* , as roots of

$$[p_j/w] = \mathbf{a}_0(1+r) [\mathbf{I} - \mathbf{a}]^{-1}$$
(3a)

$$w/(\sum_{j=1}^{n} p_j m_j) = 1.$$
 (3b)

Nguyen, using the Seton matrix with \mathbf{a} augmented by the feeding coefficient component of the matrix \mathbf{B} , writes this paradigm as the roots of the eigenvector relations

$$[p_j/w][a_{ij}+m_{ij}a_{0j}(1+r)] = [p_j/w]$$
(4a)

$$[p_j/w][m_i] = 1, \text{ normalization.}$$
(4b)

Just as (1) and (2) are easily shown to be identical, (3) and (4) are easily shown to be identical. Just as (1a) and (2a) are alternative incompatible equilibrium regimens, so (3a) and (4a) are alternative incompatible equilibrium regimens.

Using an eigenvector formulation rather than a solved-out formulation does not one whit whitewash the arbitrariness of the (2) and (4) paradigms. Whether Baumol [1] is right or half-right in his allegation that Marx thought that one could understand the essence of

the (1) or (3) solution *only after* one contemplates Marx's novel (2) or (4) paradigm, any such belief on Marx's part is an illusory belief. (Whether Baumol would be right in alleging that the chap who wrote me a letter last week thought he was trisecting the angle, we must be clear that a general angle can't be trisected by means of a compass and straight edge alone.)

To clinch the point that an eigenvector description has no power to alter the merits and demerits of economic alternatives, consider the bizarre third paradigm that appeared in Samuelson [12, 65] in response to Leontief's oral suggestion of a *reductio ad absurdum*. Instead of having a regime that marks up all outlays by the same percentage factor, let me here give an eigenvector formulation of a third Napoleonic regime, different from the regimen of *prices* and of *values*, a regimen that marks up only outlays on raw-material inputs and not on labor inputs. If 100ρ is the percent rate of markup on raw materials, (2a) and (4a) are shoved aside in favor of the following relations for ρ^* and real prices $(\pi_i/\omega)^*$:

$$[\pi_j/\omega][a_{ij}(1+\rho) + m_i a_{0j}] = [\pi_j/\omega], a \ge \text{and indecomposable}$$
(5a)

$$\sum_{j=1}^{n} (\pi_j/\omega)m_j = 1.$$
 (5b)

I do not know anyone who would not consider the system (5) a bizarre system. Whether (5) in its relation to (1) or (2) is, in Nguyen's words, "entirely arbitrary," I am content to leave moot: (5) is arbitrary enough to lack normative and descriptive interest, even though it meets Nguyen's test of using the same (a_{0j}, a_{ij}, m_i) coefficients as (1) and (2) do.

How should a new Napoleon characterize the "transformation" between bizarre (5) and bourgeois (2)? Although they both involve $[a_{ij}]$ and $[m_i a_{0j}]$ coefficients, their behavior equations are contradictory. To go from (5) to (2) or from (2) to (5) is to reject one and accept the other — much as we do when we reject phlogiston and accept Kelvin-Clausius-Gibbs thermodynamics. The form that Bortkiewicz's noble effort took, in Bortkiewicz [2], most unfortunately concealed the erase-and-replace nature of the choice between paradigms.

But, protests a follower of the new Napoleon who (to use a Baumol attribution) claims that understanding of competitive (2) can come only from *prior* contemplation of (5):

A Morishima will rise to prove the fundamental Napoleonic theorem:

"If and only if ρ exceeds zero in (5) will the profit rate R in (2) be able to be positive."

What are we the jury of scientists to say in response to these Napoleonic objections? I believe we are justified in saying that there are a multitude of *equivalent* technical tests that assure that a system satisfies the Hawkins-Simon conditions that it be "net productive" and capable of supporting a positive profit rate; and there are a multitude of such tests, which are not remotely "Marxian", and which assure that the stipulated subsistence wage vector is compatible with one of those feasible positive profit rates. Morishima's "Fundamental Marxian Theorem" is just as uninteresting as the Fundamental Napoleonic Theorem.

III. Inferiority of the Values Paradigm: Invention

It is well to give two basic examples to show that only after one has solved the bourgeois

(real prices, profit rate) paradigm can one know how to calculate the irrelevant *values* and Napoleonic paradigms.

First, consider the laws of motion of competitive capitalism in reacting to a technical change. Corn in my 1971 Bortkiewicz example can now be produced in two ways: by 4/15 of coal and 120 of labor as of old; but also now alternatively by 8/15 of coal and 25 of labor. All the other a_{0j} and a_{ij} coefficients remain unaltered, as do all the (m_i) wage-subsistence coefficients.

What will be the new actual equilibrium? The new profit rate, R^* ? The new real prices $[\mathbf{P}_1/W, \mathbf{P}_2/W, \mathbf{P}_3/W]^*$?

In case someone thinks that of relevance or of importance for its own sake, what will the new rate of surplus value r^* be? What will the new marked-up real values $[p_1/w, p_2/w, p_3/w]^*$ be that tell workers what things would cost them in a state run along the lines of the Marxian paradigm? (I won't belabor the point by asking the equally redundant questions: What will ρ^* and $(\pi_j/\omega)^*$ now be in the Napoleonic regime? "Who wants to know?" would be the appropriate response to such inquiries.)

If Baumol believed there was merit in Marx's alleged need to "explain" *profit* by rate of surplus value calculations, he'd have to be able to show that only by solving anew the (3) or (4) relations could one properly arrive at the correct new solutions to (1) or (2) relations. Of course, as Morishima's text knows, the truth is the reverse of this. The competitive regime will eschew the new invention even though it could raise the capitalists' rate of surplus value from 66 2/3% up to over 100%. Competition will make the old technology outcompete the new by offering a higher rate of profit at the equilibrium real subsistence wage, 25% versus the barely 23% of the new technology.

Conclusion: the values paradigm is indeed an irrelevancy to describe positivistically the statics and dynamics of competitive capitalism.

But should society pass up this new technology that can yield a higher real wage in the zero-profit steady state? I must explain at some length the sense in which it may be rational for society to do so.

Actually, the old technology is one that is intertemporally Pareto-efficient. Any planned state, that maximizes a Ramsey social welfare function of the new type $\sum_{0}^{\infty} U[c_1(t), c_2(t), c_3(t)]/(1.25)^t$ will end up using the old technology rather than the new.

Actually, if the Soviet Union had this choice of technologies, and its labor supply grew by 25% per period, the only way it could pay labor the stipulated needed subsistence of corn per labor unit would be by using the old technology despite its inferior productivity in terms of (undated!) socially necessary labor.¹ See Weizsäcker and Samuelson [21].

John Roemer [6] provides an instructive example of the harm to clear understanding that can come from a preoccupation with Marxian values and with steady state analysis. Roemer classifies inventions into those which are "socially desirable" and those which are not. The invention mentioned here is of the category Roemer labels "socially desirable." He then points out, correctly, that a competitive regime may fail to adopt an invention that meets his test of being socially desirable and it may insist on adopting an invention that he

^{1.} The equilibrium profit rate, $\mathbb{R}^* = .25$, whatever its role as a factor of "exploitation" or discounting, does have a technocratic characterization. Consider a vector of existent gross [coal, corn, velvets], = $[Q_i(t), Q_2(t), Q_3(t)]$. Consistent with the technological and subsistence coefficients, $(a_{0j}, a_{ij}, m_i]$, there is a balanced rate of growth $[Q_j(t)]$ is capable of, so that it becomes $[Q_i(t+1)] = \lambda [Q_j(t)]$, where λ is a scalar that is a function of the two ratios $[Q_j(t)/\Sigma_1^3 Q_i(t)]$. There is a maximum value that λ can attain, $1 + G^*$. By von Neumann-like reasoning, we easily show that $\mathbb{R}^* = G^*$

dubs socially undesirable. His readers may be forgiven for being tempted to believe that a defect in the price-profit regimen has been uncovered by Marxian insights.

Alas, Virgil nodded. An ideal planned utopia, using the most powerful techniques of optimal-control dynamic programming, often will also want to spurn new technical options that raise the equilibrium rate of surplus value, r^* , at the previous real wage rate. Thus, face China today with a new invention dubbed socially desirable by Roemer. It may well be the case that it will take another half century before rational Chinese planners could afford to adopt the invention that can raise its golden-rule zero-population-growth consumption level — and this even if its population is strictly constant and its planners are optimally optimizing present versus future consumption benefits and sacrifices. The dual variables of pre-Marx prices-and-profits may correctly tell it now to adopt the so-called socially undesirable invention, and at the proper time later tell it to adopt an invention that Roemer's pejorative terminology favors. See Samuelson [13] for demonstration that Marxian variables such as rate of surplus value cannot provide the normative efficiency that profit-including prices can give.

One example can deflate the pejorative terminology of its pretensions. Begin with situation A and then introduce new technical options B and C.

A. First 1 of corn output needs 1 input of labor and 1/2 of corn. Labor receives its stipulated subsistence wage of 1/4 corn per labor unit. Perforce the profit rate is 33 1/3% per period. The rate of surplus value, for those interested, is 100%: (1/2 corn - 1/4 corn)/(1/4 corn). Now disturb the equilibrium with two new inventions.

B. Now 1 of corn can be produced by 2.1 of labor alone. Roemer would dub this a "socially undesirable" invention. Adopting it would raise the rate of profit to 90% from 33 1/3%. But it would lower the rate of surplus value, r^* , from 100% to 90%. And, what interests us for its own sake, adopting B would lower the per-laborer corn output, producible by the system in a "golden-rule" state with a constant population, from A's 1/2 corn per laborer to B's 1/(2.1) corn.

C. The second invention, which Roemer would dub "socially desirable", enables 1 of corn to be produced by .1 of labor and .8 of corn. But, when capitalists must pay workers the real wage of 1/4 corn, C can only earn them a profit rate of 21.7/33%. Since A earns them 33 1/3%, Roemer is correct to point out that the C invention will not win its way under the laissez faire rules of the game. This even though C could yield a rate of surplus value of 700%! And, in the golden-rule state of a stationary population, C can outyield A and B in corn per capita (providing 2 corn per laborer versus 1/2 and 1/2.1 respectively).

To show that there is nothing necessarily wrong about adopting the B that Roemer labels as socially undesirable, consider a socialist utopia run by Ramsey's omniscient Philosopher-Planners who know how to solve the optimal control problems of weighing present corn pleasures against future.

Let a revolution break out, displacing laissez faire. At the same time, let B and C inventions become known. Let the labor supply be frozen to give Roemer his postulated rules of the game. What will happen?

Immediately, the optimal control solution will mandate that the "socially undesirable" invention B be adopted. In this numerical example, it must be blended with the C invention and A must be dropped. (In other examples, B and some other known technology will be mandatory until, periods later, the system has accumulated enough corn capital to be able to "afford" the C technique. When that time comes, the profit-including dual variables of

dynamic programming will give the signal to introduce C in a way that the 1867 magnitudes cannot.) Immediately the real wage imputable to labor's worth to society will rise, holding down the rise in society's own-corn-rate of interest produced by the two inventions. Then, as society optimally accumulates toward its golden-rule rendezvous the interest rate will fall so low as to make B no longer optimal to use. In this example A never comes back into use, it being "dominated" by C and B. Increasingly C comes into relative use — as it should when the corn/labor ratio becomes larger and larger. Q.E.D.

I ought to conclude this too-brief discussion with a warning. Under realistic laissezfaire, the short run impact of adopting invention B will raise the demand for labor, bidding up its real wage and holding down the rise in profit rate until the requisite increase in labor/capital ratio can be coaxed out. Neither Marx, nor Okishio, nor Samuelson, nor Roemer can predict from steady-state analysis what an invention's *transient* incidence on profit and wage rates will be.

The stables of Marxian economics are clogged with the misleading doctrine of "unequal exchange". It should not be left to non-Marxians to clean up this matter. One aspect of the misunderstanding parallels what is involved here in the Roemer taxonomic classification. Clarification is provided by a theorem which is provable only by going *outside* steady-state analysis:

When a laissez faire system cannot go spontaneously from a status quo non-goldenrule-configuration to a golden-rule configuration, such a transition necessarily involves some tradeoff of consumption sacrificed at an earlier date in return for an increment of (possibly permanent) consumption at a later date. This technological fact, which is independent of "reswitching," is as true under socialism as capitalism. Therefore, it is no valid social reproach that an invention will not be adopted now even though its adoption would permit permanent improvement later. Nor in trade theory is it a valid reproach that a positive profit rate leads to a steady-state production pattern that is "dominated in the steady state" by another golden-rule pattern.

Again the *prices* regime of Ricardo, Walras, von Neumann and Sraffa comes through with flying colors while the *values* regime waits lamely to take its cue from that of *prices*.

IV. Subsistence-vector Changes

As a second example of the inferiority and irrelevance of the values paradigm, suppose labor supply can be maintained either by the old (m_j) vector of corn only, or alternatively by a new (m_j) vector involving only the third good.

The third good in the 1907-1971 example, the luxury good velvets, involves a lower component of non-labor to labor cost than does corn. (Coal, by contrast, has a higher component of non-labor to labor cost than corn does.) Therefore, if the competitive system is given the new option of paying the subsistence wage not in corn but in terms of velvets, "it" will prefer to do so if the new subsistence velvet wage is less than the real velvet wage that previously prevailed in the 25%-profit equilibrium. By this shift the system can wrest a higher ("exploitative") rate of profit, higher than the previous 25% profit rate. So this will be done, just as in British history the white potato replaced bread from grain as the effective staple.

But what happens to the Marx rate of surplus value r^* (which is Morishima's defined coefficient of exploitation)? As the velvet real subsistence wage is lowered a little, but

enough to raise the system's R^* , we calculate that r^* falls! Have the avaricious capitalists lost their marbles? Have the ferocious tigers changed their spots in favor of sweet altruism? Of course not. It is merely that the Marx-Morishima coefficient of surplus-value exploitation is a poorly devised measure to describe the laws of motion of Darwinian capitalism and what is ordinarily meant by "exploitation." What was proposed in 1867 as an innovative improvement was actually a detour and cul-de-sac. Folderol about eigenvalues does not conceal this fact but to the objective economist only reveals it.

V. Conclusion

Each argument brought forward to justify working with an 1867 model involving a uniform rate of markup on direct labor alone, on examination, discredits itself. Using an eigenvalue formulation, the present paper confirms this basic fact.²

In the interest of brevity, I refer the reader to other articles in Collected Scientific Papers of Paul A. Samuelson [17] in which I have patiently and good temperedly reviewed the analytical issues. The reader may also be referred to Ian Steedman [19], who writes sympathetically from within the non-neoclassical tradition and has also come to similar findings.

Being fully appreciative of Professor Nguyen's valuable audit, I am required in the end to insist that, to point out that Marx's values regime uses the same (a_{0i}, a_{ii}, m_i) coefficients as are used in the conventional prices regime, does not absolve Marx of the charge that he has introduced an arbitrary detour. The crazy Napoleonic regime also uses exactly those same data and still earns my charge of gratuitous arbitrariness. It is by misuse of data that a paradigm earns rejection.

2. I owe the reader a brief explanation of the contribution by Dr. K. Valtukh [20]. His notations can be related to those used here most simply in the admissible case where all his turnover periods happen to be unity,

$$\begin{aligned} \alpha_{ij} &\equiv 1 \equiv \alpha_{j}^{1}; \\ & [P, Q_{i}, C_{i}, W, L; a_{0j}, a_{ij}, m_{i}; R] \\ & \equiv [\bar{P}, Q_{i}, Y_{i}, \bar{P}^{1}, L; 1_{j}, a_{ij}, a_{i}^{1}; r]. \end{aligned}$$

His relations, as numbered by him, then translate into my matrix notations as follows:

 $Q = aQ + C; Q, C \ge 0, a \ge 0$ (1)

$$a_0 Q = L \le L^{max}; a_0 \ge 0 \tag{2}$$

$$W = Pm \tag{3}$$

 $P = [Wa_0 + Pa](1+R) > 0.$

Logically, from (1)-(4) follow Valtukh's further relations, as translated into my notations:

$$PQ = PaQ + Pma_0L + R[PaQ + Pma_0L]$$
(10)

 $PQ = PaQ + Pma_0L + P[C - ma_0L]$ (11)

.....

$$R[PaQ + Pma_0L] = P[C - ma_0L].$$
⁽¹²⁾

On the supposition that the R^* root of (1)-(4) is positive, $C - ma_0 L$ will be a non-negative non-zero vector of profit receivers' consumption.

Note that all of the above is derived without mentioning any of Marx's 1867 novel concepts — such as a "rate of surplus value" common to all industries. Therefore, the reader is under no temptation to find optimal Dr. Valtukh's encapsulating sentence (20,184, lines 7-8): "This means that profit is nothing else but the form of surplus value." The reader can thus skip without peril the following pages in which the non-proportionability of $a_0[I-a]^{-1}$ to $a_0(1+R^*) [I-a(1+R^*)]^{-1}$ is adumbrated and some deficiencies in my ideology are bruited.

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