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A SUMMING UP *

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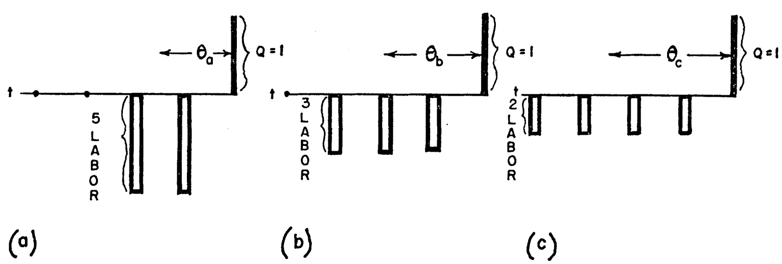
The phenomenon of switching back at a very low interest rate to a set of techniques that had seemed viable only at a very high interest rate involves more than esoteric technicalities. It shows that the simple tale told by Jevons, Böhm-Bawerk, Wicksell, and other neoclassical writers — alleging that, as the interest rate falls in consequence of abstention from present consumption in favor of future, technology must become in some sense more "roundabout," more "mechanized," and "more productive" — cannot be universally valid.

I. THE SIMPLEST AUSTRIAN AND MORE GENERAL MODELS

Figure I shows the simple picture of Böhm-Bawerk and Hayek, in which labor is applied *uniformly* for a shorter (or longer) period of time and in consequence society gets a smaller (or larger) output



In the conventional Austrian model, labor is uniformly applied prior to production of final output. In going from Ia to Ib, the average period of production, however measured, rises and reduces the total labor needed per unit of output. Ic is still more roundabout in the Austrian sense, and lowering of interest or profit rate leads unequivocably to lengthening of the period of production and to all the conventional features of the neoclassical capital-theory parables.

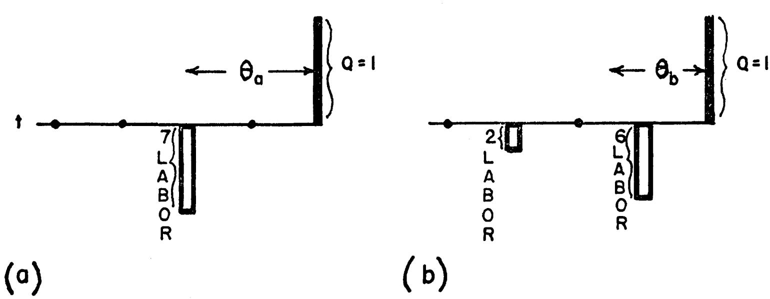
FIGURE I

* My thanks go to the Carnegie Corporation for providing me with a reflective year in 1965–66 and to Felicity Skidmore for research assistance. None of the other writers in this symposium has seen this summary, which reflects my own appraisals only.

of consumption. When the structure of production is elongated, as in going from Ia's short period of production to Ib's longer period and to Ic's still longer period, each unit of consumption good is produced with successively less total labor — from $5 + 5 = 10$, down to $3 + 3 + 3 = 9$, down to $2 + 2 + 2 + 2 = 8$ units of labor. In Ia the interest rate is thought to be high in reflection of society's small stock of goods in process. By accumulating more goods in process to fill the longer-period pipeline of Ib, society ends up with one-ninth more steady-state consumption and with a lower interest rate. By building up Böhm-Bawerk's "subsistence fund" enough to get into a Ic configuration, society enjoys still another increment of steady state consumption (arriving at nine-eighths of the Ib level or ten-eighths of the Ia level) and again with a still lower interest rate.

Readers of Böhm-Bawerk's *Positive Theory* or Hayek's *Prices and Production* will not need Descartes' rule of signs to be convinced that Ic is more roundabout or mechanized than Ib or Ia, and that as the interest rate declines the competitive system can never go back to Ia (or any other Figure I state) once it has been left behind.[1]

But now look at Figure II, which tells more simply the full story of the twenty-fifth and eighth degree polynomials of the Sraffa-Pasinetti example of reswitching. Is IIa more or less round-



Comparing the time intensity of IIa and IIb is ambiguous: is 7 labor invested for 2 periods less or more roundabout than 2 labor for 3 periods and 6 for 1 period? At interest rate above 100 per cent per period, IIa will be used; at interest rate between 50 and 100 per cent, IIb will be used; but at interest below 50 per cent, the system reswitches back to IIa!

FIGURE II

about than IIb? Our eye looks at the time-shape of labor inputs and cannot say: are 7 units of labor invested for 2 time periods

1. I have not appended to Figure I the familiar triangle of goods in process, obtainable from synchronized steady-state repetitions of each of these processes. IIa's triangle would be visibly longer than IIb's. However, when labor is not applied uniformly — as in Figure II — the structures are not simple triangles and a measure of length becomes more ambiguous.

more or less roundabout than 6 units of labor invested for 1 period and 2 units invested for 3 periods of time?

There is no obvious answer. Of course, we can calculate Böhm-Bawerk's arithmetic-average-time-period of production,[2] but such a measure presupposes simple rather than compound interest and has no longer a presumptive claim on our attention. For what it is worth, IIa has a longer conventional period of production than IIb. Yet at very high interest rates, i.e., more than 100 per cent per period, IIa will win out in the competitive market over IIb. Lest you think IIb must therefore be more roundabout, be warned that when the interest rate drops below 50 per cent per period, IIa again drives out IIb and it continues to hold its own down to a zero interest rate.[3] (Admittedly, 100 per cent and 50 per cent are high

2. In Ia this weighted-mean is given by

$$\theta = \frac{5 \text{ labor} \times 2 \text{ periods} + 5 \text{ labor} \times 1 \text{ period}}{5 \text{ labor} + 5 \text{ labor}} = 1\tfrac{1}{2} \text{ periods}$$

$$= \frac{\sum^{N} L_t \cdot t}{\sum_{1}^{N} L_t} \text{ in general}$$

$$= \frac{S}{\Sigma L_t}$$ where S is the subsistence fund, consisting of total goods in process (evaluated at their *labor* costs!) and $\Sigma\, L_t$ is total output (evaluated at labor costs).

For Ib, $\theta = \dfrac{3 \times 3 \text{ periods} + 3 \times 2 \text{ periods} + 3 \times 1 \text{ periods}}{9 \text{ labor}} = 2 \text{ periods};$
for Ic, $\theta = 2\tfrac{1}{2}$ periods.

Generally, if labor is invested *uniformly* over $(1, 2, \ldots, N)$ periods,

$$\theta = \frac{1 + 2 + \cdots + N}{N} = \frac{N+1}{2} \text{ periods.}$$

Hence, in the simple Böhm-Bawerk triangular structure of production, θ and the range N move always in the same direction (as do all higher statistical moments). Böhm-Bawerk's θ in effect neglects compound interest in favor of simple interest, ignoring interest on the interest part of the value of all intermediate goods. For IIa and IIb, we get respectively

$$\theta = \frac{2 \times 3 \text{ periods} + 0 + 6 \times 1 \text{ periods}}{8} = 1\tfrac{1}{2} \text{ periods}$$

$$\theta = \frac{0 + 7 \times 2 \text{ periods}}{7} = 2 \text{ periods}$$

3. At a zero interest rate, the process with the lowest total labor requirement, ΣL_t, the zeroth statistical moment, will win out. For two processes with tied ΣL_t, Böhm-Bawerk's measurement of mean θ, or first moment, will be decisive at low interest rates: that with lower θ will be preferred, as can be determined from simple interest calculations alone. But, as Wicksell pointed out to Böhm-Bawerk, compound interest (involving higher powers of i) means that, in general, along with the mean we must also calculate the variance, the skewness, kurtosis, and all the higher moments of the time distribution of labor invested. J. R. Hicks, *Value and Capital* (2d ed.; Oxford: Clarendon Press, 1946), Part III, Part IV, Appendix to Chap. XVII, gives a quite different definition of "the average period of production," which allegedly *always* increases as the interest rate declines. As applied to IIa and IIb, it would seem

rates, selected merely to keep the arithmetic simple. The reader can think of each period as a decade if he wants to pretend to be realistic.)

II. Why Reswitching Can Occur

To help economic intuition, suppose champagne is the end product of both IIa and IIb. In IIa, 7 units of labor make 1 unit of brandy in one period. Then 1 brandy ferments by itself into 1 unit of champagne in one more period. In IIb, 2 units of labor make 1 grapejuice in one period. In one further period 1 grapejuice ripens by itself into 1 wine. Then 6 units of labor shaking 1 unit of wine can in one more period produce 1 champagne. All champagne is interchangeable.

Now what happens at very high interests, above 100 per cent per period? Interest on interest on interest of the 2 units of labor invested for 3 periods in IIb becomes so colossal as to make the IIb way of producing champagne ridiculously dear. So, of course, IIa gets used at highest interest rates.

Now go to the other extreme. At zero interest, or negligible interest, only labor and wage cost matters. IIa takes only 7 units of labor in *all* the stages, as against IIb's 8 units in all. So again, at very low interest rates, IIa wins out competitively.

Does IIb ever give lower cost of production of champagne? Yes. For any interest rate between 50 and 100 per cent, IIb turns out to be best. Let us verify that IIa and IIb are tied at the switching point $1+i = 1+1$ corresponding to 100 per cent interest per period. Suppose each unit of labor costs $W = \$1$. Then 1 brandy costs \$7 wages + \$7 interest = \$14. And 1 champagne from IIa costs \$14 brandy + \$14 interest = \$28. Now calculate the IIb cost. 1 grapejuice costs \$2 wages + \$2 interest = \$4. 1 wine costs \$4 grapejuice + \$4 interest = \$8. And 1 champagne costs (\$8 wine + \$6 wages) + \$14 interest = \$28, the same as in IIa.

The reader can verify that IIa and IIb are again tied at $1+i$

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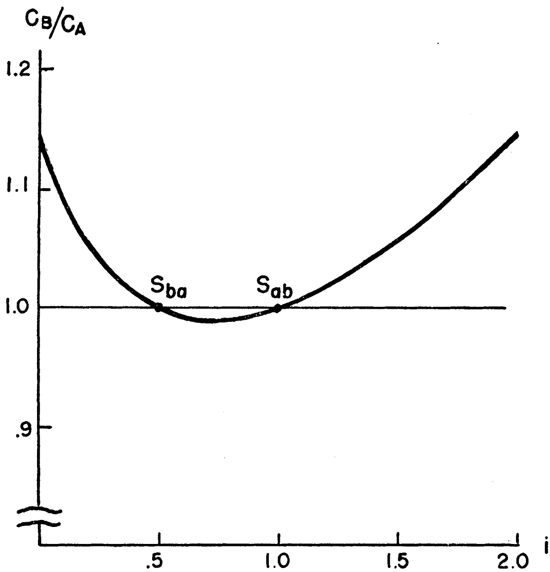
to say that IIa has a longer and longer average production period as interest drops steadily from 200 per cent to 100 per cent — even though technology has not changed at all! Actually, however, the *Value and Capital* definition cannot even be applied to the comparisons of Ia, Ib, Ic or IIa, IIb. For Hicks's definition must take into account the fact that, under perfect competition with free entry and constant returns to scale, the prices of all final, intermediate, and input goods will change with the interest rate until *net* present-discounted-values are again zero. Then *his* average is found to be always infinite! Cf. P. Samuelson, *Foundations of Economic Analysis* (Cambridge, Mass.: Harvard University Press, 1947), p. 188. John Hicks, *Capital and Growth* (Oxford: Clarendon Press, 1965), Chap. XIII is in agreement with the upshot of the present symposium.

= 1 + .5, at $15¾ each. But now try an intermediate case, $1 + i = 1 + .6$. Then brandy costs $7 (1.6) = $11.2. And champagne costs $11.2 (1.6) = $17.920 from IIa. However, from IIb it costs only $17.792: namely, grapejuice costs $2(1.6) = $3.2; wine costs $3.2(1.6) = $5.12; and finally, champagne from IIb costs ($5.12 + $6) (1.6) = $17.792, seen to be the only economic result.[4]

Figure III summarizes the effect of interest rate changes on the relative costs of producing champagne by the two methods. At



Reswitching back to IIa from IIb occurs because the plotted curve of relative costs fails to be one directional. Between switch points technique IIb is used; elsewhere IIa is used.

FIGURE III

4. The present example, like that of Pasinetti, represents a decomposable or reducible matrix. Thus, there are goods — like grapejuice or brandy — that do not require champagne as an input, directly or indirectly. Hence, the submatrixes of the two processes are also decomposable. Furthermore, brandy of one process does not need grapejuice of the other as an input. Hence, the present example does not *itself* refute the Levhari theorem, which purported to apply to indecomposable or irreducible matrixes only. However, the Levhari theorem is false, as the irreproachable counterexamples of Morishima, Sheshinski, and Garegnani show. Moreover, each of these Pasinetti-type *sub*matrixes can be made indecomposable by adding as a requirement for its first output at least a little of *its* final output. This would still leave a *total* system matrix, composed of two indecomposable submatrixes, but *itself decomposable*. This decomposability can be got rid of by the following device: Let the production of first-stage brandy and of first-stage grapejuice each require, along with labor and a little end-good champagne requirements, a little of penultimate-stage wine *and* brandy. This would convert the Pasinetti-Sraffa model into an irreproachable indecomposable matrix, and provide a legitimate counterexample to the Levhari proposition.

very high rates of i, IIb costs much more than IIa. The cost ratio, C_B/C_A, falls as i falls, reaching unity at the switchpoint S_{ab} of Figure II; it continues to fall until i is about 70 per cent per period, rising from this minimum as i falls farther. At the switch point, S_{ba}, again the cost ratio equals unity, continuing to rise to the eight-sevenths level of pure labor costs at $i = 0$.

III. Reswitching in a Durable-Machine Model

The simplest possible example of reswitching has been demonstrated for an Austrian circulating-capital model. The same kind of arithmetic can be used to show that similar reswitching can occur in a durable-machine model.

Suppose we have two machines, each producible instantaneously by 1 unit of labor. Machine A yields 18 units of output 1 period later and 54 units of output 3 periods later. Machine B yields 63 units of output 2 periods later. Which is more durable? More capital-intensive? At a zero rate of interest, the present discount value (PDV) of A exceeds that of B (since $18 + 54 > 63$). At an interest rate of 100 per cent per period, both turn out to have equal PDV and choice will be indifferent between them. At still higher interest rate, say 200 per cent per period, machine A will turn out to have the higher PDV. At interest rates between 50 and 100 per cent per period, machine B will turn out to be more profitable. But — and this is the essence of reswitching — below 50 per cent machine A will again have the preferable PDV.[5]

In passing, I should mention that Irving Fisher's technique of calculating present discount values handles all cases, that of durable capital goods and the circulating-capital model of Böhm-Bawerk's *Positive Theory of Capital.* Moreover, Fisher's tools of general equilibrium put Böhm-Bawerk's theory of interest on a rigorous basis for the first time, showing how Böhm-Bawerk's two subjective factors of time preference interact with his third factor of technical productivity to produce an equilibrium pattern of interest. It is ironical that Böhm-Bawerk should have rejected Fisher's analysis in rather churlish terms, in part because of his propensity to differentiate his product from that of all other writers and in part be-

5. Note that the numbers (7) and (6, 2) have been modified to (18, 54) and (63) so that the switching points will again be at $i = .5$ and $i = 1.0$. For the cautious reader, I append the following table of (i: PDV_A, PDV_B), drawn up on the assumption that output sells for \$1 per unit. [0: \$72, \$63; .5: \$28, \$28; ⅔: \$22.464, \$22.68; 1.0: \$15 ¾, \$15 ¾; 2.0: \$8, \$2 ⅓]. A sample calculation for $i = .5$ and $1/(1 + i) = ⅔$ goes as follows:

$$\$18(⅔) + \$54(⅔)^3 = \$12 + \$16 = \$28 \text{ and } \$63(⅔)^2 = \$28.$$

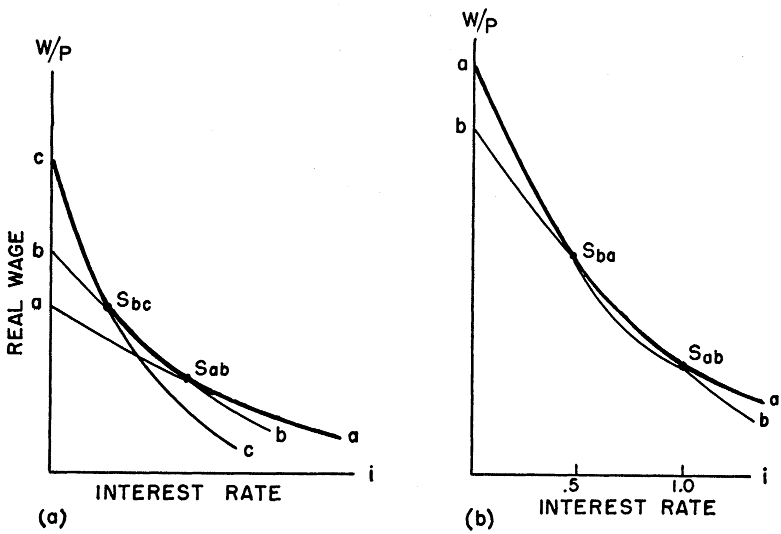
cause of the archaic nineteenth century notion that a quasi-mathematical formulation fails to come to grips with the true essence and causality of an economic problem.

For the rest of this comment, I shall stick to circulating-capital models merely for expositional brevity.

IV. The Well-Behaved Factor-Price Frontier

The fact of possible reswitching teaches us to suspect the simplest neoclassical parables. However, we shall find that all cases are well behaved in showing a trade-off between the real wage and the interest or profit rate. Thus, when Marx enunciated the law of falling rate of profit and the law of declining real wage, he was proclaiming one law too many.

For the conventional Austrian model of Figure I, the real wage always rises as the interest rate falls — for two reinforcing reasons. First, even without any change in technique (i.e., when we stay in case Ia or Ib or Ic), there gets to be less *discounting* of the wage product at lower interest rates, a phenomenon familiar to Taussig, Wicksell, and other neoclassical writers. Second, the changes in



In every case, the factor-price frontier is downward sloping, with real wage rising as interest rate drops. In the conventional Austrian case of Figure I, there is both less discounting and an increase in prediscounting gross labor output. In Figure II's reswitching example, only the first reason operates, being at first partially offset by IIb's drop in prediscounting gross labor output.

Figure IV

technique induced in Figure I's traditional model by lower interest rates happen to be always in the direction of increasing the prediscounting total of labor product. Figure IVa gives a picture of the declining factor-price frontier, which relates the real wage, W/P, and the interest rate, i, for Böhm-Bawerk's case.

But now turn to the reswitching example of Figure II. Can we still be sure that its factor-price frontier is a declining one? Assuredly there is less discounting of wage product at lower i. But it is no longer universally true that a lower i induces a change in technique which increases the prediscounting labor product. Figure IVb shows the factor-price frontier as the outer envelope of the light frontiers appropriate to each technique. The switch points are marked S_{ba} and S_{ab}, the latter being the point at which a lowering of i shifts us from IIa's technique to IIb's. Examine S_{ba}. There a lowering of i does increase W/P for both reasons — since IIa does give greater (gross) undiscounted wage product than IIb, namely product of one-seventh rather than one-eighth. But when you examine S_{ab}, the lowering of interest rate from 101 per cent per period to 99 per cent is seen to induce a change from the IIa to IIb technique, thereby lowering prediscounting product to one-eighth from one-seventh!

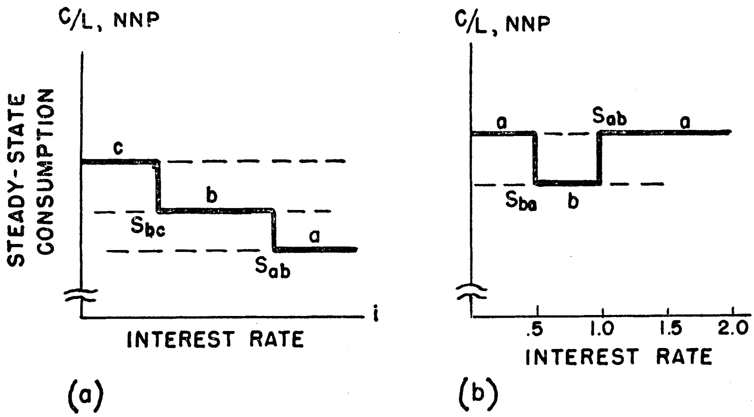
Why can we be sure that in every case the envelope frontier slopes downward, which means that the improvement in real wage from less discounting always outweighs any "perverse" disimprovement in gross product? This follows geometrically from the properties of a continuous outer envelope. Economically, it follows from the workings of ruthless competition. Under perfect competition, either workers can hire capital goods or capitalists hire workers. At a lower interest rate, or cost of capital, workers can always pay themselves a higher real wage even without changing techniques; so capitalists will have to match up. And ruthless competition will ensure that, at any given i and money wage, the price of the finished product will be at its minimum — thus, ensuring the maximum W/L out on the declining envelope.

Figure IV shows the real wage in terms of the final consumption good, champagne in our case. But the same declining frontier could be drawn for the real wage expressed in terms of any intermediate product — such as grapejuice, wine, or brandy.

Both IVa and IVb show that the neoclassical parable remains valid as far as the factor-price frontier trade-off between real wage and profit rate is concerned. But that is all that remains valid regardless of reswitching.

V. Unconventional Relation of Total Product and Interest

Now let us examine the steady-state consumption levels for each different interest rate i. If population is stationary (and technology unchanging), this steady-state consumption is the same thing as real net national product per head. (If labor grows at the geometric rate of n per year, we could generalize our analysis by plotting per capita consumption against each different i rate, an easy task skipped here.)



The left-hand Austrian model shows the conventional rise in steady-state per capita consumption as the interest rate falls, as a result of the alleged fact that roundaboutness is productive. But the reswitching example on the right illustrates that, at first, steady-state consumption may decline at lower interest rates, only agreeing ultimately with the diminishing returns tale of the neoclassical parable.

Figure V

In the nice Austrian case of the parable, Figure Va shows that lowering interest always, if anything, raises NNP and steady-state consumption. As we move from Ia to Ib to Ic, the steps marked *a, b, c* in Figure IVa are shown as rising. At $i = 0$, we are in the Schumpeter-Ramsey Golden Rule state of Bliss, with maximum NNP.

But Figure Vb shows a reversal of direction of the steps as a result of Figure II's reswitching. When i drops from 101 per cent per period to 99 per cent, NNP actually falls. Of course, diminishing returns asserts itself "eventually," in that at zero interest rate we are in the Golden Rule state of Bliss. (But note that this maximum level of consumption was also reached for interest rates above

100 per cent. It is no longer literally true[6] to say, "Society moves from high interest rates to low by sacrificing current consumption goods in return for more consumption later, but with each further dose of accumulation of capital goods resulting in a lower and lower social yield of incremental product." Actually, society can go from B to E in Figure IVb without making any physical changes at all: a reduction of profit from a 200 per cent rate per period to a 5 per cent rate, merely lowers what a critic might call the "degree of exploitation of labor" prevailing. In Figure Va the apologist for capital and for thrift has a less difficult case to argue.

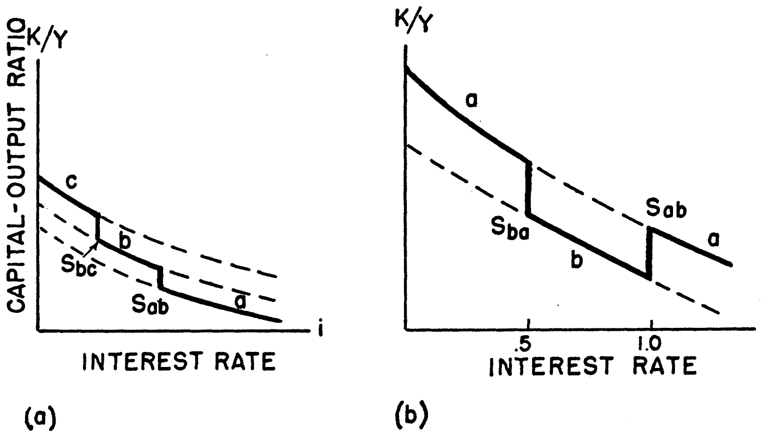
VI. Unconventional Capital/Output Ratios

Since World War II the literature on growth and development has brought the capital/output ratio into prominence. Before the war this concept was met frequently in the shape of the accelerator. And Böhm-Bawerk's average period of production, θ, was actually a primitive capital/output ratio, namely that one which would prevail if the interest rate were zero and all goods could be priced at their wage costs alone: i.e., θ can be written as the ratio of Böhm-Bawerk's subsistence fund, S, to final output, where S and NNP are both reckoned in labor terms or wage costs alone.

More accurately, we can calculate for each interest rate i, the true market cost of each intermediate and final good inclusive of interest as well as wage costs, and can calculate the aggregate value of all capital goods. This can then be divided by the true market value of all final goods, to give the capital/output ratio.

Figure VIa shows that in the simple model of Böhm-Bawerk, Hayek, and other Austrians, the capital/output ratio does rise steadily as the interest rate falls. This duplicates the behavior of the simplest J. B. Clark parable of a single homogeneous capital that, together with labor, produces aggregate output by a standard production function of Cobb-Douglas or more general neoclassical type.

6. The reversal of direction of the (i, NNP) relation was, I must confess, the single most surprising revelation from the reswitching discussion. I had thought this relation could not change its curvature if the underlying technology was convex, so that there had to be a concave, basic Fisher (intertemporal) production-possibility frontier, of the form $0 = F(K_0;\ C_0,\ C_1,\ \ldots,\ C_T;\ K_T)$, where K_0 is the vector of initial capital stocks, K_T the vector of terminal capital stocks, and C_t the vector of consumption goods available at any time t. I had wrongly confused concavity of F with concavity of the (i, NNP) steady-state locus. Note that reswitching *reveals* this possible curvature phenomenon, but is not necessary for it. Later I correct another misconception revealed by reswitching.



In the neoclassical case on the left, lowering the interest rate is expected to raise the capital/output ratio. However, the reswitching example of Figure II involves a fall in the capital/output ratio as i drops through the point S_{ab}, inducing a less rather than more capital-intensive technique.

FIGURE VI

But the reswitching model of Figure II is seen in Figure VIb to lead to a pattern of capital/output ratio that fails to move in one direction. Thus, at the switch point S_{ab}, lowering i a little induces a shift from IIa technique to IIb, giving rise to a significantly lower capital/output ratio.

Such an unconventional behavior of the capital/output ratio is seen to be definitely possible. It can perhaps be understood in terms of so-called Wicksell and other effects. But no explanation is needed for that which is definitely possible: it demonstrates itself. Moreover, this phenomenon can be called "perverse" only in the sense that the conventional parables did not prepare us for it.

Such perverse effects do have consequences. Thus, suppose you have a theory of life-cycle saving which, like Modigliani's, attaches significance to the wealth/income or capital/output ratio. Then the dynamic stability of some of your equilibria may be affected by the fact that the capital/output ratio drops as i drops. Similar stability and uniqueness problems may be raised for a Solow-Harrod growth model.

Nevertheless we must accept nature as she is. *In a general blueprint technology model of Joan Robinson and M.I.T. type, it is quite possible to encounter switch points, like S_{ab} of* Figure VIb, *in which lower profit rates are associated with lower steady-state capital/output ratios.*

VII. Reverse Capital Deepening and Denial of Diminishing Returns

We can use the contrasting models of Figure I and Figure II to demonstrate a startling possibility. In the conventional parable, people accumulate capital goods by sacrificing current consumption goods in return for more future consumption goods, with the interest rate depicting the trade-off or substitution ratio between such consumption goods.[7] So far so good. All this turns out to be essentially true in every case.

But, in the conventional model, successive sacrifices of consumption and accumulations of capital goods lead to lower and lower interest rates. This conventional neoclassical version of diminishing returns is spelled out at length in my *Economics*.[8] Unfortunately, until reswitching had alerted me to the complexity of the process, I had not realized that the conventional account represents only one of two possible outcomes. When we are in the Figure II technology, the story can be reversed: after sacrificing present consumption and accumulating capital goods, the new steady-state equilibrium can represent a rise in interest rate![9]

There is perhaps no need for me to describe in detail how the traditional neoclassical model of Figure I goes from the high interest rates that make short-period Ia optimal to lower interest rates that make Ib optimal. Suffice it to assert that, in going from the steady-state of Ia (where say 90 labor produce $90/(5+5) = 9$ units of output or consumable NNP per period) to the more roundabout steady state of Ib (where the same 90 of labor produce $90/(3+3+3) = 10$ units of NNP), there must first be a net sacrifice of consumption to below the *status quo ex ante* rate of 9 per year.

7. In all cases, $1 + i_t = -\partial C_{t+1}/\partial C_t$ along the transformation function $T = 0$ of the previous footnote; if smoothness is not present, the above equality becomes an inequality $R \leq 1 + i_t \leq L$, where L and R are the left-hand and right-hand derivatives $-\partial C_{t+1}/\partial C_t$, which may diverge little or much depending upon the technology.

8. (6th ed.; New York: McGraw-Hill, 1964), pp. 595–96.

9. This can happen whenever NNP rises with i, as in Figure Vb. The change in curvature of the steady-state relation $\text{NNP}/L = C/L = f(i)$, when f'' becomes greater than zero, was wrongly thought by me to be ruled out by the fact that $\partial^2 C_{t+1}/\partial C_t^2$ — or its finite-difference equivalent — can never be positive in a convex technology. The last fact is correct, and it has important implications. But it does not imply diminishing returns to the *steady-state* relations of Figure V. Teleologically speaking, the invisible hand of a competitive system cares nought what happens to net national product but only what happens to what I have called net net national product, which is that available to the primary factor labor alone, excluding the "necessitous return" to capital (which can be regarded as an intermediate product — like horses and their fodder — producible or available within the system at constant costs once the i rate is specified.)

In fact, Professor Solow has recently [1] proved what at first glance appears to be a remarkable theorem, showing that the present discounted value of all the *net* gains in future consumptions resulting from a switch from a process like Ia to a process like Ib (net in the sense that all sacrifices of consumption must be taken into account, with their proper discount, as subtractions) will balance out to exactly zero, if the interest rate used in discounting is that of the switch point S_{ab} between process Ia and Ib. In a bookkeeping sense, therefore, the Fisherian yield of product received in terms of product sacrificed is precisely measured by the market rate of interest. In this sense the market rate measures the "net productivity of capital" even in a model where there is no homogeneous capital good of the mecanno set or "leets" type, and where there are no smoothly substitutable factors in the relevant production functions.

Upon further reflection, one realizes that Solow's result is indeed as much a bookkeeping as a technical relationship. For what else can happen in a system where the rate of interest and the total of wage cost is constant in every period, except that the value of consumption plus the value of net capital formation equals the level of factor income? As we shall see, Solow's result is merely an instance of this general accounting relationship, as applied to a constant-returns-to-scale technology in which residual monopoly profit is always zero because the competitive equalities must prevail.

I rush now to show what happens in the reswitching model of Figure II when we move from high interest rate, say 101 per cent per period, down to 100 per cent, and subsequently to 99 per cent, thereby going from a steady-state equilibrium using the IIa technique to a new steady-state using the IIb technique. (Later the interested reader can perform for himself the reverse switch as we go down from 51 to 49 per cent, moving back from IIb equilibrium to IIa and this time duplicating the conventional consumption-sacrifice of the neoclassical parable.)

Table I shows us initially in equilibrium with the IIa technique. Up to time 2, all of society's labor (taken to be 56, the product of 7 and 8, to keep the arithmetic simple) is allocated to Ia, with 8 (or 56/7) output emerging 2 periods later from this stage 2 allocation. After time period 6, the system has moved to a IIb equilibrium, producing only 7, or $56/(2+6)$, units of output or steady-state NNP. In between society has been splashed with net consumption

1. R. M. Solow, "The Interest Rate and Transition between Techniques," a January 1966 paper to be published in a symposium.

rather than having to sacrifice consumption: thus, in the transitional periods 3 through 6, the system generates $8 + 8 + 6 + 13 = 35$ units of champagne output, which is definitely greater than $32 = 8 + 8 + 8 + 8$. Moreover, in agreement with Solow's theorem, and providing a trivial generalization of it, if we calculate the PDV of the Ib steady-state consumption stream, using the interest rate of

TABLE I

TRANSITION FROM IIa TO IIb TECHNIQUE, AND BACK AGAIN

LABOR | TIME	1	2	3	4	5	6	7	·	·	20	21	22	23	24	25	·
Stage 3	0	0	14	14	14	14	14	·	·	14	0	0	0	0	0	·
Stage 2	56	56	42	42	0	0	0	·	·	0	14	14	56	56	56	·
Stage 1	0	0	0	0	42	42	42	·	·	42	42	42	0	0	0	·
Final Output	·	·	8	8	6	6+7	7	·	·	7	7	7	2+7	2	8	·

100 per cent characterizing the switch point S_{ab} and taking into account the transitional alteration of consumption, we must get the same PDV that the system would have had if it stayed permanently in Ia.

In the Table the system is left in Ib equilibrium from period 7 to period 20. During this time we could imagine the interest rate dropping, suddenly or gradually, to 50 per cent from 100 per cent. Nothing real happens, except that the real wage goes up as a result of less discounting and also the price ratio of finished champagne rises relative to that of earlier-stage products. (Despite Ricardo and Marx, goods do not exchange in proportion to their total labor content, direct plus indirect.)

But now suppose that the system tries to accumulate capital from time 21 to 25. In going from IIb's steady-state level of 7 to IIa's steady-state level 8, the system must indeed sacrifice *consumption net*. Thus, $7 + 7 + 9 + 2 = 25$ is less than $7 + 7 + 7 + 7 = 28$.[2]

In summary, this section has shown that going to a lower interest rate may have to involve a *dis*accumulation of capital, and a surplus (rather than sacrifice) of current consumption, which is balanced by a subsequent perpetual reduction (rather than increase) of consumption as a result of the drop in interest rate. This anoma-

2. And, at either switch interest rate ($i = .5$ of S_{ba} or $i = 1.0$ of S_{ab}) there must be, at time 21, equality of PDV that would come from maintaining the old equilibrium forever after, with the PDV of the actual transition shown followed by perpetual IIa equilibrium. (Which i is then the true Solow net productivity of capital? Answer: either $i = 100$ per cent or $i = 50$ per cent will give the accountant's competitive identity of PDV, an instance of the multiplicity of Fisherian yield known ever since the 1930's.

lous behavior, which can happen even in models that do not admit of reswitching, might be called "reverse capital deepening." Whether it is empirically rare for this to happen is not an easy question to answer. My suspicion is that a modern mixed economy has so many alternative techniques that it can, so to speak, use time usefully, but will run out of new equally profitable uses and is likely to operate on a curve of "diminishing [3] returns" (at least after non-constant-returns-to-scale opportunities have been exhausted). In any case, by the time one reaches a zero interest rate (or more generally the Golden Rule state where the interest and growth rates are equal), this kind of diminishing returns must have set in.[4]

VIII. Conclusion

Pathology illuminates healthy physiology. Pasinetti, Morishima, Bruno-Burmeister-Sheshinski, Garegnani merit our gratitude for demonstrating that reswitching is a logical possibility in any technology, indecomposable or decomposable. Reswitching, whatever its empirical likelihood, does alert us to several vital possibilities:

Lower interest rates may bring lower steady-state consumption and lower capital/output ratios, and the transition to such lower interest rate can involve denial of diminishing returns and entail reverse capital deepening in which current consumption is augmented rather than sacrificed.

There often turns out to be no unambiguous way of characterizing different processes as more "capital-intensive," more "mechanized," more "roundabout," except in the *ex post* tautological sense of being adopted at a lower interest rate and involving a higher real

3. All my models involve constant-returns-to-scale, a fact not inconsistent with diminishing returns of consumption in terms of interest rate.

4. Suppose society maximizes a generalized Ramsey sum, $\sum_0^\infty U(C_t/L_t)/(1+\rho)^t$, where ρ begins above $i = 1.0$. Then the system will have come into steady-state equilibrium using IIa as in the first and last part of Table I. Now let ρ fall suddenly but permanently, say to $i = .55$. Then the system will ultimately come into the IIb equilibrium of the Table's middle part. But suppose the initial drop in ρ has been from, say 1.1, down to .4, skipping completely intermediate values. Will the market interest rate drop gradually from $i =$ initial $\rho = 1.1$ to $i =$ new $\rho = .4$? The answer is, No. The system will come *at once* into the new equilibrium, which will be identical with the old labor allocation but with higher real wage and lower profit share. There is a moral here for a system that may not show literal reswitching: as it moves from high-i equilibrium to low-i equilibrium, it may not pass at all near to the equilibrium configurations on the factor-price frontier that correspond to intermediate interest rates! Dr. Michael Bruno, visiting M.I.T. and Harvard from Israel in 1965–66, has provided valuable analysis of similar optimal dynamic programs.

wage. Such a tautological labeling is shown, in the case of reswitching, to lead to inconsistent ranking between pairs of unchanged technologies, depending upon which interest rate happens to prevail in the market.

If all this causes headaches for those nostalgic for the old time parables of neoclassical writing, we must remind ourselves that scholars are not born to live an easy existence. We must respect, and appraise, the facts of life.

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