Samuelson's "Reply on Marxian Matters" ✔

Martin Bronfenbrenner [1, 1973] has generously tried to clarify issues raised by my two JEL papers on Marxian analysis and various replies that have come into the *Journal*. All interested in the overdue task of secularizing Marxian economics will be grateful to him. To further the process of convergence toward understanding, I offer the following brief commentary on the most important points raised.

1. *Values model as an "approximation" to competitive price model?* This is an issue of some interest. But it was not at all dealt with by my two papers. All that needs to be said here is this:

i   Since "values" and "prices" agree when organic compositions of capital are equal, their results must perforce be strongly positively correlated when labor intensities are not too different.

ii   An invention that, for the same profit rate, lowers the ratio of prices to the wage, will also lower the ratio of values to the wage when the rate of surplus value is held constant in a Volume I regime of values.

iii   On the other hand, for many purposes the values approximation to prices is a shockingly poor one. Thus, in choosing among different technologies, anyone who works completely in the realm of values will find that, at *every* rate of surplus value, the technology chosen to minimize values is precisely that appropriate to a system with stocks of raw material as plentiful as in an undiluted labor-theory-of-value state. The prices regime has the advantage that it selects at each profit rate only that technology optimal for it (including, be it noted, the possibility of reswitching); also, as will be seen shortly, such minimized prices can properly signal the terms on which choice of the consumption-goods mix should be made for maximum welfare in planned optimal-control steady state.

2. *The "algorithmic transformation" from the "values" model to the "prices" model (or vice versa), is it truly a process of rejection of the former and replacement by the latter?* Here is my true crime. I pointed out the blunt truth. And this has been construed as an attack on Marx, covert or explicit.

Why is this truth thought to be an attack on Marx? And why is the simple truth to be regarded as the blunt truth? Non-Marxians and Marxians can profitably ponder over the questions. While they are doing so, let me develop two related points.

i   My vantage point in the discussion was *not* neoclassical. It was Sraffian! Or alternatively stated, it was pre-Marxian: it was not what Cobb-Douglas or J. B. Clark (or, heaven forbid, what Böhm-Bawerk) would have said; it is what Ricardo and Smith would have said, once the ground rules were explained to them about (a) stationary states, (b) profit-seeking competition, (c) etc. What I said is exactly what Joan Robinson, no neoclassicist, has been saying all along—only I said it at such tedious length and in such pedantical detail that, one would have thought, no reader could find anything to quarrel with except its boringness.

ii   Wassily Leontief (whom the besieged editor of this *Journal* asked to serve as a referee of one or another submission) has suggested to me that the valid "erase-and-replace" point could not help being recognized as valid if I would show how to transform from some third fantastic model to either of the "values" or "prices" models. I fear Leontief is an optimist, but let me put the matter to the test.

Contrast a "values model" (with its equal percentage markup on labor outlays) with a bizarre "gibberish model" (defined to involve equal percentage markup on raw-materials outlays only). I banish to an extended footnote[1]

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[1] Assume technically that 1 labor and 1 coal produces 1 corn; and 1 labor produces 1 coal. Let the minimum-subsistence wage be 1/6 corn per 1 labor. The "values" regime is then implied to be: rate of surplus value = 200 percent, (value of coal)/wage = 3, (value of corn)/wage = (1)3 + 3 = 6. In short, $(X_1, X_2) = (3,6)$. Now for the gibberish regime, where Y_1 stands for the "gibberish equilibrium value" in each of the two departments relative to the wage: $Y_1 = 0 + 1 + g0 = 1$; $Y_2 = (1)(1) + 1 + g(1)(1)$; and $Y_2 = 6$, if and only if the equalized rate of gibberish, g, is 500 percent or $g = 5.0$. Now, do we need a Bortkiewicz or an aficionado of matrix algebra to tell us how we get from $(X_1, X_2) = (3,6)$ to $(Y_1, Y_2) = (1,6)$? Of course not. We drop (thud!) the X_1 equations and replace them by the gibberish-de-

the arithmetic of the *"algorithm* of transformation," which, despite Bronfenbrenner's tentative wording ("... such an algorithm, if [!] it exists ..."), cannot help but exist. Here, I merely summarize the algorithm's exact logical nature: "To go from the regime of 'gibberish equilibrium' to that of 'values equilibrium,' (a)

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fined Y_1 equations. Or, if we use Bortkiewicz's route, we write $Y_1 = X_1(Y_1/X_1)$. Q.E.D. [Incidentally, Bronfenbrenner's own suggestion for the transformation process, as given in his footnote 4, is *identical* to the Bortkiewicz-Seton-Samuelson algorithm once he completes it by solving his two relations for his unknown p_1/p_2 conversion ratio, leading to Bortkiewicz's quadratic equation for the 2-department case.]

A different point: When Bronfenbrenner writes, "Marx tried nobly with inadequate mathematical background—as had David Ricardo a generation before him—to work out the transformation algorithm for his system," he incorrectly diagnoses where the difficulty of the problem is. Thus, Ricardo's tergiversations in connection with the labor theory of value [I love thee. I love thee not. I love ...] had little to do with his inability to grapple with the correct algebra of equalized-profit rates. It had to do with the first point that I have termed a non-issue for this *JEL* discussion: Ricardo debated with himself as to whether it was or wasn't a good enough *expository approximation* to make total costs proportional to labor costs alone. As far as I can recall, Ricardo never wrote down an incorrect cost-cum-profit relation in those cases where he chose to grapple with the non-simplified case. The issue is not one of esoteric matrix operations, but only of clear thinking.

Lest the present exchange set back the cause of mutual understanding, let me comment on Bronfenbrenner's two ambiguities in Marx "most relevant to our present discussion." First, in every tableau of prices that Marx derived *after* his transformation from values to prices, Marx writes down (as he should and must!) *unequal* rates of surplus value. Therefore, why muddy the clear waters with this query? Second, it is of no significance what scale units are used so long as any analyst is free to equate between the two regimes any one (but only one!) of the following alternatives: (*i*) sum of surpluses, (*ii*) sum of industries' gross outputs evaluated in each regime's relative evaluation, or (*iii*) a chosen numeraire good's unity reference point. Where Marx went unambiguously wrong was in equating *both* (*i*) and (*ii*), which is impossible save in my Santa Claus case of equal-internal-organic compositions. The validity of my contention on this point, be it noted, stands even if we drop a minimum-subsistence wage as unrealistic.

ignore the behavior equations of gibberish, and (b) write down the behavior equations postulated for values."

Could anything be more plain? Or less controversial?

3. "I *find my paradigm easier to understand, you find your paradigm easier to understand, and there is no commensurability between them?*" Here is a point that must be handled with delicacy. Perhaps one can specify cases where such argumentation can be validly made. Without pronouncing on that issue, I have to state that nine out of ten times when such arguments are used, they are misused.

One Indo-European language may be as good as another for many purposes, but it is not the case that Anglo-Saxon units of measure are as good as the metric system. A map maker who uses Euclidean geometry for areas of the globe cannot claim that his geometry is as good as that of an opponent who happens to have been brought up on spherical geometry. When Thomas Kuhn published his *Structure of scientific revolutions* in 1962, I formed the impression that his treatment of the incommensurability of different paradigms did not do justice to the degree to which one paradigm in the physical sciences often unambiguously "dominates" another. (Dr. Kuhn has since made clear that he is of a similar opinion.) But, all such surmises aside, I knew that in the social sciences the Kuhn paradigm about paradigms would be rampantly misused. This prediction has, alas, been abundantly fulfilled.

With specific application to the present subject, there is really no room for subjective judgment as to whether two *different* "transformations" can *both* be correct. This is not a matter on which, depending upon which side of the Berlin Wall you happen to have been educated, you are entitled to choose your preferred algorithm.

And what is quite a different point, if one is trying to describe correctly the arbitrage conditions that will prevail under competitive capitalism, where capitalists are free to move their funds from one department or industry to another, no one can validly assert "given my training and background and mathematical adequacies, I find it more convenient to stick with a values model than with a prices model

—however *you* may feel." (Marx, himself, did not make such a defense for a values Tableau. Successfully or unsuccessfully, he was *pursuing* the correct procedure.)

Bronfenbrenner's comparing the number of apostates from one tradition to another is germane to some issues but not, I think, to a logical procedure that leads to self-contradiction.

4. *Exception to profit-wage tradeoff?* To narrow the area of dispute, let me agree with Bronfenbrenner that it is only in a competitive closed economy with labor the only non-producible factor that simultaneous worsening of the real wage and the profit are impossible. Without diagrams, we realize that an open economy, which faces an adverse turn in its terms of trade, can experience both lower profit rate and lower real wage; and if we add further primary factors to labor—land, etc.—the correct theorem becomes "The profit rate cannot fall in a closed competitive economy at the same time that all real remunerations are also falling." To derive important results like these requires Sraffian-Leontief methods and not those of Volume I.

5. *Semantics of word "exploitation."* On the issue of the terminology of "exploitation" under bourgeois pricing, I agree with Bronfenbrenner (and Lerner and Lange earlier) that an ethical observer could chose to deplore as "exploitation" any distribution of GNP that fails to give labor 100 percent of the product.

Normative Versus Descriptive Evaluatings

Any discussion designed to clarify misunderstanding should also, if it can, advance the state of the subject. Before turning to Bronfenbrenner's six writers, I shall go into some welfare-economics issues.

Sometimes one hears it said, "Granted that the Volume III bourgeois prices model is a more correct realization of the logic of the competitive capitalist system, still the Volume I values model [even if it should be deemed redundant in providing the macroeconomic underpinning of the prices' microeconomics] is more applicable in the rational planned society that will supersede capitalism."

Von Weizsäcker and I [3, 1971] have demonstrated this to be false. Let me describe here, using the corn-coal example, a new and different theorem asserting the optimality in the om-

niscient good society of the "prices" pricing and the non-optimality of the "values" pricing.

Marx often spoke scathingly of Robinson Crusoe economics, a view one can sometimes sympathize with. In the writings of von Wieser, Pareto, Barone (and later denied by von Mises), there is an alternative to Crusoe—the ideal communist state. Let me stick with Sraffian-Marx coal-corn technology, but consider a Ramsey optimal-control Planner who maximizes social utility from now and forever. Thus, he omnisciently maximizes

$$u[c_1^0, c_2^0] + \frac{1}{1+\rho} u[c_1^1, c_2^1]$$
$$+ \frac{1}{(1+\rho)^2} u[c_1^2, c_2^2] + \cdots \quad (1)$$

out to infinity, where (c_1^t, c_2^t) are the coal and corn consumed per capita in the tth period. (The form of $u[,]$ is almost immaterial: it could be $\log c_1 + \log c_2$, or $-c_1^{-1} - c_2^{-1}$, or $c_1^{.3} c_2^{.4}$, etc.)

Now how shall the Planner "evaluate" coal and corn? At the (3,6; or 6/3) of the values regime, or at the (2, 6; or 6/2) of the prices regime? The following theorem is valid:

Theorem of rational evaluation. An optimal planned state will (generally) come ultimately into a stationary state. In such a steady state, its rational evaluation of corn-to-coal must be at a P_2/P_1 of the Vol. III prices regime and not at any $Value_2/Value_1$ of the Vol. I values regime.

To be specific, assume $\rho = 0$ so that there is no time preference. Assume for expositional exaggeration that available labor doubles every period. Then every technocrat, whether he be Marxian or anti-Marxian, knows the golden-rule theorem of wider-than-neoclassical applicability: the profit rate must be uniformly equal to the growth rate in every department, 100 percent, (and, hence, the rates of surplus value must be unequal for efficiency).

Lest anyone think this to be a special truth associated with the golden rule, let me point out that if $\rho = 50$ percent systematic time preference, and labor grows at $g = 33\frac{1}{3}$ percent per period—so that $(1+\rho)(1+g) = (1.50)(1.33\frac{1}{3}) = 1 + 1.0$—the same bourgeois pricing of 2 corn-for-each-6 coal is optimal for so-

ciety's consumption decisions. And using the 3 corn-for-each-6 coal of the Vol. I regime would result in perpetual loss of social welfare!

I omit proofs of this Ramseyan "turnpike" theorem. Note that it holds for an anti-marginalist fixed-coefficient technology.[2]

Reactions to Quoted Writers

It is a bit odd to be commenting on abortive articles that have not been accepted for publication. But here are some paragraphs on each of Bronfenbrenner's six writers, numbered in his ordering.

1. Yes, monetary analysis is important. But there seems no need to bring that issue into the present discussion.

2. In a fixed-coefficient technology, no supply-of-capital-goods-cum-marginal-productivity relations are available to determine a unique profit rate and set of real wages. I fail to see why *that* realization should make a modern

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[2] See my forthcoming paper entitled "Optimality of Profit-including Prices Under Ideal Planning." Mention is made there of some singular cases, discussed by Sutherland, where oscillations around the turnpike may prevail. Also, note that if consumption involves completely fixed coefficients, as when $u \equiv \min[c_1/b_1, c_2/b_2]$, these optimality conditions become vacuous.

The optimality of equal-profit-rate pricing is all the greater if, as in the third section of P. Sraffa's *Production of commodities by means of commodities* [2, 1960] there is more than one technology blue print available. Suppose, above 200 percent profit rate and below 50 percent, method A gives lowest bourgeois prices. Between 50 percent and 200 percent let method B do so. Now, if $\rho = 2.5$ in equation (1), method A will ultimately be optimally used. But, if suddenly ρ changes to .45 in an unforseen but permanent way, method A will still be used (but presumably with more coal and less corn consumed in the new steady state). Alternatively, suppose we change from $\rho = 2.5$ to $\rho = .75$, known to hold "for a long time" and then known to be followed by a permanent change in ρ down to $\rho = .45$. What should ideally happen is an omniscient Planner's optimal program? He will shift from A asymptotically to B, hold to B for a long time, and then optimally shift back to A. Why, when he is in B, does he put up with the lower plateau of consumption and instantaneous utilities? Because he lacks enough raw materials for the A regime (and, actually, if he had them, he'd want at $\rho = .75$, to dip into them or decumulate them). Not only are steady state price ratios optimal, but as well their dynamic version (inclusive of forseeable capital-gains terms) would be optimal for the Socialist Planner.

Marxist prefer the Vol. I equal-rates-of-surplus-value model to the Sraffian Vol. III prices regime with equal profit rates. If class power is to supply the missing equation for distribution, it can do so for pre- and post-Marxian equation systems with not the slightest loss of efficiency.

3. I agree that it is needful to distinguish between (i) the undiluted-labor-theory-of-value case, where the rates of profit and of surplus value agree in both being zero, and in which all exchange evaluations (whether of "values" or prices, of Vol. I or Vol. III) are *equal* to total necessary labor hours (direct-plus-indirect, live-plus-dead), and between (ii) the case of Vol. I tableaux in which there is a positive rate of surplus values and in which *proportionality* to labor contents (rather than equality) prevails. However, terminologically I did refer to case (ii) as well as to case (i) by the term "values" for the reason that it is case (ii) with which Marx begins in his Vol. III, Ch. IX tables of transformation.

4. My own analysis stipulated that Marx's technical transformation slip was unimportant (and could be non-existent in my preferred case of equal-internal-organic compositions—unimportant in comparison with the misconception that macroeconomic profit rates *have to be based* on macroeconomic rates of surplus value, and *not* vice versa). But I ask the informed reader to decide—once this point has been granted, and all the mysteries of matrix mumbo jumbo are understood—in what sense the matrix expression $a_0[I-a]^{-1}(1 + s.v.)$ is truly better at "establishing economic categories as social categories expressive of social relations and structure" than is the matrix expression $a_0(1 + P_r) [I-a(1 + P_r)]^{-1}$. On reflection, is there not seen to be involved in such a false perception an almost-comical fetishism and word play?

5. The quoted assertion ". . . from a Marxian point of view, there is no way of understanding price formation except by way of the value concept" is left with only religious content once Marxians perceive for themselves that they can as well go from prices to values as vice versa.

6. Sraffa (and Marx[3] before him) proved

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[3] See the discussion in my footnote 1 of Bronfenbrenner text's first alleged ambiguity in Marx.

that, when organic compositions differ [whether measured by $c_i/v_i \neq c_j/v_j$ or $c_i/(c_i + v_i) \neq c_j/(c_j + v_j)$], equal rates of profit imply necessarily unequal rates of surplus value, which can be shown to be as true of a proper durable-capital model as of a raw-materials model. Also, the point has already been made that any workers' society which makes its allocation and consuming decisions on the basis of positive equal-rates-of-surplus-values will be inefficiently and unnecessarily embracing deadweight loss of welfare. Even if the case contemplated is that of a decision-making workers' cooperative, which is a small island in the vast sea of competitive capitalism, then it is still true that the cooperative will lower its members' welfare by picking a_{ij} coefficients different from those of the bourgeois Sraffa analysis.

In conclusion, let me applaud the movement toward secularizing Marx, the economist. As Joan Robinson has said, he deserves the compliment of being taken seriously as a scholar.

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REFERENCES

1. BRONFENBRENNER, M. "Samuelson, Marx, and their Latest Critics," *J. Econ. Lit.*, March 1973, *11*(1), pp. 58–63.
2. SRAFFA, P. *Production of commodities by means of commodities.* London and New York: Cambridge University Press, 1960.
3. VON WEIZSÄCKER, C. C. and SAMUELSON, P. A. "A New Labor Theory of Value for Rational Planning Through Use of the Bourgeois Profit Rate," *Proceedings of the National Academy of Science*, 1971, 68, pp. 1192–94.