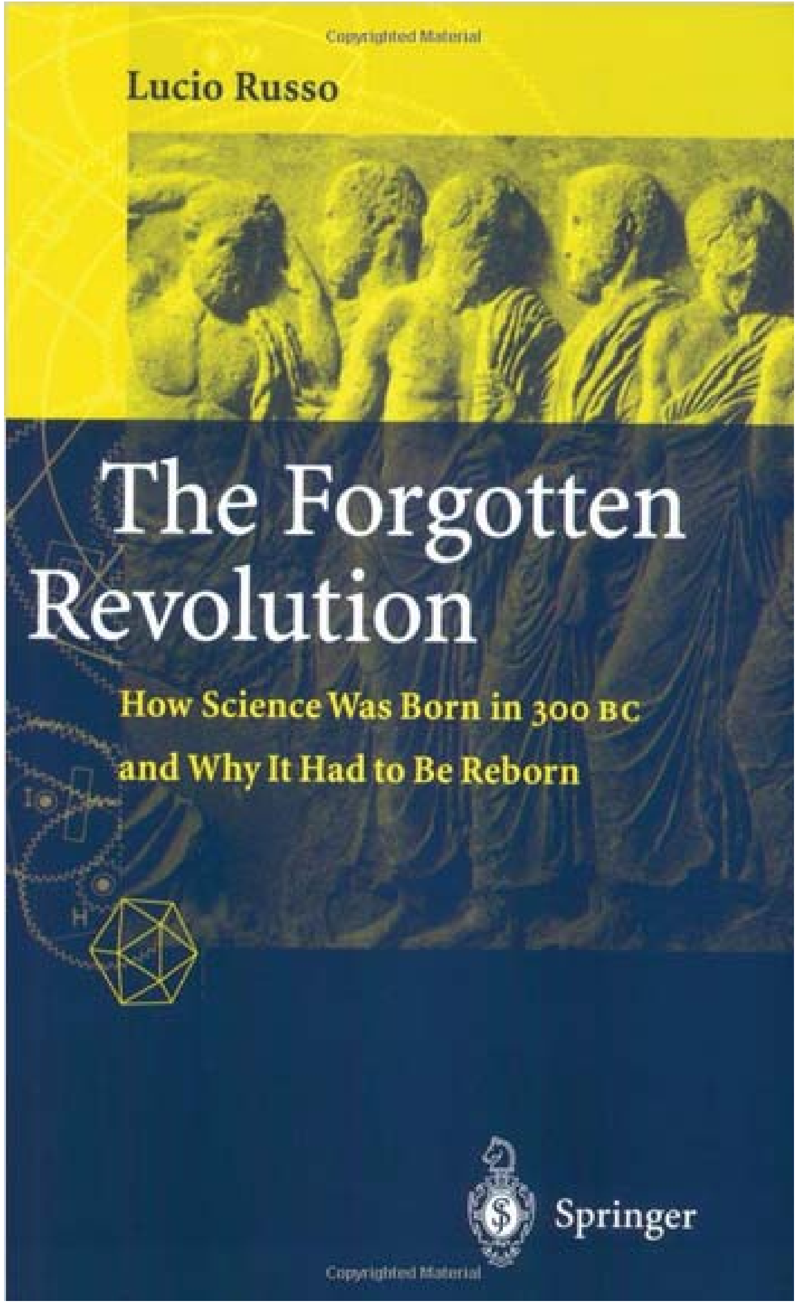

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Lucio Russo
The Forgotten Revolution
How Science Was Born in 300 BC
and Why It Had to Be Reborn
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Contents

Introduction

The period from the late fourth to the late second century B.C. witnessed, in Greek-speaking countries, an explosion of objective knowledge about the external world. While Greek culture had reached great heights in art, literature and philosophy already in the earlier classical era, it is in the so-called Hellenistic period that we see for the first time — anywhere in the world — the appearance of science as we understand it now: not an accumulation of facts or philosophically based speculations, but an organized effort to model nature and apply such models, or *scientific theories* in a sense we will make precise, to the solution of practical problems and to a growing understanding of nature. We owe this new approach to scientists such as Archimedes, Euclid, Eratosthenes and many others less familiar today but no less remarkable.

Yet, not long after this golden period, much of this extraordinary development had been reversed. Rome borrowed what it was capable of from the Greeks and kept it for a little while yet, but created very little science of its own. Europe was soon smothered in the obscurantism and stasis that blocked most avenues of intellectual development for a thousand years — until, as is well known, the rediscovery of ancient culture in its fullness paved the way to the modern age.

What were the landmarks in the meteoric rise of science 2300 years ago? Why are they so little known today, even among scientists, classicists and historians? How to they relate to the post-1500 science that we're familiar with from school? What led to the end of ancient science? These are the questions that this book discusses, in the belief that the answers bear on choices we face today.

This is so for several reasons. A better understanding of ancient science and how it relates to its modern counterpart can shed light on the internal structure of science, on its links to technology and other aspects of modern civilization, and on the origins of, and possible remedies for, the contemporary rift between the humanistic and scientific worlds. But what makes ancient science an even more relevant topic, and at the same time helps explain the low esteem in which it has been held in the last two centuries, is its tragic end. The naïve idea that progress is a one-way flow automatically powered by scientific development could never have taken hold, as it did during the 1800s, if the ancient defeat of science had not been forgotten. Today such dangerous illusions no longer prevail absolutely, and we may have a chance to learn from the lessons of the past. Those who engage in defending scientific rationality against the waves that buffet it from many directions would do well to be forearmed with the awareness that this is a battle that was lost once, with consequences that affected every aspect of civilization for a thousand years and more.

Another reason to delve into Hellenistic science is historical. As we shall argue, the rise of the scientific method was part of a more general trend: roughly speaking, in Hellenistic times the creation of culture became a conscious act. Not only do we see physicians conducting controlled experiments, scientists using mathematics and mechanics to build better weapons, painters applying geometry to their art, but even the notion of language changes: poetry becomes a playground for experimentation, while words are consciously assigned precise new meanings in technical fields, a procedure that would not become familiar again until the nineteenth century. The material component of prescientific societies is largely defined by their technology; but once technology starts to be consciously developed through science, the two become inseparable, and science takes on a vital role, down to the very way a society sees itself.

In sum, an appreciation of the original *scientific revolution* is essential for the understanding of Hellenistic civilization; in turn, the role it played in that civilization can help us better analyze key historical questions, such as Rome's legacy, the causes of urban and technological decline in the Middle Ages, and the origins, features and limitations of what is called the early modern *scientific renaissance*. In this sense the subject of this book is not so much History of Science as simply History — "history via science", so to speak, just as one may study history through the "material civilization", or through literature, or, more traditionally, though a political and military lens. In the case of the Hellenistic period and its aftermath, the approach via science and technology seems to me particularly fruitful.

Reader's Advisory

The reader who peruses the Table of Contents will notice that the book weaves together many threads, offering general formulations but also a wealth of examples. That the subject matter overlaps with so many distinct specialties means there is no hope of giving a complete picture of the literature. Therefore the bibliography's 340 works fall roughly into two types: on the one hand, many of the articles and books of twentieth- and nineteenth-century scholarship I have drawn on, and which I feel are most important or helpful — sometimes as an entry point to the bibliography on a specific subject. On the other hand, the goal of some citations and references is to illustrate a widespread opinion; in those cases the choice is not necessarily of the best works, but of the most popular and therefore most representative. Several of these are encyclopedic works.

Citations of works in the bibliography are given in brackets, together with page numbers (sometimes for multiple editions; or else an edition-invariant method of location may be used instead).

The 200 or so ancient texts referred to, plus another hundred medieval and early modern works, are collected in a separate List of Passages, where the reader unfamiliar with the conventions of classicists may turn for additional help. Both in that list and in the text, the references are as explicit as possible, often including both the chapter/section number and (as the first not otherwise marked arabic numeral) the page number in the reference edition. Although "Plato, *Republic*, VI, 510c" will easily be found in any edition or translation, since they all correlate with the reference edition (Henri Estienne, Geneva, 1578), the situation for many other texts is not so neatly standardized. In such cases, at the cost of perhaps being thought too fussy, I have felt it better to spell out the edition to which the page numbers refer, or to offer in other ways what to a specialist might be redundant information.

All chapters and sections are interconnected, and not as independent as their titles might suggest. The reader who chooses to dip into the text here and there will be in turn informed, challenged to reflection, occasionally amused or amazed, perhaps infuriated; but for the full benefit of logical argumentation, the book is best read sequentially. Nevertheless, a comprehensive subject index and a network of cross-references will help those who are primarily interested in a particular topic.

Acknowledgments

I would probably not have been able to bring this work to fruition had it not been for the support of two great classicists: Carlo Gallavotti, who many years ago read my first articles on Hellenistic science and who, with

his feedback, extended to me crucial words of encouragement; and Bruno Gentili, in whom I found a valuable ally in subsequent years.

As the theses contained in this book were maturing, I had the chance to teach several times a course on the History of Mathematics. My work owes much to the enthusiasm and intelligence with which many of my students embraced the study of questions raised in the course, grasping their importance and topicality in defiance of current fads.

I thank Marcello Cini for his attentive reading of the original manuscript, for bringing it to the attention of the Italian publisher Feltrinelli, and for his comments.

I am indebted to many friends and colleagues who gave me feedback, suggestions and criticism to early versions of these pages or the articles where some of the material first appeared. They all taught me something, but I wish to extend my warmest thanks to Donatella Capocaccia, Giovanni Ciccotti, Ferdinando Degan, Henri Epstein, Giorgio Fusco, Giovanni Gallavotti, Alberto Gandolfi, Sandro Graffi, Mario Iannuzzi, Giovanni Jona-Lasinio, Marco Lippi, Gabriele Lolli, Enzo Olivieri, Luigi Radicati, Archie Roy and Giovanni Stelli.

In preparing the second Italian edition I tried to take into account the many comments I got from reviewers, colleagues, friends and students. I thank them all, but especially Fabio Acerbi, Riccardo Caneba, Giovanni Cerri, Filippo Franciosi, Emanuele Narducci and Paolo Radiciotti.

1
The Birth of Science

1.1 The Erasure of the Scientific Revolution

Given the central and widely recognized role science plays in our civilization, one might think that the birth of science would be regarded as a crucial juncture in human history. Instead, its importance is almost never perceived. Histories of scientific thought tend to obscure the revolutionary state of knowledge in the age of Archimedes — the Hellenistic period — toning down the differences between it, the natural philosophy of classical Greece two centuries earlier, and even the prescientific knowledge of ancient Egypt and Mesopotamia. The omission is even more glaring in histories of Antiquity: one can typically find more information about Archimedes or Aristarchus of Samos in a book about the Renaissance, in connection with their rediscovery, than in a work on classical civilization.

A person who studies the modern age thinks of the Renaissance or the seventeenth century with eyes set on the future, toward contemporary civilization. She therefore cannot ignore the importance of the "rebirth of science". The student of Antiquity, on the contrary, often has (and in the past even more so) the tendency to contrast the Hellenistic period either with the supposed perfection of classical Greece or with Rome. He thus runs the risk of judging it either by the standards of an earlier civilization or by those of a civilization to which science remained foreign; in either case, from the point of view of a prescientific culture.

The result is that most authors were led to identify the birth of scientific method with what, not by accident, is called the Scientific Renaissance, and that until the nineteenth century the civilization that gave us science

was not even considered worthy of a name: it was just a "period of decadence" of Greek civilization.

Droysen was the first historian to reevaluate this extraordinary period and give it a name, in his *Geschichte des Hellenismus*.[1]

In the last half-century things have become clearer; today one can find very interesting works on various aspects of Hellenistic civilization.[2] But in general these are specialized works that have not changed much the general picture available to the educated public, to whom the Hellenistic period often continues to appear as one whose cultural heritage is for us less essential than that of the classical period.

There seems to have been in fact an erasure of Hellenistic civilization, and in particular of the scientific revolution that took place in the third century B.C., from our collective historical conscience, not unlike the phenomenon of repressed memories. Our culture, though built on the twin foundations of history and science, resorts to various expedients to hide from itself the historical importance of the birth of science.

Let's consider three great beacons of the scientific revolution: Euclid of Alexandria, Archimedes of Syracuse, Herophilus of Chalcedon. What does an educated person know about them?

About Herophilus, nothing.[3] About Archimedes one remembers that he did strange things: he ran around naked shouting *Heureka!*, plunged crowns in water, drew geometric figures as he was about to be killed, and so on. The childish store of anecdotes associated with his person and the meager diffusion of his works give the impression that Archimedes has more in common with figures of myth and legend than with other thinkers. So he is remembered, yes, but as a legendary character, outside of history. One ends up forgetting that he was a scientist of whom we still have many writings and whose results continue to be part of scientific education at many levels — from the formula for the volume of a sphere, learned in elementary school, to university-level notions of mechanics and mathematical analysis that were born with his work.

Euclidean geometry has remained throughout the centuries the framework for basic mathematical teaching.[4] But Euclid himself has been taken out of history. In his case the mechanism is opposite the one used for

[1] [Droysen].

[2] Some of them will be cited later. Among the works of broad scope on the Hellenistic age I still consider [Rostovtzeff: SEHHW] fundamental, while [Green] is a good representative of more recent tendencies. Regarding Alexandria, in particular, much information and above all a useful collection of testimonies can be found in [Fraser].

[3] We will return to him in Chapter 5.

[4] In view of the failure of attempts to base teaching on axiomatic systems devoid of geometric content, the tendency today is increasingly not to teach the deductive method in high school at all; but I do not think that such teaching can be fairly classified as mathematical.

Archimedes: instead of being depicted in legend and in anecdotes, he is offered to us without any historical context, laying down "Euclidean geometry" as if it were something that had always been there at mankind's disposal. If you are not convinced of this, try asking your friends what century Euclid lived in. Very few will answer correctly, in spite of having studied Euclidean geometry for several years.[5] And yet Euclid has been one of the most read authors in the history of humanity; his most famous work, the *Elements*, has been studied without interruption for twenty-two hundred years: from 300 B.C. to the end of the nineteenth century. There is probably no author as well-studied (though not at first hand nowadays) about whom we know so little in general.

Another mechanism leading to the erasure of Hellenistic civilization, and particularly of the century of greatest scientific development, the third century B.C., is the vague attribution of results, especially scientific or technological, to "the Ancients". For example: one always says that the diameter of the Earth was measured "in Antiquity", that "the Ancients" discovered the principle of hydrostatic pressure, that the organ goes back "to Antiquity", that Copernicus had a precursor "in Antiquity". We will see many other examples later.

The difficulty one experiences in trying to frame historically the facts and individuals of the third century B.C. is tied to our profound ignorance of that period, which has been almost obliterated from history.

First of all, there remains no sustained historical account of the period between 301 (when the *Bibliotheca historica* of Diodorus Siculus breaks off[6]) and 221 B.C. (the beginning of Polybius's *Histories*, which also reached us incomplete). Not only do we have no historical works dating from the Hellenistic period, but even the subsequent work of Livy is missing its second ten books, which contained the period from 292 to 219 B.C. The tradition preserved the history of classical Greece and that of the rise of Rome — the periods that remained cultural reference points in the late Empire and in the Middle Ages, whereas the history of the century of scientific revolution was forgotten with the return of civilization to a prescientific stage.

Secondly, almost all writings of the time have been lost. The civilization that handed down to us, among so many intellectual achievements, the very idea of libraries and of the zealous preservation of the thinking of the past, was erased together with its works. We have a few scientific works transmitted through Byzantium and the Arabs, but Europe preserved none. A little has been recovered: a few papyrus fragments

[5]This at least is the result of a little personal survey conducted among my friends and colleagues.

[6]At the end of Book XX; of later books we have only fragments.

found in Herculaneum[7] comprise all we can read of the hundred or so books written by Chrysippus, who was according to many testimonies the greatest thinker of his time; a fundamental work, Archimedes' *The method*, was fortuitously discovered in 1906 by Heiberg (on the famous palimpsest subsequently lost and found again in 1998); and thanks to recent papyrus finds we can read Menander. But these favorable cases are few.

The seriousness of the destruction of Hellenistic works has usually been underestimated in the past, due to an assumption that it was the best material that survived. Unfortunately, the optimistic view that "classical civilization" handed down certain fundamental works that managed to include the knowledge contained in the lost writings has proved groundless. In fact, in the face of a general regression in the level of civilization, it's never the best works that will be saved through an automatic process of natural selection. That the same tradition that preserved in their totality the 37 books of Pliny's *Natural history* overlooked the few pages of Archimedes' seminal treatise *The method* is in itself a proof that the tendency is exactly the opposite. Late Antiquity and the Middle Ages favored compilations, or at least books written in a language still understandable to a civilization that had returned to the prescientific stage. Thus we have Varro's work on agriculture and Vitruvius' on architecture, but not their Hellenistic sources; we have Lucretius' splendid poem on nature, but not the works of Strato of Lampsacus, who according to some indications may have originated natural science in the true sense. Even among real scientific works, some of which were preserved by the Byzantines and Arabs, two selection criteria seem to have been at work. The first was to give preference to authors of the imperial period, whose writings are in general methodologically inferior but easier to use: we have, for example, Heron's work on mirrors, but not the treatise that, according to some testimonies, Archimedes wrote on the same subject. Next, among the works of an author the ones selected are generally the more accessible, and of these often only the initial portions. We have the Greek text of the first four, more elementary, books of Apollonius' *Conics*, but not the next four (of which three survived in Arabic); we have Latin and Arabic translations of the work of Philo of Byzantium on experiments in pneumatics, but none of his works on theoretical principles. We will see further examples of these selection criteria.

A third reason for our ignorance is that until recently there had been no systematic excavation of the centers of Ptolemaic Egypt. Even in the

[7]Herculaneum and Pompeii had an intense interchange with the Hellenistic world until their sudden destruction in 79 A.D. The Vesuvius eruption thus had the effect of saving precious testimonies of Hellenistic art and culture from the loss that took place elsewhere in the late Empire and early Middle Ages.

case of Alexandria, the submerged remains of the ancient city only began to be explored systematically in 1995. Much of our knowledge of Ptolemaic Egypt comes form papyri found in the last hundred years. These are fortuitous finds, in general discarded sheets used as "waste paper" by embalmers.

Fourthly and finally, apart from some diplomatic and military events that come to our ken through the Roman pen and from the paltry legal data and the like that we glean from inscriptions, our knowledge about Hellenistic states other than Egypt is virtually nil. Our lack of information about the Seleucid kingdom, which included Mesopotamia, is particularly jarring, because there are several indications that its contribution to scientific development may have been comparable to Ptolemaic Egypt's. Our ignorance derives only in part from the perishability of parchment and papyrus, which will not last for millennia except under exceptional climates such at that of certain areas of Egypt. Hellenistic Mesopotamia still used cuneiform writing on clay tablets, a much more durable material; but this fortunate circumstance does not appear to have been exploited to any great extent. The historian Rostovtzeff writes:

> We know rather more of Babylonia than of the other eastern parts of the empire. A few Greek inscriptions, the ruins of some buildings of Hellenistic date and, most important of all, thousands of cuneiform tablets of the same period mostly from Babylon and Uruk have been found. Very few of these have been read and published and even fewer translated...[8]

Perhaps what we have called "erasure" is a phenomenon profoundly characteristic of our culture. Not only are cuneiform tablets not being read, but even the Hellenistic writings that have come down to us in Greek are often not found in accessible editions.[9]

We will try to identify the origins of this erasure in this book. And if on the one hand the scarcity of sources makes it hard to prove any thesis whatsoever, on the other hand one should not be astounded if some of the current and earlier interpretations turn out to be misguided. If we face Hellenistic scientific culture without doing our best to forget it, we may encounter surprises and be forced to modify many longstanding ideas about "Antiquity".

[8][Rostovtzeff: SE], p. 187.

[9]For example, there is no critical edition of the fragments of Eratosthenes. The only attempt in that direction, by G. Bernhardy, dates from 1822. For scientific works there is no collection of classics comparable to the various existing authoritative series devoted to literary or philosophical works.

1.2 On the Word "Hellenistic"

To give a sense to the claim that science was born during the Hellenistic age, it is well to agree beforehand about the meaning of "Hellenistic" and of "science". This section and the next define these two terms.

We start by locating in time and space the civilization that concerns us and some of the protagonists of the scientific revolution. The Hellenistic age, in the terminology introduced by Droysen and accepted by later historians, starts in 323 B.C., with the death of Alexander the Great.[10] His empire broke apart after that, giving rise to several political entities, which were at first governed in the name of the emperor by various pretenders to that title and later became autonomous kingdoms. The three main states were:

– Egypt, with the new city of Alexandria (founded by Alexander in 331 B.C.) as its capital, and ruled by the Ptolemaic dynasty, which also governed Cyprus, Cyrenaica, and in the third century B.C. Phoenicia and Palestine;
– the Seleucid state, with Antioch as its capital, comprising Syria, almost all of Asia Minor, Mesopotamia, Persia, and after 200 B.C. also Phoenicia and Palestine;
– the Antigonid state, comprising Macedonia and some cities in Greece.

There were also smaller states, such as the kingdom of Pergamum, ruled by the Attalid dynasty, the Pontus, and Bithynia. One Hellenistic state of which we know little, but which probably had a major role as a channel between Hellenistic culture and Indian and Chinese cultures was Bactria, which overlapped with today's Afghanistan, Uzbekistan, and Tajikistan.

Hellenistic civilization was not solely the product of Greeks who dwelt in regions that had formed Alexander's empire; it also enjoyed the contributions of autonomous Greek cities, which were spread all over the Mediterranean. Among the important autonomous centers were Rhodes, Syracuse and Massalia (Marseilles).

Hellenistic science boomed in the third century B.C. and has often been called Alexandrian because it had its main center in Alexandria, in Egypt. Among the reasons for this supremacy were the policies of its early rulers, particularly Ptolemy I Soter, who was in power from 323 to 283 B.C., and Ptolemy II Philadelphus, who ruled from 283 to 246.

[10]It might seem more logical to make the Hellenistic period start with Alexander's expedition or his reign, given that its essential new element was the fulfilment of Alexander's program of Hellenization of the territory of the ancient empires. The difference of a few years matters little, of course, but the (slightly morbid) choice of a starting point suggests that even Droysen shared to some extent the prejudice about "Hellenistic decadence".

It was in Alexandria that Euclid worked and taught, around the end of the fourth century B.C.. Also there, in the first half of the next century, lived Ctesibius, creator of pneumatics and founder of the Alexandrian school of mechanics, and Herophilus of Chalcedon, founder of scientific anatomy and physiology.[11] The activity of Aristarchus of Samos, famous above all for having introduced heliocentrism, dates from the same period.[12] It was also most likely in Alexandria that Archimedes (287–212) studied, and even while in Syracuse he remained in constant communication with Alexandrian scientists. Among the scientists of the second half of the third century was Eratosthenes, head of the Library at Alexandria, who, among other things, carried out the first true measurement of the size of the Earth. Chrysippus, who will interest us in particular for his contributions to logic, lived during the same century in Athens, which continued to be the main center for philosophical studies. The activities of Philo of Byzantium, who continued the work of Ctesibius, probably date from the second half of the century. At the turn of the next century there was the work of Apollonius of Perga, to whom we owe in particular the development of the theory of conic sections.[13] The greatest scientist of the second century B.C. was Hipparchus of Nicaea, who was active in Rhodes and studied mainly astronomy.

Starting with the year 212 B.C., which witnessed the plunder of Syracuse and the killing of Archimedes, Hellenistic centers were defeated and conquered by the Romans. During the second century B.C. scientific studies declined rapidly. Alexandria's scientific activity, in particular, stopped abrutly in 145–144 B.C., when Ptolemy VIII (Euergetes II), who had just ascended the throne, initiated a policy of brutal persecution against the city's Greek ruling class. Polybius says that the Greek population of Alexandria was almost entirely destroyed at that time;[14] Athenaeus gives a lively description of the subsequent diaspora of the city's intellectuals;[15] other sources give a few more details.[16] Our information is not enough to reconstruct the causes of the persecution. Subsequently, Euergetes II

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[11]It is certain that Ctesibius was active during the reign of Ptolemy II Philadelphus; see, for example, [Fraser], vol. II, p. 622. We will come back to the problem of dating Herophilus.

[12]Ptolemy tells us that οἱ περὶ Ἀρίσταρχον ("Aristarchus's collaborators" or "the school of Aristarchus") made an observation in 279 B.C. (*Almagest*, III, i, 206, ed. Heiberg, vol. I.1). We also know from Aetius (in Stobaeus, *Eclogae* I, xvi §1, 149:6–7 (ed. Wachsmuth) = [DG], 313b:16–17) that Aristarchus had been a disciple of Strato of Lampsacus, who headed the Peripatetic school until 269 B.C.

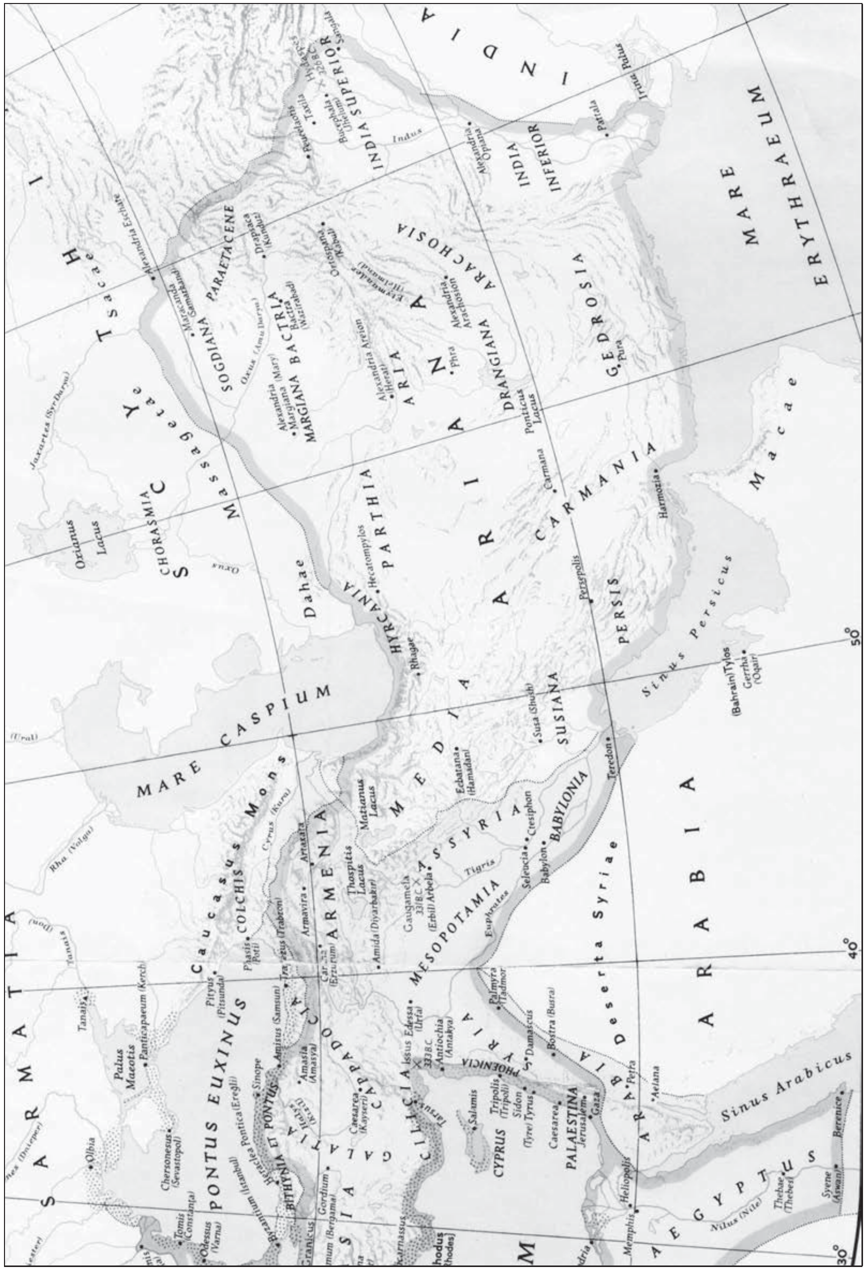
[13]For the dating of Apollonius see G. J. Toomer, *Apollonius of Perga*, in [DSB], vol. I, 179–193.

[14]Polybius, *Historiae*, XXXIV, xiv = Strabo, *Geography*, XIV, xx §19.

[15]Athenaeus, *Deipnosophistae*, IV, 184b–c.

[16]For instance, Valerius Maximus tells us that the king ordered the gymnasium surrounded and all those within killed (*Factorum et dictorum memorabilium libri IX*, IX, ii, ext. 5). The few other sources we have on the persecution are collected in [Fraser], vol. II, pp. 216 ff.

For both maps: Gray strip indicates boundary of Alexander's empire in 325 B.C. Darker land indicates Roman empire in 116 A.D. Dots near the coast indicate



Greek and/or Punic settlements. Adapted from the *National Geographic Magazine*, December 1949. Used with permission of the National Geographic Society.

continued to pursue a policy hostile to the Greek community in Alexandria, turning to the indigenous ethnic groups for support.[17] Since he had enjoyed Roman support even before ascending the throne (when, exiled by his brother, he had taken refuge in Rome[18]), it is reasonable to think that he became a proxy for Rome’s Mediterranean expansionism,[19] which at the time was particularly violent.[20]

Rome’s expansion ended in 30 B.C. with the annexation of Egypt, thus completing the unification of the whole Mediterranean under Roman rule. This event is usually considered as the end of the Hellenistic era, which was followed by the “imperial period”. From our point of view, however, it is not a particularly significant date: although the golden age of science had tragically come to an end over a century earlier, with the end of scientific activity in Alexandria and the conquest of the other main centers by the Romans, Hellenistic culture survived during the imperial age. The former kingdoms, in fact, were not assimilated linguistically or culturally, and from the technological and economic point of view there was perhaps more continuity with the preceding period than similarities with the Latin-speaking West. For this reason one sometimes continues to use the term Hellenistic to refer to the culture of the part of the Roman Empire where Greek remained the dominant language.

After the interruption caused by the wars with Rome, the Pax Romana allowed a partial resumption of scientific research in the first and second centuries A.D. — the time of Heron, Ptolemy and Galen — after which the decline was unstoppable. For another couple of centuries, Alexandria remained the center of what scientific activity there was. The last scientist worthy of note may have been Diophantus, if, as has often been thought, he lived in the third century A.D.[21]

The activity documented in the fourth century A.D. is limited to compilations, commentaries and rehashings of older works; among the commentators and editors of that time we will be particularly interested in Pappus, whose *Collection* brings together many mathematical results that

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[17] The Alexandrians managed to chase him away, but he reconquered the city in 127 B.C.

[18] Polybius, *Historiae*, XXXI, xx.

[19] This impression appears to be confirmed by an inscription in Delos, which contains the dedication of a statue to a general of Euergetes II, on the part of the Roman merchants, in acknowledgement *of the privileges granted them when Alexandria was taken by the king Ptolemy Euergetes* (that is, Euergetes II). The dedication does not refer to the events of 145–144, but to those of 127. The inscription ([OGIS], 135) is reported in [Fraser], vol. II, p. 217.

[20] Recall that in 146 B.C. the Romans had razed Carthage and Corinth to the ground.

[21] There are good reasons to place him instead as early as the first century A.D.; see [Knorr: AS]. In any case, the deciphering of cuneiform texts has caused a drastic revision in our estimate of Diophantus’ originality, since it shows that the methods he describes had long been in use in Mesopotamia.

have not reached us otherwise, and Theon of Alexandria, whose editions of Euclid's *Elements* and *Optics* have survived through the centuries.[22] The definitive end of ancient science is sometimes dated to 415, the year in which Hypatia, the daughter of Theon and herself a mathematician who wrote commentaries on Apollonius, Ptolemy and Diophantus, was lynched for religious reasons by a fanatical Christian mob in Alexandria.

Because only a few works and fragments, often not exactly datable, are left from the extraordinary wealth of Hellenistic science, we will describe its essential characteristics without always following a timeline.[23] We will concentrate on the third and second centuries B.C., but when documents from that period are lacking we will use later ones. In using documents from the imperial period great caution is necessary, because, as we shall see, scientific methodology had regressed profoundly. When we discuss certain political and economic aspects of the scientific revolution it will of course be essential to differentiate between the period of independence of Hellenistic states and the Hellenistic tradition within the Roman Empire.

1.3 Science

A coarsely encyclopedic organization of knowledge risks appearing to validate the existence of a multitude of sciences, each equally worthy, each characterized by its particular object of study: chemistry, computer science, ornithology, mathematics, trichology, and so on. In this model it is enough to define an object of study and choose a name (possibly of Greek origin) in order to create a new science, understood as a container in which are to be placed all the true statements concerning the specific object chosen. Occasionally, in fact, some have felt that just a bit of Greek is enough, without even the object of study: thus were born, for example, parapsychology and ufology.[24]

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[22] Heiberg identified Theon's edition with the one transmitted in almost all our manuscripts of the two works of Euclid, but this identification has been contested; see [Knorr: PsER], [Jones], [Knorr: WTE].

[23] Among general books on the history of ancient science it's worth mentioning [Enriques, de Santillana], which still makes interesting reading, though many specific arguments are outdated; the succint [Heiberg: GMNA], which summarizes the contents of extant works; [Farrington]; [van der Waerden: SA]; and the lectures in [Neugebauer: ESA], those about Mesopotamia being especially interesting. [Pauly, Wissowa] is an irreplaceable reference work on ancient science and indeed on classical civilization, while [Sarton] can still be useful for its bibliographical references.

As anthologies of sources we cite [Cohen, Drabkin] and [Irby-Massie, Keyser].

For quick and trustworthy information about individual scientists, ancient and modern, one can use [DSB].

[24] Since UFO stands for "unidentified flying object", the word ufology means approximately "knowledge about unknown flying objects", and is therefore a "science" whose content is void by definition. Similar considerations hold for parapsychology.

In this view, the history of science is the union of the histories of all particular sciences, each being understood as a timetable of "acquisition of truth" in the particular field considered. Naturally, those who adopt this view have little interest in the history of science: that is the case with many historians, who spare for science a nod or brief mention, if that.

Although there have been much more complex philosophical elaborations, the coarse model just described was widespread among scientists at least until the first decades of the twentieth century. The constant and rapid modification of scientific principles, particularly in physics, eventually made untenable the view that science is a collection of statements holding true with certainty. Indeed, this view forces one to consider nonscientific all superseded theories. So long as it was a matter of bodies of knowledge that, more often than not, dated from earlier centuries, their demotion had been accepted painlessly enough; but with the new pace of scientific development the same criterion would imply exclusion from the ranks of science of all but the most recent results. This seemed unacceptable to scientists, probably because it would have meant that their own results would inevitably be some day relegated to the category of nonscience. It became clear, in other words, that a good definition of science must allow one to regard as scientific even mutually contradictory assertions, such as the principles of classical mechanics and those of relativistic mechanics.

At the same time, the usefulness of the term "science" evidently lies in the possibility of telling scientific knowledge apart from other valid types of knowledge, such as historical knowledge or empirical technology. Since what distinguishes science from other forms of knowledge is not the absolute validity of scientific assertions, the question remains:

What is science?

At first glance one might think of two different methods for answering this question: either describing the characteristics of science as it arose historically, or approaching the problem theoretically. But a slightly closer analysis easily shows that each of the two methods presupposes the other. One cannot approach the problem of characterizing the scientific method without being familiar with the science that did in fact evolve through the centuries, that is, without knowing the history of science. On the other hand, any history of science must obviously presuppose a definition, if perhaps tacit or even unconscious, of science.

The only way to avoid this apparent vicious circle is probably to follow a spiral path, alternating between both methods so they justify each other in turn.

Since our primary aim is historical rather than philosophical, and since it is better to work with explicit rather than hidden assumptions, we will present and illustrate in this section a definition of science without discussing its validity. The definition's aim is simply to pin down the object of study of the next few chapters, and to clarify our criterion for selecting the works that will be regarded as scientific. Once this definition has done its job, helping us identify a corpus of relatively homogeneous works, we will turn in Chapter 6 to the problem of characterizing science, asking what were the origins and features of the Hellenistic scientific method as it developed historically. I believe that a better understanding of the method used by ancient scientists has essential relevance to the history of modern science (this will be fleshed out with examples in later chapters) and that it can be an important source of insight in the discussion of current science (a point that lies beyond the scope of this work).

To reach our definition of science, we start by observing that some theories that everyone regards as scientific, like thermodynamics, Euclidean geometry, and probability theory, share the following essential features:

1. *Their statements are not about concrete objects, but about specific theoretical entities.* For example, Euclidean geometry makes statements about angles or segments, and thermodynamics about the temperature or entropy of a system, but in nature there is no angle, segment, temperature or entropy.

2. *The theory has a rigorously deductive structure;* it consists of a few fundamental statements (called axioms, postulates, or principles) about its own theoretical entities, and it gives a unified and universally accepted means for deducing from them an infinite number of consequences. In other words, the theory provides general methods for solving an unlimited number of problems. Such problems, posable within the scope of the theory, are in reality "exercises", in the sense that there is general agreement among specialists on the methods of solving them and of checking the correctness of the solutions. The fundamental methods are proofs and calculation. The "truth" of scientific statements is therefore guaranteed in this sense.

3. *Applications to the real world are based on correspondence rules between the entities of the theory and concrete objects.* Unlike the internal assertions of the theory, the correspondence rules carry no absolute guarantee. The fundamental method for checking their validity — which is to say, the applicability of the theory — is the experimental method. In any case, the range of validity of the correspondence rules is always limited.

Any theory with these three characteristics will be called a scientific theory. The same term will be used for some other theories, which we may call "of a higher order". They differ from the theories we have been considering in that they possess no correspondence rules for application to the real world — they are applicable only to other scientific theories. That

is the most common case in contemporary mathematics. Although some who work at the higher levels may tend to lose sight of it, the relationship between theory and reality does not change in any essential way: albeit indirect, it is nonetheless guaranteed by the same mechanism of formation of theories.

Exact science will mean to us the totality of scientific theories.

A simple criterion to verify whether a theory is "scientific" is to check whether one can compile an exercise manual; if that is not possible, it's certainly not a scientific theory.

The immense usefulness of exact science consists in providing models of the real world within which there is a guaranteed method for telling false statements from true. Whereas natural philosophy failed in the goal of producing absolutely true statements about the world, science succeeds in guaranteeing the truth of its own assertions, at the cost of limiting itself to the realm of models. Such models, of course, allow one to describe and predict natural phenomena, by translating them to the theoretical level via correspondence rules, then solving the "exercises" thus obtained and translating the solutions obtained back to the real world. There is, however, another possibility, much more interesting: moving freely within the theory, and so reaching points not associated to anything concrete by the correspondence rules. From such a point in the theoretical model one can often construct the corresponding reality, thus modifying the existing world. (See Figure 1.1.)

Thus scientific theories, even if created for the purpose of describing natural phenomena, are able to enlarge themselves by means of the deductive method, and as a consequence they usually develop into models of areas of technological activity. *Scientific technology*, characterized by purposeful planning done inside some scientific theory or other, is intrinsically connected to the methodological structure of exact science, and cannot but arise together with the latter.

One of the goals of this work is to corroborate this last assertion — which openly contradicts the common notion that science in "Antiquity" lacked technical applications — by analyzing Hellenistic science and technology. We will also try to clarify all the methodological characteristics mentioned so far by examining the first scientific theories, which arise precisely in Hellenistic times.

Every scientific theory has a limited realm of use; it can in general be used to model only phenomena that are not "too far" from those that motivated its creation. Theories that prove inadequate in describing new sets of phenomena must be replaced for the purpose; but they remain scientific theories according to our definition, and can continue to be used inside their own sphere of validity.

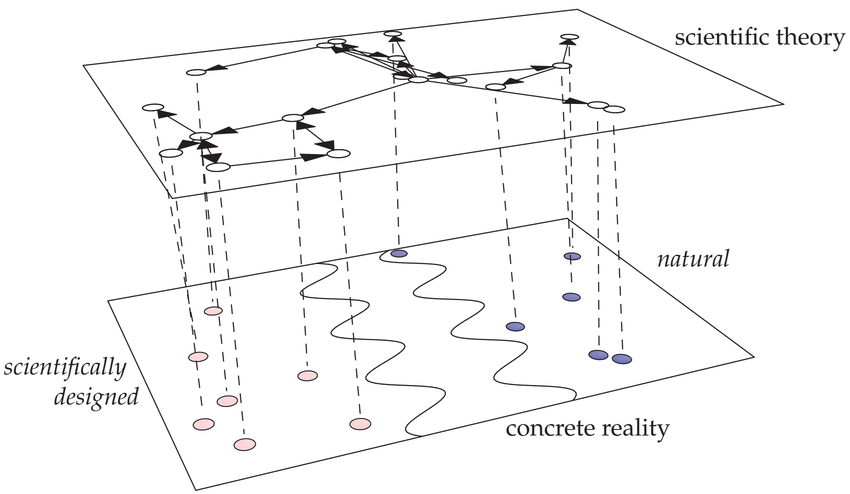


FIGURE 1.1. The role of scientific technology. Dark-shaded circles on the concrete (lower) plane represent objects from nature or prescientific technology. Their counterparts on the theoretical (upper) plane are linked via logical deductions (arrows) to many other constructs, which may or may not have a concrete counterpart. Some of these theoretical constructs give rise, via correspondence rules (dashed lines), to new concrete objects (lightly shaded circles on the lower plane).

The structure of science is enriched by links of various types between different theories; sometimes one theory manages to include another, but more often there is partial overlap between their spheres of applicability.

Two essential aspects of exact science, closely connected to one another, are its methodological unity and its extreme flexibility in considering new objects of study. The scientificness of a discipline does not depend on the kind of thing it studies, but on whether scientific theories can be applied to that thing, and the answer of that question is a historical given. For example, the study of chemical reactions, which had been purely empirical for centuries, acquired the character of exact science as soon as it started to approach the problem using a scientific theory (based on postulates such as the well-definedness of elements, their quantitative preservation and their combination in fixed proportions).

The most significant divisions of exact science are those based not on the phenomena under study, but on the theories brought to bear, each of which generally applies to an enormous set of phenomena seemingly unrelated to one another (other than through that theory).

Science will mean to us primarily exact science. The so-called empirical sciences are to an extent similar to exact science, and distinct from various

types of prescientific knowledge, above all because their development is based on the experimental method and is carried out by specialists whose work, unlike philosophical speculation on the one hand and professional activities on the other, has the purpose of simply acquiring knowledge. One can talk about theories in connection with the empirical sciences, inasmuch as these sciences too are based on the construction of specific theoretical concepts, but these empirical theories do not satisfy the second property in our definition of a scientific theory, lacking as they do the rigorously deductive structure that characterizes exact science. Empirical theories, because they cannot be extended via the deductive method, can be used only as models for a specific set of phenomena and do not produce results exportable to other spheres. Therefore it is possible and convenient to classify empirical sciences by their concrete objects of study, in contrast with the situation for exact science.

The assignment of a privileged status to current scientific theories, as if they represented the standard of truthfulness, is a distorting lens that has often in the past led historians of science to misevaluate and misinterpret ancient science. This can best be clarified with an example. Among many possible quotations we select one from Max Jammer:

> Even Archimedes, the founder of statics, has little to contribute to the development of the concept of force. His treatment of mechanics is a purely geometric one[.][25]

Archimedean statics is a scientific theory that allows the solution of pretty much the same problems as modern statics, which was born from the translation of Archimedes' theory into a Newtonian language, where the concept of force plays a fundamental role. But the concept of force is not a necessity of nature, as demonstrated by the several formulations of mechanics that do not involve it at all. To regard it as a limitation of the Archimedean theory that it contributed little to the development of the concept of force is like regarding it as a limitation of the Greek language that it contributed little to the development of the word "horse".[26]

If we conceive history of science as the history of successive episodes of acquisition of truths bearing directly on natural phenomena, we cannot but be led to the practice, often adopted by historians, of confining all mention of science to inessential remarks or footnotes; but if scientific theories are conceived as theoretical models of sectors of human activity, they clearly acquire fundamental historical interest. On the one hand, their

[25] [Jammer: CF], p. 41.

[26] The culminating irony is that the search for "purely geometric" formulations of mechanics has been a constant from Lagrange to Einstein, whose general theory of relativity has allowed a "purely geometric" formulation of the theory of gravitational forces.

study can supply precious information about the sectors of activity for which the models themselves were created and used; on the other, they are cultural products that can be situated in relation to other aspects of the civilization that created them.

At the other extreme, the contextualization of scientific theories can have the side effect of obscuring what is special about scientific knowledge. This is one of the outcomes of a process that probably started with Thomas Kuhn's famous book[27] and culminated in the complete relativism of many authors. Kuhn's work comprises many of the ideas we have discussed, but what he calls a scientific paradigm is a much more general notion than a scientific theory having the three properties we listed on page 17. It includes forms of knowledge, such as Pythagorean mathematics, Aristotelian physics and various medieval theories, which are excluded from the definition we use. One reason to keep the distinction in sight is that a paradigm such as Aristotelian physics may provide a useful system to frame known reality,[28] but cannot be used to plan different realities, since it lacks a rigorously deductive structure and is thus unable to extend itself via the deductive method. Therefore there is no obvious relationship between technology and science in the very broad sense that Kuhn gives the word. Also the problem of the birth of science cannot be posed in the scope of Kuhn's terminology.

The definition of science proposed here will appear overly restrictive to many. There is no question that it excludes many important conceptual constructs that are often called scientific. The use of a restrictive definition is not intended to deny the importance of other cognitive methods — among which are those used in this book. Its purpose is to focus on a particular intellectual instrument, which, as we will attempt to show, is inherited from Hellenistic culture and was essential in building what we call modern civilization.

1.4 Was There Science in Classical Greece?

The thesis that science, in the particular sense we have given this term, is a product of Hellenistic civilization obviously should not be taken to mean that no element of the scientific method appeared before 323 B.C. — the conventional boundary, which for our purposes should perhaps be moved slightly earlier. Many characteristics of science certainly appeared in the

[27] [Kuhn: SSR].

[28] Directly perceptible "physical" properties are better described by Aristotelian physics than by later science. See [Bozzi].

preceding period, especially in Greek geometry and astronomy during the fifth and fourth centuries. Nonetheless, we will try to show that:

- the method that we have called "scientific" was not fully present in the ancient empires, nor yet in fifth century Greece or in the works of Plato and Aristotle;
- the boom in scientific theories took place during the third century B.C. and was an essential feature of Hellenistic civilization;
- if one must identify a turning point in the process of formation of the new method, the best candidate seems to be the foundation of Alexander's empire.

The assertion that classical Greek culture had not created science needs clarification.

Usually the comparison between modern and ancient scientific thought is established primarily in terms of modern physics and the ideas of the Greeks, most often presented as a conceptual evolution that, starting with the Ionian school, seems to essentially end with Aristotle. Framing the comparison in these terms allows one to pay homage to "Greek thought", to which we all recognize ourselves heirs, while maintaining an obvious, if implicit, attitude of benevolent superiority. Today's physicist, talking about atoms, is often aware of using a term introduced by Leucippus and Democritus almost twenty-five hundred years ago. She recognizes the merits of these ancient thinkers who, although lacking our experimental means and refined conceptual tools, nonetheless intuited a theory that foreshadows the modern one. This acknowledgement is gladly made, because it allows one to display one's humanistic culture, while savoring a pleasant sensation of superiority, based on the belief that the old atoms, being born of pure philosophical imagination, had in fact very little in common with the homonymous objects of modern physics. The debt to ancient science explicitly acknowledged by modern science generally stays within similar limits. Even a scientist of vast learning like Heisenberg, in sketching a comparison between Greek thought and modern physics, after having dwelt at length on pre-Socratic thinkers (with interesting things to say) jumps from Aristotle to modern science, without devoting a single word to the development of ancient exact science, which took place chiefly after Euclid.[29]

From now on we will instead discuss Hellenistic science, referring only occasionally to its classical antecedents. This is because these antecedents are not really relevant to our subject. The atomic theory of Leucippus and

[29] [Heisenberg], Chapter 4.

Democritus, for example, has of course tremendous interest for the history of thought, but it does not seem to be a scientific theory in the sense we gave this expression in the preceding section, because, as far as we know from surviving fragments, no theorems of atomic theory were proved by the ancient atomists, nor any true experiments carried out.

However, we stress the following points:

- Explanations of phenomena by means of theories that involve nonobservable entities, such as the atoms of Leucippus and Democritus, is a step of enormous importance toward the construction of scientific theories.
- Many of the ideas destined to become keystones in science, Hellenistic and modern alike, were born from the Greek thought of the classical period. This is the case with mechanistic determinism, which seems to go back to Leucippus,[30] and the distinction between primary and secondary qualities, which appears in Democritus[31] and became an essential foundation for the formulation of quantitative theories of phenomena such as sound, color and the chemical properties of substances.
- Even some more specific notions that are often considered scientific already appear in the thought of the so-called pre-Socratic thinkers.[32] Science is indebted to the Greeks not only for the general notion of atoms and for the word, but also for ideas such as the chaotic motion of atoms[33] (which, developed in the Hellenistic period and revived in modern times, was essential in the creation of the kinetic theory of gases) or the presence of "hooks" that allow atoms to connect together,[34] a didactic image still used in elementary chemistry books.

For another example, consider the "bucket experiment", one of whose variants consists in spinning a bucket full of water in a vertical plane very

[30]As reported by Aetius (Stobaeus, *Eclogae* I, iv §7, 72:11–14 = [DG], 321b:10–14 = [FV], II, 81:3–6, Leucippus B2).

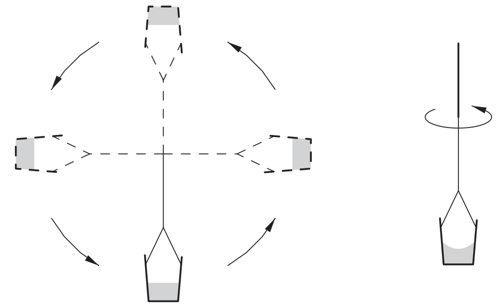
[31]See for instance Stobaeus, *Eclogae* I, xvi §1, 149:10–16 = [DG], 314b:1–10 = [FV], II, 112:28–32, Democritus A125.

[32]Among the philosophers traditionally called pre-Socratic we will be particularly interested in Democritus, who in fact survived Socrates by several years.

[33]See, for example, Diogenes Laertius (*Vitae philosophorum*, IX §31 = [FV], II, 70:26 – 71:5, Leucippus A1), where the idea is attributed to Leucippus. It would be interesting to know the origin of the notion of chaotic motion of atoms. A superb passage in Lucretius about the disordered motion of dust lit by a sunbeam (*De rerum natura*, II:112–141) hints at the type of phenomena that might have suggested the idea. Lucretius mentions the disordered and extremely fast motion of atoms as the ultimate cause of the progressively slower motion of larger particles. It is interesting to compare the lucid explanation reported by Lucretius with the vitalist explanation given in 1828 for a similar phenomenon by the famous discoverer of "Brownian motion"; see [Brown].

[34]The existence of atoms with hooks was postulated by Democritus, as we know from Aristotle's (lost) book *On Democritus*, a passage of which is reported by Simplicius (*In Aristotelis De caelo commentaria*, [CAG], vol. VII, 294:33 – 295:24 = [FV], II, 93:37 – 94:2, Democritus A37).

fast: the liquid does not fall out. Or, if the bucket is kept upright and spun around its own axis, the surface of the water takes on a characteristic concave shape. Either way one sees that the equilibrium configuration of



the water depends not only on the bucket's position with respect to the ground, but also on its state of movement, making it possible to assign an absolute meaning to the statement that the bucket is "in motion", at least if the motion is rotational.

Such remarks, which today we phrase in terms of centrifugal "force", must have made an important contribution to the birth of dynamics, but they are not true experiments, just qualitative observations, so it's not surprising that they predate the rise of scientific theories: indeed they must go back to deep Antiquity. The first documented use of the bucket experiment for theoretical ends seems to be due to Empedocles.[35] Centrifugal "force" was brought to bear in a cosmological context by Anaxagoras, among others; he explained the origin of our world by invoking the separation of the various types of matter caused by the centrifugation of an immense vortex.[36] The very idea of vortices in cosmology was to remain a constant throughout the history of thought: Kant's and Laplace's theories on the formation of the solar system seem to have been influenced by it.

Certainly, many ideas of the pre-Socratic philosophers seem to be akin to the later, Hellenistic, scientific method. However, in no case is there documentation for the use in the classical period of full-fledged hypothetico-deductive theories or the experimental method.[37]

[35]See Aristotle, *De caelo*, II, xiii, 295a:13–22 = [FV], I, 295:31–37, Empedocles A67. According to the passage, Empedocles used the bucket experiment to make some argument about the immobility of the Earth.

[36]Simplicius, *In Aristotelis Physicorum commentaria*, [CAG], vol. IX, 35:13–17 = [FV], II, 36:19–24, Anaxagoras B9.

[37]The tale that Pythagoras made experiments with sound, studying for instance the change in the pitch of a string as the tension varies, is widespread, but the earliest source we have for it dates from about 100 A.D. (Nicomachus of Gerasa, *Manual of harmonics*, 6). This report is unreliable, not

To show the qualitative leap from Aristotelian natural philosophy to Hellenistic science, we recall that Aristotle wrote:

> If, then, [a force] A moves B a distance D in a time T, it does not follow that E, being half of A, will in the time T or in any fraction of T cause B to traverse a part of D that is to the whole of D as A is to E.... Otherwise one man would move a ship, since both the motive force of the ship-haulers and the distance that they all cause the ship to traverse are divisible into as many parts as there are men.[38]

Without reconstructing all of Aristotle's reasoning, we focus on certain key features of the method he uses to approach the problem of motion (and other "physics" questions). Aristotle's problem is determining the quantitative relationship between force, time, and displacement. With the scientific method one can solve such problems in one of two ways: either by supposing a relation given "in principle" (in which case experiment plays an essential role in checking whether the real phenomena whose model one wishes to build do in fact occur in the way predicted by applying the correspondence rules to the stated principle), or else by deducing the desired relationship inside a preexisting scientific theory, using the deductive method. But Aristotle cannot use either the deductive method or experiments, for he does not have, and does not wish to build, a scientific theory. The forces, times and displacements that he talks about are not in fact entities internal to a theory, but he conceives them as concrete objects, whose mutual relations can be understood via philosophical speculation.

He mentions an empirical datum (the impossibility of a single person moving a ship), but the decisive argument is that the portion of the force under consideration acts differently depending on whether it is in isolation or as part of a whole, because in the second case the part exists only potentially. For all intents and purposes, the empirical fact is mentioned just by way of illustration. The real game is to deduce quantitative statements about particular physical phenomena directly from general philosophical principles, derived from qualitative observations of nature.

Archimedes' confutation of Aristotle's argument, reported by Plutarch and Proclus, was very persuasive. According to the tradition they transmit, Archimedes designed, within his scientific theory of mechanics, a device that enabled a single man — himself or King Hiero II, depending

only for chronological reasons and because of the general tendency neo-Pythagoreans like Nicomachus had of backdating all knowledge to Pythagoras, but also because the same experiments are attributed not to Pythagoras but to his followers by Plutarch (*De animae procreatione in Timaeo*, 1020F–1021A) and by Porphyry (*In Harmonica Ptolemaei commentarius*, 119:13 – 120:7, ed. Düring). Iamblichus copied the story from Nicomachus (Iamblichus, *Vita pythagorica*, §§115–119) but on another occasion he follows Porphyry's version (*In Nicomachi arithmeticam introductionem*, 121).

38 Aristotle, *Physica*, VII, v, 250a; loosely based on a translation by R. P. Hardie and R. K. Gaye.

on the version — to drag into the water a ship towed up onto dry land in the Syracuse harbor; Proclus specifies it was a full-laden three-masted ship.[39] The machine itself carried out the division of force that Aristotle had judged impossible, and which indeed had probably never been achieved before for a ship. This was a very effective way to demonstrate the superiority of "scientific" method, in the sense already explained, over natural philosophy. Rather than reflecting the world in philosophical speculation, the scientific method had changed the world, by allowing the design of a machine that eliminated the impossibility observed by Aristotle.

The methodological value of the experimental demonstration narrated by Proclus and Plutarch, which stands out most clearly in comparison with the Aristotelian quotation, does not of course depend on whether Archimedes explicitly wished to confute Aristotle[40] or whether the reported details are historically accurate. The essential point is that, since we know that Archimedes had in fact developed the possibility of designing machines with high mechanical advantage, the story is not a legend without foundation. It reflects on the one hand the type of achievement made possible by Archimedean mechanics, and on the other a widespread interest in this new technology, and in these respects it is completely believable.[41] But instead, the ship episode is usually recounted in the context of the legendary and anecdotal treatment of Archimedes' persona, which deprives it of its true meaning.

One often reads that Greek scientists invented statics but not dynamics. That is, they knew the equilibrium conditions of bodies, but not their laws of motion. Such statements give the impression that ancient scientists, because of their "contemplative" nature, spent their time observing objects in equilibrium, without ever moving them. This impression can hardly be reconciled with the tale of Archimedes designing and using a machine that enabled a single person to drag a ship. The truth is that in the third century B.C. "our" dynamics had not been developed; but the quantitative theory of machines such as winches and cogwheels with mechanical advantage, which had most certainly been developed, must be considered as a form of dynamics, since the point of such machines is not just equilibrium. The notion that Archimedes invented statics but not dynamics comes from the fact that our statics essentially coincides with his, but the same cannot be

39 Proclus, *In primum Euclidis Elementorum librum commentarii*, 63, ed. Friedlein. The same episode is told by Plutarch in a slightly different way (Plutarch, *Vita Marcelli*, xiv §8).

40 C. Mugler argues for this conscious reference to Aristotle's passage; see [Archimedes/Mugler], vol. I, Introduction, p. xi).

41 The origin of the tradition was probably not a true experimental demonstration, but the wonder aroused by the machine designed by Archimedes to launch the huge ship *Syracusia* (Athenaeus, *Deipnosophistae*, V, 207b).

said about our dynamics. Archimedes' mechanics — literally, his "science of machines" — was nonetheless a scientific theory, which dealt with both equilibrium and motion, even though, like all scientific theories, it applied only to phenomena that lie within a limited realm.

The situation was probably analogous to that of our thermodynamics of reversible transformations. Since we only know how to define the thermodynamical state of a system when it is in equilibrium, we only know how to study thermodynamical transformations by approximating them by a series of equilibrium states. In this way we study thermodynamic cycles that model for example what happens inside an internal combustion engine; the model, within certain limits, applies, but that does not mean that our internal combustion engines remain in equilibrium, nor has anyone ever thought of naming "thermostatics" the study of such evolutions through states of equilibrium.

Likewise, the main mechanical problem of the third century B.C. was the study of machines that, while carrying out work, could be thought of as if the forces in question were at all times "almost in equilibrium". That is indeed the case of a pulley that lifts a weight slowly. Problems regarding mechanical systems of that type (in particular the calculation of their mechanical advantage[42]) can be solved using Archimedean mechanics. Our "classical mechanics" is an improvement on the Archimedean theory because it subsumes it and can be applied in many cases where the preceding assumptions are not valid. But this difference is of the exact same nature as the difference that makes, say, relativistic mechanics an improvement on the classical version. The essential qualitative leap, from natural philosophy to science, has already taken place with Archimedes. After that it's "just" a matter of developing theories that can model increasingly more general classes of phenomena; the path is already laid out, as shown by the fact that several Hellenistic scientific theories, such as hydrostatics, geometric optics, and the theory of simple machines, have been absorbed essentially without change into modern science.

We will come back to successive developments in Hellenistic mechanics and their relationship with Newtonian dynamics in Chapters 10 and 11.

1.5 Origins of Hellenistic Science

Why was science born precisely at the same time as Hellenistic civilization, with Alexander's conquests?

[42] We will return to this point in Section 3.3.

Probably one important factor was the new relationship established between Greek civilization and the ancient Egyptian and Mesopotamian civilizations. The Greek cultural tradition, which in the classical period had created historiography, theater, political democracy and the masterpieces of literature and art that we all know, and also natural philosophy as we have already discussed, was obviously essential. But what did the creators of this stupendous civilization have to learn from the Egyptians, for example? We must let sink in the (long ignored) fact that, despite all the achievements of their culture, the Greeks of the classical age were still behind the Egyptians and Mesopotamians from the technological point of view. Recall what Charles Singer wrote in the epilog of the second volume of his *History of Technology*:

> Whatever view be taken of the beauty and interest of the art, literature, ethics, and thought of Greece and Rome, it can no longer be held true that their technology was superior to that of the ancient empires.... The curve of technological expertness tends to dip rather than to rise with the advent of the classical cultures. This will become apparent if the relevant chapters of volume I be compared with the corresponding chapters in the present volume.... Greece and Rome ... rose to their might by the destruction of the more ancient civilizations that they displaced.... [T]he rise of the Hellenic and Roman peoples represents a 'heroic age' which, like many heroic ages, was primarily a victory of barbarians over an effete but ancient civilization.[43]

This is one of the conclusions of an influential work on the history of technology, filled with articles by the greatest experts in their fields and thus deserving of careful consideration. But one is struck by the constant and mechanical merging of Greece and Rome into an indivisible unit. It is impossible to see in what sense Greece might have destroyed older civilizations, or in what sense the Hellenes can be called barbarians. Moreover, it's easy to document (and we shall do so) that Egypt's technological level rose under the Ptolemies. Singer's conclusion seems to have been reached by melding together three elements of wildly unequal worth:

- the conclusion — interesting and very valuable, in that it draws upon a huge fund of historical research on numerous technological areas — that the technology of the ancient empires was superior to classical Greece's (the point that concerns us) and Rome's;
- the fairly obvious fact that Rome rose to its might by the destruction of more advanced civilizations;

[43] [HT], vol. II, pp. 754–755.

– the clichés that lie at the root of the uncritical association of Greece and Rome into an indivisible unit and of the use of "Greek civilization" to refer basically to the classical era, ignoring the originality of Hellenistic civilization.

We can in any case take it as certain that the Greeks who moved into Egypt and Mesopotamia at the time of Alexander's conquests found there a level of technology higher than their own. The technological development of all three cultures — classical Greece, Egypt and Mesopotamia — having proceeded by a gradual accumulation and transmission of empirical knowledge,[44] it is natural that the extra millennia would give the two older civilizations a technological advantage, unsurmountable except in the presence of a qualitative methodological leap.

The Greeks had always been interested in the traditions of older civilizations, with which they had been in contact for centuries. It is not by accident that the beginnings of Greek mathematics are credited to Thales and Pythagoras, both of whom were said to have lived in Egypt (and Pythagoras also in the East). But in the Hellenistic period the contact becomes much closer.[45] The Greeks who moved to the new kingdoms that arose from Alexander's conquests had to administer and control these more advanced economies and technologies with which they were not familiar; their one crucial advantage and guide consisted in the sophisticated methods of rational analysis developed by the Greek cultural tradition during the preceding centuries. It is in this situation that science is born.

Actually there are indications that at the time Alexander formed his empire many features of the scientific method were already in place. Since no scientific work from that period has survived, this is difficult to prove, but the progress achieved by scientists such as Eudoxus of Cnidus a few decades before Alexander seems to show elements of continuity with the following period. However, although on the basis of surviving documents this continuity seems to be well-attested regarding individual instruments internal to mathematics and astronomy, the scientific explosion, that is to say the creation of many different scientific theories understood as models of the real world based on systems of explicitly specified assumptions, seems to be new to the Hellenistic era.

[44]Of course the pace of technological development never stayed constant. Mesopotamia, for example, enjoyed a surge in the development of water, agricultural, and building technologies during the fourth millennium B.C., with the appearance of the first cities. But this and similar bursts are to be taken in a relative sense; they required many centuries. We will return to this question at the beginning of Section 7.2.

[45]To an extent that is impossible to quantify, this change preceded, and even helped motivate, Alexander's campaigns: interactions between Greece and the territories of the ancient empires had been intensifying throughout the fourth century, again thanks to increased migration.

Note that the application of the scientific method requires the ability to use simultaneously two levels of discourse, one internal to the theory and one concerning concrete objects, and to move between the two levels by means of what we've called "correspondence rules". It is enticing to conjecture that this ability was favored, in the territories belonging to Alexander's empire, by the simultaneous presence of two cultures and by the ability developed by Greek immigrants to use both at the same time according to their goals, in particular by reworking into their conceptual framework the large mass of empirical knowledge inherited by the Egyptian and Mesopotamian cultures.[46]

One example of the ability of Hellenistic science to provide a rational framework within which the knowledge of ancient civilizations could be used to advantage is given by the organization, under the Ptolemies, of the immense labor of waterworks that consisted in the regulation of the Nile floods. The Egyptians had millennia of experience with this problem; it was the very problem that had led to the creation of Egypt as a unified state.[47] The Ptolemies organized the necessary labor by using many Egyptian experts, but entrusting the general administration of the project to Greek engineers. We shall see what these engineers were able to achieve.

[46]Incidentally, in later eras, an analogous mastery of two cultures — that of one's ethnic group and the majority culture of the surrounding population — has been a characteristic of the Jews, to whom we also owe many key scientific results.

[47]Karl Marx remarked that the Egyptian state, and indeed the state structures of many ancient riverside civilizations (in Mesopotamia, in the Indus Valley, by the Yellow River), arose from the need to coordinate the labor of irrigation and dam building. This observation was the starting point for Karl Wittfogel's monumental studies on "hydraulic civilizations" and "hydraulic despotism" (see [Wittfogel]). Beyond his (highly ideologized) theories, the essential role played by hydraulic problems in the formation of states is widely recognized nowadays. The fact that the Greeks, in a few years, had surpassed in hydraulic works the most ancient "hydraulic civilizations" shows clearly how powerful the new scientific method was.

2
Hellenistic Mathematics

2.1 Precursors of Mathematical Science

The term "mathematics" is seldom defined by historians of the subject: for instance, Boyer's *History of Mathematics* explicitly avoids the task, saying merely that "much of the subject [...] is an outgrowth of thought that originally centered on the concepts of number, magnitude and form."[1] Were we to take this as the basis for a definition, mathematics would not only go back to paleolithic times — and indeed long before, since one can talk about the "mathematical abilities" of various animals, and research has been done on the issue — but would encompass even the Neapolitan *smorfia*, a series of rules for extracting from dreams information supposed to be helpful in predicting winning lottery numbers. This too, one must admit, deals with questions centered on the concept of number.

But mathematical *science*, in the sense we have given the word, arises in the Hellenistic period. Of course Hellenistic mathematics does not come out of nothing. Earlier mathematics can be divided, roughly speaking, into two phases. The first, extremely long, phase includes the mathematics of Old Babylonia and of Egypt under the Pharaohs. The second consists of a period of approximately two and a half centuries in which classical Greece created what we will call *Hellenic mathematics*, to distinguish it from Hellenistic mathematics.

The first period started in the paleolithic, with the ability to count,[3] and saw the accumulation of a remarkable body of knowledge, as in Egypt

[1] [Boyer], p. 1.

during the Pharaonic era; there, for the first time, appeared specialized writings with mathematical content.[4] These writings can be called mathematical only in that their object is solving problems that we would call arithmetical or geometric; they completely lack the rational structure that we associate with mathematics today. They contain recipes for solving problems — for example, calculating the volume of a truncated pyramid or the area of a circle (the latter being, of course, unintentionally approximate) — but there is no sign of anything like a justification for the rules given. At that stage, then, fairly elaborate notions beyond the integers had already been developed, including many plane and solid figures, area, and volume; problem-solving methods were passed down the generations; but the correctness of the solutions was based solely on experience and tradition. This was very far from being a science in the sense we have given the word. It was simply a part of that enormous store of empirical knowledge that enabled the Egyptians to achieve their famous technological feats; it was methodologically homogeneous with the rest of such knowledge, and transmitted in the same way.[5]

Otto Neugebauer, one of the twentieth century's most accomplished scholars of ancient exact science, wrily remarked:

> Modern authors have often referred to the marvels of Egyptian architecture [in connection with their mathematics], though without ever mentioning a concrete problem of statics solvable by known Egyptian arithmetical procedures.[6]

There is nothing surprising about the lack of applications of Egyptian mathematics to statics or other theories with technological interest. Since mathematical and technological knowledge alike were purely empirical, either could be applied only to directly related, concrete, specific problems; there was no scientific theory within which technological planning could be carried out, so there could not have been what we have called scientific technology. The quantities considered in mathematical problems known from Pharaoh-era Egypt are not internal to a theory, but instead had immediate and concrete interest: the number of bricks needed for a building of a given shape and size, or the area of a field.

[3] Animal bones have been found in today's Lebanon, dating from 15000 to 12000 B.C., with series of notches arranged into groups of equal cardinality. Thus the recording of tallies far predates the invention of writing, which for that matter seems to have arisen precisely from the evolution of a bookkeeping system (see Section 7.2).

[4] For a review of sources, see [Gillings].

[5] The papyrus Anastasi I gives some perspective on the role of "mathematical" knowledge in the context of the competencies required of a scribe. See [Gardiner].

[6] [Neugebauer: ESA], p. 151.

Similar considerations apply to Old Babylonian mathematics, though it had reached a higher level.

The bridge between the empirical knowledge of Pharaoh-era Egypt and ancient Mesopotamia, on the one hand, and the sophisticated mathematical science of the Hellenistic period, on the other, is Hellenic mathematics. Without it, the transition would have been unthinkable. During these two and a half centuries Greek culture assimilated the Egyptian and Mesopotamian results and subjected them to a sharp rational analysis, closely linked to philosophical inquiry. Greek tradition names two pioneers in these investigations: Thales, who supposedly started developing, at the beginning of the sixth century B.C., the geometry that he had learned in Egypt, and Pythagoras, who founded his famous political and religious association during the second half of the same century.

An ancient tradition, attested by (among other things) the name "Pythagorean theorem" given to the famous theorem of geometry, holds that the deductive method arose at the very beginning of Hellenic mathematics. This belief goes back at least to the *History of geometry* written by Aristotle's disciple, Eudemus of Rhodes, according to whom Thales proved that a diameter divides a circle into two equal parts, and that opposite angles at a vertex are equal.[7] But it is not possible that statements so apparently obvious could have been among the first to be demonstrated. The usefulness of the deductive method must have been noticed first in proving nonobvious statements. Only when a well-developed deductive system is attained can the demand arise for demonstrations of such apparently obvious statements as the ones attributed to Thales.[8]

In fact Eudemus systematically backdated mathematical results, in a process made explicit by Proclus in at least one case:

> Eudemus, in his *History of geometry*, attributes to Thales this theorem [that triangles having one side and two adjacent angles equal are congruent], because, in his opinion, the method with which Thales is said to have determined the distance of ships in the sea depends on the use of this theorem.[9]

It is clear from this passage how Eudemus confused the logical order, which requires a theorem to be proved before it can be applied, with the historical order. In fact, for the application he mentions, it is not necessary

[7]These statements by Eudemus (whose work has perished) are reported in Proclus, *In primum Euclidis Elementorum librum commentarii*, 157:10–11; 299:1–3, ed. Friedlein = [FV], I, 79:8–9+13–15, Thales A20.

[8]This argument is developed in [Neugebauer: ESA], p. 148.

[9]Proclus, *In primum Euclidis Elementorum librum commentarii*, 352:14–18, ed. Friedlein = [FV], 79:15–19, Thales A20.

to know the theorem: it is enough to be convinced (and only in the particular case in question) of the truth of the *statement* of the theorem, without knowing its proof; and without the proof one cannot, of course, talk about its being a theorem. The error made by Eudemus is still made today. One often reads that the "Pythagorean theorem" was known in Mesopotamia in the Old Babylonian period; actually what was known was its empirical basis, the fact that the square on the hypotenuse has the same area as the sum of the squares on the sides. The idea of proofs and theorems had not been invented in Old Babylonia, nor yet in Pythagorean times.

Without getting into the history of Hellenic mathematics,[10] we will show with some examples how it was not a science — not only at the time of Thales and Pythagoras, but even much later.

A remarkable example of the state of Hellenic mathematics in mid-fifth century is afforded by Zeno's paradoxes, which are so famous (particularly the one with Achilles and the turtle) that we need not repeat them here.[11] Why are they thought of today primarily as a philosophical subject, although they deal with the concept of a continuous quantity, which is essential in mathematics? Above all because Zeno talks about space and time and not about their mathematical model, which had not been constructed then. The instruments used to analyze the notions of space and time are ordinary language and philosophy; the structure of a scientific theory, in the sense defined in Section 1.3, is still lacking. Zeno's paradoxes certainly influenced significantly the evolution of the concept of a continuous magnitude, which eventually resulted in the sophisticated theory expounded in Book V of Euclid's *Elements*, but once the building had been erected there was no place in it for that type of reasoning.

Another important example, traditionally attributed to the Pythagorean school, is the discovery of the incommensurability of the side and diagonal of a square. This is often quoted as a demonstration of the irrationality of the square root of 2, but the original argument should not be blurred with its later development. One reconstruction of the early state of affairs is the following.[12] We know that the early Pythagoreans thought that

[10]Perhaps because of the complete absence of primary sources, Hellenic mathematics has induced much more writing that its Hellenistic heir. Among the books devoted wholly or mostly to Hellenic mathematics, we mention [Lasserre], [Szabó], [Knorr: EEE], and [Fowler]. For Greek mathematics of both periods, the standard reference is [Heath: HGM], while [Loria] can also be useful. An anthology of original texts can be found in [GMW], whose two little volumes are all the Loeb Classical Library spares for Greek mathematics.

[11]The main source for Zeno's paradoxes is Aristotle, *Physica*, VI, ix, 239b–240a. This passage and all other relevant sources are reported in [FV], I, 247–258. The paradoxes are discussed in [Heath: HGM], vol. I, pp. 271–283.

[12]Reconstructions of this episode are based primarily on two sources. The older one is a passage of Aristotle (*Analytica priora*, I, xxiii, 41a:26–27), which says that the diagonal is not commensurable

every segment was made up of finitely many points.[13] If we construct a square whose side is made up of an odd number of points, say k, we can ask whether the number n of points of the diagonal is even or odd. Since the square of n, by the Pythagorean theorem, is $2k^2$, and therefore even, and only an even number can have an even square, it can easily be concluded that n is even. But someone must have noticed that if n were even its square would be a multiple of 4, whereas $2k^2$ is not such a multiple if k is odd; therefore n must be odd. Since both reasonings appear convincing, but an odd number cannot be even, they did not know what to conclude.[14]

The result is an impasse, analogous (from the viewpoint of the culture of the time) to Zeno's paradoxes. Because the Pythagoreans attached great importance to the opposition between even and odd numbers,[15] it is reasonable to assume that the difficulty just described arose among them, as tradition maintains. But if the reconstruction above is correct, the Pythagoreans had not proved anything by contradiction; they had simply *reached* a contradiction (to their chagrin!), while trying to find the parity of the diagonal. Note that to get to this point it is not necessary to know the Pythagorean theorem in full generality, but only in the case of an isosceles right triangle, and the validity of this case can easily be verified by counting half-squares in a square array.

Unlike Zeno's paradoxes, the argument just discussed, which most likely dates from the last quarter of the fifth century,[16] was later incorporated into mathematical science, where it provides the base for the proof of a theorem. But to reach a theorem there must be a qualitative leap allowing

[with the side] because, if it were, odd and even numbers would coincide. The second source is a (probably spurious) passage in Euclid's *Elements* (X, 408–411, ed. Heiberg, vol. III), containing a complete proof of the incommensurability, consistent with Aristotle's brief remark.

[13]This can reasonably be deduced from several elements: the fact, reported by many sources, that the Pythagoreans based geometry on the integers; the Pythagorean theories of "figurative numbers" (for which see [Heath: HGM], vol. I, pp. 76–84, and [Knorr: EEE], Chapter 5); and above all Aristotle's assertion that the Pythagoreans attributed a magnitude to the units that made up material bodies (Aristotle, *Metaphysica*, XIII, vi, 1080b:16–21 + 1083b:8–18 = [FV], I, 453:39 – 454:9, Pythagoreans B9, B10). Sextus Empiricus seems to still be thinking in Pythagorean mode when he says that it is impossible to bisect a segment formed by an odd number of points: *Adversus physicos* I (= *Adv. dogmaticos* III = *Adv. mathematicos* IX), §283; *Adversus geometras* (= *Adv. math.* III), §§110–111.

[14]The reconstruction given here, apart from the modernized notation (the use of letters to denote numbers is not part of the Greek tradition) and the use of "points" as in the Pythagorean tradition, follows in essence the sources that report the theorem on the incommensurability of a square's side and diagonal. It is generally accepted that incommensurability was first discovered in this case, particularly because Plato and Aristotle always talk about it in connection with this example. A different conjecture about the origin of the idea of incommensurability is argued for in [von Fritz].

[15]Philolaus, as quoted by Stobaeus (*Eclogae*, I, xxi §7c, 188:9–12 = [FV], I, 408:7–10, Philolaus B5); Aristotle, *Metaphysica*, I, v, 986a:18+23–24. Evenness and oddness were still at the basis of arithmetic for Plato (*Gorgias*, 451a–b). For a discussion of the Pythagorean ideas about even and odd numbers, see [Knorr: EEE], pp. 134–142.

the circumvention of the impasse. Only by abandoning the Pythagorean notion that a line segment is made up of a succession of points — and therefore abandoning the Pythagorean program of basing explanations about real objects on the concept of an integer — can one attain the idea that two segments may not admit a common subdivision — may be incommensurable. The impasse can then be transformed into the proof by contradiction, known to Aristotle, that the side and diagonal of a square are incommensurable.[17] There is no evidence for classifying this step as being Pythagorean. That neither Plato nor Aristotle, in any of several passages where they discuss the problems posed by incommensurability, ever attribute its discovery to the Pythagoreans is a strong indication that neither should we; that neo-Pythagoreans did so attribute it[18] is sufficiently explained by the likelihood that the realization of the difficulty dated to Pythagoreans.

Even after the essential step of transforming the impasse into a proof by contradiction, the result remains purely negative: the statement of an impossibility, insufficient to serve as the basis for a theory of continuous magnitudes. The mathematicians of the beginning of the fourth century knew several pairs of incommensurable magnitudes, as we know from Plato,[18a] but they did not have a "theory of irrationalities". As attested by the very word "irrational", they did not say that the ratio between the side and the diagonal of a square is irrational, but rather that there is no such ratio.[19]

Because the Pythagoreans probably thought that statements about the parity of the number of points on the diagonal referred to something in

[16]The dating of the first difficulties caused by the incommensurability between side and diagonal of a square is discussed in [Knorr: EEE], Chapter II, where it is noted that the first author who might have faced the problem was Democritus (born ca. 460 B.C.). But the evidence of Democritus' possible interest in this question is very weak and debatable, being based only on a doubtful interpretation of the title of a lost work. Information given by Plato and Aristotle makes it likely that the problem first arose shortly before 400 B.C.

[17]The purpose of the Aristotle passage cited in footnote 12 is precisely to illustrate what we call proof by contradiction.

[18]It is only in late sources that the discovery of incommensurability is attributed to the Pythagorean school. The main one is a fragment, surviving in Arabic, of Pappus's commentary on Book X of the *Elements*, which says moreover that the first member of the school who divulged the discovery perished in a shipwreck ([Pappus/Junge, Thomson], pp. 63–64). The same story can be found in a scholium to Book X of the *Elements* (scholium 1 in [Euclid: OO], vol. V, p. 415), which perhaps goes back to Proclus. There are also a passage in Proclus, certainly anachronistic, that attributes outright to Pythagoras a theory of irrational numbers (Proclus, *In primum Euclidis Elementorum librum commentarii*, 65:19–21, ed. Friedlein = [FV], I, 448:23–24, Pythagoreans B1), and various mutually contradictory assertions of the neo-Pythagorean Iamblichus.

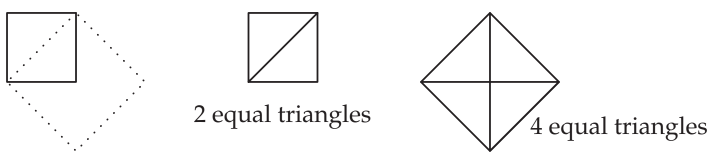
[18a]In fact, an infinity of such pairs; see note 46 on page 45.

[19]The Latin word "irrationalis" is a literal calque based on the Greek ἄλογος, which originally expressed the nonexistence of a ratio (λόγος), and took on its modern meaning in Hellenistic times.

the real world, there may be some truth to the tradition that they regarded the discovery of incommensurability (or rather, of the impasse that led to the notion of incommensurability) as a dramatic event. If we discover that a scientific theory is contradictory, it's no big deal: we change theories. But what can we do if we discover, or think we have discovered, a contradiction in reality itself?

The widespread idea that the discovery of incommensurability shook the foundations of mathematics is based on the assumption that in the fifth century B.C. mathematics already existed in our sense; it must have arisen by analogy with the shaking of the foundations of mathematics at the turn of the twentieth century.[20] What could have been shaken at that time was the original Pythagorean philosophical framework, which wished to be the foundation (in a sense very different from the one used by today's scholars) of much more than our mathematics. The studies that we call mathematical today would have continued without much trouble, precisely because they did not have a monolithic foundation.

Our third example of Hellenic mathematics comes from Plato, who was very interested in the methods used by geometers of his time, and who presents in his works mathematical arguments of great interest.[21] As an example of a demonstration expounded by Plato, we recall the famous passage in the *Meno* where it is shown that, given a square, the square built on the diagonal is double. The proof consists in observing that the



second square is formed of four triangles, each of which is equal to half the initial square.[22] This presupposes assumptions that are not made explicit (among them: a square can be built on a given side, and the four triangles into which a square is divided by the two diagonals are equal). In other words, the truth of a geometric statement is deduced logically from other statements chosen *ad hoc* among those that are visibly true. Precisely because a proof such as the one in the *Meno* is not embedded in a theory, but stands on its own, independent of any other geometric line of argument,

[20]This point is emphasized in [Knorr: EEE], p. 307.

[21]Fabio Acerbi has shown that one can recognize in Plato (*Parmenides*, 149a:7–c:3) an example of a proof by complete induction, a method that is usually considered to have been introduced in modern mathematics. See [Acerbi: Plato].

[22]Plato, *Meno*, 84e–85b.

it can, as in this case, be understood by a young slave completely ignorant of geometry.

In the *Timaeus* Plato explains the growth of a young body as follows:

> [Consider] the young constitution of the whole animal, which has the triangles of the elements new.... Since the triangles coming in from outside, which make up food and drink, are older and weaker than its own triangles, it overpowers them and cuts them up with its new triangles, making the animal grow by nourishing it with many similar elements.[23]

What does Plato mean by "triangle"? It is clearly something real, not a theoretical entity such as we, after twenty-three hundred years of Euclidean geometry, naturally think it should be. Indeed, Plato states elsewhere that mathematical objects are endowed with a higher level of reality than that of their perceptible images;[24] this was certainly an important thought in the process leading to the conscious construction of theoretical entities.

The Hellenic period — or at least most of it — can be considered as a long gestation of mathematical science, in which ever more refined logical instruments were being accumulated, but mathematics had not yet reached the stage of a science in the sense we have given the word, since there was not a single logically coherent and connected corpus of knowledge inside which any student whatsoever could solve an unlimited number of "exercises".

Probably a key person in the transition from the Hellenic to the Hellenistic period was Eudoxus of Cnidus, who according to the traditional boundary falls at the end of the Hellenic period. Because all his works have perished, it is hard to ascertain whether he was a precursor or the founder of mathematical science of the sort that appears in the *Elements*. We will revisit the relationship between Euclid and his fourth-century predecessors in Section 2.6.

In the next few sections, without intending to outline a history of Hellenistic mathematics, we will illustrate some of its fundamental aspects by means of examples, taken primarily from Euclid[25] and Archimedes.[26]

[23]Plato, *Timaeus*, 81b–c.

[24]Plato, *Republic*, VI, 509c–511a.

[25]The critical edition of his works is [Euclid: OO]. An English translation of the *Elements* and a rich historical and critical apparatus can be found in [Euclid/Heath].

[26]All his surviving works can be found in [Archimedes/Mugler]. Another very useful book is [Dijksterhuis: Archimedes], which contains a detailed exposition of the contents of his existing works.

2.2 Euclid's Hypothetico-Deductive Method

Three types of problems came up in Hellenic mathematics. First, it was noticed how certain apparently obvious statements about geometric figures could imply others, much less obvious. This revealed the usefulness of the deductive method; but of course, as already remarked in Aristotle,[27] one could not prove everything without causing the proof of any statement to involve an infinite regression. Second, paradoxes such as Zeno's and the impasse found by the Pythagoreans had made apparent the high degree of subtlety of concepts such as space, time, and infinity, and of the relations between discrete and continuous magnitudes; it had also shown how inadequate everyday language is for dealing with such questions. Finally, there was the question of the unclear relationship between the concepts of mathematics and the real world.

In Euclid's *Elements* we can see for the first time the solution to these problems, which was reached by establishing mathematics as a scientific theory — more precisely, by explicitly defining the theory's entities (circles, right angles, parallel lines, and so on) in terms of a few fundamental entities (such as points, lines, and planes)[28] and by listing the statements about such entities that must be accepted without proof.

In the *Elements* there are five statements of this type, called "postulates" (αἰτήματα):[29]

1. [One can] draw a segment from any point to any point.
2. [One can] continuously extend a segment to a line.
3. [One can] draw a circumference with arbitrary center and radius.
4. All right angles are equal to one another.
5. If a line transversal to two lines forms with them in the same half-plane internal angles whose sum is less than two right angles, the two lines meet in that half-plane.

Any other statement regarding geometric entities can and should be accepted as true only if it can be supported by a proof (ἀπόδειξις), that is, a chain of logical implications that starts from the postulates (and the "common notions") and leads to the given statement. This method is known to anyone who has studied mathematics in high school (at least that was the

[27] Aristotle, *Analytica posteriora*, I, ii, 71b:26–28.

[28] In the *Elements* even these fundamental entities are "defined", and the presence of these "definitions" (which are mere tautologies or purely illustrative statements) appears to contradict the thesis of the present discussion. This important question will be the subject of Section 10.14, where we will be able to study it in light of the material contained in intervening chapters.

[29] There are also five "common notions", that is, statements that are not about the specific entities of geometry. However, the authenticity of the "common notions" has often been contested. See, for example, [Euclid/Heath], vol. I, pp. 221 ff.

case until a while ago), because it was inherited by modern mathematics. Note the privileged role played, from the postulates on, by lines and circles. The reason for this choice is clear: these two entities play a special role because they are the mathematical models of what can be drawn with ruler and compass. Euclidean geometry arises explicitly as the scientific theory of the objects that can be drawn with ruler and compass. Euclid's first three postulates are nothing but the straightforward transposition, into the context of mathematical theory, of the usual operations carried out with these basic instruments. Of course, there is a tremendous difference between mathematics and drawing. A compass cannot draw circumferences of arbitrary radius — in fact it cannot draw a true circumference at all. Mathematical science arises from the replacement of the ruler and compass by an ideal ruler and compass, theoretical models of the real instruments, capable of the operations described by the first three postulates; for this theoretical model, both its origin and the correspondence rules that allow its application are perfectly clear.

The difference between the first three postulates, which affirm the constructibility of lines and circumferences, and the last two, whose nature is more theoretical, is reflected in the propositions that make up the treatise, which are of two types: "problems" (προβλήματα) and "theorems" (θεωρήματα).[30] The first type consists in the description of a geometric figure with specified properties, followed by the figure's construction and a proof that the figure constructed does satisfy the desired properties. The first proposition of the *Elements*, for instance, solves the problem of constructing an equilateral triangle. The theorems, by contrast, consist in the statement that certain properties imply others, and can be followed by the demonstration alone. One famous theorem, for example, states that the square built on the hypotenuse of a right triangle is equivalent to the sum of the squares built on the other two sides.[31] In the *Elements* this theorem is immediately preceded by the problem of building the squares (and proving that the construction works).[32] In fact, Euclid never uses a geometric figure unless he has given its construction and demonstrated the validity thereof.

[30] The distinction between problems and theorems is discussed at length by Proclus (*In primum Euclidis elementorum librum commentarii*, 77–81, ed. Friedlein), and appears twice in Pappus (*Collectio*, III, 30:3–24; VII, 650:16–20). Euclid does not differentiate between the two types of propositions in these terms, but the distinction is clear from the formula that closes the demonstration, which is either "as was to be shown" (ὅπερ ἔδει δεῖξαι) or "as was to be done" (ὅπερ ἔδει ποιῆσαι).

[31] Euclid, *Elements*, I, proposition 47.

[32] Euclid, *Elements*, I, proposition 46.

2.3 Geometry and Computational Aids

Mathematics has always been used to obtain quantitative results, and its theoretical structure has always been influenced, if often unconsciously, by the way in which these results are obtained. Today we have digital computers. What were the computational aids of classical and Hellenistic Antiquity? For calculations with integers abaci of various types were used, but we know very little about the classical versions. Our ignorance of, and the usual attitude toward, the subject is well illustrated by this quotation from an authoritative history of technology (italics mine):

> The form of the Greek abacus is obscure, but the *more developed* Roman type is well known. . . [33]

The other computational aid, used above all for noninteger quantities, was geometry. Every problem about continuous magnitudes was cast into geometric language, the data being represented by lengths of segments. Knowing how to solve the problem meant knowing how to construct geometrically a segment whose length represented the solution; this length was then measured. The instruments used in geometric constructions were primarily the ruler and the compass, which thus became not just drawing instruments but analog computational tools.[34] The use of analog computational tools may seem strange to us, accustomed as we are to digital computers, but remember that until a few decades ago engineers did a large part of their calculations with slide rules, whose precision is less than that attainable with the ruler and compass of Hellenistic mathematics. Two features of ruler and compass solutions made them particularly useful. First, their relative error was very small (of the order of the ratio between thickness and length of the lines drawn): no technical application could want better. Second, the construction was easily reproducible in solving an equal problem with different numerical data. Today we consider independent three activities that were indissolubly linked in the practice of Hellenistic mathematics: deductive reasoning, calculation, and drawing.

[33][HT], vol. III, p. 501.

[34]The problems that can be solved in this way are those that we would express in terms of algebraic equations of the first or second degree. For example, the determination of the fourth proportional of three given segments (Euclid, *Elements*, V, proposition 12) is equivalent to the calculation of a ratio, once one segment is chosen as the unit of measurement. The determination of the proportional mean of two given segments (Euclid, *Elements*, V, proposition 13) amounts to taking a square root. Obviously the algebraic formulation is not necessary for applications: every problem solvable by taking a square root can be solved equivalently by finding a proportional mean.

Forgetting that ruler and compass were the main computational aids of Hellenistic mathematics can lead one badly astray. Boyer writes in his *History of Mathematics*:

> The Greek definition and study of curves compare quite unfavorably with the flexibility and the extent of the modern treatment. Indeed, the Ancients overlooked almost entirely the part that curves of various sorts played in the world about them. Aesthetically one of the most gifted people of all times, the only curves that they found in the heavens and on the earth were combinations of circles and straight lines. [35]

Finding combinations of circles and lines in heaven and on earth meant successfully reducing the solution of problems both earthly and astronomical to calculations that could be performed with elementary instruments such as ruler and compass.[36] Boyer might as well have charged contemporary scientists with infinite intellectual poverty because, in using digital computers, they are unable to imagine anything other than combinations of zeros and ones.

Moreover the Greeks, from the Hellenistic period on, did study curves that cannot be drawn with ruler and compass. They knew, for instance, that the quadratrix of Hippias (the curve described by the intersection of a segment in uniform translational motion with one in uniform rotational motion) allows one to square circles and trisect angles. However, they considered this a pseudo-solution, or “sophistic solution”, to these problems. Why? Clearly because it transferred the difficulty from the original problem to that of building a machine that could carry out in practice the two required synchronized motions, tracing the intersection point. The task was certainly feasible, but not with the same precision with which segments and circles could be traced with ruler and compass, and above all not in such an easily reproducible way and with a precision so easy to check.

The preference on the part of Hellenistic mathematicians for ruler and compass solutions has often been considered an intellectual prejudice. That misses the point; geometers who proposed “sophistic” solutions such as the one just described were much in the same position as someone today who might propose to solve a physics problem not by finding a theoretical method translatable into an algorithm that can be implemented on digital computers, but by using an analog “computer” that measures

[35][Boyer], p. 173 (1st ed.), p. 157 (2nd ed.).

[36]How far one can get with “combinations of circles” was clear in Hellenistic times, and is even clearer today to anyone who has studied Fourier series expansions. This point will be taken up again in Section 3.8.

the desired physical magnitude by reproducing the phenomenon under study. Such a procedure can certainly be useful, but no one would say that it provides a true solution for the problem.

Archimedes introduced in his *Arenarius* a numbering system whose expressive power equals not only that of our positional system,[37] but even that of today's exponential notation. Despite the creation of an analogous system by Apollonius[38] and the introduction of zero,[39] there was no consequent spread of the positional notation system; its use remains largely limited to the base sixty system used in astronomy and trigonometry.[40] It can be conjectured that the efficiency of geometric algorithms contributed to the slow rate of diffusion of "algebraic" computation methods. This impression is supported by the fact that both Archimedes and Apollonius developed their (essentially equivalent) methods in close connection with the problem of representing very large numbers, and so invented exponential representation before positional representation pure and simple. Clearly the geometric method was so efficient that the need for improving it came up chiefly when very large ratios were involved, a case which geometry does not serve well (how can one represent with segments two numbers that differ by several orders of magnitude?).[41]

The efficiency of algorithms based on ruler and compass was closely tied to the possibility of making accurate drawings on papyrus. The link between theoretical structures and material instruments is illustrated by the very different route taken by mathematics in Mesopotamia. There, as we have said, clay tablets were used all the way down to the Hellenistic period, and they do not allow accurate drawings. This meant that Mesopotamian mathematics had to be based on numerical, rather than geometric, methods. The scarcity of sources, already mentioned in Section 1.1,[42] prevents us from following Hellenistic-era Mesopotamian mathematical

[37]The first versions of positional notation arose in Old Babylonia; they were the result of millennia of unplanned evolution (as discussed further in Section 7.2) and had not, until Hellenistic times, led to a completely coherent and ambiguity-free system. Archimedes' creation, by contrast, was consciously designed, with full knowledge of the conventional nature of such systems.

[38]A précis of Apollonius' system can be found in Pappus, *Collectio*, II, 6–28.

[39]Zero was being systematically used in Mesopotamia with sexagesimal notation around 300 B.C. Its possible earlier history in Babylonian mathematics is unclear; see [Neugebauer: ESA], p. 29. Its present symbol, transmitted by Indians and Arabs, appears in Ptolemy's trigonometric tables and in papyri from the Ptolemaic era (where it is modified by a line above or other decorations); see [Neugebauer: ESA], p. 13–14. A late mention of the role played by the number zero in arithmetic can be found in Iamblichus (*In Nicomachi arithmeticam introductionem*, 17–19). Iamblichus' word for zero is οὐδέν, from whose first letter may have derived the symbol we use.

[40]Because of its usefulness in astronomical calculations, the positional system was imported into India together with astronomy (see Section 2.8).

[41]Another case where geometric algorithms fell short was that of trigonometric tables; see p. 53.

[42]Among the clay tablets dating from the Seleucid era many have been found to have mathematical content, but again the great majority of these have never been published or translated.

developments as well as we'd like, but the few cuneiform texts that have been translated make it clear that a transition from prescientific mathematics to mathematical science took place during the Hellenistic period in the Seleucid kingdom, just as it did in the Mediterranean world whose main center was Alexandria.[43]

Although in the rest of this chapter we will continue to deal mostly with mathematics characterized by the use of geometric methods, we should keep in mind that this is not all there was to mathematical science in the Hellenistic period. The different strand of mathematics pursued in Mesopotamia led not only to the introduction of zero but also to certain algebraic methods which, after being taken up to some extent in Alexandria during the imperial age (first by Heron and especially by Diophantus), reappeared in different guise after centuries of dormancy, as part of new developments in India and China.

2.4 Discrete Mathematics and the Notion of Infinity

Two classes of objects of study in the *Elements* are the integer numbers (ἀριθμοί) and the magnitudes, or continuous quantities. As an example of a theorem about integers, we recall the famous proof that there are infinitely many primes. The statement of Euclid's theorem (IX, proposition 20) is:

> There are more prime numbers than any preset multitude of prime numbers.

The proof is the following. Given any "multitude" (finite set) of prime numbers, let k be the number obtained by multiplying them all together[44] and adding 1 to the result. Clearly, k cannot be a multiple of any of the given prime numbers (which are assumed to be different from 1). Thus, if m is a prime factor of k distinct from 1, m cannot be one of the given prime numbers. Thus we have found a prime number not included among those originally preset.

It is often said that in Antiquity the concept of infinity was not used in mathematics. For example, Morris Kline writes:

> In Greek science the concept of the infinite is scarcely understood and frankly avoided.... The concept of a limitless process fright-

[43][Neugebauer: ESA], p. 48.

[44]Actually Euclid considers not the product, but the least common multiple, of the numbers. Since the numbers are prime, the result is the same. Euclid's choice makes possible the geometric interpretation provided by the illustrators of our manuscripts, who, representing each prime number by a segment, represent in the same way also their least common multiple.

> ened [the Greeks] and they shrank before 'the silence of the infinite spaces.' [45]

Similar statements can be found in many histories of mathematics, such as the one already cited by Boyer.

But the argument of *Elements* IX, 20 is a rigorous demonstration of the infiniteness of a set. Euclid, knowing very well the subtlety of the concept of infinity, which had been clear at least since Zeno's time, manages to obtain a rigorous proof without ever dealing directly with infinity, by reducing the problem to the study of finite numbers. *This is exactly what contemporary mathematical analysis does.* That Euclid does not use the word "infinite" is of course irrelevant. In any case, the word "infinite" is not a novelty introduced by modern mathematicians; it is the literal translation of Greek ἄπειρος, which, after a long and complicated history, was eventually used in mathematics in its current meaning of "infinite" (by Apollonius of Perga, for example).[46] We will return to this question in Section 11.9, where we try to pinpoint the origin of the opinion that Kline and so many others have held.

2.5 Continuous Mathematics

The use of "magnitudes", or continuous quantities, in addition to integers, gave rise to a difficult problem. Consider segments. To operate with these "magnitudes" one must know how to carry out the basic arithmetic operations. Addition poses no problem: if a and b are two segments, the sum $a + b$ is the segment obtained in a natural way by extending the first segment a length equal to that of the second. Differences are defined analogously. These rules correspond to what one effectively does in order to add or subtract noninteger quantities using the geometric method. For multiplication things were also simple: the product ab was thought of as a rectangle whose sides were represented by a and b.[47] But what meaning could be assigned to the ratio $a:b$? Of course, the operation of addition

[45] [Kline], p. 57.

[46] See, for instance, *Conica*, II, proposition 44, where Apollonius, after showing how to construct a diameter of a conic, concludes: "In this way we will find infinitely many diameters" (ἀπείρους διαμέτρους). This use of ἄπειρος in the sense of actual infinity in a mathematical context appears already in Plato's *Theaetetus*, 147d. Theaetetus reports a conversation between the mathematician Theodorus and his students (of whom he was one), dealing with squares that are multiples of the unit square but whose sides are not multiples of the unit length (and therefore are incommensurable with it). They remark that such sides are infinite in number (ἄπειροι τὸ πλῆθος).

[47] This way of regarding products returns magnitudes nonhomogeneous with the factors, so it makes expressions such as $a + ab$, where a and b are lengths, nonsensical. This introduces a kind of automatic "dimension control".

between magnitudes induces in an obvious way the operation of multiplication of a magnitude by a natural number: if k is a natural number, the product ka can be defined as the sum of k magnitudes, each equal to a. If two integers, k and h, can be found such that $ka = hb$, the ratio $a:b$ can be defined as the ratio between integers $h:k$. In other words, the ratio $a:b$ can be defined as a fraction. When there are no two integers h and k satisfying $ka = hb$, the magnitudes a and b are called incommensurable. In this case (which happens, for example, when a and b represent the side and diagonal of a square, as we saw in Section 2.1), it is not clear what can be understood by a "ratio" between the two magnitudes. Yet the theory of similarities — which we would not want to restrict to the case of commensurable magnitudes — requires that we assign a meaning to proportions such as $a:b = c:d$ even when the magnitudes are incommensurable.

This problem is solved by Euclid with his definition of proportion (*Elements*, Book V, definition 5, Heath translation):

> Magnitudes are said to be in the same ratio, the first to the second and the third to the fourth, when, if any equimultiples whatever be taken of the first and third, and any equimultiples whatever of the second and fourth, the former equimultiples alike exceed, are alike equal to, or alike fall short of, the latter equimultiples respectively taken in corresponding order.

Expressing this in an algebraic language more familiar to us, we say that the proportion $a:b = c:d$ holds when, for any chosen natural numbers h and k, one of the following statements is true:

(A) $ka > hb$ and simultaneously $kc > hd$;
(B) $ka = hb$ and simultaneously $kc = hd$;
(C) $ka < hb$ and simultaneously $kc < hd$:

In this way we manage to define equality between ratios using only multiplication by natural numbers, even in the case of incommensurable magnitudes.

For a long time, this definition of a ratio was criticized by modern mathematicians, who, for reasons to be explained later,[47a] did not realize the need for such complexity. The idea was finally grasped by Weierstrass and Dedekind, who founded the modern theory of real numbers essentially by translating Euclid's definition into the language used nowadays. The translation,[48] into the terms used by Dedekind, is essentially this: If we define a *real number* as any possible "Euclidean ratio",[49] the Euclidean

[47a] See Section 11.4, especially page 350.

[48] Obviously, this translation into modern language led to significant modifications. In particular, the modern notion of the algebraic structure of the set of real numbers is new and comes from the

definition boils down to saying that a real number is uniquely determined by its behavior regarding every pair (h, k) of integers; that is, it is identified by the set of pairs for which cases A, B, or C obtain. The sets corresponding to cases A and C are called by Dedekind "contiguous classes of rational numbers", and are clearly sufficient to identify the ratio $a:b$, that is, the real number. The first works on the "modern" theory of real numbers date from 1872.[50]

On the subject of Euclid's proportions, Heath wrote:

> The greatness of the new theory itself needs no further argument when it is remembered that the definition of equal ratios in Eucl. V, Def. 5 corresponds exactly to the modern theory of irrationals due to Dedekind, and that it is word for word the same as Weierstrass's definition of equal [real] numbers.[51]

As other authors whom we will encounter have done in similar cases, Heath — one of the foremost modern historians of ancient science — regards it as the greatest glory of Greek mathematicians that they managed to anticipate modern theories. Here he seems almost to suggest that the "word for word" correspondence he notes may have to do with Euclid's ability to anticipate results that would come two thousand years later, rather than with Weierstrass's "word for word" use of Euclid's definition, which he knew well. (Let's not forget that the *Elements* were the textbook at the foundation of Weierstrass's and Dedekind's early mathematical education.)

It may seem that Euclid's definition of proportions, like the modern definition of real numbers that derives from it, is impossible to apply, since it requires the consideration of infinitely many integers. The question can be clarified by examining an application of Euclid's definition. Consider his proof that the ratio of two circles equals the ratio of the squares on their diameters.[52] The existence — in the sense of constructibility with ruler and

modern primacy of algebraic over geometric aspects. But these changes, though important, do not contradict the fundamental fact that the "modern" theory appeared with the recovery of Euclid's definition.

[49]Physicists and engineers know very well that even today a real number is just a ratio of homogeneous magnitudes (and so they know that in their formulas the arguments of functions such as sines and exponentials must be ratios of homogeneous magnitudes). This awareness seems sometimes to elude mathematics students.

[50]They are an article by Dedekind and one by Heine, both based on the ideas of their teacher, Weierstrass.

[51][Heath: HGM], vol. I, pp. 326–327.

[52]Euclid, *Elements*, XII, proposition 2. "The ratio of two circles" may sound odd to modern ears, but the notion of area arose precisely as the ratio between a given figure and another chosen as a unit of measurement. Euclid always talks about ratios between plane figures (or segments, or solids) rather than about areas.

compass — of the mathematical objects involved has already been established: that of circles is the content of the third postulate, and that of squares was proved earlier.[53] The definition of proportions is used to state a relationship between geometric objects already constructed, one whose validity can be demonstrated in a finite number of logical steps, as can be checked by reading Euclid's proof. Thus there is an important difference between Euclid's ratio between magnitudes and today's real numbers: whereas modern mathematicians have introduced axioms about the set of all real numbers and have often considered real numbers whose existence is proved thanks to these axioms and is not supported by constructive procedures, Euclid considers only ratios of constructible magnitudes.

2.6 Euclid and His Predecessors

According to a widespread opinion, the contents of the *Elements* had appeared before Euclid in similar treatises, since lost. This belief rests largely on the fact that most of the statements of theorems contained in the *Elements* had indeed been known before Euclid, and that many proofs had been accomplished, including complex ones.[54] Much effort has been expended, often fruitfully, on the task of tracing the origins of the material contained in the various books of the *Elements*. But from our point of view the main feature of Euclid's work is not the set of results presented, but the way in which these results connect together, forming infinitely extensible "networks" of theorems, drawn out from a small number of distinguished statements. To judge the originality of the *Elements*, therefore, one must ask whether a similar structure (without which one cannot extend the theory by doing "exercises": that is the whole point!) had been achieved prior to Euclid.

In the surviving fragments on pre-Euclidean mathematics there is no evidence for sets of postulates similar to Euclid's. The works of Plato and Aristotle, moreover, offer an explicit description of what the "principles" accepted by mathematicians as the initial assumptions of their science were like at the time. Plato writes that "those who work with geometry, arithmetic, and the like lay as 'hypotheses' evenness and oddness, figures, the three kinds of angles and similar things."[55] Aristotle, in a passage where he discusses the role of principles in the deductive sciences, makes

[53]Euclid, *Elements*, I, proposition 46.

[54]For example, Archytas gave a construction for two proportional means (which amounts to the extraction of a cube root), and Eudoxus proved the formulas for the volume of the pyramid and cone. Both proofs date from the first half of the fourth century.

[55]Plato, *Republic*, VI, 510c.

a distinction between the principles common to all sciences and those particular to each. As an example of the first type he mentions the assertion "Subtract equals from equals and equals remain", which appears in the *Elements* exactly as one of the "common notions". Immediately before that he had written: "Particular [principles] are 'The line is such-and-such', and likewise for straightness."[56]

There is an obvious difference between the type of "geometric principles" exemplified by Plato and Aristotle, which surely could not serve as the basis for proving theorems, and the postulates contained in the *Elements*.

As to the premises actually used in the demonstration of geometric theorems, several passages from Plato and Aristotle attest to a deductive method much more fluid in the choice of initial assumptions than that transmitted by the *Elements* and later works.

The logical unity of the *Elements*, or of a large portion of it, is clearly not due to chance; it is the result of conscious work on the part of the same mathematician to whom we owe the postulates. There is no reason to suppose that this unity is not an innovation due to Euclid, and a very important one at that.

2.7 An Application of the "Method of Exhaustion"

As an example of an application of what in modern times was named the "method of exhaustion" — that is, of Hellenistic mathematical analysis — we recall how Archimedes computed the area of a segment of parabola, in his *Quadrature of the parabola*. This is probably the simplest of the surviving proofs of Archimedes (and therefore the most popular), but it will suffice for giving an idea, if not of Archimedes' ability to solve difficult problems, at least of his level of rigor. Readers who dislike detailed mathematical arguments may proceed to the first full paragraph of page 52.

Let a parabola be given. If A and B are points on it, the part of the plane comprised between the segment AB and the arc of the parabola joining A and B is called the segment of parabola with base AB. The point C of the arc of parabola that lies farthest from the line AB is called the vertex of the segment of parabola.[57]

Archimedes' proof is based on a postulate, fundamental in nature and discussed at the beginning of the work, and on three technical lemmas.[58]

[56] Aristotle, *Analytica posteriora*, I, x, 76a:40.

[57] The vertex of a segment of parabola depends on the base AB, and should not be confused with the vertex of the parabola, which is usually a different point.

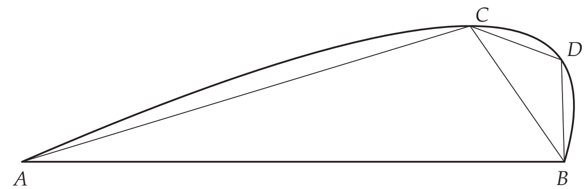
[58] Our exposition differs slightly from the original one, which contained more propositions.

Postulate. If two areas are unequal, there is some multiple of the difference that exceeds any previously fixed area.

Lemma 1. If C is the vertex of the segment of parabola of base AB, the area of the triangle ABC is more than half the area of the segment of parabola.

From the technical point of view, the fundamental ingredient in the proof lies in the following lemma, whose proof, together with that of the preceding lemma, is given in the Appendix.

Lemma 2. If C is the vertex of the segment of parabola of base AB and D is the vertex of the segment of parabola of base CB, the area of triangle CBD is one-eighth that of triangle ABC.



Archimedes' basic idea is to cover the segment of parabola with infinitely many triangles. He starts by inscribing the triangle ABC into the segment of parabola of base AB, thereby dividing the latter into three parts: the triangle ABC and two segments of parabola, in each of which we can inscribe another triangle following the same procedure (like triangle CBD in the figure). The procedure can of course be iterated, leading to ever smaller triangles. Let S be the area of the initial segment of parabola, A_0 the area of the triangle ABC inscribed in it, and $A_1, A_2, A_3, \ldots$ the total area of the triangles inscribed at each successive iteration. Since at each iteration the number of triangles doubles, whereas the area of each, by Lemma 2, becomes 8 times smaller, it is clear that $A_1 = \frac{1}{4}A_0$, $A_2 = \frac{1}{4}A_1$, and so on. At this point Archimedes proves another lemma:

Lemma 3. If $A_0, A_1, A_2, \ldots, A_n$ form a finite sequence of magnitudes, each of which is four times the next, we have

$$A_0 + A_1 + A_2 + \cdots + A_n + \tfrac{1}{3}A_n = \tfrac{4}{3}A_0. \qquad (*)$$

We will not reproduce the proof of this lemma; we merely note that it is a particular case of a property of geometric progressions that is very well known today.

Archimedes can now compute the area S of the segment of parabola by proving the following theorem:

Theorem. $S = \frac{4}{3}A_0$.

The theorem is proved by contradiction, by showing that S cannot be either more or less than $\frac{4}{3}A_0$. Suppose that $S > \frac{4}{3}A_0$, and call E the difference

$S - \frac{4}{3}A_0$, so that

$$S = A_0 + A_1 + A_2 + \cdots + A_n + \varepsilon_n = \tfrac{4}{3}A_0 + E,$$

where we have indicated with ε_n the area of the part of the segment of parabola not covered by triangles after n iterations. If n is large enough, the area ε_n, which at each step gets smaller by a factor greater than 2 (by Lemma 1), will end up smaller than E (by the postulate). Therefore

$$A_0 + A_1 + A_2 + \cdots + A_n > \tfrac{4}{3}A_0.$$

But this inequality is false, because it contradicts $(*)$. Thus we have excluded the case $S > \frac{4}{3}A_0$.

Suppose instead that $S < \frac{4}{3}A_0$. Using the postulate again, we see that if n is a sufficiently large integer the area $\frac{1}{3}A_n$ must be less than the difference $\frac{4}{3}A_0 - S$. Using $(*)$ we deduce that

$$\tfrac{4}{3}A_0 < A_0 + A_1 + A_2 + \cdots + A_n + \tfrac{4}{3}A_0 - S,$$

that is,

$$S < A_0 + A_1 + A_2 + \cdots + A_n.$$

This inequality, too, is clearly false, since the right-hand side represents the area of a portion of the segment of parabola of area S. This concludes the proof of the theorem.

We note (and it will be clearer to those who read the Appendix) that the proof depends crucially on the study of triangles, which don't appear at all in the formulation of the problem. They are used merely as a tool. This example makes it clear why Hellenistic mathematicians laid out with great care such simple theories as that of triangles, presented in the *Elements*: they were useful tools for tackling even problems whose original statement had no connection whatsoever with the auxiliary theory. Triangles were studied so that figures could be triangulated. We will encounter a similar use of circles as a tool in the study of planetary orbits.

Every real number different from zero has a multiple that is greater than an arbitrary fixed real number. In modern axiomatizations of the reals, this property is assumed true and is called the "Archimedean postulate". The postulate that Archimedes actually stated is different: the magnitudes that he (and Hellenistic mathematicians in general) considered in fact form a non-Archimedean set (in the modern terminology), in that the magnitude of a segment, though nonzero, has no multiple that exceeds the magnitude of a square. In the parlance of Hellenistic mathematicians, two magnitudes *have a ratio*, and are called *homogeneous*, if each has a multiple that

exceeds the other.[59] Archimedes explicitly postulates that the difference between any two inequivalent surfaces is homogeneous with (has a ratio with) any other surface.[60]

Archimedes' surviving writings may give the impression that the level of scientific works transmitted through late Antiquity and the Middle Ages was not as low as we claimed on page 8. In fact, the selection criteria we mentioned are confirmed even in this case, because some of Archimedes' works have reached us only through exceptionally lucky circumstances. In spite of their author's fame, some of his writings (such as the *Quadrature of the parabola*) apparently hung on for several centuries in a single copy, a codex prepared in Byzantium in the ninth century, at the initiative of Leo the Mathematician. This manuscript, now lost, found its way to the Norman court in Sicily in the twelfth century and thence to the hands of Frederick II, Holy Roman Emperor; after the battle of Benevento (1266) it ended up in the Vatican Library. It still existed in the fifteenth century, when copies were made in France and Italy, but there the trail ends.[61] Another manuscript, which contained different works and was probably given to the pope at the same time, is lost track of earlier, in the fourteenth century. From this second manuscript was derived a Latin version of the treatise *On floating bodies*. Our only other source for the works of Archimedes is the already mentioned palimpsest (page 8) discovered by Heiberg in 1906, subsequently lost, and recently found again.

If we had none of his works, our knowledge about Archimedes would be limited to remarks transmitted by authors such as Plutarch, Athenaeus, Vitruvius and Heron. We would be exactly in the same situation we are with respect to, say, Ctesibius: a scientist who, to judge from the same sources, appears no less interesting than Archimedes. Circumstances such as the preservation for six centuries of a codex owned successively by Byzantines, Normans, German emperors, Angevins, popes and Florentine humanists are hardly replicable. In how many other cases have we been less fortunate?

2.8 Trigonometry and Spherical Geometry

We conclude this chapter on Hellenistic mathematics with a brief mention of plane and spherical trigonometry. We make this choice not because of

[59] Euclid, *Elements*, V, definition 4.

[60] A more general version of the postulate, applying not only to surfaces but also to lines and solids, appears in Archimedes, *De sphaera et cylindro*, 11:16–20 (ed. Mugler, vol. I).

[61] See [Dijksterhuis: Archimedes], Chapter 2. For a full discussion of the transmission of Archimedes in the Middle Ages, see [Clagett: AMA].

the subject's intrinsic importance — Apollonius' theory of conics or the methods introduced in Archimedes' *On spirals* would be more nourishing fare — but because it affords the opportunity of illustrating a method used by many historians of science.

Until a few decades ago there was widespread agreement that "the Ancients" did not know trigonometry. In fact, the few results comprised in this elementary part of mathematics, such as the addition formulas, were developed quite early, and trigonometric tables were compiled for use in astronomy. The only difference between ancient and modern trigonometry is in the choice of the fundamental function, which was then the chord, rather than the sine. The two choices are clearly equivalent: one can pass from one function to the other using the obvious formula

$$\text{chord}\, a = 2 \sin \frac{a}{2},$$

since the sine of an angle is half the chord subtended by twice the angle.

It is not possible to find the chord associated to an arc of given length using the methods typical of geometric algebra; that is, one cannot compute the chord function using ruler and compass (this task being one of the possible formulations of the famous problem of squaring the circle).[62] That this impossibility did not block the development of trigonometry, but instead channeled it toward methods other than geometric algebra, such as numerical tables written in positional notation, shows that the use of ruler and compass was a matter of convenience rather than an intellectual prejudice.

In the fourth century A.D., trigonometry was imported into India together with astronomy, to which it had become an indispensable technical adjunct.[63] Indeed, at various times during the century, Alexandrian astronomers and mathematicians decided to emigrate to India, pressed by their ever more precarious situation in Alexandria.[64] It seems, for example, that the Paulisa who authored the Indian astronomical work *Paulisa siddhanta* was the astronomer Paulus, a refugee from Alexandria.

Indian mathematicians, having to use half-chords often, decided to pick the half-chord as the main variable. (They eventually even transferred to

[62]Obviously one can draw, with ruler alone, the chord corresponding to a given arc, but computing the chord function with ruler and compass would require constructing with these instruments the chord of an arc whose length is the same as that of a given segment. The construction of such an arc is effectively equivalent to the inverse operation of rectifying a circumference.

[63]It seems certain that the astronomical methods developed in the West were first introduced in India in the second century A.D., but these were Mesopotamian arithmetic methods used in Greek astrological texts; see [Neugebauer: HAMA], vol. I, p. 6. The use in Indian astronomy of geometric methods, which required trigonometric functions, started in the fourth century.

[64]Hypatia's end early in the next century (see page 15) shows that this was not at all a bad idea.

it the Sanskrit term for the chord or bowstring itself, *jiva*. The Arabs, instead of translating this term, transliterated it with consonants that could also be interpreted as *jaib*, meaning "bosom of dress, cavity"; this was subsequently translated into Latin as "sinus", with the same meaning.[65]) The novelty consisted of a trivial change in variables, which eliminated factors of 2 in some formulas but did not alter in any way the theorems of Hellenistic trigonometry; the latter were recovered intact on the other side of the vacuum represented by the Romans and the barbarian post-Roman kingdoms, through the eventful mediation of Indians and Arabs.

One historian of science wrote:

> The development of our system of notation for integers was one of the two most influential contributions of India to the history of mathematics. The other was the introduction of an equivalent of the sine function in trigonometry to replace the Greek tables of chords.[66]

And another:

> Since he did not know the ordinary trigonometric functions (sine, cosine, etc.), Ptolemy employs to this end the so-called calculus of chords, based on the properties of the chord considered as a function of the arc that subtends it. It would fall on Arabic mathematicians (and this was one of their most notable feats . . .) to bring to light the unquestionable advantages obtained by replacing this calculus with true trigonometry in the modern sense of the word.[67]

We see that the opinions of the two scholars diverge: the "feat" of dividing the chord by two is attributed to the Arabs by one and to the Indians by the other (who is better informed). On one issue, however, there is total agreement: real trigonometry (at least "in the modern sense of the word") only appeared when instead of using the chord people started using half the chord!

This example, however banal, is instructive, because it illustrates vividly an attitude that, although declining in connection with trigonometry, is alive and well in many other cases, as we shall see. It consists in "confirming" the originality of modern science using the following circular reasoning. It is implicitly assumed that modern science is of higher quality than ancient science — indeed, that it is the only true science, which "the Ancients" may have "foreshadowed" at best. As a result, whatever led to the current formulation, even if it's just a renaming of a concept or a

[65][Rosenfeld], p. 11.

[66][Boyer], p. 237 (1st ed.), p. 215 (2nd ed.).

[67][Geymonat], vol. I, p. 354.

division by two, is regarded as the crucial step in the development of the science in question, perhaps with the qualification "in the modern sense of the word". Armed with that characterization, one concludes, sure enough, that "the Ancients" had not yet developed that science, and so convinces oneself of the correctness of the initial assumption.

One sees this attitude applied to the method of exhaustion, which is usually presented as a "precursor" of modern methods of passage to the limit.[68] Readers of Section 2.7 will have noticed that Archimedes does not employ limits only in the sense that he fails to use a *word* that matches ours exactly; a proof in modern analysis needs only to have "limit" replaced by its definition to become equivalent to his in every way.

Returning to Hellenistic mathematicians, it should be stressed that they also developed spherical geometry and trigonometry, subjects for which our main sources of information are the *Sphaerica* of Theodosius (who straddled the second and first centuries B.C.), the homonymous work of Menelaus (first century A.D.), and Ptolemy's *Almagest* (page 79), of the second century A.D.[69] The mathematics developed in these treatises, although of course instrumental to astronomy and mathematical geography, has great theoretical interest. It includes not only formulas of spherical trigonometry (which can be useful to astronomers or geographers), but also, particularly in the work of Menelaus, a theoretical development of intrinsic spherical geometry, constructed in analogy with the plane geometry of Euclid's *Elements*.[70] In particular, the theory of spherical triangles (subsets of the surface of the sphere bounded by three arcs of great circle) is developed in analogy with the theory of triangles contained in Book I of the *Elements*, based on postulates of spherical geometry — some closely analogous to those of Euclid's plane geometry and some very different. As we shall see, those investigations would become important again many centuries later.

[68]For instance, the *Encyclopaedia Britannica* says: "Although it was a forerunner of the integral calculus, the method of exhaustion used neither limits nor arguments about infinitesimal quantities" (15th edition, Micropaedia, sub "exhaustion, method of").

[69]There are hints, however, that spherical geometry arose even before Theodosius, during the heyday of astronomy and mathematical geography, its two most obvious applications. See note 27 on page 273.

[70]Theodosius' work, by contrast, uses mostly stereometric methods; that is, theorems about spherical geometry are demonstrated as theorems of solid geometry, using the three-dimensional space where the surface is immersed. But even Theodosius sometimes uses methods of intrinsic spherical geometry.

3
Other Hellenistic Scientific Theories

3.1 Optics, Scenography and Catoptrics

One of the first Hellenistic scientific theories was optics (ὀπτική), that is, the "science of sight", and the first known treatise on the subject is Euclid's *Optics*. In this work, Euclid deals with optics stricto sensu; according to the nomenclature of the time (which we follow in this section), the term included all that has to do with direct sight, but did not include reflection (which was the object of the science called *catoptrics*[1]) or refraction.

The fundamental entities of the theory are *visual rays* (ὄψεις), finite in number and extending in a straight line from the eye. The assumptions of the theory establish simple correspondences between visual perceptions and the beams of visual rays that intercept the objects seen. In particular, they reduce the apparent size of an object to its angular dimension.

The emergence of optics illustrates well where the novelty of the scientific method lies. The term "visual ray" had been in use for a long time[2] and the law that visual rays propagate in a straight line was well known.[3] Plato, recalling that an object seems to vary in size depending on how far it is from the viewer, states that no store should be placed on apparent sizes: they are illusions, while true sizes are those that, being measurable,

[1]Euclid's *Optics* and a pseudo-Euclidean *Catoptrics* appear in [Euclid: OO], vol. VII.

[2]Visual rays should not be confused with light rays. The relative roles of visual rays and light, both of which are necessary for seeing, is clarified in many sources, such as Plato, *Republic*, VI, 507c–508a. Then as now, in order for an object to be seen two conditions must be satisfied: the object must be lit and we must be looking at it.

[3]Plato, *Parmenides*, 137e:3–4, implicitly uses sight to define straightness.

can be studied by science.[4] On the other hand, in Euclid's *Optics*, a chain of theorems based on a few assumptions is enough to show that visual perceptions, too, can be analyzed using the scientific method. In this specific case things are particularly simple, since in terms of internal structure the theory can be considered a part of geometry.[5] What changes radically are the correspondence rules: visual rays, which within the theory can be identified with segments, no longer correspond to lines drawn with the ruler, but to possible directions of sight.

Optics had an important role as a bridge between geometry and sciences that relate to vision.

First of all, it was an important preliminary tool for astronomy. In the *Arenarius*, Archimedes describes a measurement of the apparent size of the sun,[6] not at all a trivial task if a reasonable degree of precision is desired. Optics was also a necessary ingredient in the design of all visual instruments, such as those used in topographical measurements and the astrolabe.

The "science of sight" also had important applications to the figurative arts. One connection between optics and painting was *scenography*, originally the technique of creating theater stage sets (which were apparently introduced by Sophocles[7]). Geminus defines scenography as the part of optics required for drawing views of buildings;[8] Vitruvius explicitly mentions the use of the geometry of visual rays to give a three-dimensional appearance to buildings painted on theater backdrops.[9]

Alas, no works on scenography and very little of Hellenistic painting has survived, but the ties between optics, scenography and painting can be partly recovered from literary testimonies and Roman-era frescoes and mosaics, which were invariably of Hellenistic inspiration.

Although Vitruvius says Anaxagoras and Democritus were the first to write works on scenography based on the geometry of visual rays,[10] the earliest documented paintings that benefited from the new techniques for rendering three-dimensionality date from Alexander's reign: in particular, Aetion's *The marriage of Alexander and Roxane* (a description of which we owe to Lucian[11]) and a painting by Apelles showing Alexander holding a

[4]Plato, *Republic*, X, 602c–603a.

[5]Aristotle had already made remarks relevant to this: *Analytica posteriora*, I, xiii, 78b:37; *Physica*, II, ii, 194a:7–12.

[6]Archimedes, *Arenarius*, 137–140 (ed. Mugler, vol. II).

[7]Aristotle, *Poetica*, iv, 1449a:18–19.

[8]Geminus, in [Heron: OO], vol. IV, 106:15–16. Several passages of Geminus and Anatolius were published with Heron's works because they appeared in the Byzantine collection where *Heron's Definitions* (on which more in note 226, page 322) were preserved.

[9]Vitruvius, *De architectura*, VII, preface §11.

[10]Vitruvius, loc. cit. = [FV], II, 14:35 – 15:5, Anaxagoras A39.

lightning bolt in the temple of Diana. Pliny, stressing that the bolt seemed to come out of the painting,[12] is probably referring to one of the first successful attempts to represent the third dimension. He also states that the great Apelles was surpassed in his ability to render the distance of objects by the less famous Asclepiodorus;[13] this is yet another indication that toward the end of the fourth century new techniques of perspective were being perfected.[14]

The Pompeii frescos clearly reveal the use of effective geometric rules for three-dimensional rendering — not only in the depiction of buildings (in particular in the so-called "second style") but in optical illusions that would only be taken up again in the *trompe l'œil* of the baroque period.[15] The notion of the vanishing point, too, is well attested, *pace* the many who have denied that perspective was known in Antiquity.[16] Lucretius observes that a long portico appears like a cone toward whose vertex the ceiling, the floor and the side walls converge.[17] Sextus Empiricus and Geminus give the same example,[18] while Vitruvius writes:

> Likewise scenography is the sketching of the front and sides that recede and the correspondence of all lines toward the center of the compass.[19]

The connection between scenography and optics, mentioned by Geminus, is confirmed by looking at Euclid's treatise: already one of the first propositions asserts that parallel lines are not seen as parallel.[20]

The debate over whether perspective was known in Antiquity has been going on for centuries. It goes right back to the Renaissance painters[21] and is still alive. If by perspective we mean primarily the systematic use of central perspective as codified in the fifteenth century, its existence in

[11]Lucian of Samosata, *Herodotus or Aetion*, chap. 4–6 (in, e.g., Loeb Classical Library, vol. 430).

[12]Pliny, *Naturalis historia*, XXXV §92.

[13]Pliny, *Naturalis historia*, XXXV §80.

[14]For Hellenistic painting see, for example, [Bianchi Bandinelli] or [Robertson], vol. I (where the discussion of Hellenistic perspective is on pp. 587–588).

[15]Some of the chief examples of these effects can be found in the villa at Oplontis, where excavations started in 1964 have not yet been concluded.

[16]See [Veltman] for a bibliography on the subject.

[17]Lucretius, *De rerum natura*, IV:426–431.

[18]Sextus Empiricus, *Adversus logicos* I (= *Adv. dogmaticos* I = *Adv. math.* VII), §244; Geminus, in [Heron: OO], vol. IV, 102:4–8.

[19]"Item scaenographia est frontis et laterum abscedentium adumbratio ad circinique centrum omnium linearum responsus" (Vitruvius, *De architectura*, I, ii §2).

[20]Euclid, *Optics*, proposition 6.

[21]Piero della Francesca starts his *De prospectiva pingendi* underscoring the need to recover this ancient technique and listing ancient painters who had used it, while some of his contemporaries held instead that the Ancients did not know perspective. Note that the word "perspective" comes from the Latin "perspectiva", itself a translation of the Greek ὀπτική.

Antiquity is contested by many on the grounds that the Pompeii frescoes generally seem to employ what is now called herringbone perspective (having mutually inconsistent vanishing points along an axis). There is dispute even about whether Euclid's *Optics* contains rules that have immediate application to the projection techniques used in the Renaissance. The work certainly contains the prerequisites of a theory of perspective, but, being about optics and not scenography — that is, dealing with our sight of objects rather than with the preparation of plane drawings that generate particular visual effects — it does not develop this line of applications. Yet in spite of the complete loss of all treatises on scenography, explicit applications of optics to central perspective are mentioned in surviving ancient works,[22] and are in fact evident in some frescos, such as the one discovered in 1961 in the "Room of the Masks" in the House of Augustus on the Palatine Hill in Rome, which dates from around 30 B.C. (see Figure 3.1).[23] Moreover central perspective is only one of the possible applications of ancient optics; it is designed to optimize the visual impression a painting makes when seen from a particular point, but it is not the best technique in every case.[24] For instance, it is not well suited to big wall paintings; even modern artists have often ignored central perspective in large murals, to avoid the glaring deformations that would appear in peripheral areas to those not looking from the unique "correct" viewpoint.

Ancient optics and scenography had several other uses in the figurative arts. Proclus writes:

> Optics ... uses visual rays and the angles they form; it is divided into optics proper, which explains the appearance of objects at a distance, including the convergence of parallel lines, ... catoptrics, ... and scenography, which shows how, in images, what is seen might be made not to appear out of proportion or deformed, according to the distance and the heights of the things drawn.[25]

[22]Ptolemy (*Geography*, VII, vi–vii) provides instructions for drawing in perspective a world globe with parallels and meridians (compare [Andersen]). An even more interesting example, noted by A. Jones in a conference cited in [Knorr: PLP], is a passage in Pappus' commentary on Euclid's *Optics* that deals with the vanishing point, identifying the point through which one should draw the lines of a plane in order that they should appear parallel to a given line from a given point of view (Pappus, *Collectio*, VI, proposition 51). Elsewhere in his commentary Pappus makes remarks on linear perspective that are absent from Euclid's *Optics* itself.

[23]This fresco is examined, in the context of a history of perspective, in [Ghione, Catastini].

[24]This was rightly stressed in [Panofsky]. In the same essay the author argued that Euclidean optics led ancient painters to the use of a perspective distinct from, but not inferior to, the one used in the Renaissance. Although the "angular perspective" he postulates is unconvincing, his ideas may help explain features of certain works. It should be kept in mind, however, that he was unaware of works such as the fresco in the House of Masks.

[25][Proclus/Friedlein], 40:10–21.

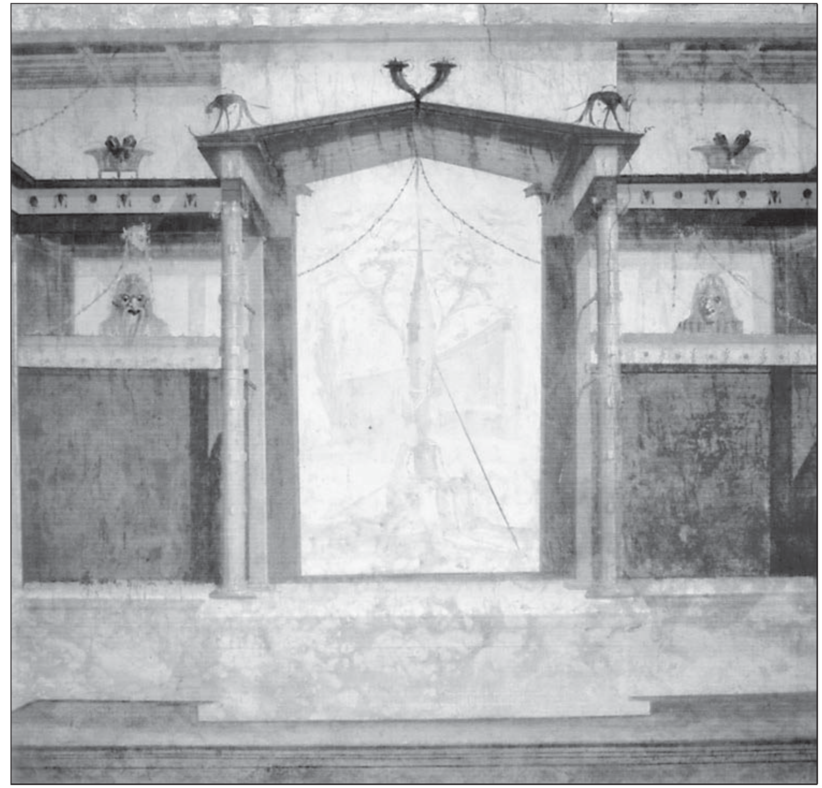


FIGURE 3.1. Fresco from the Room of the Masks, House of Augustus (*Domus aurea*), Palatine Hill. Courtesy of Ministero per i Beni e le Attività Culturali, Soprintendenza Archeologica di Roma.

We have from Geminus a complete description of scenography, as a technique useful to painters who wished to create the illusion of three-dimensionality, to sculptors (especially the makers of very large statues), and to architects wishing to obtain certain effects from a particular point of view.[26] Art historians are well aware that such techniques were indeed used in many cases by Greek architects and sculptors, and that they were rediscovered only in the seventeenth century.

Catoptrics, based on the well known law of reflection, was used to design mirrors of various types, including "burning mirrors", which were parabolic mirrors capable of concentrating the parallel rays of the sun into a point (called, for this reason, the focus of the parabola, this being

[26]Geminus, in [Heron: OO], vol. IV, 106:14 – 108:9. The reference to colossal statues clearly has in mind the need to plan them so they appear well-proportioned to those standing below.

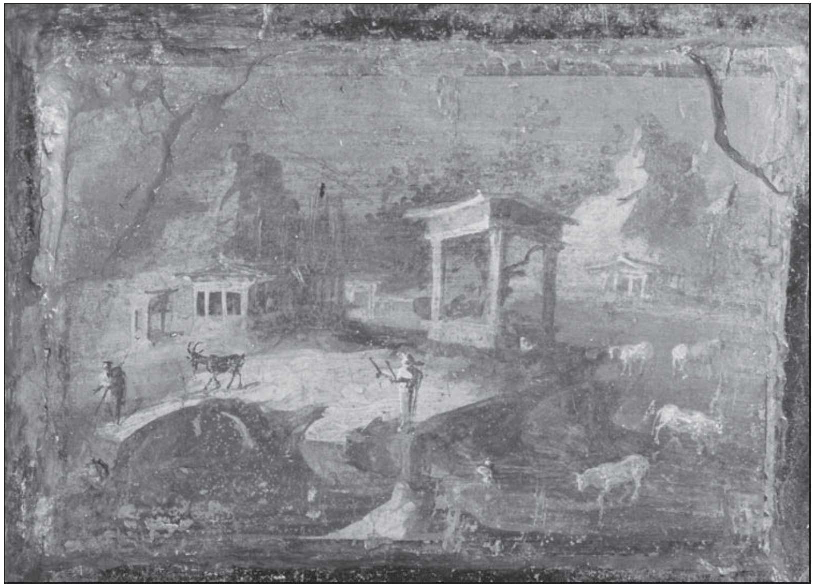


FIGURE 3.2. Painting from Pompeii, now at the Museo Archeologico Nazionale, Naples (#4344). Courtesy of Ministero per i Beni e le Attività Culturali, Soprintendenza Archeologica delle province di Napoli e Caserta.

the Latin word for hearth). A widespread tradition, first documented in Galen[27] and later in various Byzantine sources, associates burning mirrors with Archimedes, who supposedly built them during the siege of Syracuse to set fire to the Roman ships. But this is unlikely for several reasons: the mirrors would have to be very large; they would not be very effective weapons; and there is no mention of them in the accounts of the siege of Syracuse left by Polybius, Livy and Plutarch, all of whom wrote about war machines built by Archimedes. This has led many to implicitly reject as legendary not only the use of mirrors to burn ships, but burning mirrors themselves; yet there is nothing legendary about the latter. Diocles,[28] possibly Apollonius of Perga,[29] and before them Dositheus[30] in mid-third

[27] Galen, *De temperamentis*, III, ii.

[28] Diocles' *On burning mirrors*, from the second century B.C., survived in Arabic and was found and edited by Toomer; see [Diocles/Toomer].

[29] Apollonius' work on burning mirrors is mentioned in the *Fragmentum mathematicum Bobiense* ([MGM], 88:8–12). But Toomer, based on the comparison between the citations and Diocles' text, thinks that the work in question is that of Diocles, and was erroneously attributed to Apollonius by the author of the fragment; see [Diocles/Toomer], p. 20.

[30] As we know from Diocles (*On burning mirrors*, 34, ed. Toomer).

century B.C. had studied these applications. Unfortunately we know very little about the actual use of such devices.

The tradition that associates burning mirrors with Archimedes may be founded on his works. We know from Diocles that Archimedes' main correspondent in Alexandria, Dositheus,[31] studied parabolic mirrors, having obtained only partial results;[32] thus the problem was very likely brought to Archimedes' attention. The existence of an Archimedean *Catoptrics* is attested by Apuleius[33] and by Theon.[34] It is reasonable to think that in this book Archimedes, who wrote theoretical works on parabolas and paraboloids, would mention the caustic properties of parabolic mirrors — in fact Apuleius, listing some of the book's contents, explicitly mentions concave mirrors able to concentrate the sun's rays on one point.[35] One can see how, combining such writings of Archimedes with the recollection of his contribution to the defense of Syracuse, which included the construction of ballistic weapons capable of setting fire to ships from far away, the traditional belief may have arisen.

The most interesting surviving theoretical result about reflection is probably a theorem in Heron's *Catoptrics* saying that a light ray that leaves a point A and reaches a point B after reflection in a plane mirror has equal angles of incidence and reflection because it follows the shortest path from A to B that touches the mirror.[36] The simple proof is based on the observation that the path of the ray does not change in length if the leg from A to the incidence point is replaced by its mirror image. See Figure 3.3. Thus the reflection law can be deduced from a minimization principle — the oldest such principle known. Archimedes had already deduced the law of reflection from the principle of reversibility of optical paths.[37]

The earliest extant work that includes a systematic account of refractive

[31] Archimedes addressed to him his works *On the sphere and cylinder*, *On conoids and spheroids*, and *On spirals*.

[32] Diocles says that Dositheus has solved only "practically" (or something like that: the Arabic text is unclear) the problem of building a mirror that would make the sun's rays converge to a point ([Diocles/Toomer], p. 34).

[33] Apuleius, *Apologia*, xvi.

[34] Theon, *Commentary on the Almagest* (on I, iii) = [Theon/Rome], II, 347:5 – 348:1.

[35] Apuleius, loc. cit.

[36] *De speculis*, iv = [Heron: OO], II.1, 324–328. This work, preserved anonymously in Latin and reproduced in [Heron: OO], vol. II.1, pp. 301–365, is believed to be a translation of Heron's *Catoptrics*. The proof is also reported by Olympiodorus (sixth century A.D.), *In Aristotelis Meteora commentaria* III, in [CAG], XII.2, 212:5 – 213:20 = [Heron: OO], II.1, 368–372. The *De speculis* and Olympiodorus agree in stating the result as *economy implies equal angles* and in giving a proof that is recognizably the one reproduced here, but somewhat garbled in each case (the *De speculis* in fact details the proof of the opposite implication).

[37] Archimedes' proof is reported in a scholium to the pseudo-Euclidean *Catoptrics*: [Euclid: OO], vol. VII, p. 348, sch. 7.

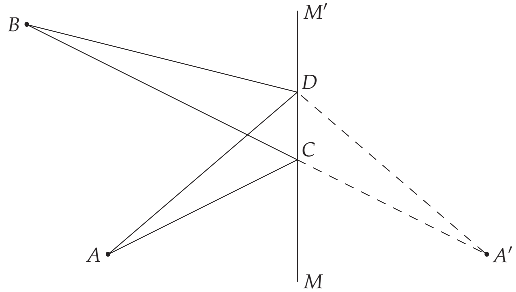


FIGURE 3.3. The shortest path from A to B (via the mirror MM') must touch the mirror at a point C such that $\angle ACM = \angle BCM'$. This is because only the path ACB, upon reflection of the first leg, gives a straight line $A'CB$. Another path such as ADB gives a broken line $A'DB$, necessarily longer than $A'CB$.

phenomena is Ptolemy's *Optics*.[38] But studies of refraction started much earlier; even Ptolemy's observation that a heavenly body is seen elsewhere than in the true direction where it lies because of atmospheric refraction seems to go back to Hellenistic times.[39]

Ptolemy's *Optics* also tabulates the refraction angles corresponding to various incidence angles for air-water, air-glass, and water-glass interfaces.[40] Apparently Ptolemy thought that the refraction angle varies with the incidence angle according to what we call a quadratic function. He does not state the functional dependence explicitly, but the values he gives show constant second differences for each interface (see Figure 3.4 for the water-air case). His values match reality with remarkable accuracy in the central range, but are far off at the extremes, especially when the incidence angle is 80°. Clearly these values come from two procedures: careful experimentation on the one hand, and subsequent extrapolation (or "correction") on the other, based on the *a priori* belief that the second differences should be constant. The two procedures represent such disparate attitudes toward experimental data that it is plausible to attribute them to different people, possibly from distinct periods.

[38]All we have of this work is an incomplete and often obscure Latin translation made in the twelfth century from an Arabic version. The Latin translation was first published in [Ptolemy/Govi]; the critical edition is [Ptolemy/Lejeune].

[39]This is likely, because the observation (Ptolemy, *Optics*, V §§23–30, 237:20 – 242:7, ed. Lejeune) also appears in authors from the imperial period and from late Antiquity whose sources seem to be independent of Ptolemy: namely Cleomedes, *Caelestia*, II §6, 82:174 – 83:177 (ed. Todd), and Sextus Empiricus, *Adversus astrologos* (= *Adv. mathematicos*, V), §82. Other mentions of refraction appear in works from the early imperial period, such as Seneca, *Naturales quaestiones*, I, vi §5.

[40]Ptolemy, *Optics*, V §§7–21 = 227:1 – 237:7 (ed. Lejeune).

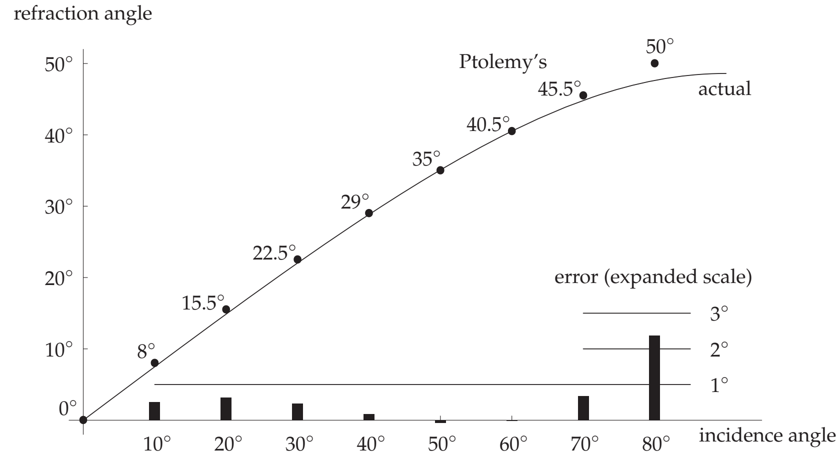


FIGURE 3.4. Angle of refraction versus angle of incidence for air and water. The Ptolemaic values for the refraction angle (marked by dots) have constant second differences, that is, the differences between consecutive values form an arithmetic progression: 8°, 7.5°, 7°, 6.5°, 6°, 5.5°, 5°, 4.5°.

In Book V Ptolemy examines refraction between two media separated by a plane or cylindrical surface. At that point the text stops and the translator adds that the rest of the work could not be found. What did the missing part of Book V contain? One might hope to learn this from the statement of contents and purpose usually present at the beginning of such works, but unfortunately Book I is missing too.

3.2 Geodesy and Mathematical Geography

Herodotus attributes to the Egyptians the introduction of *geometry*, in the original sense of measuring of the land, and specifies that it arose from the need to estimate, for taxation purposes, by how much plots of land were eroded by the Nile.[41] When Greek geometry embarked on its spectacular course of development, its concrete applications, such as to surveying and topography, were reclassified under the rubric of *geodesy*.[42] Unfortunately there is meager direct documentation about the evolution of these techniques from the empirical stage, common to many ancient civilizations, to Hellenistic science-based surveying and cartography.[43]

[41]Herodotus, *Histories*, II §109.

[42]See, for example, Aristotle, *Metaphysica*, III, ii, 997b:26–28, where geodesy is distinguished from geometry by its concrete nature.

[43]An important, if late, source on geodesy is the *Dioptra* of Heron. See [Dilke] for a synthesis of the available information.

The basic notion of triangulation — the graphical determination of the distance to an inaccessible feature by comparing the direction of the lines of sight from two points lying a known distance apart — is very old. It was present in Hellenic mathematics from the beginning.[44] But the transformation of this idea into effective surveying techniques had to await the creation of instruments for viewing from a distance and the development of trigonometry. The first documented use of trigonometric methods goes back to the surviving astronomical work of Aristarchus of Samos, in the first half of the third century B.C.; his calculations of the distance to the sun and to the moon are clearly bold extensions of topographic triangulation methods to an astronomical scale.[45]

Geminus, in the first century B.C., describes geodesy by listing the tasks involved in determining distances and differences in height through the use of instruments such as rulers, plumb lines, squares and dioptras for looking through.[46] Vitruvius mentions only the dioptra as an instrument used in measuring differences in height, and another Greek instrument having the same purpose: the chorobate (a water-filled level).[47] We will return later to the dioptra described by Heron.

Town planning affords some indirect evidence about the development of surveying techniques. Greek town planning goes back to Hippodamus of Miletus (fifth century B.C.), but it was probably in early Hellenistic times that the establishment of many new and large cities with their infrastructure stimulated the development of effective surveying instruments. Such instruments would also have been needed for planning works such as the citadel of Pergamum, which required hill terracing in addition to building construction.

Chorography, whose purpose according to Polybius is to determine the location of sites and the distances between them,[48] seems to be a middle step between the techniques used in town planning and in mathematical geography.

The difference between Hellenistic mathematical geography and the purely descriptive geographical works of classical Greece, Rome, and the

[44] Compare the Proclus passage mentioned in Chapter 2, footnote 9.

[45] Aristarchus of Samos, *On the sizes and distances of the sun and moon* = [Heath: Aristarchus], Appendix. Aristarchus uses trigonometric methods in the sense that he computes the ratios between the sides of a triangle whose angles are known. Of course his values are not exact, nor can he use tables of approximate values of trigonometric functions (which did not exist at the time), but he determines small intervals which he can show contain the ratios that interest him.

[46] This fragment from Geminus appears in [Heron: OO], vol. IV, 100:4 – 102:8.

[47] Vitruvius, *De architectura*, VIII, v §§2–3.

[48] Cited in Strabo, *Geography*, X, iii §5. It may be significant that at the beginning of his work Strabo compares geographers to architects who plan buildings or cities (*Geography*, I, i §13). There is no direct evidence for topographical maps.

Middle Ages is a good illustration of the difference between scientific and prescientific societies.

We know Hellenistic mathematical geography through a single work, Ptolemy's *Geography*, but that is enough to show that it was as scientific as today's.[49] It is a typical scientific theory with correspondence rules, whereby each spot on the surface of the earth corresponds in the model to a point on a spherical surface, identified by two spherical coordinates: latitude and longitude. Ptolemy is also familiar with cartography; he knows several projections (including modified conic projections, whose mathematical properties he uses) in order to represent the earth on plane charts, preserving all the information present in a spherical representation. He records the latitude and longitude of about eight thousand places, from Ireland to Southeastern Asia.

Mathematical geography, too, arose early in the Hellenistic period. The quantitative description of the whole known world becomes an acutely felt need following the sudden expansion of the Greek world due to Alexander's conquests. Already around 300 B.C., Dicaearchus, a student of Aristotle, had taken the first step toward the creation of mathematical geography by identifying a parallel of latitude, that is, by selecting a series of locations all having the same latitude, from Gibraltar to Persia.[50]

Eratosthenes of Cyrene drew the first scientific map of the known world, going from Gibraltar to India and from Somalia to the North polar circle. His work already relied on the spherical coordinates we use.[51] The latitude of a place — the Greek word was κλῖμα, which originally meant "inclination" and later gave our "climate" — is easy to fix, say by measuring with a sundial the angle the sun rays make with the vertical at noon, on the day of the solstice. Another way is to obtain it from the ratio between the duration of day and night at the solstice.[52] Fixing longitude has always been much harder, until the advent of chronometers and radio, and we don't know what method Eratosthenes might have employed. Possibly it was the one mentioned by Ptolemy at the beginning of his work, whereby one finds the difference in longitude between two places by determining the difference in latitude and estimating the angle that the line between the two makes with the meridian.[53] In the case of cities linked by a sea

[49] In fact, "modern" mathematical geography is none other than Ptolemy's, recovered by Renaissance scholars.

[50] The attribution of the introduction of the first parallel to Dicaearchus is based on a passage of Agathemerus (*Geographiae informatio*, proem, 5 = [Wehrli], vol. I, fr. 110).

[51] The spherical shape of the earth was known at least as early as Parmenides, in the first half of the fifth century.

[52] Compare [Szabó, Maula], part II.

[53] Ptolemy, *Geography*, I, iii.

lane, this information would have been known approximately to seafarers plying the route.

Eratosthenes' most famous result was the measurement of the earth's meridian.[54] Earlier numbers, reported by Aristotle with no mention of the method by which they were obtained,[55] had been estimates rather than measurements. The admiration earned by Eratosthenes' feat was so widespread that Pliny, centuries later, could still hear its echoes.[56]

Eratosthenes' method, as given by Cleomedes (and as expounded in numerous textbooks and works of scientific popularization), is the following.

It was known that Syene (today's Aswan) is located almost on the tropic line: during the summer solstice, the sun passes almost exactly overhead at noon. Therefore, if someone in Alexandria at the same time measures with a sundial the angle made by the sun's rays with the vertical, he has the angle formed by the vertical lines through the two cities. If she also knows the distance between them as the crow flies, she can divide it by the angle to deduce the distance corresponding to one degree of great circle. The difficulty in knowing in Alexandria when it was noon in Syene is overcome by assuming that Syene is directly south of Alexandria, so that noon occurs simultaneously in both places.

We will return to the technical details in Section 10.2. For now we make only a methodological observation.

Today Eratosthenes' method seems almost banal to many people who can easily explain it with the help of a drawing. Yet it is inaccessible to prescientific civilizations, and *in all of Antiquity not a single Latin author succeeded in stating it coherently*. The difficulty lies not in the geometric reasoning, in itself very simple, but in understanding that by reasoning about a drawing one can derive conclusions about the whole earth. Someone who goes back and forth in thought between a drawing and the world is using, most often unconsciously, correspondence rules — precisely what we have singled out as an essential characteristic of scientific method. Indeed, only by making explicit all the underlying assumptions (which in the case of Eratosthenes were those of optics and of geometry, the roundness of the earth and the smallness of the earth compared with the distance to the sun) can one create a theoretical model that, being approximately applicable to the earth, is also amenable to depiction by a drawing and so furnishes a logical bridge between the two.

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[54] This was described by Eratosthenes in his *On the measurement of the earth*, which is lost; we know about his method chiefly through Cleomedes, *Caelestia*, I §7, 35:48 – 37:110 (ed. Todd). We will come back to Cleomedes' account in Section 10.2.

[55] Aristotle, *De caelo*, II, xiv, 298a. Probably the method involved comparing the elevation of stars at different places.

[56] Pliny, *Naturalis historia*, II §§247–248.

Eratosthenes' method is a brilliant example of the power of scientific method: by going back and forth between the real world and the model, he gained information about the unknown regions of the earth, which no Ancient had ever seen.

In the second century B.C., mathematical geography progressed thanks above all to Hipparchus of Nicaea. It was he who, in critiquing the method of his predecessors, had the idea of finding differences in longitude using astronomical methods, by measuring differences in local time for the same lunar eclipse.[57]

The study of geography was taken up again in the imperial age, in close connection with astronomy and spherical geometry, by Marinus of Tyre, whom we know only through Ptolemy's criticism, and by Ptolemy himself (both second century A.D.). But whereas Eratosthenes had found one degree of meridian to be worth 700 stadia, a quite accurate number also accepted by Hipparchus a century later,[58] Marinus and Ptolemy adopted instead the value 500 stadia.[59] Although we do not know why the world shrank like this in the imperial age, an error of this magnitude must mean that either the values of certain key geographic data current in Ptolemy's time were much less accurate than they had been 400 years earlier, or that such data were misunderstood or misapplied. Either possibility should not surprise us, given that Ptolemy was separated from the golden age by centuries during which there had been no continuity in the transmission of information.[60] Now we must ask ourselves how it came about that Marinus and Ptolemy, who knew the method used by Eratosthenes and

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[57] Strabo, *Geography*, I, i §12. Strabo mentions also solar eclipses, obviously by an oversight, which nonetheless has propagated down the centuries (see, for example, the *Encyclopaedia Britannica*, 15th edition, Micropaedia, sub "Hipparchus").

[58] Strabo, *Geography*, II, v §7.

[59] Apparently this was not just a matter of inconsistent units: Ptolemy really did believe the meridian was shorter than formerly thought. That the shorter value had already been adopted by Marinus is mentioned in Ptolemy, *Geography*, I, xi. Using Ptolemy's data, a person setting out to travel westward along the latitude of (say) Palos, Spain, would expect to cover 17,000 km ($\frac{5}{7}$ of the actual value of 24,000 km) before returning home. If his goal were to reach Asia by traveling due west from Spain, he would estimate the length of the voyage by subtracting from 17,000 km the breadth of Eurasia (about 10,000 km). It turns out that Ptolemy's error did not affect the size of the known continents, which he reports with reasonable accuracy; thus the calculated difference (17,000 − 10,000 = 7,000 km) would be about half the true value (24,000 − 10,000 = 14,000 km). This helps explain why Columbus grossly underestimated the length of a westward route to Asia.

[60] Studies at Alexandria were tragically interrupted by the persecution unleashed by Euergetes II in 145 B.C. (page 11). The Library survived, and became the main element of continuity between the golden era and the revival of the imperial age. But the scarcity of intellectuals after the persecution was such that the position of the head of the Library fell to a certain Cida "from the corps of lancers" (ἐκ τῶν λογχοφόρων), as we know from a papyrus (P. Oxy. 1241, II, 16). It is easy to see how this situation led in the imperial age to that passive dependence on written authorities that became even more acute later on and that is sometimes backdated to the golden period of Alexandrian science, through the conflation of two profoundly different cultural climates.

had access to the same information about Syene, did not try to repeat the simple measurement of the inclination of the sun. We might also ask why Columbus did not repeat the measurement himself, instead of traveling throughout Europe looking for information about the size of the earth in libraries. Or yet why neither Galileo nor any of his contemporaries did it. Clearly Eratosthenes' method hides a further difficulty, which escapes those who think it trivial. We will come back to this in Section 10.2.

3.3 Mechanics

We alluded in Section 1.5 to the main features of Aristotelian mechanics. By contrast, the mechanics (literally, "science of machines") we encounter in the first Hellenistic treatise we possess on the subject, Archimedes' *On the equilibrium of plane figures*, already has the characteristic structure of a scientific theory in our sense. That work deals with two kindred problems: the law of levers and the location of barycenters of plane figures.[61]

Archimedes' interest in the theory of levers is clearly aimed at the study of machines and in particular at the calculation of their mechanical advantage. Unfortunately very little remains of contemporary theoretical writings on this subject; Archimedes' other treatises have perished.[62] But we can reconstruct some features of third century B.C. mechanics by combining information from three sources: the one surviving Archimedean book; documents — particularly works of military technology — that mention machines actually built; and treatises written centuries later, above all Pappus' *Collection* and Heron's works. Among these the most useful is the *Mechanics*,[63] which describes the five *simple machines* (the winch, the lever, the pulley, the wedge/ramp and the screw) as well as a number of composite machines designed for various uses. The pseudo-Aristotelian *Mechanics*, which share many features with Heron's *Mechanics*, also has

[61]Mach's criticism that Archimedes deduced the law of levers from inadequate symmetry considerations (in [Mach], Sections I.3 and I.5) reflects a lack of understanding of the function of the Archimedean postulates. Mach considers only the first two postulates, whereas in deducing the law of the lever Archimedes makes essential use of the sixth. This is emphasized by O. Toeplitz, W. Stein and E. J. Dijksterhuis, who showed how much subtler the Archimedean analysis was than Mach's (see, for example, [Dijksterhuis: Archimedes], pp. 291–295). Even as keen an intellect as Mach's can fall into the trap of assuming that the long time elapsed grants us an automatic superiority over Hellenistic scientists.

[62]The one remaining piece is probably an excerpt from a longer work, *Elements of mechanics* (Στοιχεῖα τῶν μηχανικῶν), which Archimedes himself seems to cite under this title (*De corporibus fluitantibus*, II, 25:25; ed. Mugler, vol. III).

[63]This work was found in Arabic translation by Carra de Vaux and published in [Heron/Carra de Vaux]. The standard critical edition, based on several manuscripts, is due to Nix and appears in [Heron: OO], vol. II.

interesting information. We postpone until Chapter 10 a discussion of the successive theoretical developments, and treat here only the problem that gave rise to the *science of machines*.

The main mechanical problem of the time can be described as follows. Suppose we wish to raise a weight W to a height h. Instead of doing it directly, one can use a machine that, upon application of a force F, raises the desired weight to the desired height, the point of application of F moving in the process a distance d in the direction of F. In today's language, the principle of conservation of energy implies that the weight cannot be lifted unless the product Fd, now called the *work* performed by the force, exceeds the product Wh. If Fd does exceed Wh (and friction is sufficiently small), the weight can be lifted, and moreover, by using appropriate devices, one can choose the direction and the place where the force is applied, as well as the decomposition of the work between the two factors: one can apply a small force along a long distance or a large force along a short distance. In particular, one can lift the weight W using a force F less than W. The ratio W/F is the *mechanical advantage* of the machine.

The problem, given a maximum available force F and the need to lift a weight W, is to design a machine having the appropriate mechanical advantage and configuration, so the weight can be lifted by applying the available force at a convenient point and in a convenient direction. All devices of this type can ultimately be traced back to the simplest such device, the lever, which Archimedes uses as the starting point of his scientific theory of mechanics.

Of course problems of this type had always been around and had often been solved practically, as far back as paleolithic times, when levers and wedges/ramps were already in use. At the time of the ancient empires pliers (pincers) were also known, and the pyramids could not have been built without the help of many machines. Classical Greece knew the pulley and the winch, the latter having first been used, in all probability, for shipbuilding or in the theater. This long evolution of empirical mechanics was based on the slow accretion of craftsmen's experience. The qualitative leap made possible by science lay in that now one could compute the mechanical advantage theoretically, and so for the first time design a machine from first principles. This leap surely took place as early as the third century B.C. Pappus[64] and Plutarch[65] tell us that Archimedes had solved the problem of lifting a given weight with a given force; in other words, he know how to design a machine with a specified mechanical advantage. There is no reason to doubt these sources, since the theoretical

[64]Pappus, *Collectio*, VIII, 1068:20 (ed. Hultsch).

[65]Plutarch, *Vita Marcelli*, xiv §7.

bases of such a solution are given by Archimedes in his extant work and several applications of his designs were reported by various authors. We also know that the same period witnessed the introduction, perhaps due to Archimedes himself, of a piece of technology still used today for many problems of this type: the gear.[66]

Hellenistic mechanics is closely connected to geometry. Diogenes Laertius states that Archytas (first half of the fourth century B.C.) was the first not only to introduce concepts from mechanics in the study of geometry (using lines generated by moving figures to construct the two proportional means between magnitudes), but also to treat mechanical questions using mathematical principles.[67]

The close link between geometry and mechanics, understood as two scientific theories, is clear in Archimedes. First of all, his *On the equilibrium of plane figures*, which founds the study of simple machines, borrows from geometry not only the general form of the deductive scheme, but also many particular technical results. More surprising to us today is that Archimedes uses the laws of mechanics to find theorems of geometry. In his *Quadrature of the parabola*, the rigorous proof we gave in Section 2.7 is preceded by a heuristic discussion based on the principle of the lever. Likewise, the volume of the sphere is found by imagining the balancing of a spherical and a cylindrical object, each placed on one plate of a balance. This procedure is explained systematically in *The method*,[68] where Archimedes expounds the two distinct methods he uses, respectively, for discovering mathematical results and for proving them rigorously. The geometric method is used only as a second step, to prove propositions already identified as plausible. For the discovery of propositions he uses instead the mechanical method, which he considers more intuitive. *The method* glows with the intellectual honesty of someone who is trying to communicate not only the proofs of his results but also the mental route that led to them, and it is of great interest for this and other reasons, such as the importance the author attaches to what we might call physical intuition and because it shows how essential it is, even for a genius, to use familiar methods in finding new scientific results, however tenuous the

[66]See Section 4.1.

[67]Diogenes Laertius, *Vitae philosophorum*, VIII §83. The construction given by Archytas for the two proportional means is reported by Eutocius in his commentary to Archimedes' *On the sphere and cylinder* (pp. 62–64 in [Archimedes/Mugler], vol. IV). Plato reproached Archytas for having contaminated geometry with mechanics (Plutarch, *Quaestionum convivalium libri iii*, 718E–F).

[68]Archimedes, *The method*, 77–127 (ed. Mugler, vol. III). The palimpsest (see page 8) is incomplete and some pages are largely unreadable. Heiberg conjecturally filled in extensive gaps, and his work was the base for the English translations in [Archimedes/Heath] and [Archimedes: GSM]. Recent work has led to significant revisions; see [Netz, Saito, Tchernetska].

objective connection between these methods and the initial problem might seem after the fact.

Many widespread ideas about the relationship between mathematics and physics should perhaps be revised in the light of the realization that the original proof of the now familiar formula for the volume of a sphere was in fact one of the first results of mechanics.

3.4 Hydrostatics

As far as we know, scientific hydrostatics was born with Archimedes' *On floating bodies*. And it was born already with much the same form it has today. Indeed, Archimedes makes it a scientific theory by laying its foundation in the form of a postulate:

> If contiguous portions of liquid lie at the same level, the portion that is more compressed pushes away the portion that is less compressed. Each portion is compressed by the weight of liquid that lies vertically above it, as long as the liquid is not enclosed in something and compressed by something else.[69]

The second half of the postulate has generally been misunderstood, both because a key word was twisted in the Latin translation that was until 1906 the only accessible version, and because Archimedes never uses the statement in this one surviving work, which deals with bodies that float on an open liquid. But note that the so-called *principle of communicating vessels* (though also not deduced explicitly in this work) clearly follows from the postulate, and may even have suggested its formulation.[70]

As a theorem arising from this postulate, Archimedes derives the famous principle that bears his name and that we all learn in school: Any

[69]Archimedes, *On floating bodies*, I, 6:2–8 (ed. Mugler, vol. III).

[70]If two open vessels joined by a horizontal tube are in equilibrium, portions of liquid lying at the same level are under the same pressure, whether they be contiguous (by the first part of the postulate and the assumption of equilibrium) or not (by transitivity). Now consider a portion of liquid in each container, both portions being at the same level and not *compressed by anything else*, only by the liquid above them: the equality of pressures just derived implies (by the second part of the postulate) that the columns of liquid above these two portions are equal. Therefore the surface of the liquid is at the same level in both containers.

If the communicating tube is not horizontal, the deduction is a bit more involved, but it can be derived as an exercise by anyone who has read carefully the first few propositions of *On floating bodies*, book II.

The principle of communicating vessels has generally been attributed to Heron, who uses it in the *Pneumatica* and in the *Dioptra*. But it was certainly known empirically before Archimedes; Plato mentions that water will flow through a wool thread from the fuller to the emptier of two cups (*Symposium*, 175d:6–7), implicitly supposing the two cups to be identical and placed on the same table.

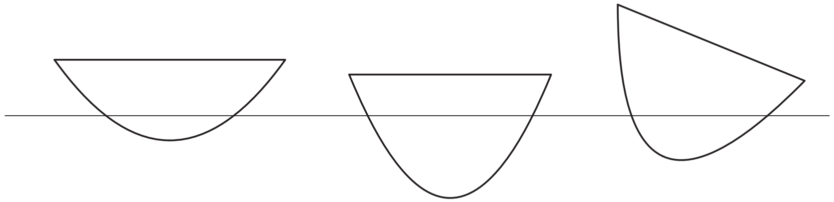

FIGURE 3.5. A low-form solid paraboloid of revolution (left) always floats upright. A tall-form one will float upright if it is dense enough (middle), but for lesser densities its equilibrium position is tilted at an angle (right) that depends on the density and the form factor. Archimedes proved these facts, calculating the threshold between the two regimes and the stable angle in the second case. (For the tall shape illustrated here, the threshold density is $\frac{1}{4}$ of the liquid's; the figure on the right has density $\frac{1}{9}$ of the liquid's.)

object is buoyed up with a force equal to the weight of liquid it displaces. But, contrary to the impression we generally get in school, Archimedes' hydrostatics is not limited to this statement. The typical problems that Archimedes solves in his treatise are finding the waterline for solids in equilibrium in a homogeneous liquid, and above all whether the equilibrium position is stable. The most interesting results along these lines are about an arbitrary floating solid in the shape of a right paraboloid of revolution (that is, a paraboloid truncated perpendicularly to the rotation axis). The stability of equilibrium in the upright position is studied as a function of two parameters — the form factor, which says how "fat" the paraboloid is, and the density of the solid. To summarize the results: If the paraboloid is fat (shallow) enough, upright equilibrium is always stable. If it's skinny, upright equilibrium is stable only if the density exceeds a certain value, while less dense paraboloids will stabilize at a certain angle of tilt that depends on the density (Figure 3.5).[71]

This study would be regarded today as an application of bifurcation theory, and according to Dijksterhuis it "deserves the highest admiration of the present-day mathematician, both for the high standard of the results obtained, which would seem to be quite beyond the pale of classical mathematics, and for the ingenuity of the argument".[72] Evidently, in present-

[71]To be precise, the position with the axis vertical is stable regardless of density if and only if the height of the segment of paraboloid is less than $\frac{3}{2}$ times the parameter of the generating parabola (the *parameter* is the distance between focus and directrix). Archimedes also determines, for taller segments, the threshold density and the angle of tilt at any given density above the threshold; both increase with the form factor. Eventually the paraboloid is so tall that it floats on its side for density values in a certain range; Archimedes shows that this happens when the height exceedes $\frac{15}{4}$ of the parameter.

[72][Dijksterhuis: Archimedes], p. 380.

day opinion the pale of "classical mathematics" does not even reach as far as the few surviving works of its most famous luminary.

That Hellenistic scientists were conscious of the "theoretical model" character of scientific theories is made clear by the use of not one but two such models in what we know of Archimedean hydrostatics. Indeed, the first book of *On floating bodies* derives from the postulate already quoted the fact that the surface of the oceans is spherical, whereas in the second book the surface of the liquid is implicitly assumed to be flat from the start: Archimedes does not spend a single word in justifying this assumption as an approximation of the "true" spherical shape. Obviously we are dealing with two different models, appropriate for phenomena at different scales.

The function of hydrostatics is clear. Inside the theory we have elegant and difficult mathematical problems. As for the real-life objects that the theory models, what can these objects be, about which one wants to calculate the waterline and the stability of equilibrium theoretically, before placing them in a homogeneous liquid? Clearly these are problems of naval architecture. Archimedes not only solves them, but even frames his study in an elegant and efficient hypothetico-deductive structure that allows the reduction of other problems analogous to those discussed to "exercises" internal to the theory (though perhaps solvable only approximately). This makes possible, in particular, the theoretical design of ships. [74] This interplay is the essence of scientific method.

Important precursors of Archimedean hydrostatics appear in the fragments of Democritus and in Strato of Lampsacus,[75] whereas Aristotle's ideas in this respect were much more distant.[76]

3.5 Pneumatics

In the case of pneumatics, as in many others, we find an important qualitative precursor of Hellenistic science in pre-Socratic thought. The earliest clear reference we have to the effects of atmospheric pressure was made

[74]The connection with ships is not far-fetched if we consider that Archimedes' proofs apply also to an elliptic paraboloid, so long as one considers roll and pitch separately.

[75]According to Democritus, the upward motion of "light" bodies is explained by the fact that, though having weight, they are pressed toward the top of the atoms of the surrounding fluid, if these atoms are heavier (Simplicius, *In Aristotelis De caelo commentaria*, [CAG], vol. VII, 569:5–9 = [FV], II, 100:3–6, Democritus A61). Similar ideas are attributed to Strato of Lampsacus, who wrote a work *On the vacuum* (relevant testimonia are collected and discussed in [Wehrli], vol. V, fr. 54–67 or [Diels: Strato], pp. 110–119; see also [Rodier], p. 57; two of the main ones are Simplicius, *In Aristotelis De caelo commentaria*, [CAG], vol. VII, 267:30 – 268:4 and Aetius, in Stobaeus, *Eclogae*, I, xiv, 143:6–9 (ed. Wachsmuth) = [DG], 311b:23–26).

[76]According to Aristotle (*Physica*, IV, i; *De caelo*, I, iii; *De caelo*, IV), levity is a quality opposite to gravity (weight); bodies that have that quality naturally tend to go up.

by Empedocles.[77]

However, the first unequivocal testimonia about pneumatics, understood as the science of compressible fluids, date from the first half of the third century B.C., when Ctesibius of Alexandria wrote at least two works on the subject: one, perhaps more theoretical, called *Demonstrations in pneumatics* (Πνευματικὰ θεωρήματα),[78] and a more applied one, the *Commentaries* (Ὑπομνήματα),[79] where a great many machines were described. Strato of Lampsacus, too, probably made significant contributions to the birth of pneumatics, but these are harder to document.

Unfortunately no work of Strato or Ctesibius has survived. Apart from some indirect references,[80] our knowledge of this ancient science is based essentially on the *Pneumatica* of Philo of Byzantium, who continued Ctesibius' investigations, and on the homonymous and much later work by Heron (first century A.D.). The work that bears Philo's name is represented by an Arabic text in 65 chapters, describing as many devices, and by Latin manuscripts.[81] The latter match the first 21 chapters of the Arabic text, albeit with notable omissions; it is reasonable to assume that this material was written by Philo, though our texts are very corrupt. By contrast, the part we have only in Arabic must be a compilation from heterogeneous sources,[82] because it is highly uneven in terms of technical sophistication and subject matter: while many of the devices it talks about are essentially amusements (as in the homonymous work by Heron), the last few chapters describe water-wheels and water-lifting machines — applications of great economic value which, as we shall see in Section 4.6, appeared in the early Hellenistic period.

[77] Aristotle, *De respiratione*, 473a:15–474a:6 = [FV], I, 347:13 – 349:6, Empedocles B100. (For a translation see, for example, [Empedocles/Inwood], pp. 138–139.) The *clepsydra* of this text is a gadget used to transfer liquids between containers — an inverted funnel whose wide mouth is closed by a perforated plate. In its normal usage, the clepsydra is partly immersed in the liquid; after it fills up to the surrounding level, the narrow opening at the top is capped with a finger and the instrument is lifted with the liquid inside. The girl of this passage, instead, immerses the clepsydra with the top opening covered, and the water cannot come in; as she lifts her finger, the air rushes out. The process is discussed, explained and illustrated in Heron, *Pneumatica*, I, vii = [Heron: OO], vol. I, 56–60.

[78] This work is cited by Philo of Byzantium in *Belopoeica*, 77:12 = [Marsden: TT], p. 152. The word θεωρήματα here can mean either "theorems" or, more likely, "demonstrations", in the sense of demonstrative experiments.

[79] Vitruvius, *De architectura*, X, vii §4.

[80] For Ctesibius we have very important testimonia in Vitruvius and Athenaeus, besides those in the works by Philo and Heron about to be discussed.

[81] See [Philo/Prager] for an English translation of the Arabic and Latin texts; Prager's introduction is probably the most interesting modern article on ancient pneumatics. The standard reference on the subject is [Drachmann: KPH].

[82] Prager attributes to Philo only chapters 37–39, besides those surviving in Latin ([Philo/Prager], p. 66).

The Greek word pneuma (πνεῦμα, translated *spiritus* in Latin) had a wide range of meanings, which changed significantly through the centuries and between cultural environments. The early meanings were air, breath, breathing, spirit, etc. In Stoic thinking, pneuma is also a continuous medium that underlies exchanges between the various parts of organisms[83] and of the universe.[84] But Heron, at the beginning of his *Pneumatica*, states that pneuma is just air in motion;[85] this simple meaning of the term may have been sharpened precisely through the development of pneumatics.

Although the total loss of Ctesibius' works and the corruption of Philo's text greatly limit our knowledge of theoretic aspects of Hellenistic pneumatics, we do have some insight on one important methodological characteristic of it. The objects described in the early, and reputedly authentic, chapters of Philo's works are intended neither to amaze nor, for the most part, to perform useful functions; they are simple experimental devices designed to demonstrate particular phenomena, such as those related to the syphon principle. Some of Philo's demonstrative experiments are still used in school today to teach the experimental method: for example, the effect that burning a candle inside a submerged dome has on the water level inside the dome.[86] Heron, most probably drawing from Ctesibius or Strato of Lampsacus, devotes the entire introduction of his *Pneumatica* to arguing that macroscopic regions of vacuum cannot exist in nature, but can be approximated artificially — as in fact he demonstrates later in the work on several occasions. In particular, Heron explains that the natural distribution of particles and void in air can be changed in both directions under the application of external forces, although the air opposes such changes with an elastic reaction.[87] The elastic properties of air had already been described and used by Ctesibius, as we know from Philo.[88]

[83] In animals, according to several authors, the transmission of information from sensory organs to the central unit and from there to the muscles is mediated by pneuma; see for example Stobaeus, *Eclogae*, I, xlix, 367:17 – 368:20 (ed. Wachsmuth) = [SVF], II, 826 (quoting Iamblichus) and Calcidius, *Ad Timaeum*, ccxx = [SVF], II, 879.

[84] Pneuma — in this function overlapping with the *aether* of some authors — is the medium for interactions between the various parts of the universe, thanks to its *tension* (τόνος) and its characteristic *tensional motion* (τονικὴ κίνησις); in particular it allows the transmission of light (see for example Cleomedes, *Caelestia* I §1, 3:68–74 (ed. Todd) = [SVF], II, 546). The reference [Sambursky: PS] may be useful in connection with Stoic physics.

[85] Heron, *Pneumatica*, I, introduction = [Heron: OO], vol. I, 6:6–7.

[86] Philo of Byzantium, *Pneumatica*, viii = [Philo/Prager], p. 136. Even Philo's explanation for the rise (some of the air "perishes", or is consumed) continues to be offered by many introductory science books, in spite of its incompleteness. In this the books follow an uninterrupted tradition over two thousand years old. We will return to Philo's explanation at the end of Section 5.7.

[87] Heron develops these notions throughout the work's proem ([Heron: OO], vol. I, 2–28); see in particular 6:23 – 7:16 and 26:23 – 28:11.

[88] Philo of Byzantium, *Belopoeica*, 77–78 = [Marsden: TT], pp. 152–154.

Pneumatics, appearing from its foundation as a theory of phenomena that can only be caused artificially, lies very far from natural philosophy and certainly from Aristotelian philosophy.[89] This provides corroborative evidence for the revolution in thought illustrated in Section 1.4 with examples from mechanics.

One important technical application of pneumatics was the pressure pump. Vitruvius has left us a description of it taken from the *Commentaries* of Ctesibius, to whom he attributes the pump's invention.[90] (He refers the reader to the same work on the subject of several other air-operated machines.[91]) The design is that of today's two-piston, two-phase pumps (see Figure 4.10 on page 124). Its construction was made possible by the introduction of a new element, the valve, which became crucial in all later technology.

3.6 Aristarchus, Heliocentrism, and Relative Motion

Starting in the fourth century B.C., scientific astronomy developed in close connection with mathematics. The greatest astronomers we know of were Eudoxus of Cnidus (whose mathematical accomplishments we have already mentioned), Callippus and Heraclides of Pontus in the fourth century; Aristarchus of Samos, Conon of Samos[92] and Archimedes[93] in the

[89] Aristotle gave several "demonstrations" of the impossibility of the vacuum (*Physica*, IV, vi–ix, 213a–217b). Like Archimedes with the ship (page 25), Ctesibius overcomes Aristotle's objections by designing machines that create phenomena not observable in nature. It was once thought that Heron offered the impossibility of a vacuum as an explanation for why syphons work (*Pneumatica*, I, ii = [Heron: OO], vol. I, 36:8–18). This view turns out to be based on the emendation of a question mark into a colon by philologists eager to shoehorn Heron into the confines of Aristotelian orthodoxy. This at least is what Prager concluded after comparing the edited text with the Biblioteca Marciana codex of the *Pneumatica*; see [Philo/Prager], p. 21.

[90] Vitruvius, *De architectura*, X, vii §§1–3. A variation, used as a hydrant, is described by Heron (*Pneumatica*, I, xxviii). A description of a vacuum pump contained in the Arabic text of the *Pneumatica* of Philo of Byzantium (chapter lxiv) seems to be by Arabic hands and has scarce technical value.

[91] The clocks and pumps that Vitruvius describes are, he says, the "most useful" among the devices in the Ctesibian *Commentaries* (ibid., §5); the others he dismisses as very ingenious but meant solely for amusement. The contents of the first chapters of the *Pneumatica* of Philo, whose main source was certainly Ctesibius' work, may suggest that some of those "useless" devices might in reality have been designed for experimental demonstrations (see page 77).

[92] Conon is best known because Callimachus, in a famous short poem translated by Catullus, mentions him as having explained the motion of heavenly bodies (*Coma Berenices* = Catullus poem 66:1–7; a fragmentary Greek text has been found on papyrus: Callimachus, fr. 110 Pfeiffer). One imagines that he made important contributions to science, since Archimedes mentions him admiringly more than once (*Quadratura parabolae*, 164:1–12; *De sphaera et cylindro*, I, 9:12–15; *De lineis spiralibus*, 8:12–20, ed. Mugler in each case) and Apollonius of Perga stresses the importance of some of his theorems on conics (*Conics*, preface to Book IV; we quote the passage on page 200).

[93] The astronomical activities of Archimedes are attested by references in his extant works and

third; Apollonius of Perga (better-known for his treatise on conic sections) between the third and second; and Seleucus and Hipparchus in the second century B.C. After that astronomical research stops.

Of all the astronomical works of the scientists mentioned so far, only two remain, both altogether minor: Aristarchus' *On the sizes and distances of the sun and moon*, already mentioned,[94] and Hipparchus' *Commentary on the Phenomena of Aratus and Eudoxus*, containing a critical commentary on Aratus' poem and saved thanks to the latter's popularity. To these one can add a famous passage in Archimedes' *Arenarius* describing the heliocentric theory of Aristarchus of Samos. The information contained in these writings is meager. Aristarchus' surviving work, indeed, gives us a sense of his scientific method and of the use of trigonometric methods, but it is essentially a geometric work, unrelated to the fundamental problem of astronomy: the description of the motion of heavenly bodies. Also barren is Hipparchus' commentary on the poem of Aratus, which merely furnishes angular coordinates of fixed stars. In sum, the only contemporary source of insight into early Hellenistic models for describing the motion of planets is the passage in the *Arenarius*, and that's just a brief digression, a mention of an astronomical argument embedded in a different context. (We will examine it shortly.)

The only important astronomical work that has come down to us from Antiquity was written in the imperial period by Claudius Ptolemy (second century A.D.), and is the major work of this author. Called *Syntaxis mathematica* (*Mathematical treatise*), it is better known under the name the Arabs gave it, *Almagest*.[95]

Two results of Hipparchus, recoverable from the *Almagest*, will suffice to give an idea of the level astronomy had reached in his time in terms of accuracy of measurements. Hipparchus discovered the precession of the equinoxes, and he probably measured the mean distance to the moon, finding it to be 59 earth radii.[96]

by a passage of Hipparchus reported by Ptolemy (*Almagest*, III, i, 195).

[94] See page 66 and footnote 45 thereon.

[95] The critical edition of the *Almagest* is Heiberg's (Leipzig, 1898–1903), whose pagination we follow; for a recent translation see [Ptolemy/Toomer].

[96] Certainly Hipparchus made the observation (reported also in Plutarch, *De facie quae in orbe lunae apparet*, 921D) that lunar parallax can be measured. The distance of 59 terrestrial radii is obtained in the *Almagest* (V, xiii, 416) through a procedure marred by many errors that miraculously cancel out. Toomer suggests that this was a value obtained by Hipparchus and known to Ptolemy ([Toomer: HDSM]). The mean distance between the centers of the two bodies is just over 60 earth radii; thus the approximation reported by Ptolemy is very good, and indeed exceptionally good if it refers to the distance between the surfaces, which is the datum that can be measured directly. (Compare Aristarchus' *On the sizes and distances of the sun and moon*, proposition 11, where he had taken the distance as measured "to the center of the moon from our eye", though there was no need for him to be specific, since he assumed the radius of the earth to be negligible in comparison.)

The difficult problem of trying to reconstruct the fundamental ideas of astronomy in the third and second centuries B.C. will occupy us in Chapter 10. For now we make just a few observations about the heliocentrism of Aristarchus. As related by Archimedes,[97] Plutarch[98] and Simplicius,[99] among others, Aristarchus formulated a theory according to which the earth revolves yearly about the sun and rotates daily about an axis tilted with respect to the plane of its orbit. The Plutarchan passage says that by postulating these two earthly movements Aristarchus was trying to *save the phainomena* (φαινόμενα σῴζειν) — that is, explain what is seen in the skies. (We use the Greek word *phainomenon* instead of the modern spelling *phenomenon* in order to stress the original meaning: anything that is seen or appears to the senses. In current English, *phenomenon* tends to be used in a more "objective" sense — see page 381 — or to imply a marvel.) Since the description of the apparent motion of sun, moon and fixed stars cannot be affected in any way by heliocentrism, the phainomena in question must have concerned the planets. Archimedes mentions in the *Arenarius* that Aristarchus had certain demonstrations (or illustrations) of the phainomena.[100] These evidently consisted in showing how the complex planetary motions, with their stations and retrogressions, could be obtained from the combination of two simple uniform circular motions around the sun: the earth's revolution and an analogous movement for the planets.

The Aristarchan "demonstrations" could become particularly effective if illustrated by a mechanical model of planetary motion. We know that Archimedes did build such a model, a moving planetarium that reproduced the apparent motion of the sun, the moon and the planets. Some have wondered how Archimedes managed to do this, presupposing that in his contraption the sun and the planets moved independently, their mechanisms hinged on a fixed earth. A purely geocentric construction would indeed be hard-pressed to account for the observed motion of planets, but it would only be attempted by someone who did not know about Aristarchus' theory and its power to save the phainomena. Since this theory came down to us precisely through the *Arenarius*, to imagine that Archimedes made no use of it is to place the dogma that heliocentrism was rejected after Aristarchus ahead of all the evidence.[101]

[97]Archimedes, *Arenarius*, 135:8–19 (ed. Mugler, vol. II).
[98]Plutarch, *De facie quae in orbe lunae apparet*, 923A.
[99]Simplicius, *In Aristotelis De caelo commentaria*, [CAG], vol. VII, 444:31 – 445:5.
[100]τὰς γὰρ ἀποδείξιας τῶν φαινομένων ... ἐφαρμόζει (Archimedes, *Arenarius*, 136:1–2, ed. Mugler, vol. II).
[101]See, for example, [Neugebauer: HAMA], vol. II, p. 652, note 7, where the author admits he "do[es] not see how the daily ... motions of sun and moon can be combined with the planetary retrogradations ... in one spherical model", and concludes from this that Archimedes' model did

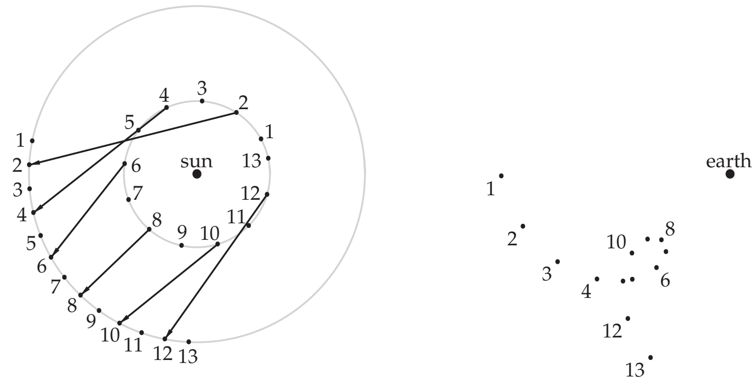

FIGURE 3.6. Planetary retrogressions. On the left, a schematic heliocentric view: because the earth (inner orbit) has a shorter period than the outer planet, the direction in which the planet is visible appears most of the time to move counterclockwise, but at times, as between points 6 and 10, it moves clockwise. This is also seen in the diagram on the right, which plots the difference between the sun-planet and sun-earth vectors — that is, the vector shown by the arrows on the left. From the "fixed" earth the planet appears to move against the background of stars first westward (clockwise), then eastward (during part of the loop), then westward again. The bottom panel of Figure 3.7 on page 90 shows what the actual track of a retrograde planet in the sky looks like.

The idea that Archimedes' was a geocentric planetarium probably arose because the testimonia are unanimous in saying that his device showed the motion of the sun and the planets around the earth. But what does this mean? First note that the aim of explaining planetary motions as seen from the earth is not achieved by a machine that shows only the earth and planets revolving around the sun; instead the *earth* must be kept fixed as the device operates, in order to make obvious the motions actually observed in the sky.[102] (Compare the top two diagrams in Figure 3.6.) What makes a planetarium heliocentric is the crucial feature that the mechanical linkage between the various planets and the earth is performed through a single

not show "the most characteristic features of planetary motions, [namely] stations and retrogradations, and the inner planets must be ignored altogether" — in direct contradiction with the sources, which say that all five planets were accounted for with their irregular motions (see below).
[102]Naturally, if one only wants to show "true" motion, with no concern for what is actually observed, a stationary-sun device is enough. But this attitude seems to me far removed from that with which Aristarchus built a theoretical model that could save the phainomena; it is of more recent vintage. In fact, it's typical of today's teaching that children are indoctrinated from a tender age with the notion that the earth revolves around the sun (not to mention electrons revolving around the nucleus) before being aware of any directly observable fact that can be explained by this motion.

hinge placed in the sun, and this is sufficient to generate relative motions sometimes in one direction and sometimes in the other. Most likely Archimedes' planetarium was of this type. This conjecture has an important source of support in the main testimonium about the issue, from Cicero:

> Archimedes' invention is admirable in that he figured out a way to make a single *conversio* reproduce the diverse and various trajectories, in motions that contrast among themselves.[103]

The word *conversio* can mean rotation, inversion, reversal. It would be an appropriate term for a hinge allowing the production of retrograde motion. In any case this emphasis on the singleness of the mechanism on which the various contrasting motions depend would be inconsistent with a mechanical model based on a Ptolemaic-type algorithm.

We are told of other moving planetaria from Antiquity in a passage of Pappus[104] to the effect that some were built that were powered by a hydraulic mechanism, and again by Cicero,[105] who mentions a planetarium constructed by Posidonius in the first century B.C. While we do not know that anyone ever managed to build a Ptolemaic-type mechanism able to represent planetary motion, after the "Copernican revolution" the construction of moving planetaria — of heliocentric type, of course — became possible again.

Thus the history of planetaria suggests that the heliocentric theory was not abandoned right after Aristarchus, as is generally assumed,[106] but after the time of Hipparchus, when scientific activities stopped. We will return to this point in Chapter 10.

[103]"... in eo admirandum esse inventum Archimedi, quod excogitasset, quem ad modum in dissimillimis motibus inaequabiles et varios cursus servaret una conversio" (Cicero, *De re publica*, I, xiv §22). Cicero is relaying observations contained in a lost work of Sulpicius Gallus, who saw the Archimedean planetarium at the home of his coconsul Marcus Marcellus, who in turn had inherited it from his grandfather, the Marcellus who sacked Syracuse. Cicero brings up elsewhere the same idea of a single *conversio* on which all the motions depend (*Tusculanae disputationes*, I, xxv §63).

[104]Pappus, *Collectio*, VIII, 1026, 2–4 (ed. Hultsch).

[105]Cicero, *De natura deorum*, II, xxxiv §88.

[106]The common idea is that Aristarchus was too far ahead of his time to have had a lasting influence on the course of science, and support for it is generally found in the accusation of impiety supposedly leveled at him because of his heliocentrism. The belief that Aristarchus was accused of impiety originates with the seventeenth century philologist Gilles Ménage, who, obviously influenced by the prosecutions of Giordano Bruno and Galileo, changed a passage in Plutarch (the same one cited in note 80 above) by emending an accusative into a nominative and vice versa. Later editors, perhaps regarding as inevitable the link between heliocentrism and impiety, have almost without exception accepted the emendation to Plutarch's text, which became canonical in Ménage's "modernized" version. For more information on this enlightening episode, see [Russo, Medaglia].

Nor was Aristarchus the first person to say that the earth moved. Already Heraclides of Pontus, in the fourth century B.C., asserted the daily rotation of the earth,[107] and the same is said of the Pythagoreans Hicetas and Ecphantus.[108] These are important precedents, of which Aristarchan heliocentrism is a natural development. If the alternation of day and night can be explained by a movement of the earth, it is natural for astronomers to explain the retrograde motion of planets in an analogous way. Saying for the first time that the earth moves, on the other hand, requires a profound transformation in the concepts of space and motion. It is not by accident that Ptolemy, who shares Aristotle's conception of space, rejects not only heliocentrism but the rotation of the earth as well; nor that, in modern times, the two were accepted at the same time.[109]

Prescientific societies have always talked about rest and movement of bodies in an absolute sense, without ever feeling the need to specify with respect to what reference system the motion is considered. From the modern point of view this was possible through the implicit use of a reference system fixed on the earth. Aristotle still believed that space is absolute and comprises places of differing natures, and that one such place, having the property of attracting heavy bodies, had become (thanks to that property) the center of the world. This concept of space is closely connected to the idea that the absolute state of motion of bodies is observable.

If, while accepting the idea of absolute space, you dare hypothesize an (absolute) motion for the earth, as Heraclides did before anyone, the very idea of absolute motion cannot but face a crisis. For then you must accept that you are moving at over a thousand kilometers per hour (the approximate speed due to terrestrial rotation at the latitudes of Hellenistic cities) without noticing it. You realize then that your observations do not indicate the "true" state of motion of bodies, but only the relative motion between observer and observed.[110]

Did granting the earth movement lead to a relativistic idea of motion in Hellenistic times? A positive answer can be deduced from many sources.

[107]This is attested in quite a few sources; see especially Simplicius, *In Aristotelis De caelo commentaria* ([CAG], vol. VII, pp. 444:31 – 445:5; 519:9–11; 541:27 – 542:2). These passages are translated in [Heath: Aristarchus], pp. 254–255. Other testimonia about Heraclides' astronomical doctrines are listed in [Wehrli], vol. VII, fr. 104–117.

[108]See, for example, [Heath: Aristarchus], pp. 187–188, 251–252. The meager information we have about these Pythagoreans is due to Diogenes Laertius, Aetius and Hippolytus.

[109]Recall that the objection to Copernicism based on biblical authority did not apply to terrestrial revolution, only to rotation.

[110]Of course in our classical mechanics not all reference systems are equivalent, and a reference system fixed on the earth is not inertial. But since the motions of the earth, compared with those experienced in everyday life, have small acceleration despite their enormous velocity, this subtlety can be neglected to a first approximation.

The remark that what is seen depends only on the relative motion between observer and observed is already in Euclid's *Optics*.[111]

Archimedes, discussing Aristarchus' theory in the *Arenarius*, abstains from objecting to it on physical grounds (though he does take issue with the mathematical formulation; see page 87). He uses the theory only to deduce from the absence of stellar parallax an estimate for the diameter of the sphere of fixed stars. Thus he is interested in terrestrial motion not in an absolute sense, but with respect to the fixed stars. Naturally, if one believes, as Archimedes did, that there is a sphere of fixed stars,[112] the existence of such a naturally privileged reference system lessens the importance of relativism (and perhaps obviates the need to state it), but, as a matter of principle, it's one thing to refer motions to "natural" reference bodies such as the stars, and quite a different one to consider absolute motion (relative to the void). The essential point is that Archimedes thought there was no way to check whether the earth moves relative to the stars by means of earth-based experiments.[113]

Ptolemy, expounding in the *Almagest* his own theory of the earth's immobility, attacks the contrary opinion, concerning in particular the daily rotation. The opinion he reports and opposes is not the assumption that the earth rotates, but the relativistic statement made by "some people" that the rotation can be ascribed indifferently to the earth or the heavens, or even to both, so long as both rotations have the same axis and their difference (the relative motion) is the one actually observed.[114] He recognizes that this is compatible with all astronomical phainomena. To refute the theory and assert the earth's immobility, he must resort to arguments from natural philosophy , taken in large measure from Aristotle. This is not an isolated case: authors from the imperial age often frame Hellenistic scientific results by stating first their own arguments, taken from philosophers of the classical period. We'll see in Section 10.15 a similar situation regarding geometrical concepts.

Many Hellenistic works, or works based on Hellenistic sources, illustrate the relativity of motion. The most famous locus is perhaps that of Lucretius about the passengers of a ship to whom it seems that the ship is stationary and the land is moving.[115]

[111] Euclid, *Optics*, proposition 51. The passage is quoted in Section 6.3, page 178.
[112] However, the existence of the sphere of fixed stars had already been called into question before Archimedes, in the fourth century B.C. We will return to this point in Section 3.7.
[113] Indeed Archimedes discusses the compatibility of Aristarchus' hypotheses with the absence of measurable parallax effects and accepts as possible the explanation based on the enormous distance to the stars, without taking into consideration any Ptolemaic-type arguments.
[114] Ptolemy, *Almagest*, I, vii, 24.
[115] Lucretius, *De rerum natura*, IV:387–390.

It should be noted that, just as thinking that the earth moves naturally leads to relativistic views, such views, in turn, can make the question of whether the earth moves appear inconsequential. Thus the preceding paragraphs can explain why post-Aristarchan Hellenistic sources, starting with Archimedes, appear to modern eyes so indifferent to the question of heliocentrism as to generate the belief that heliocentrism was suddenly abandoned.[116]

Confirmation for this explanation is provided by a passage of one of the greatest modern scholars of ancient astronomy, John L. E. Dreyer, who analyzed with great skill all the available evidence about Aristarchus and later astronomers:

> Aristarchus is the last prominent philosopher or astronomer of the Greek world who seriously attempted to find the physically true system of the world. After him we find various ingenious mathematical theories which represented more or less closely the observed movements of the planets, but whose authors by degrees came to look on these combinations of circular motion as a mere means of computing the position of each planet at any moment, without insisting on the actual physical truth of the system.[117]

This passage is very instructive. Dreyer evidently thinks of the physical truth of an astronomical theory as something other than its ability to predict the observable position of each planet at any moment. This suggests that the idea of judging the validity of a theory solely on the basis of its power to save the phainomena (i.e., in the case of astronomy, represent the observable positions of celestial bodies)[118] had not yet been fully recovered in Dreyer's time (the *History of astronomy* from which the quotation is taken is from 1906). What does Dreyer regard as the physical truth of an astronomical system? Its ability to determine the "true motion" of planets, we surmise, given the prevailing belief at the turn of the twentieth century in an absolute space with respect to which motions should be identified by astronomers.[119] Dreyer, not finding the same concept of absolute space in

[116] According to Sextus Empiricus the motion of the earth was accepted by the "followers of Aristarchus" (οἱ περὶ ᾿Αρίσταρχον), so that Aristarchus was not isolated (*Adversus physicos* II (= *Adv. dogmaticos* IV = *Adv. mathematicos* X), §174). We will return to the developments of heliocentrism in the second century B.C. in the next section and in Chapter 10.
[117] [Dreyer], p. 149.
[118] See Section 6.3 for a more extensive discussion.
[119] Galilean relativity, as is well known, was refuted by Newton. The idea of absolute space was then reinforced by the theory of the ether, which still held sway in the early 1900's. Relativistic views reacquired the upper hand only thanks to Einstein, whose first work on special relativity (*Zur Elektrodynamik bewegter Körper*, 1905) came out just before Dreyer's *History of astronomy* and, given its technical character and difficulty, could hardly have had an immediate influence on the ideas presented in a historical work such as Dreyer's.

the ancient astronomers, draws the facile (and unwarranted) conclusion that this absence was a fault.

The attitude of historians of science changed after the idea of absolute space was definitively laid to rest. Neugebauer, having collected many ancient passages that illustrate the relativity of motion, concluded that "statements about obvious cinematic equivalences are a commonplace in ancient literature".[120] But the oldest reference he found regards Heraclides of Pontus. Neugebauer calls Heraclides' idea "relativistic" and considers it obvious, but ideas of this type were certainly not obvious before Heraclides, and they stop being obvious again from the end of the Hellenistic period down to Dreyer's time at least. Clearly, notions like the possibility of a free choice of reference system are not only extremely hard to acquire: once acquired, it is also extremely hard to shake them off in order to appreciate their depth.

3.7 From the Closed World to the Infinite Universe

As everyone knows who has observed the night sky for a few hours, the stars seem to move all together, keeping fixed their mutual distances and so the shape of the constellations. This naturally suggests the thought that what is going around, making a full turn per day, is the whole sky, imagined as a material sphere in which the individual stars are embedded. The rotating sphere of fixed stars, besides giving a straightforward explanation for the most obvious of astronomical observations, seems to provide also a natural limit for the extension of the cosmos, imagined as a sphere whose center is the earth. This image of an enclosed and spherical universe, which goes back perhaps to Pythagoras and was certainly held by Parmenides, is also present in Plato's and Aristotle's works, and was accepted by Ptolemy, who handed it down to the Arabic and European Middle Ages. Nevertheless, this was *not* the cosmology of all the "Ancients", as many think.

In Hellenistic times the idea that the earth moves had important cosmological consequences, which modified profoundly the picture just limned. If, indeed, one is bold enough to imagine that the motion of the stars is merely apparent, and that it is the earth that turns around daily, the sphere of the fixed stars loses its function. It is not an accident that both Aristotle and Ptolemy, who believed in the earth's immobility, also believed in a rigid sidereal sphere, and that the first person to challenge the notion of that sphere was apparently the same who first asserted that the earth rotates — Heraclides of Pontus, who maintained that the universe is infinite and that every celestial body is a world in itself (and even has its own atmosphere).[121]

[120][Neugebauer: HAMA], p. 695.

An interesting argument in favor of an infinite universe is reported by Lucretius:[122] If the universe were finite, all masses would already be concentrated in its center. This supposes that gravity affects all bodies, not just "heavy" ones. (In this connection see Section 10.7.)

Aristarchan heliocentrism brought to bear a new argument that increased enormously the traditional dimensions of the cosmos. The supporters of heliocentrism, indeed, had to explain why ever our motion around the sun causes no observable parallax — that is, why the appearance of the constellations does not change as our vantage point moves relative to them throughout the year. According to Archimedes, Aristarchus overcame that objection by assuming that the radius of the earth's orbit is to the radius of the sphere of fixed stars as a sphere's center is to its radius.[123] This "ratio" (λόγος) between the earth and the sphere of fixed stars (also mentioned by Geminus, Cleomedes, Ptolemy etc.[124]) is what Archimedes criticizes, on the grounds that the ratio between two lengths is necessarily nonzero.[125]

This issue calls perhaps for a mathematical parenthesis. The aim of the *Arenarius* may have been precisely to defend the "Archimedean postulate" (see page 51 in Section 2.7), by showing that one can assign a finite, nonzero ratio to any two nonzero lengths (or other homogeneous magnitudes). To accomplish this it was necessary to work out a numbering system able to express even the largest imaginable ratio between homogeneous magnitudes, such as the ratio between the volume of the sidereal sphere and that of a grain of sand; this is what Archimedes does in his tract. The triumph of Archimedes' views on commensurability took away the rationale for the task undertaken in the *Arenarius* and rendered the work hard to understand (it has always been felt to be strange). Obviously, what Aristarchus (and the other authors mentioned) intended in saying that two lengths are in the same ratio as a point is to a circumfer-

[121]Aetius, in Stobaeus, *Eclogae*, I, xxi, 182:20–21 and I, xxiv, 204:21–25 (ed. Wachsmuth) = [DG], 328b:4–6 and 343b:9–14. The latter passage attributes the same opinions to the Pythagoreans as well; these may have been the same Pythagoreans, such as Hicetas and Ecphantus (page 83), who asserted that the earth moves.

[122]Lucretius, *De rerum natura*, I:984–997.

[123]*Arenarius*, 135:14–19 (ed. Mugler, vol. II).

[124]See, for example, Geminus, *Eisagoge eis ta phainomena* (= *Elementa astronomiae*), XVII §16; Cleomedes, *Caelestia*, I §8, 38:1–5 (ed. Todd); Ptolemy, *Almagest*, V, xi, 401:22 – 402:1. Aristarchus likewise postulates that the ratio between the earth and the orbit of the moon is equal to the ratio between a point and a circumference (*On the sizes and distances of the sun and moon*, hypothesis 2 = [Heath: Aristarchus], p. 352).

[125]*Arenarius*, 135, 19–22 (ed. Mugler, vol. II).

ence was to translate in mathematical terms an assumption that the ratio is too small to be measured or estimated from observed data. To admit the Aristarchan statement by assumption is not simply equivalent to saying that one length is negligible with respect to the other in calculations; it constitutes rather an attempt to construct a model in which lengths form what we would call a non-Archimedean set. In particular, to think that the stars lie on a "sphere" whose radius is incommensurably greater than any observable length is but a step away from introducing a mathematical model where the "celestial sphere" is a conventional and useful way to represent the set of directions. This step was effectively taken, as seen from the fact that Geminus, in his compilation dating probably from around 50 B.C., introduces the "so-called sphere of fixed stars", explaining its conventional nature and warning the reader not to suppose it to have a physical existence, since the stars are at different distances from us.[126]

Archimedes was utterly victorious in his assertion that all lengths have nonzero ratio. But if the history of mathematics for two millennia followed the path shaped by this view, it is not to be concluded that older formulations like that of Aristarchus were necessarily erroneous or lacked all possibility of coherent development. Indeed, the goal of constructing geometries that admit points "at infinity" came to be achieved in modern projective geometry.

To go back to astronomy: Not surprisingly, another known proponent of heliocentrism, Seleucus,[127] likewise did without the sidereal sphere and believed in an infinite universe.[128]

Because the further something is the slower it appears to move, the new distances suggested for the stars also left room for the possibility that the stars were not in fact fixed. Thus it is no wonder that Hipparchus also conjectured that the apparently fixed stars were in fact mobile. According to Pliny, Hipparchus compiled his catalog of stars precisely so that later generations might deduce from it the displacements of stars and the possible appearance of *novae*.[129] Clearly, Hipparchus too did not believe in a material sphere in which the stars are set. His catalog achieved its aim in full: the stellar coordinates listed therein were incorporated into Ptolemy's work[130] and so handed down until such a time when a change

[126]Geminus, *Eisagoge eis ta phainomena* (= *Elementa astronomiae*), I §23, a good recent edition being [Geminus/Aujac]. The exact same approach is adopted today by, for instance, the *Encyclopaedia Britannica* (15th edition, Micropaedia, sub "celestial sphere").

[127]Plutarch, *Platonicae quaestiones*, 1006C.

[128]The opinion of Seleucus is reported by Aetius together with that of Heraclides of Pontus, in the first passage cited in note 121 above.

[129]Pliny, *Naturalis historia*, II §95.

[130]See page 284 for the relationship between the catalogs of Hipparchus and Ptolemy.

in the positions of the "fixed" stars could be detected. Changes were first noticed in 1718 A.D. by Halley, who, probably without realizing that he was completing an experiment consciously started two thousand years earlier, recorded that his measured coordinates for Sirius, Arcturus and Aldebaran diverged noticeably from those given by Ptolemy.

Now, one can hardly relinquish the sphere of fixed stars without concluding that the daily motion of stars is merely apparent — that is, without recognizing that the earth moves.[131] Thus the preceding testimonia, too, corroborate the view that the earth's motions were not discarded by Aristarchus' Hellenistic successors, and in particular that Hipparchus did not consider the earth immobile.

Once the stars are conceived as extremely distant bodies not all at the same distance, they can be ascribed other important properties, above all an enormous size. We cannot follow the debate on this point in astronomical works of the time, but we can perhaps hear some distant echoes of it. For example, Cleomedes, though a believer in geocentrism and the sidereal sphere,[132] wonders how the earth would appear from a star; since he knows that the sun is much bigger than the earth and that the stars are vastly more distant than the sun, he deduces that seen from the sun the earth would appear minuscule, and that from a star it would not be visible at all. It follows that the stars, which we can see, must be much bigger than the earth. Cleomedes also says that the sun, seen from a star, would look as the stars look to us.[133] The statement that the stars are larger than the earth is also found in other authors.[134]

The notion of the universe as a multicentered structure, where a great many (or infinitely many) worlds coexist, was held also on other grounds. We will return to this in Section 10.7.

3.8 Ptolemaic Astronomy

The only well known Greek astronomical theory is the one Ptolemy expounds in the *Almagest*. We defer to Chapter 10 a comparison between early Hellenistic ideas and Ptolemy's on such subjects as space and motion, limiting ourselves here to some observations on the mathematical model used in the *Almagest*. Everyone knows that Ptolemy's planetary

[131]The examples of Aristarchus, Copernicus and Kepler show that the reverse implication is false. The earth's motions can be imagined to coexist with a rigid sidereal star, kept for tradition's sake though stripped of its function of explaining the rigid motion of the heavens.

[132]However, Cleomedes insists that the void is infinite, and regards it as somehow real and existing beyond the sky.

[133]Cleomedes, *Caelestia*, I §8, 38:19 – 39:31 (ed. Todd).

[134]Cicero, *De re publica*, VI, xvi §16; Proclus, *In Platonis Rem publicam*, II, 218:5–13 (ed. Kroll).

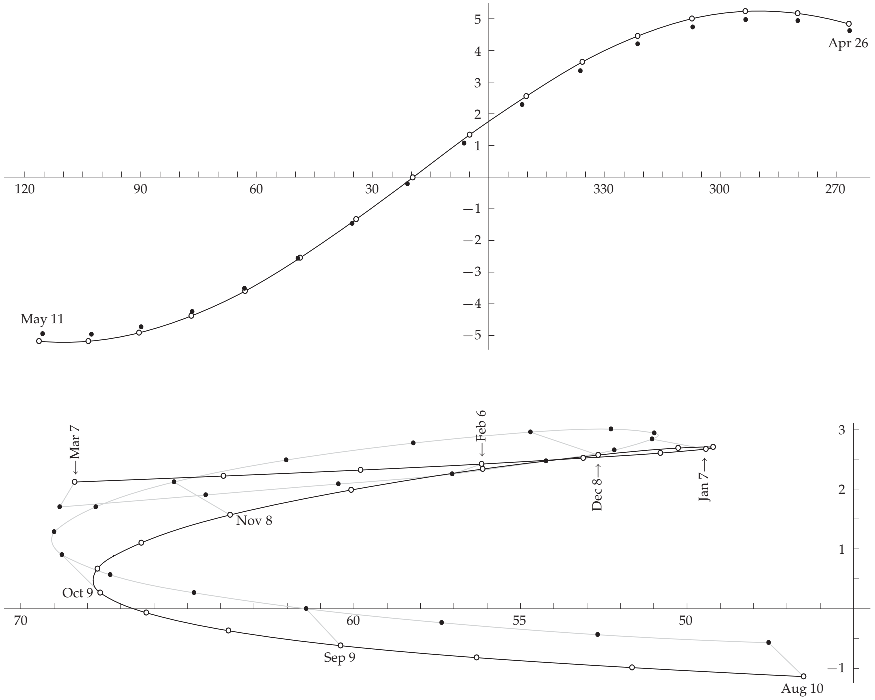

FIGURE 3.7. Agreement between *Almagest* algorithm (black dots) and modern calculations (white dots linked by dark curves). Top: moon at 6 A.M. Alexandria time on consecutive days in 126 B.C. Bottom: Mars, every 10 days from August 8, 297 to March 7, 296 B.C., a period that includes a full retrograde loop. The moon and Mars exemplify opposite extremes in the accuracy of the approximation; Martian motion was known not to be in very good agreement with the theory, but its qualitative features were nonetheless correctly modeled, as here. The data are taken from [Neugebauer: HAMA], vol. I, pp. 97, 188, 220. (Axes show longitudes and latitudes in degrees; the vertical axis is expanded in each case).

theory is based on the composition of circular motions. This technique had been used in astronomy by Apollonius of Perga, and it was in fact much older: the first algorithm of this type that we know about was worked out by Eudoxus, who described planetary motions as obtained from a succession of concentric spheres rotating uniformly, each with an axis of rotation marked by two points fixed on the previous sphere.

Because "epicycles" is still a byword for clumsy and backward attempts at science, we spell out the two reasons why the method was supremely well-adapted to the purposes to which it was put.

First, accounting for the observed motion of planets as the composite of several uniform motions on circular orbits (the first centered on the earth and called the *deferent* in medieval terminology, and each of the others, called *epicycles*, centered on the point obtained on the preceding circumference) is equivalent to a modern expansion in Fourier series, and allows an efficient description of observed data with increasing precision as the number of epicycles grows. The analogy between Fourier series expansions and developments in epicycles was observed by Schiaparelli,[135] but one can imagine that the thought occurred earlier. One can conjecture, in fact, that in this important observation Schiaparelli was preceded by the mathematicians who developed the idea of Fourier series expansions — starting with Daniel Bernoulli in the eighteenth century, who also studied planetary motion and surely knew about developments in epicycles. A formal demonstration of the equivalence has been given by Giovanni Gallavotti.[136]

Second, since the main computational tool of Hellenistic mathematics was geometric algebra performed with ruler and compass, decomposition into circular motions was the most efficient possible system for computing the observable position of planets.[137] If, for example, the motion of a planet is described as a combination of three uniform circular motions, in order to compute the position at a given instant it is only necessary to draw three arcs of circle and measure out three angles obtained by multiplication.[138] Thus the calculation is reduced to a very few arithmetic operations and six elementary geometric operations, realizable with two simple instruments: compass and protractor.

In Ptolemy's work the number of circular motions needed to obtain good agreement with experimental data (cf. Figure 3.7) is reduced to two, one deferent and one epicycle, through the introduction of *eccentrics* (the center of the deferent does not coincide with the earth) and *equants* (the circular motion is not considered to be uniform, but to have uniform angular

[135] "And we shall understand also the necessity and reason for this multiplicity of spheres, which has been wrongly criticized by those who have not grasped its function, and which arouses derision and condescension in our contemporaries, who, without knowing it, use epicycles by the dozen and by the hundreds in their planetary theories, under the name of periodical terms of infinite series" ([Schiaparelli], vol. II, p. 11).

[136] In [Gallavotti: QPM], which also contains an interesting translation into modern mathematical terms of the main ideas from the systems of Hipparchus, Ptolemy and Copernicus.

[137] Just a few epicycles suffice to account for the position of the planets to within the precision afforded even by modern experimental data.

[138] It's worth noting that, since multiplications were carried out in sexagesimal notation and angles were measured, accordingly, in degrees, the result of multiplying an angular velocity by time is immediately reducible to a "plottable" angle of less than 360 degrees. If the same task is performed (as today) in decimal arithmetic using the same degree units, every such multiplication must be followed by an extraction of the remainder under division by 360.

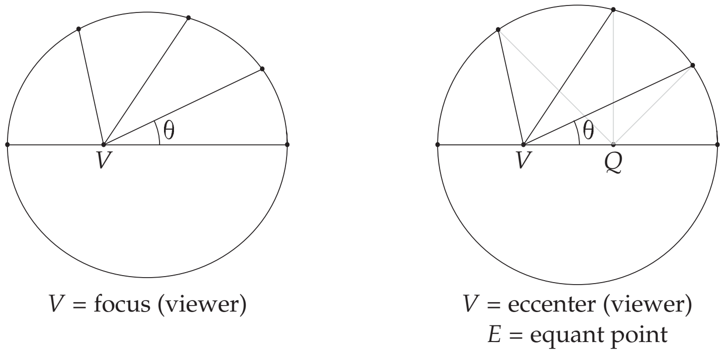

FIGURE 3.8. A two-dimensional sketch shows how eccentrics and equants help model orbits. Left: a Keplerian orbit with the planet's position marked at regular intervals. Note the significant variation in the rate at which the central angle θ is swept. Right: A judicious Ptolemaic-type construction with one eccentric circle and equant suffices to approximate θ with remarkable accuracy — the maximum deviation is $0°26'$ for the Keplerian orbit shown here, of eccentricity 0.436 (greater than that of any solar planet).

velocity with respect to a point, the equant, which differs from the center of the circle).[139] Although these modifications disrupt the symmetry of uniform circular motion, they have the essential merit of buying flexibility without sacrificing computational ease: the use of eccentrics and equants amounts to a simple displacement of the point on which each of the two fundamental instruments — compass and protractor, respectively — is to be centered. This shows that the use of circular motions was not due, as has often been said, to reasons of symmetry or esthetics, but to the concrete need for an algorithm that could reduce the necessary calculations to simple operations realizable with elementary instruments.[140]

Much as in the case of mathematics (Section 2.3), the link between theoretical scientific structures and material instruments is evidenced by the fact that in Mesopotamia the use of clay tablets as the writing material took astronomy in a different direction from Alexandrian astronomy.[141] Mesopotamian astronomy did not employ geometric constructions like

[139] On the actual use of eccentrics and equants one can read [Neugebauer: HAMA] or, for a briefer treatment, any history of astronomy.

[140] We cannot conclude that Hellenistic scientists had a preference for circles because of their "perfection", just because this idea is present in authors of the classical and imperial periods — the more so because imperial-age authors, who no longer mastered the Hellenistic scientific method, often framed scientific results in conceptual schemes belonging to prescientific natural philosophy.

[141] By Alexandrian astronomy we mean, as in the case of mathematics, the homogeneous scientific tradition developed in Hellenistic times in the Greek-speaking Mediterranean world, and having its greatest center in Alexandria.

Ptolemy's epicycles, but the study of numerical regularities. Although much is still unknown, it is clear from cuneiform tablets deciphered in the twentieth century[142] that, whereas the astronomy of Old Babylonia was prescientific and qualitative, Hellenistic-era mathematical astronomy in Mesopotamia had a level similar to that of contemporary geometric-based Alexandrian astronomy.[143] It is also clear that, though the mathematical methods were distinct, there were cultural exchanges. This is shown not only by the fact that scientific astronomy arose in both civilizations simultaneously, but also by mutual influences. For example, Alexandrian astronomers adopted the Mesopotamian sexagesimal system, and the Mesopotamian astronomer Seleucus is referred to as a follower of Aristarchus of Samos.

[142] The astronomical tablets deciphered prior to 1955 have been published in [Neugebauer: ACT].

[143] For a survey of what is known regarding the history of Mesopotamian astronomy in Hellenistic times one can turn to [Neugebauer: HAMA], pp. 347–558.

4
Scientific Technology

In this chapter we will try to verify in the case of Hellenistic civilization the relationship between exact science and scientific technology that we have characterized theoretically. The consequences of this new technology to production and the economy will be considered in Chapter 9.

Alas, the information we have on Hellenistic technology is very limited: literary sources are almost completely silent on the subject,[1] and archeological data, though having grown in the last decades, yield information that is fragmentary, casual and often not at all easy to interpret. Today's secondary literature contains excellent works on particular sectors of technology, but among general works even the best are by now completely obsolete.[2]

Given this situation, our objective will be simply to document through examples the existence of scientific technology in Hellenistic civilization, and to get a qualitative idea of its level.[3]

[1]The meagerness of the source material is such that the useful anthology [Oleson, Humphrey, Sherwood] gathers in one volume all the passages deemed relevant — not just for the Hellenistic period but for for the whole Greco-Roman world.

[2]I refer to the *History of technology* edited by C. Singer and others ([HT]) and to the *Studies in ancient technology* in nine volumes by R. J. Forbes ([Forbes: SAT]). The main limitation of these works (which are admirable in many respects) is not the presence of ideological prejudices — though they often are present, as we will have more than one occasion to point out — but that they were written before a good part of the archeological evidence now available came to light. These works have contributed for several decades to the supremacy of the "primitivist" view, according to which Antiquity as a whole was a stagnant period in all sectors of technology.

[3]The prevalence of statistical methods has discredited in the eyes of many historians of culture the use of examples as an insufficiently "scientific" method. But a single example is enough to prove a theorem of existence in mathematics, and a single counterexample is sufficient to prove

4.1 Mechanical Engineering

Vitruvius lists twelve authors of works on mechanics: Archytas (whose works have all perished), Archimedes (who, according to Plutarch, wrote nothing on the subject), Ctesibius and Philo of Byzantium, whom we have already discussed, and eight others we know nothing about.[4] Athenaeus mentions a work on mechanics by one Moschus, not included in Vitruvius' list; we know nothing else about him either.[5]

The anonymous *Laterculi Alexandrini*, dating probably from the second century B.C. and found on a papyrus,[6] includes rosters of men who had reached the pinnacles of fame on various accounts: legislators, painters, sculptors, architects and *mechanikoi*, or mechanical engineers. The choice of categories demonstrates an interest in technology that has long been denied in connection with all of the "classical world".[7] The selection is so stringent that only five sculptors are named: Phidias, Scopas, Praxiteles, Myron and Polycletus.[8] It's obviously some kind of "hall of fame" intended to enshrine the exponents of human genius, in the same vein as the famous catalogs of seven wonders of the world. One of the engineers listed is Abdaraxus, "who built the machines in Alexandria".[9] These machines must have been so famous at the time that the author of the *Laterculi* judged any further specification superfluous. Yet the name Abdaraxus has not reached us through any other source, nor have any clues that might help us understand what machines are meant.

All of this suggests that our ignorance about Hellenistic mechanical engineering may reflect not indifference to the subject on the part of writers from the third and second centuries B.C., but rather the selection process of later ages — imperial, late ancient and early medieval — characterized by a marked lack of interest in technology.

Despite the virtual silence of our sources, we can tell from the meager information available that, not surprisingly, the birth of mechanics — the science of machines — was accompanied by a newly developed ability to

that an assertion is false. Of course, after one demonstrates through examples the *existence* of a cultural phenomenon — in this case, scientific technology — there is still the problem of assessing its magnitude; but this too cannot be addressed solely through statistical surveys.

[4]Vitruvius, *De architectura*, VII, preface §14.

[5]Athenaeus, *Deipnosophistae*, XIV, 634b.

[6]See [Diels: LA], where the extant text (the papyrus is much damaged) is transcribed and discussed.

[7]Fraser notes that the inclusion of mechanical engineers in this list "is of importance as showing the rather unexpected interest taken in such engineers and as indicative perhaps of a new prestige acquired by them" ([Fraser], vol. I, p. 426).

[8][Fraser], vol. I, p. 456.

[9]Each entry on this list of engineers found in the *Laterculi Alexandrini* is followed by the respective achievements in a nutshell. The whole passage is reproduced in [Fraser], vol. II, p. 617.

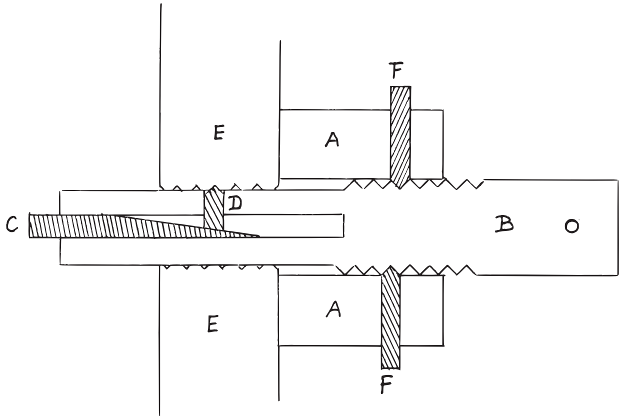

FIGURE 4.1. Cutter for screw holes, from [Drachmann: MTGRA], p. 138. First a cylindrical hole is made in the plate *E* where the screw will engage. Into this hole goes the smooth end of the tool stem *B*, whose other end includes the template screw. A wedge *C* fixes the position of a tip *D* which, as *B* is turned, cuts the groove into *E*, shallow at first and then deeper in subsequent passes, as *D* is pushed out. Four guiding pegs *F* engage the stem *B* as it turns within its casing *A*.

design and build a great many machines. At least two important novelties arose in early Hellenistic times: the use of several new foundational technological elements, such as screws and cogwheels, and the appearance of composite machines of high mechanical advantage, whose design and construction were stimulated by the possibility of computing advantages theoretically.

Augers for wood-drilling and other forerunners of the screw are very ancient, but cylindrical bolts with nuts are first found in presses, which were probably introduced in the early Hellenistic period.[10] Heron shows a method for manufacturing such bolts, which consists in wrapping about a cylinder a right triangle made of metal foil, the short side being parallel to the axis of the cylinder; the hypotenuse thus coils into a cylindrical helix that serves as a guide for the grooving.[11] He also explains how one grooves the hole where the screw will engage, using a tool with preexisting (male) grooving identical to the screw's; see Figure 4.1.[12] Both methods are based on scientific design, without which precision screws cannot be made.

[10]See Section 5.3 for this dating.

[11]Heron, *Mechanica*, II §5.

[12]Heron, *Mechanica*, III §21. The reconstruction of this tool is due to Drachmann, who, after a carefully study of the extant Arabic manuscripts, proposed the emendations necessary for the text to yield sense, and even built a model. See [Drachmann: MTGRA], pp. 135–139. Earlier attempts at understanding the text, in [Heron/Carra de Vaux] and [Heron: OO], were not as successful.

Cylindrical objects having a helical groove were used for different purposes, as we shall see. The theoretical properties of cylindrical helices were studied in a lost work of Apollonius of Perga, *On the cylindrical helix*[13] (Περὶ τοῦ κοχλίου; the curve's Greek name means "snail"). According to Proclus, Apollonius proved, among other things, that the cylindrical helix is *homeomeric*; that is, given any two points *P* and *Q* on it, there exists a rigid motion that leaves the curve invariant and moves *P* to *Q*. Thus the curve can slide along itself without changing shape; this is precisely the property that makes it useful in the construction of bolts and nuts.

It seems that cogwheels, too, were made for the first time in the early Hellenistic period.[14] They opened to engineers many novel possibilities, including the transfer of torque between perpendicular axles (as in the machine depicted on page 121) and the achievement of high mechanical advantages through reduction gear trains (see Figure 4.2).[15] The reduction gears that we still use in many devices, from bicycle derailleurs to timepieces, are direct descendants of the Alexandrian inventions, recovered through the study of ancient works — particularly those of Heron of Alexandria — by Arabs and Renaissance Europeans. They are not objects thrust on us by nature or by logic, as many seem to believe, but cultural products inherited from Hellenistic civilization.

4.2 Instrumentation

Archeological finds and surviving descriptions are so rare that we cannot hope to know a good part of Hellenistic measuring instruments. But we have enough information to form an idea of the prevailing qualitative technological level. We will mention only two types of measuring instruments: surveying instruments and timepieces.

Surveying instruments. The main surveying instrument of Pharaoh-era Egypt had been the surveyor's cross or *groma*, which consisted of two perpendicular wood beams and allowed the recognition of right angles. It was inherited essentially unchanged by classical Greece and by Rome. The difference between prescientific and scientific technology is well illustrated by the comparison between the groma and a Hellenistic instrument

[13]Proclus, *In primum Euclidis Elementorum librum commentarii*, 105:1–6, ed. Friedlein.

[14]Our earliest documentation about gears is from the first half of the third century B.C. See [Drachmann: MTGRA], pp. 200–203; [Price: Gears]; [Sleeswyk]. As we shall see, gears were used in Hellenistic times in water mills and in machines for lifting water, among others.

[15]Reduction gears (also called demultiplier gears) and their mechanical advantages are examined in Heron, *Mechanica*, II §§21–28.

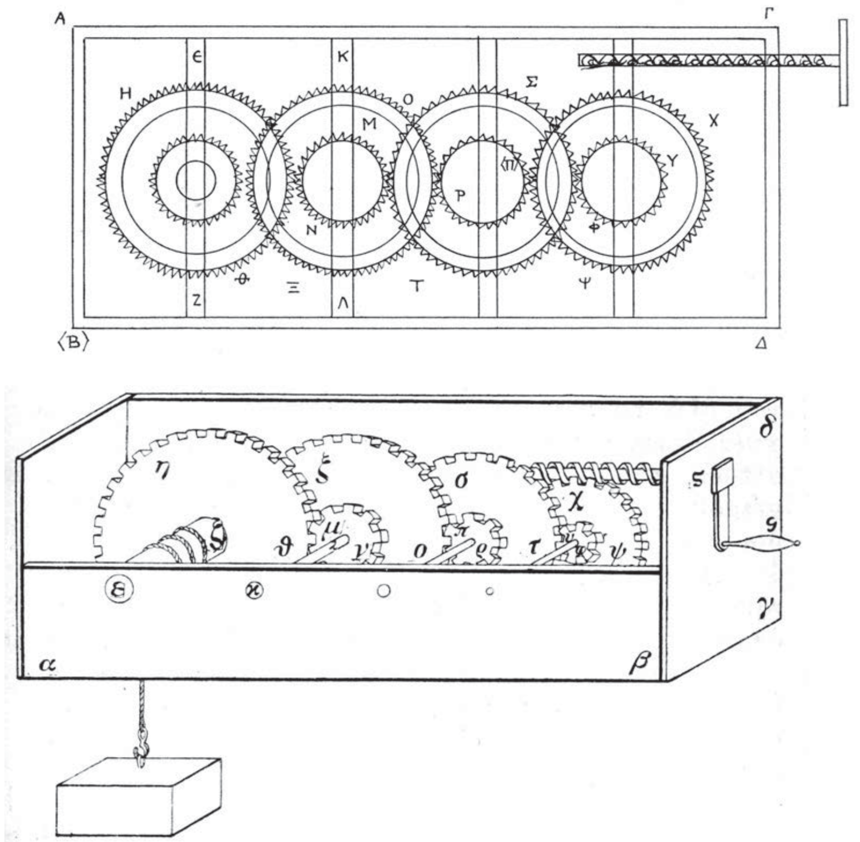

FIGURE 4.2. Weight-lifting reduction gear (*barulkos*) from Heron's *Mechanics*, as preserved in Greek in his *Dioptra*.[16] The author explains in detail, referring to the letters in the diagram, how the device allows a force of 5 talents to lift a weight of 1000 talents. Top: diagram as it appears in the Mynas codex (fol. 82r); taken from [Drachmann: MTGRA], p. 25. Bottom: a modern editor's rendition of the same object, based on the textual description and the manuscripts' figures; taken from [Heron: OO], vol. II.1, p. 263. (Manuscript drawings suffer more than the text in the copying process, and important features — here, the gear ratios and the engagement of the endless worm screw on the top right with the wheel — often become corrupt and have to be reconstructed from the text. But the placement of the wheels in the same plane as their axles is a convention motivated by the need to allow easy copying with ruler and compass. For another comparison see page 127.)

[16]Heron, *Dioptra*, xxxvii, in [Heron: OO], vol. III, 306–310 = *Mechanica*, I §1, in [Heron: OO], vol. II.1, 256–266. The text and illustration are also preserved in Pappus, *Collectio*, 1060–1068.

used for the same end: the *dioptra* described by Heron (Figure 4.3).[17] Here the perpendicular axes are drawn on a graduated disc; one axis, at whose ends there are slits for looking through, is free to rotate on the plane of the disc, around its center. The disc itself not only can rotate freely about its center, but it is attached at right angles to the diameter of a cogged semicircle made of brass, which can pivot about a horizontal axis; an endless worm screw allows one to fine tune and secure the position. The whole device described so far is attached by three pivots to a vertical cylinder that can itself turn about its axis; here again fine tuning and fixing depend on a worm, which meshes with a cog coaxial with the cylinder. Heron mentions several uses for the dioptra, particularly in astronomical measurements and in the surveying necessary for the excavation of underground galleries. Unfortunately the description we have is incomplete and we don't know whether the missing part dealt with important components of the instrument.

Heron also describes in the *Dioptra* a level, consisting of a horizontal wooden pole, about two meters long, containing inside a tube with ends turned up, to which two vertical glass tubes could be connected. When the instrument was filled with water, the principle of communicating vessels ensured that the water would reach the same level in both glass tubes. The tubes were endowed with small sliding brass plates that could be kept in place at the water level by means of screws. Each plate had a peephole for alignment.

Instruments such as these are incompatible with the widespread belief in a constant rate of progress and in the primitive state of classical technology. Those historians of technology who, imbued with these beliefs, nonetheless studied Heron's dioptra have become aware of the existence of an error; however, they generally attribute the error to Heron himself, for failing to keep his designs in line with the age in which he happened to live. For example, in the authoritative *History of technology* edited by Singer et al., we read:

> Heron's dioptra remains unique, without past and without future: a fine but premature invention whose complexity exceeded the technical resources of its time.[18]

In fact by Heron's time the dioptra was already centuries old, as can be seen just by reading the *Almagest* (see note 23 on page 272). As for the future of the dioptra, it contained if nothing else the theodolite, born in the sixteenth century of the study of Heron's work.

[17]Heron, *Dioptra* = [Heron: OO], vol. III, pp. 187–366.
[18][Price: Instruments], p. 612.

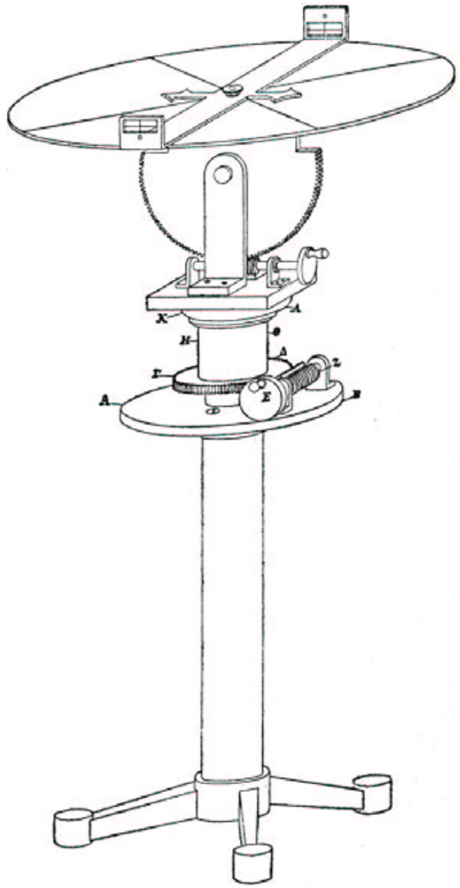

FIGURE 4.3. The dioptra. From [Heron: OO], vol. III, p. 193.

Timepieces. The main Hellenistic instrument for measuring time was the water clock. Its ancestor was the water clepsydra of Pharaoh-era Egypt, which was simply a container with an orifice in the bottom. The time elapsed from the filling of the container could be read off from the level of the water against a scale drawn on the interior. A clepsydra of this type, from about 1400 B.C., was found in Karnak.[19]

The Egyptian clepsydra served to give an idea of the passage of time, particularly at night, but it cannot be said to have been a true measuring instrument, for two reasons. First, the rate at which the water flows out depends on the pressure and therefore decreases as more water escapes.

[19] See [Borchard], pp. 6–7.

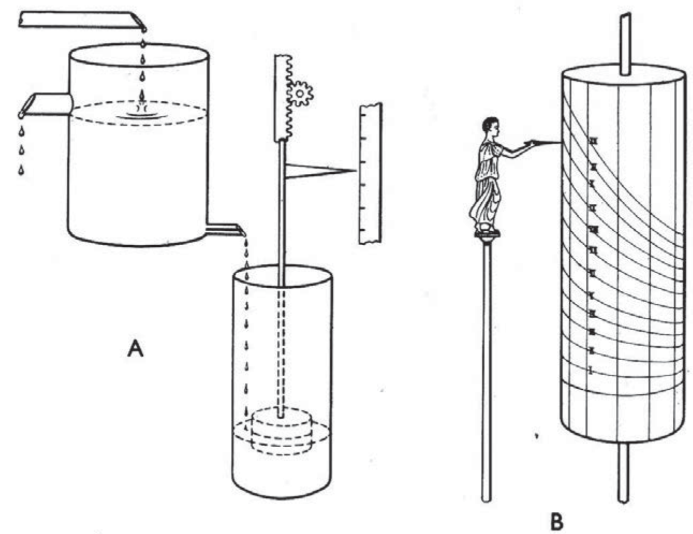

FIGURE 4.4. A: Ctesibius water clock. B: cylinder to adapt reading to the seasonal length of the hour. From [Price: Instruments], p. 601.

Second, the size of the hole cannot be regarded as constant except in the short term, because of corrosion and accretions that tend to constrict it. The Egyptians had partly overcome the first difficulty — but not the second — by using containers shaped as truncated cones rather than cylinders; as far as we know this was a purely qualitative correction. A further complication was caused by the unit of time used then, the hour, which changed from day to day and between day and night, being defined as one-twelfth the time between sunrise and sunset and vice versa.

Classical Greece made no essential improvements to the Egyptian clepsydra: the water timers used in trials and mentioned by Aristotle[20] were clepsydras, if anything simpler than the Egyptian ones.[21]

The first real clocks appeared in Alexandria in the first half of the third century B.C., thanks to Ctesibius, whom we encountered in Section 3.5. We know through Vitruvius that he solved brilliantly all the problems above, transforming the old clepsydra into a true measuring instrument.[22] The water reservoir had two openings, a small one in the bottom and a larger escape hole on the wall; it was continually refilled at a rate intermediate

[20] Aristotle, *Atheniensium respublica*, lxvii §§2–3.

[21] Because their function was merely to set a uniform limit to the duration of each party's address, these timers had no intermediate marks, much like the hourglasses that come with games today.

[22] Vitruvius, *De architectura*, IX, viii §§2–14.

between the flow rates of the two openings, so the water remained at the level of the upper hole, ensuring constant pressure.[23] The lower orifice was drilled in gold or a precious stone, to avoid corrosion and accretions. The water that flowed out of the bottom was collected in another container, where a float moved a pointer by means of gears, allowing the reading of the water level against a scale.[24] In one model multiple reading scales were set on a cylinder that rotated so as to display the scale appropriate to the time of year, thus adjusting for the variable duration of the hour.

Other scientists, including Heron,[25] studied water clocks. A remarkable design is described and attributed to Archimedes in an anonymous work preserved in Arabic.[26] This clock, unlike that of Ctesibius, is not refilled continuously; the main reservoir is filled but once a day and emptied at a constant rate, so that its level can be read out by the use of a float connected to a display by appropriate means (Figure 4.5). Water descends from the main reservoir into a second chamber through a pipe that ends in a conical flare that opens downward; inside this flare a conical float valve fits snugly. When the water in the lower chamber is not at its top level, the float valve lets in just enough water to bring the level back up to where the valve is pushed against its socket. Thus the water in the lower chamber remains always practically at its maximum level, and so flows out (through a hole near the bottom) at a constant rate; that is also the rate of emptying of the main reservoir. The float valve, if indeed due to Archimedes, is one of the earliest feedback control devices.

Among the causes of error in water clocks considered in Antiquity was the variation in the flow rate of water caused by temperature changes.[27]

Much has been written about the transformation the concept of time underwent in the modern age because of clocks,[28] but in general it has not been supposed that anything like the modern scientific concept of time

[23] This technical solution suggests that the notion that hydrostatic pressure depends on the height of liquid (stated in the first postulate of Archimedes' *On floating bodies*; cf. page 73) was already clear to Ctesibius.

[24] Like the dioptra described by Heron, the gears built by Ctesibius are considered "premature" by Price ([Price: Gears], p. 53).

[25] A fragment of Heron's treatise on water clocks can be found in [Heron: OO], vol. I, p. 456.

[26] The work is translated and discussed in [Hill: CWC]. The clock is also described in [Lewis: TH], pp. 364–366.

[27] This observation appeared in a lost treatise by Theophrastus, *On waters* (Περὶ ὑδάτων), and is mentioned by Plutarch (*Quaestiones naturales*, 914A) and by Athenaeus (*Deipnosophistae*, II, 42a–b). Indeed the higher viscosity of cold water (to use the modern terminology) slows down the flow noticeably. Athenaeus explains the slowdown with the words διὰ τὸ πάχος, "because of the *pachos*", a word that can mean both thickness and density, the two concepts seemingly being merged in his mind. A similar explanation is given by Plutarch, who is perhaps Athenaeus' direct source.

[28] It suffices to recall the classic pages of Marc Bloch on the medieval concept of time ([Bloch: FS], pp. 73–74) and the essay [Koyré: MPUP].

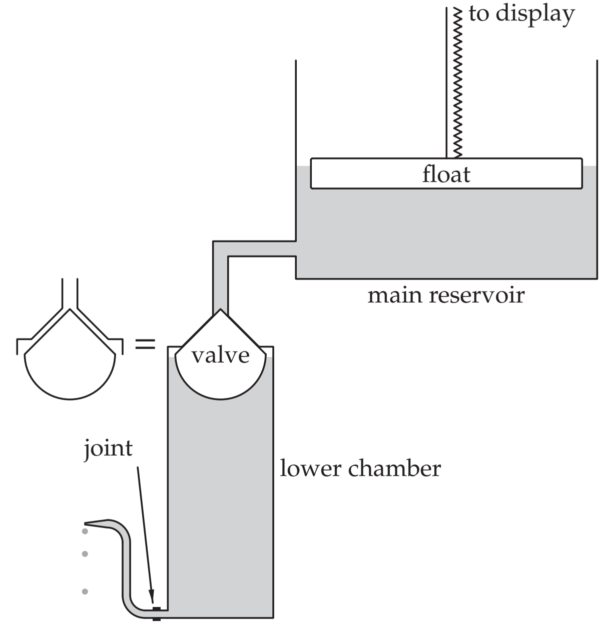

FIGURE 4.5. Water clock attributed to Archimedes. The pressure at the bottom of the lower chamber is kept constant by means of a float valve. A joint lets the output pipe be directed vertically, horizontally or diagonally, thus allowing for seasonal adjustments in the flow rate to compensate for the variable length of the hour (the rate is controlled by the difference between the water level in the lower chamber and the height of the outer spout).

might have been known to the scientists to whom we owe the invention of clocks. For example, Sambursky not only talks of the "inability [of ancient scientists] to comprehend time as an independent variable"[29] but writes:

> Galileo's work was revolutionary ... [also] in treating time as a *mathematical* quantity which could be used in calculations ... His proofs are accompanied by graphs showing portions of time as sections on a straight line. This geometrical representation of time by Galileo was a step of first-class historical significance.[30]

In fact, the use of time as an independent variable, geometrically represented, enabled Hellenistic scientists to define many curves kinematically. Archimedes, in his *On spirals* (neglected by Sambursky but studied with

[29] [Sambursky: PWG], p. 185.

[30] [Sambursky: PWG], p. 239. Sambursky's book cannot be ignored because it is the only one, as far as I know, devoted to Greek "physics" (a subject generally regarded as nonexistent).

attention by Galileo) explicitly considers the "axis of times" and uses it as a fundamental geometrical entity. Time as an independent variable plays an essential role in the *Almagest*, and to cite just one more example, Heron makes interesting considerations on the subject in his *Mechanics*(note 24 on page 334).

Did the use of clocks change the concept of time in everyday life as well? Besides the previously mentioned use of timers at trials, some indication of a movement in this direction already in the fourth century B.C. is provided by the case of the prostitute nicknamed Clepsydra, who "timed her favours by the water-clock, stopping when it was emptied".[31]

4.3 Military Technology

One area in which the close relation between Hellenistic science and technology is particularly visible is military technology. The descriptions given by Polybius[32] and Plutarch of the siege of Syracuse (212 B.C.) bear witness to the terror roused by "technological warfare". Plutarch tells us that when Archimedes

> began to ply his engines, he at once shot against the land forces all sorts of missile weapons, and immense masses of stone ... [Some ships were] lifted up into the air by an iron hand or beak like a crane's beak and ... plunged ... to the bottom of the sea; ... A ship was frequently lifted up to a great height in the air (a dreadful thing to behold), and was rolled to and fro, and kept swinging, until the mariners were all thrown out, when at length it was dashed against the rocks, or let fall.[33]

Plutarch presents the technology the Syracusans used as a set of inventions contrived by Archimedes' isolated genius. Generally speaking, this childish version has been transmitted uncritically down to our days. But the "machines of Archimedes" should instead be understood within the framework of the progress achieved by military technology starting from early Hellenistic times. This is clear from Diodorus Siculus' report of the siege of Rhodes by Demetrius Polyorcetes, in 305 B.C., which reads in part:

> Demetrius ... built an engine called the helepolis [destroyer of cities], which far surpassed in size those which had been constructed before it. ... The whole structure was movable, mounted on eight great

[31]Athenaeus, *Deipnosophistae*, XIII, 567c–d (Gulick translation).
[32]Polybius, *Historiae*, VIII, v–vii.
[33]Plutarch, *Vita Marcelli*, xv §§1–3 (Dryden translation).

> solid wheels ... [and was] nine storeys high.... The three exposed sides of the machine he covered externally with iron plates.... On each storey there were ports on the front, in size and in shape fitted to the individual characteristics of the missiles that were to be shot forth. These ports had shutters, which were lifted by a mechanical device and which secured the safety of the men on the platforms who were busy serving the artillery...[34]
>
> [On some ships intercepted by the Rhodians] were also captured eleven famous engineers, men of outstanding skill in making missiles and catapults.[35]
>
> [The Rhodians] placed all their ballistae and catapults upon the wall. When night had fallen, at about the second watch, they suddenly began to strike the helepolis with an unremitting shower of the fire missiles, and by using other missiles of all kinds, they shot down any who rushed to the spot. ... The night was moonless; and the fire missiles shone bright as they hurtled violently through the air; but the catapults and ballistae, since their missiles were invisible, destroyed many who were not able to see the impending stroke. It also happened that some of the iron plates of the helepolis were dislodged, and where the place was laid bare the fire missiles rained upon the exposed wood of the structure. Therefore Demetrius ... finally assembled by a trumpet signal the men who were assigned to move the apparatus and by their efforts dragged the machine beyond range. Then when day had dawned he ordered the camp followers to collect the missiles that had been hurled by the Rhodians ... they counted more than eight hundred fire missiles and not less than fifteen hundred catapult bolts.[36]

Mobile siege towers along the lines of the helepolis later grew to have up to twenty floors.[37] The machines designed at the time of the siege of Rhodes included ones capable of grabbing and lifting enemy matériel, just like the "Archimedean" machines used in Syracuse to lift the ships 93 years later.[38]

War machines such as siege towers and ramrods had a long history already by the time that concerns us: the Assyrians were experts in their construction and that know-how was inherited by the Persians, who transmitted it to the Greek world. According to Diodorus Siculus, the oldest

[34]Diodorus Siculus, *Bibliotheca historica*, XX, xci §§1–6 (Geer translation).
[35]Diodorus Siculus, *Bibliotheca historica*, XX, xciii §5 (Geer translation).
[36]Diodorus Siculus, *Bibliotheca historica*, XX, xcvi §3 – xcvii §2 (Geer translation).
[37]Vitruvius, *De architectura*, X, xiii §5.
[38]For designing such a machine Callias got the post of public engineer in Rhodes (Vitruvius, *De architectura*, X, xvi §3).

type of "catapult" — essentially a modified bow operated with the whole body — was built in 399 B.C. by Dionysius I of Syracuse,[39] but it may go back even further.[40]

In Hellenistic times military technology, and particularly artillery, went through a phase of rapid progress, starting with the invention of the torsion catapult, a weapon that, unlike the bent-spring variety (which could evolve purely empirically from the bow), was based on a new principle: torsion elasticity.[41] The first exemplars, which cast bolts, seem to go back to the siege of Perinthus by Phillip II of Macedonia, in 340 B.C., while torsion catapults able to throw stones were probably first used by Alexander, during the siege of Tyre in 332 B.C. The power of these weapons grew rapidly during the third century (see Figure 4.6).[42]

Missiles of a range of weights could be thrown by the different machines. We have found in various arsenals shot ranging from 10 minae (4.4 kg) to 150 minae (66 kg).[43] As to their reach, specialists disagree: some believe it was under 200 meters, whereas others think it may have exceeded 300 meters.[44]

The effectiveness of Hellenistic catapults is demonstrated by the fact that fortification techniques changed following their introduction (much as happened after the introduction of firearms).[45] Fortified walls not only became thicker and started being surrounded by moats, but were complemented by towers capable of hosting catapults. Progress in artillery, however, left behind advances in defense, as seems to be demonstrated by a rapidly increasing number of victorious sieges.[46] There is an obvious correlation between the increasing difficulty in defending the perimeter of a city and the development of the great Hellenistic states.

One salient novelty of Hellenistic military technology was that the new weapons were not just the fruit of the ingenuity of individual artisans or generals, but were designed with the participation of the greatest scientists of the time.

[39] Diodorus Siculus, *Bibliotheca historica*, XIV, xlii §§1–2; XIV, l §4.

[40] [Marsden: HD], pp. 48–64; [Milner], pp. 209–210.

[41] The main work on the history of Hellenistic artillery is [Marsden: HD]. The surviving Hellenistic treatises on the subject are collected and translated in [Marsden: TT].

[42] The main source for the rapid technological development of artillery weapons in the first half of the third century B.C. is Heron's *Belopoeica* (which is included in [Marsden: TT]), based on a homonymous book by Ctesibius.

[43] See [Marsden: HD], pp. 81–83. Much heavier stone balls have been found, but it is thought that they were meant not to be thrown but to roll down the walls onto the enemy's siege train.

[44] McNicoll discusses all the available data, reaching the conclusion that ranges between 350 and 400 meters could be achieved ([McNicoll], p. 5).

[45] See [Winter], [Marsden: HD], pp. 116–163, and [McNicoll].

[46] A table showing a sharp drop in the number of cases of successful resistance between 322 and 303 B.C. is given in [McNicoll], p. 47.

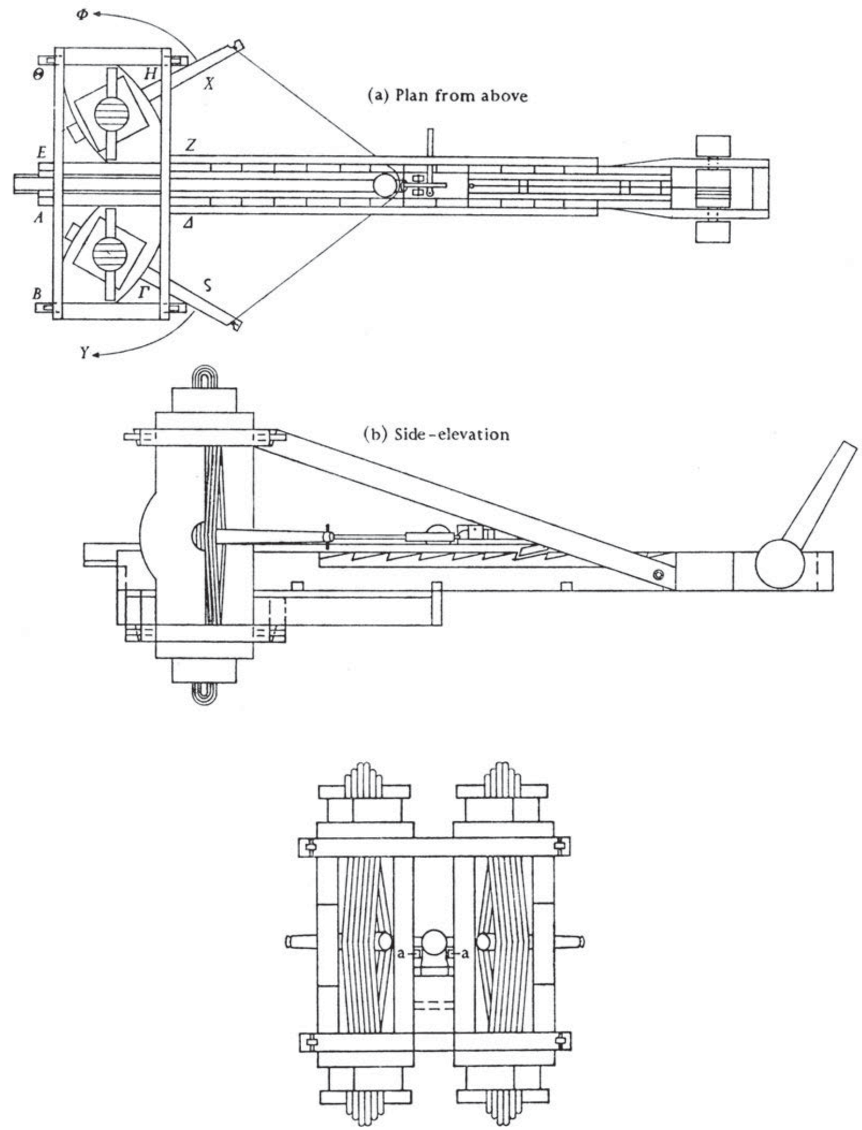

FIGURE 4.6. Torsion catapult, from [Marsden: HD], p. 35. Marsden declares these weapons "the standard artillery of the Mediterranean world from about the middle of the third century B.C. to the end of the first century A.D." (ibid., p. 33).

In torsion catapults elastic energy was stored in bundles of animal fibers (sinews or hairs) but the third century B.C. also saw experimentation with weapons based on alternative elastic materials whose properties were then being investigated. Thus Ctesibius designed and built artillery weapons based on metal alloy springs and others based on air springs. The latter were similar in structure to torsion catapults, but propulsion was effected by air compressed inside two metal cylinders by means of pistons. Contact surfaces were treated for airtightness, so when the launching arm was pulled back, pushing the pistons in, such high pressures could be achieved that upon release the stone shot would be hurled "a very respectable distance". According to Philo of Byzantium, to whom we owe the preceding description,[47] one would often see sparks between piston and cylinder. The creation of such weapons must naturally have gone hand-in-hand with Ctesibius' research on techniques for treating materials and on the compressibility properties of air.

At Rhodes, one of the most active centers of artillery studies, scientists also built a repeating catapult.[48] Modern historians who refer to these contrivances usually add that they remained in the model stage, but there is no good reason to think so. Repeating crossbows similar to the repeating catapults described by Philo, albeit less powerful and sophisticated, were in use centuries later in China and still played a role in the Sino-Japanese war of 1894–95.[49]

Romans had no contribution to make to the development of military technology, prior to the anonymous author of *De rebus bellicis* (fourth century A.D.). Even this writer uses only Greek terms for all war machines.[50]

In Western Europe's early Middle Ages, the ability to build effective artillery weapons was completely lost. Late medieval trebuchets were far less effective than the ancient catapults, the power of which could only be appreciated again after 1904, when models were first built on the basis of the indications provided by the ancient treatises, through the efforts of the German general E. Schramm.

The introduction of firearms in the modern age concerned primarily large-bore guns used against fixed fortifications; as a personal weapon, the arquebus took centuries to supplant the pike.[51] So the role of gunpowder was to replace the catapult, the technology of which had been lost.

[47]Philo of Byzantium, *Belopoeica*, 77–78 = [Marsden: TT], pp. 152–154.

[48]Philo of Byzantium, *Belopoeica*, 73–77 = [Marsden: TT], pp. 144–152.

[49][Marsden: TT], p. 178.

[50]One wonders if this person was in fact the first Roman to be interested in technological innovations or the first Greek speaker to realize the advantages of writing in Latin for the purposes of selling technology to Roman generals.

[51]See, for example, [Braudel], pp. 392–393.

Wherever there is science, military technology has been an important motivation and application for it. Hellenistic mechanics and early-modern mechanics both arose in connection with the main military applications of their day: catapults and firearms, respectively. In the latter case, mechanics had nothing to contribute to the energy imparted to the projectile, which depends on a chemical reaction outside the scope of the quantitative science of the time. Therefore scientists concentrated on the motion of the missile after it leaves the weapon's barrel, and, as is well known, the discovery of the laws of motion in free fall was decisively stimulated by the problem of determining bullet trajectories. By contrast, at the time of the catapults, the projectile was impelled by an elastic force, which could be investigated and modified using the scientific methods of the time. Thus, although we do not know why there was no interest in studying trajectories (and how can we be sure there wasn't?), it is clear from a reading of the Hellenistic treatises on military technology that Archimedean mechanics had vital applications to artillery.[52]

That science was relevant to the military technology actually used is proved not only by the efficiency of scientifically designed weapons, but by the mushrooming of texts on military technology. Clearly, the knowledge gained by scientists could not be used by builders unless mediated by specialized texts. Despite the obviously sensitive nature of such information, we know of several military treatises, teaching the construction of artillery weapons (*belopoeica*), siege machines (*polyorcetica*), and so on. Among the authors we know about are Philo of Byzantium, Biton and Athenaeus.[53] By Philo we have three texts: *Belopoeica*, *Paraskeuastica* (on defense works) and *Polyorcetica*.[54]

The descriptions contained in all these Hellenistic treatises, especially in Philo's *Belopoeica*, are of great interest because they shed light on the general level of mechanical technology in the third century. Among the technological innovations they rely on are universal joints, used for aiming, and flat-mesh conveyor belts, used in loading repeating catapults. (Universal joints were later named *Cardan joints*, after the sixteenth-century Italian scientist Gerolamo Cardano, while conveyor belts have sometimes been attributed to Leonardo da Vinci, who drew them much as described

[52]See, for example, Philo of Byzantium, *Belopoeica*, 59 = [Marsden: TT], p. 123. We will return in Chapter 10 to the question of whether a science of dynamics in the modern sense existed in Hellenistic times.

[53]Also called Athenaeus the Mechanic, and not to be confused with the homonymous author of the *Deipnosophistae*.

[54]These texts made up the three books of a single, vast treatise on "mechanics". The *Belopoeica* is included in [Marsden: TT]; the other two works, which survived only in part, were edited by H. Diels and E. Schramm ([Philo/Diels]).

by Philo of Byzantium.[55])

Authors of military technological works based on Hellenistic mechanics include several engineers who were active at the Pergamum court: we have the short book that Biton dedicated to Attalus I in the third century B.C.[56] In imperial times Heron, too, wrote works on military technology, two of which have survived.[57]

The most fascinating surviving pages on the relationship between mathematics and military technology are probably those of Philo of Byzantium. Talking about the construction of catapults, he says:

> Later, through the analysis of former mistakes and the observation of subsequent experiments, the fundamental principle of the construction was reduced to a constant element, the diameter of the circle holding the spring. This was first done by Alexandrian technicians, who benefited from large subsidies from fame-seeking kings who supported craftmanship and technique. That everything cannot be accomplished through pure thought and the methods of mechanics, but much is found also by experiment, is proved especially by what I'm about to say.[58]

Thus Hellenistic scientists had already enunciated explicitly the relationship between mathematics and experiments that is usually considered typical of the Galilean method.

Soon after this passage Philo gives the formula for the diameter of the opening that the spring (tension rope) goes through, and hence the diameter of the spring itself, as a function of the weight of the projectile that one wishes to throw a given distance; the diameter is proportional to the cube root of the weight, the proportionality constants being given by Philo. The famous problem of the *doubling of the cube* (extraction of cube roots) thus reveals its practical interest in the task of "calibrating" catapults. An ingenious instrument, the *mesolabe*, was designed by Eratosthenes to perform the extraction.[59]

[55]Leonardo da Vinci, Madrid Codex I, folios 5 and 10. The manufacture of conveyor belts was one of the main obstacles in modern attempts to reconstruct ancient repeating catapults. In 1904 E. Schramm succeeded in creating efficient weapons using bicycle chains, which were then a recent innovation.

[56]Biton, *Construction of war engines and catapults* = [Marsden: TT], pp. 66–103.

[57]The already cited *Belopoeica* and the *Cheiroballistra* (in [Marsden: TT]).

[58]Philo of Byzantium, *Belopoeica*, 50:21–29 = [Marsden: TT], pp. 107–109.

[59]The description Eratosthenes gives of his mesolabe was preserved by Eutocius (together with other solutions of the problem of doubling the cube) in his commentary *In Archimedis sphaeram et cylindrum* = [Archimedes/Mugler], vol. IV, pp. 64–69. Eratosthenes mentions the usefulness of his instrument in designing catapults.

4.4 Sailing and Navigation

Hellenistic civilization, like Hellenic civilization before it, was that of a group of harbor cities connected by sea. Therefore sailing and navigation techniques were crucial to its economy. What were these techniques? Did they have any relation to science?

We will restrict ourselves to a few observations on two forms of sailing that were long considered to have been unknown in Antiquity: windward sailing and open-seas sailing.

Did the "Ancients" know how to sail into the wind? Given that Greek and Latin have specific expressions for this action (ποδιαῖον ποιεῖσθαι, *facere pedem*) and that classical authors, including some of the best-known, mention it many times, even giving fairly detailed explanations of how it was done,[60] this would seem to be an open-and-shut question. Yet even here the primitivist position has had its defenders.[61]

From our point of view, a particularly interesting testimonium is that of the pseudo-Aristotelian *Mechanics*, giving a "scientific" — and correct — explanation of how close-hauled sailing works.[62] Here everything indicates that practice preceded theory,[63] but the need to include even sailing techniques into the framework of mechanics is an interesting example of the interaction between science and craft.

To sail the open seas, more than a way to roughly locate the cardinal points (for which a compass is useful if the skies are overcast) it is essential to have

- a coordinate system, that is, a scientific theory of geography;
- reliable charts; and
- a method to locate the ship with respect to the coordinate system.

With these theoretical instruments, even without a compass, one can correct for deviations from course caused by currents, leeway, storms and poor orientation on starless nights. By contrast, if one has a compass but is not able to determine the ship's position, the inevitable errors add up unchecked and the ship goes off course. The usefulness of the compass is

[60]See [Casson: SS], pp. 273–278, where several sources are quoted. We mention only Lucian, *Navigium*, 9; Pliny, *Naturalis historia*, II §128.

[61]The relevant article in the *History of technology* dismisses the skeptics and states that Roman ships could beat to windward, though only in "the most limited sense" ([Lethbridge], p. 574) until the invention of the lateen sail — of unknown date but probably due "to the Graeco-Romans" — made the task easy (ibid., p. 583–584).

[62]Pseudo-Aristotle, *Mechanica*, 851b:7–14. See [Casson: SS], p. 276, footnote 24, which points out that the explanation is correct.

[63]It is interesting that Philostratus attributes the discovery of techniques for windward sailing to the Phoenicians (Philostratus, *Heroicus*, i §2).

that it reduces the magnitude of the necessary corrections, but the corrections are always needed and yet cannot be made without the theoretical instruments listed above.

Spherical coordinates were relearned when a copy of Ptolemy's *Geography* reached the West, in the fifteenth century. The recovery of Hellenistic navigation instruments, including the plane astrolabe,[64] allowed mariners to determine latitude on the open sea through astronomical observations.[65] It was these "rediscoveries" that allowed the long open-sea voyages that had been impossible in the Middle Ages.

All of this would seem to suggest that the seafaring people who created spherical geometry and trigonometry, mathematical astronomy, mathematical geography, cartography and the astrolabe might also know how to use these instruments for sailing, if only because the tie between Greek astronomy and navigation was clearly present from the earliest, prescientific, days.[66] Yet until not long ago it was believed that the "Ancients" sailed only within sight of the coast, because this is what people did in the Middle Ages, when all the scientific theories needed for ocean sailing had been lost.

In fact, we know from the literature that Eudoxus of Cyzicus sailed several times between Egypt and India not by skirting the shore but along a direct ocean route from the Gulf of Aden.[67] There is also the very famous exploratory voyage in the North Atlantic made, probably in the late fourth century B.C., by the Massalian Greek Pytheas, which was described in his book *The ocean* (Περὶ ὠκεανοῦ). From fragments of this book and other information preserved by several authors,[68] we know that Pytheas reached places where the sun stays up all night in summer (such as the island of Thule, six days of sailing north of Britannia)[69] and even the *frozen ocean*

[64]That the plane astrolabe was a Hellenistic instrument known to Ptolemy was established in [Neugebauer: EHA]. Until then it was thought that its invention dated from much later.

[65]Measuring longitude is much harder, but one can do without it by first reaching the desired latitude and then sailing along that parallel to one's destination.

[66]The first "astronomical" work we know about is the *Nautical astrology* attributed to Thales (Simplicius, *In Aristotelis Physicorum libros commentaria*, [CAG], vol. X, 23:29–32 = [FV], vol. I, 80:3–8, Thales B1) or to Phocus of Samos (Diogenes Laertius, *Vitae philosophorum*, I §23).

[67]These trips, dating form the time of Euergetes II, were narrated by Posidonius and are mentioned in Strabo, *Geography*, II, iii §4. In that period the interest of the Ptolemies in sailing in the Indian Ocean is demonstrated by the naming, toward the end of the second century B.C., of a royal officer ἐπὶ τῆς Ἐρυθρᾶς καὶ Ἰνδικῆς θαλάσσης (with authority over the Red Sea and the Indian Ocean). See [Rostovtzeff: SEHHW], vol. II, p. 928 and neighboring pages (last four pages of Chapter VI).

[68]The fragments and testimonia are collected in [Pytheas/Roseman] and [Pytheas/Bianchetti].

[69]In Pliny (*Naturalis historia*, II §186), Pytheas is reported to have written that Thule's days and nights last six months; in the more reliable Cleomedes (*Caelestia*, I §4, 25:208 – 26:231, ed. Todd), that around the solstice the day lasts a month. According to Diogenes Laertius (*Vitae philosophorum*, IV §58), a certain Bion was the first to say there are places where day and night last six months.

(polar pack ice).[70] Strabo scolds Eratosthenes for using data from Pytheas, whom he considers a fibber, but in our days the credibility of Pytheas has been confirmed not least by the fragments transmitted by Strabo.

Trips in the Atlantic, toward the West, are mentioned by Diodorus Siculus, Plutarch and others.[71] Strabo even talks of attempts to circumnavigate the globe.[72]

Even fantastic tales such as Lucian's *True story* or Photius' summary of the lost novel *The incredible things beyond Thule* by Antonius Diogenes are echos of Hellenistic oceanic voyages produced in an age that no longer had the means to replicate them.[73]

Open-sea voyages certainly employed the theoretical instruments mentioned earlier. Did science also give sailors useful technological products? It may not have been by chance that Pytheas, the explorer of the North Atlantic, was a Greek from Massalia, a city recorded by Strabo as having been famous for the manufacture of instruments useful in navigation.[74] Strabo also mentions that in Massalia and in Cyzicus, as in Rhodes, the secrets of mechanical arts were guarded with particular care;[75] this may help explain our lack of information on the subject.

Another area where technology was useful to navigation was in the digging of canals. We mention here just the reactivation, around 275 B.C., of the old canal that connected the Mediterranean with the Red Sea;[76] in the imperial age it was no longer passable[77] and it took about two thousand years for navigation from one sea to the other to become possible again.

[70]Strabo, *Geography*, I, iv §§2–3; II, iv §1.

[71]Diodorus Siculus, *Bibliotheca historica*, V, xix–xx; Plutarch, *Vita Sertorii*, viii. Diodorus talks of a great island, many days to the west, with mountains and navigable rivers. The Carthaginians, he reports, had founded a colony there and even considered moving there en masse if their city were in grave danger. Many testimonia about Atlantic trips are collected and discussed in [Manfredi].

[72]Strabo, *Geography*, I, i §8.

[73]The protagonist of Photius' novel travels from the *Scythian Ocean* to the *Eastern Ocean* and from there, skirting the *Outer Sea*, he reaches Thule, an island in the North Atlantic (probably Iceland for Pytheas; but Ptolemy's Thule has been identified with the Shetlands: [Ptolemy/Toomer], p. 89, note 66). Lucian's tale, as the author explains, is a spoof of travelogues he regards as untrustworthy; though it appears as a grotesque mass of obvious falsehoods, we can be sure from the authors satirical intent that many elements of the narration were present in supposedly realistic works. At the same time, we know that Lucian's contemporaries no longer lent credence even to Pytheas' trip.

[74]Strabo, *Geography*, IV, i §5.

[75]Strabo, *Geography*, XIV, ii §5.

[76]The canal (of which archeological evidence still remains) joined a branch of the Nile with the Red Sea. It may have been dug as early as the Pharaohs and reactivated already once by King Darius of Persia. According to Strabo (*Geography*, XVII, i §25) and Diodorus Siculus (*Bibliotheca historica*, I, xxxiii, §§9–11), who mention the Pharaoh-era and Persian precedents as failed attempts, the canal was first placed in operation by Ptolemy II Philadelphus. But it was already known to Herodotus (*Historiae*, II §158) and is mentioned in an Iranian inscription by Darius.

[77]Already Pliny does not seem to be aware that the canal ever worked, though he mentions the

4.5 Naval Architecture. The Pharos

Little is known about techniques of naval architecture. We do know that the third century B.C. witnessed radical changes in this area, including an unexpected race toward ever larger ships.[78] It seems that the *cataphracts* used by Antigonus II of Macedonia against Ptolemy II Philadelphus in the naval battle near Cos (ca. 260 B.C.) were as big as fifteen quadriremes — and the race was not even on the final stretch.

In classical times the main type of vessel had been the trireme. Its name, in both Greek and Latin, is composed of the words for "three" and "oar", but the exact meaning of the term has been a subject of debate since late Antiquity. Some maintained that there were three rowers per oar, but the prevailing view today is that there were three stacked oar banks. In Hellenistic times there appear multiremes with rapidly increasing numbers, culminating with the forty-reme built by Ptolemy IV Philopator; such large numbers clearly require a different interpretation. The commonest opinion today is that they indicate the total number of rowers manning all oars comprising a vertical column. All we can be sure of is that these new terms indicate ships much bigger than their predecessors, and this sudden, drastic increase indicates qualitative changes in shipbuilding technology. One of the motivations for increasing the size of men of war, even at the expense of maneuverability, was probably the diffusion of artillery: growing use was made of ships as floating platforms that could accommodate catapults and other war engines.

Merchantmen also got bigger. Hiero II of Syracuse had a cargo ship built, the *Syracusia*, which Moschio described in a book of which generous portions are quoted by Athenaeus.[79] Thus we know that the ship, whose construction had required as much wood as sixty quadriremes, had on board, among other things, a gymnasium, a library, hanging gardens and twenty horse-stalls. Just before the *Syracusia*, Athenaeus discusses other ships of similar dimensions, built in Alexandria by the Ptolemies.

No remains of these enormous ships have been found, but undersea archeology has amassed a consistent record on smaller ships. One of the first important finds took place in 1954, near the islet of Grand Congloué, offshore from Marseilles.[80] It consisted of the remnants of a Hellenistic ship of the mid-second century B.C., about 23 meters long and lead-plated. We now know that lead plating, used to protect the hull from barnacles,

attempts made to build it (*Naturalis historia*, VI §§165–166).

[78] See [Casson: AM], [Casson: SS], [Morrison].

[79] Athenaeus, *Deipnosophistae*, V, 206–209.

[80] [Benoit].

was common in ships of the time; yet as late as the seventeenth century British and Dutch ships had no such defense.

The theoretical calculation of waterlines, which as we saw occupied Archimedes in his *On floating bodies*, probably gained in importance from shipbuilding innovations in terms of materials and size. One couldn't just multiply the dimensions of an existing ship by some factor in order to get fifteen or sixty times the tonnage: the task required theoretical planning, based among other things on mechanics and hydrostatics, novel theories of the time. The relationship between science and the building of these large ships is explicitly documented in at least one case: Athenaeus says that Archias of Corinth, the architect in charge of building the giant *Syracusia* for Hiero II, worked under the guidance of Archimedes.[81]

One would wish to know more about these enormous Hellenistic ships. We know that at Rhodes the penalty for spying on shipyards was death,[82] and that ships of the Rhodian fleet carried many specialists in technical services, including some of high rank. Yet we know neither what kinds of secrets were so zealously kept nor the duties of naval technicians.

One application of Hellenistic technology to navigation was the construction of the Pharos, the great lighthouse at Alexandria, around 280 B.C. Its total height was about 95 meters. The first level, with a square cross section, reached half that height; then came an octagonal tower and at the top there was a cylindrical room with the lantern, which was the true "wonder": its light, according to Flavius Josephus, could be seen 300 stadia (48 km) away.[83] This number seems right because it is approximately the maximum permissible by the curvature of the earth,[84] and one can suppose that, as for modern lighthouses, that was the limiting factor (else such a tall structure would be useless). To project the light that far a reflector would seem to be necessary; in fact we know that one was used, because Arab visitors to the locale talk of reflecting metal surfaces that lasted down to their time.[85]

The installation of the Pharos was considered so useful that other *pharoi* were erected at every important port of the Hellenized Mediterranean.

[81] Athenaeus, *Deipnosophistae*, V, 206d. Athenaeus also says that this ship was lead-plated (ibid., 207a).

[82] Strabo, *Geography*, XIV, ii §5.

[83] Flavius Josephus, *Bellum judaicum*, IV, x, 613.

[84] The distance to the horizon from a point at height h is $\sqrt{2Rh}$, where $R = 6366$ km is the radius of the earth. If h and d are the heights above sea level of the light and of the sailors who were supposed to see it, the light, if bright enough, would be visible at distances up to $D = \sqrt{2Rh} + \sqrt{2Rd}$. We know that h was around 95 meters; taking 10 to 15 meters as reasonable values for d, we get between 46 and 48.5 km for D, in excellent agreement with the value reported by Josephus.

[85] A brief survey of Arabic sources on the Pharos can be found in [Fraser], vol. II, p. 46. Its main structure still stood in the fourteenth century, when it collapsed after a series of earthquakes.

But Greek sources contain no overall description or a single technological detail relative to the Pharos, even though it was regarded in its own time as one of the seven wonders of the world. This confirms how reticent our sources are about technological products, in a case where the product itself is not in doubt. Because the only extant descriptions of the Pharos are by Arab historians, who visited it long after it had ceased to function, we know very little of its technology. Yet some conjectures can be made on the basis of its purpose and contemporary knowledge. First, we can imagine that the reflector consisted, as it would today, of a parabolic mirror, all the more so because the relevant theory arose precisely around the time of the construction of the Pharos.[86]

Even if it cannot be proved directly that scientists had a hand in designing the Pharos, it cannot be a coincidence that the first reflector in history appeared in Alexandria in the first half of the third century B.C., exactly when and where we first see scientists interested in conics and in catoptrics — the latter being precisely the scientific theory created for designing mirrors. (A longstanding misattribution gives Kepler the credit for first applying the classical theory of conics, but there were several earlier applications, e.g., to cartography, as attested by Ptolemy's *Geography*.)

Because a light beam in a fixed direction is not very useful in guiding ships, one can also suppose that the Pharos had a rotating light or reflector. This would also explain the cylindrical shape of the light room, replicated in every lighthouse ever known.

In the Middle Ages, catoptrics was lost, and with it the ability to build lighthouses. At best attempts were made to keep some of the remaining ancient ones in operation. Lighthouses started being built again in the twelfth century (Genoa got one in 1139), but these were vain attempts to imitate the ancient pharoi. In the *History of technology* that we have often cited we read:

> It was, however, only in the closing years of the seventeenth century that lighthouse-construction began in earnest, and on new and original lines that were to lead to the modern types of structure.[87]

[86]The scarcity of sources does not allow one to document this statement as precisely as one might desire. But we know that optics and the theory of conics were developed (particularly in Euclid's work) toward the end of the fourth century B.C. and that the theory of conics was applied to mirrors. A burning mirror is nothing but a parabolic reflector "in reverse", according to the principle of reversibility of optical paths. Since the focal property of parabolas was applied to the construction of burning mirrors around the middle of the third century (as we know from Diocles' comments on Dositheus: see footnote 32 on page 63), and since Archimedes knew the principle of reversibility of optical paths (see footnote 3.1 on page 63), the design of parabolic reflectors had certainly become possible by the middle of the third century. Since the Pharos was built around 280, it is likely that parabolic reflectors in fact predate burning mirrors.

[87][Goodchild], p. 524.

The "new and original lines" consisted in the use of reflectors based on the theory of conics. And the date when these structures reappeared might be guessed approximately without any recourse to historical documents about modern lighthouses. It is enough to know that a part of the ancient theory of conic sections, comprising the focal property of parabolas, was recovered in the first half of the seventeenth century (chiefly by Bonaventura Cavalieri[88]), and to make allowance for the time needed to overcome the technical problems attending the practical construction of lighthouses.

4.6 Hydraulic and Pneumatic Engineering

In the area of water engineering, the full extent of the practical relevance of Hellenistic scientific knowledge can hardly be overlooked.[89] Hellenistic aqueducts are remarkably well-documented by archeology. Remnants of water supply systems have been found at several sites, although many technical features are not yet well understood and in some cases even a Hellenistic dating has been challenged.

One of the main characteristics of Hellenistic aqueducts is the frequent use of pressure pipes, which overcame depressions of the terrain thanks to the principle of the inverted syphon (such pressure pipes are simply called "syphons" in the archeological literature). For a while the use of syphons was denied or regarded as exceptional, but they occur in at least seven of the nine aqueducts known for sure to be Hellenistic.[90] The relationship between the idea of the inverted syphon and the science of hydrostatics is obvious. The most remarkable syphon was at Pergamum (Figure 7.4 on page 208); it pushed water uphill to a height of perhaps 190 meters from the deepest point, and the pressure at the bottom must have been almost 20 atmospheres.[91] Note that we only know the technological level of these waterworks because of twentieth-century archeological finds.

[88]In *Lo specchio ustorio overo trattato delle settioni coniche...*, Bologna, 1632. Chapter 32 is titled "How the aforementioned mirrors [the burning mirrors of Archimedes] can be used to shine a light beam far away at night."

[89]For information on ancient hydraulic engineering, see [Bonnin], [Tölle-Kastenbein], [Hodge: RAWS], [Wikander: HAWT].

[90][Lewis: HP], p. 646. A much longer list of Hellenistic aqueducts with syphons can be found in [Hodge: A], p. 43, but for some of them a Roman dating has been proposed.

[91]See [Garbrecht]. Hellenistic engineers did not have pressure gauges and so could not measure the pressure at the bottom. But by the principle of Archimedes (note 73 on page 70), they could certainly calculate the pressure in the static case: if a U-shaped tube is full of water and in equilibrium, the pressure at the bottom is that of a column of water as tall as the difference in height between the bottom and the highest water level (which is the same on both sides because of the principle of communicating vessels). In an inverted syphon the downhill leg starts higher than the uphill leg ends and the pressure at the bottom corresponds to a height somewhere in between, as can be seen from considerations that could hardly have escaped a reader of Archimedes. Thus the

A very important technology, involving both hydraulic and mechanical engineering, is water lifting.[92]

The oldest machine for lifting water, the *shaduf*, is documented in Mesopotamia around 2300 B.C. and in Egypt around 1600 B.C., and is still used in many oriental cultures. It consists of a bucket at the end of a rod. The rod pivots about its middle, at a height of some five feet, and the other end has a clay block that serves as a counterweight. When the bucket is lifted to a certain height, the contents can be poured into a runoff channel.

In the Hellenistic period water lifting technology was revolutionized by the appearance of completely new devices, all of which date from no earlier than the third and no later than the second century B.C. The field witnessed no further progress in ancient times.

With the new machines, not only could much more water be lifted, but most importantly, unlike the shaduf (which had to be operated with some skill by a human), the necessary action can be "automated", being reduced to a continuous rotational motion that can be supplied by an animal or a natural energy source. The simplest device of this type is the *tympanum* or waterwheel described by Vitruvius.[93] It is a hollow cylinder (Figure 4.7)

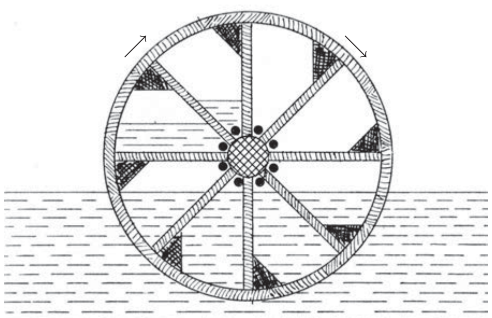

Figure 4.7. The tympanum. From [Drachmann: MTGRA], p. 150. Black dots and triangles are holes in the side walls, which otherwise are drawn as if transparent.

divided radially into wedges (usually eight) and set on a horizontal axis, the lowest part being immersed in the water that must be lifted; the openings are so placed that each wedge lets water in when it's submerged and lets it out when it rises above the axis. There is one defect: the water cannot be raised a height greater than the radius of the tympanum. To lift the water more one would use a series of buckets that were filled and emptied automatically. The buckets could be attached to the edge of a

engineers of the time could bracket the pressure within an upper and a lower bound, which in the case of Pergamum would differ by about 15%.

[92]For a complete and up-to-date survey of the field, see [Oleson: WL].

[93]Vitruvius, *De architectura*, X, iv §§1–2. The same wheel is cited in Chapter 61 of the Arabic version of Philo of Byzantium's *Pneumatica*.

wheel (so the height of lifting would be the diameter of the wheel), of, if a greater height was needed, the buckets could be attached to a chain connecting two wheels, one placed at the departure level of the water and one at the delivery level. As in the case of the tympanum, the raised water was poured into a runoff channel.[94] These machines could be arranged in series to lift water a great way (see Figure 9.2 on page 254).

The preceding machines have vertical wheels revolving around a horizontal axle. But for using animal power it is much easier to rotate a horizontal wheel about a vertical axle. Hellenistic gears allowed a solution to the problem, leading to a device now called the *sakiyeh* by the Arabs, who still use it (see Figure 4.8).

In the sakiyeh (then called simply μηχανή, "machine") the lifting is done by a tympanum or a bucket chain. The tympanum or the wheel that carries the buckets is solidly attached to a smaller coaxial wheel, which stays dry; this second wheel has cogs suitably meshed with those of a third, horizontal, wheel, which is pushed around its axis by an animal. A sakiyeh is first documented in a second century B.C. tomb fresco in Alexandria, where it is shown pushed by two oxen.[95] The use of animal power to lift water must have been very widespread, as it still is today.[96]

Another machine for lifting water introduced in Hellenistic times was the water-screw or Archimedean screw (κοχλίας). This well known tool is of a miraculous simplicity. The water flow it delivers is continuous; no vestige is left of the age-old use of buckets. The water is lifted directly, inside a tilted tube, by a helicoidal surface that fits snugly within the tube and rotates with it (Figure 4.9).

Both sakiyeh and water-screw appear as products of the new scientific technology. Indeed, they rely on the two new elements of mechanical technology mentioned earlier, the gear and the screw, and moreover they are the fruit of theoretical design. Thus, the helicoidal surface of the water-screw does not seem related either to earlier instruments or to natural objects having similar functions; at the same time, it is a natural object of investigation in the context of Hellenistic geometry.[97] This by itself would be enough to suggest that its origin is connected to scientific thought. But in fact we need not make conjectures: the invention of the water-screw is

[94]The bucket chain is described in the (certainly corrupt) Arabic text of Chapter 65 of Philo of Byzantium's *Pneumatica*. Remains of a water-lifting system of this type, dating from the second century B.C., have been found in Cosa in central Italy; see [Oleson: WL], pp. 258–261.

[95]See [Oleson: GRWL], pp. 184–185 or [Oleson: WL], p. 270.

[96]Philo of Byzantium, presenting an air-operated device of his invention for lifting water, declares it to be much better than the methods based on animal traction (*Pneumatica*, v, 84, ed. Prager).

[97]We have already mentioned on page 98 Apollonius of Perga's *On the cylindrical helix*, which, though of a later date, is significant in this context.

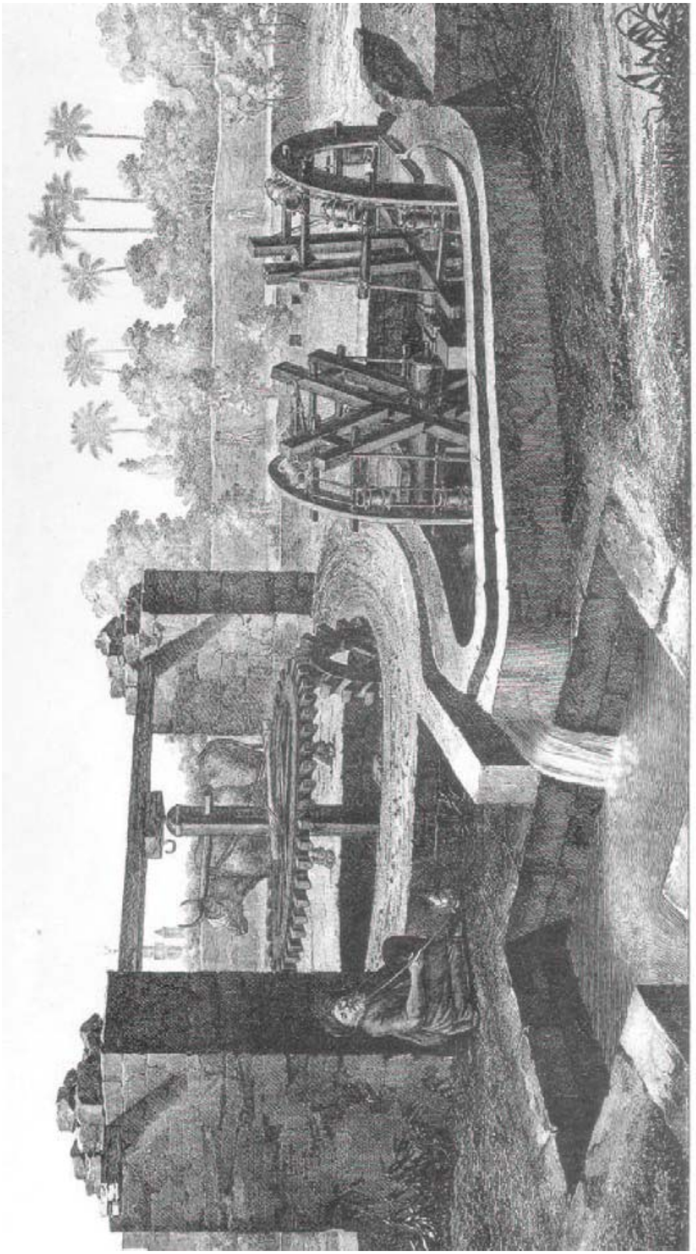

Figure 4.8. A sakiyeh-driven bucket wheel, as used in Egypt in the early nineteenth century. From *Description de l'Égypte, État moderne*, vol. II of plates, plate V, detail.

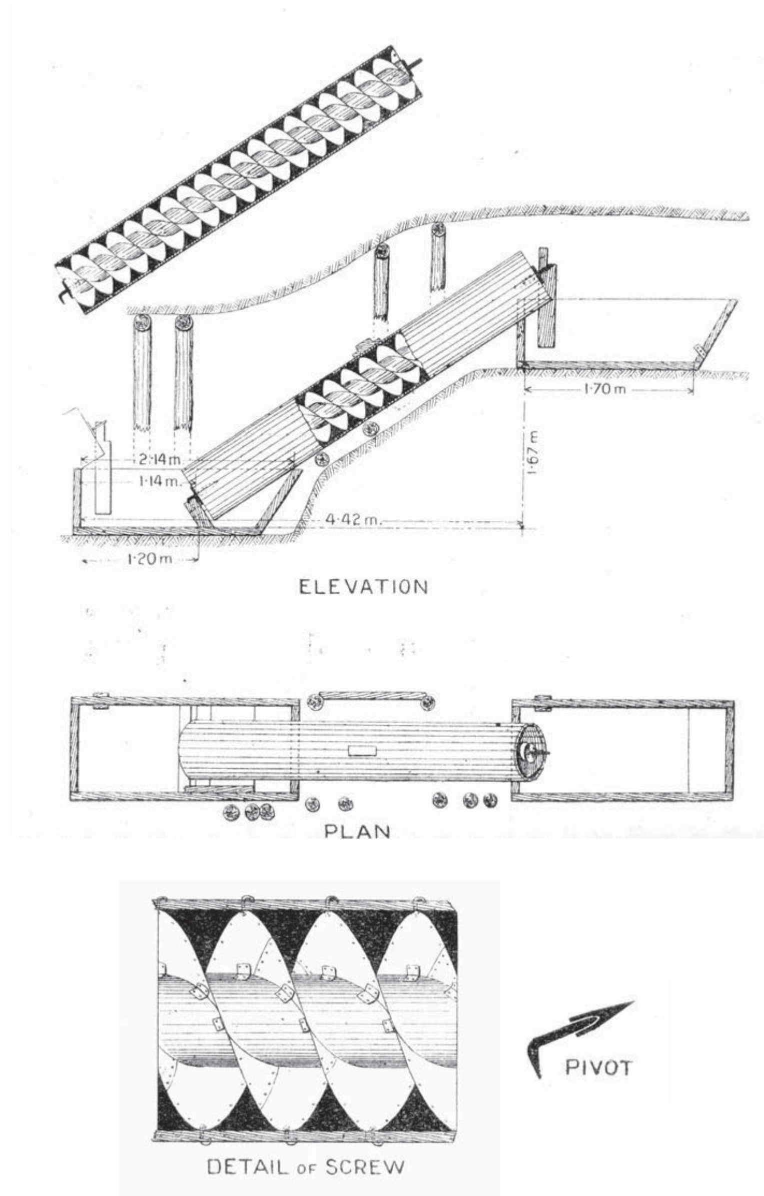

FIGURE 4.9. Roman-time Archimedean screw found at Centenillo Mines, Spain. From [Palmer], p. 330. Scale of plan and elevation, ca. 1:80; details of screw and pivot, ca. 1:20.

attributed to Archimedes by concurrent testimonia of Diodorus Siculus[98] and Athenaeus.[99] Its timing is further confirmed by the utter absence of material, documentary or pictorial evidence for earlier use.[100] Yet the tendency to cast as legendary all that has to do with Archimedes, without even considering the evidence, has led to persistent attempts to backdate it to Pharaoh-era Egypt.[101] The water-screw did become important to Egyptian irrigation under the Ptolemies (see page 252), but the copious iconographic documentation from earlier times, including much that is related to irrigation and flood control, fails to show Archimedean screws or screws of any type.

One further technological product of Hellenistic science useful for lifting water was Ctesibius' pump, already discussed on page 78. This device must still have been widespread in Roman times, because archeological digs have yielded twenty-five examples from early imperial times: thirteen in wood, eleven in bronze and one in lead (see Figure 4.10).[102] The bronze pieces bespeak skill in treating metal surfaces with the precision necessary to ensure a fit between piston and cylinder, as in the air catapult (page 109): that is, metal grinding techniques were already known.[103]

4.7 Use of Natural Power

The design of machines that could be operated through simple rotational motion, like those just described for lifting water, allowed the replacement of human by animal power, but also suggested the desirability of using

[98] Diodorus Siculus, reporting what is in all probability information from Agatharchides (second century B.C.), discusses the use of the water-screw to irrigate the Nile delta and attributes its invention to Archimedes (*Bibliotheca historica*, I, xxxiv §2). Diodorus returns to the water-screw in V, xxxvii §§3–4 (a passage thought to be based on Posidonius), mentioning its use in draining Spanish mines, and promises to discuss "in detail" (ἀκριβῶς) all of Archimedes' inventions in the books devoted to his time (these books have been lost).

[99] According to Moschio's book on the *Syracusia* (as quoted in Athenaeus, *Deipnosophistae*, V, 208f), that ship's bilge water "was pumped out by a single man with the water-screw, an invention of Archimedes".

[100] See [Oleson: WL], pp. 242–251.

[101] Thus the influential *History of technology*: "According to legend, Archimedes invented and constructed many machines … but … the evidence is vague. The only machine which is both associated with his name and precisely known is the 'Archimedean screw' for irrigation. Here it seems that legend has incorrectly attributed to him a contrivance that had long been in use in Egypt" ([Gille: Machines], p. 633). But this thesis had already been refuted in [Feldhaus] and in [Rehm]: "That the Archimedean screw … is an invention of the man whose name it bears is as well-attested as it can possibly be" ([Rehm], p. 146, note 28).

[102] [Oleson: WL], p. 272, note 96.

[103] This is stressed in [Woodbury], p. 30. Compare also [Philo/Prager], p. 12, note 30. Earlier it was thought instead that the need for ground metal joints disqualified an instrument from being contemporary with Philo ([Drachmann: KPH], p. 50).

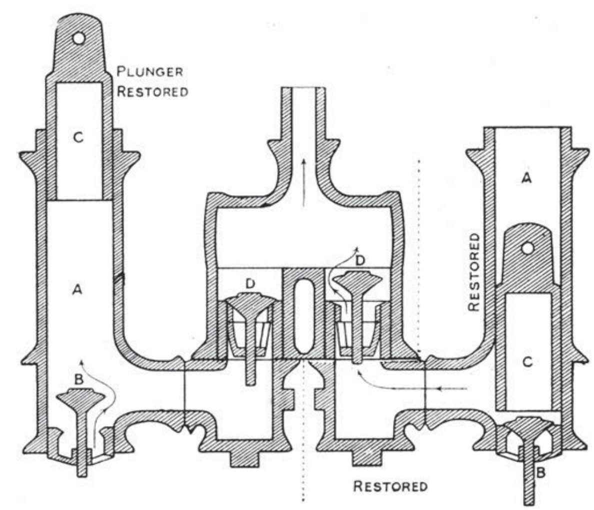

FIGURE 4.10. A bronze pump from the early imperial period found in Bolsena, Italy, boasts beveled spindle valves. The one-third to the right of the dotted line, as well as the left piston, have not been found. From [Walters], p. 121.

inanimate power sources. In Hellenistic times several such sources were used.

Running water. The earliest water mill we know of was connected with the palace of King Mithridates VI of Pontus (120–63 B.C.) at Cabeira, but the testimonium gives no details.[104] Next we read in an epigram of Antipater of Thessalonica (first century B.C.):

> Rest your hands from the mill, flour-women: sleep in, even if the cocks crow to announce dawn, for Demeter has assigned the labor of your hands to the nymphs; and they, tumbling down the very edge of the wheel, spin the axle, which with its curved rays [or cogs] turns … the Nisyrian millstones.[105]

This agrees best with a vertical water mill;[106] this is anyway suggested by the absence of good evidence for the use of horizontal water mills in Antiquity.[107] The type of configuration is of some importance: a vertical wheel is more efficient than a horizontal one, but it requires that the ro-

[104] Strabo, *Geography*, XII, iii §30.

[105] *Anthologia graeca*, IX, 418. The Greek for "tumbling down the very edge of the wheel" is κατ' ἀκροτάτην ἁλλόμεναι τροχιήν; the meaning "top" instead of "edge" is also possible.

[106] [Wikander: WM], p. 375: "A few scholars still take the epigram as referring to a horizontal-wheeled mill, but to me an unprejudiced analysis unequivocally favors the overshot variety."

[107] [Wikander: WM], p. 376.

tation of the paddle-wheel (about a horizontal axis) be converted to the rotation of the millstone (about a vertical axis); thus we have evidence for right-angle transfer gears. For centuries the vertical mill was called Vitruvian and attributed to the Romans, for no better reason than that Vitruvius described it without explicitly mentioning its Hellenistic origins.[108]

Marc Bloch, in his essay on water mills, says they were invented in the "Mediterranean East",[109] among other reasons because otherwise there would be no justification for Vitruvius to use a Greek word for them. The vertical water mill might have originated from the idea of using a sakiyeh in reverse.[110] Interestingly, Bloch finds it odd that this geographical area should have been the cradle of such an invention, because Mediterranean streams are not very reliable power sources due to seasonal variations. He seems not to take into account that among the "Mediterranean people" were the creators of both mechanics and hydraulics. Bloch, a founder of the "history of material civilization", was one of the greatest twentieth-century historians, but as a medievalist — a specialist in the study of a prescientific society — he was not in the best position to be the first to clarify the long misunderstood link between Hellenistic science and scientific technology.

The testimonia of Strabo, Antipater and Vitruvius show that water mills (vertical, to boot) existed in the first century B.C. The gears needed to build such mills were used in the sakiyeh, which is documented from the second century B.C. (see page 120); they may actually go back to the first half of the third century.[111] Most scholars think the invention of the water mill dates from around 100 B.C., but recently M. J. T. Lewis has argued that this was one of the first products of Hellenistic science, owed to Alexandrian scientists of the first half of the third century B.C.[112]

Wind. The origins of the windmill are obscure. The earliest documented specimen was built for Caliph Omar I (634–644) by a Persian who said he could do it. However, it is likely that in Sistan (the southwestern part of today's Afghanistan) windmills predated Islam.[113]

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[108]Vitruvius, *De architectura*, X, v. Forbes ascribes the invention of the vertical mill to "a Roman engineer of the first century B.C." ([Forbes: Power], p. 595), and states without supporting arguments that the Antipater epigram refers to an unspecified water mill rather than to a vertical mill (ibid., p. 593).

[109][Bloch: Moulin], p. 539.

[110]As noted in [Bloch: Moulin], p. 541, such an origin would explain why Vitruvius discusses wind mills together with water-lifting devices (*De architectura*, X, iv–v): he would simply be keeping the order adopted by his source, and that in turn would be the order of historical development.

[111][Lewis: MH], pp. 56–57.

[112][Lewis: MH], pp. 33–61. A summary of the arguments is given in [Lewis: HP], pp. 644–645.

[113][Hill: E], p. 784.

Heron, in discussing a pipe-organ moved by a wheel with paddles, describes the wheel by saying it's similar to an *anemourion* (ἀνεμούριον), evidently something the reader was expected to be familiar with.[114] The word (a compound whose first part means wind) is not otherwise attested as a common noun, but the context makes it clear that it means something that uses the wind to create rotational motion: in the manuscripts the passage is illustrated with drawings showing the organ with its paddle-wheel, which moves a piston (Figure 4.11).

The same word was in Antiquity also a place-name — given, for example, to two promontories in Cilicia.[115] In this capacity its likely meaning would of course be "windmill", a conspicuous landscape feature (unless there was a homonym that happened to mean "windy mountain").

Many scholars have felt that the Heronian passage can be disregarded because it is not confirmed by other writings. Heron presumably mentioned the anemourion in a moment of distraction, forgetting that it had not been invented yet. We know that he was given to such lapses.

Steam. Heron describes two steam engines: a demonstration model (the *eolipile*, an enclosed round vessel with curved vents through which steam came out, causing it to revolve: see Figure 4.12), and a device used to open the doors of a temple when a fire was lit.[116] Heron's steam engines have generally been considered oddities built just to amaze, but if we consider their construction in the framework of a civilization that had started to employ water power, they can be seen instead as arising from a search for energy sources that do not depend on geographical accident (see Section 4.9 for a fuller discussion).

It should be stressed that modern steam engines are not at all, as is often implicitly assumed, an invention independent of the Hellenistic engines; there is a continuous line of descent. Heron's expositions were studied carefully by Leonardo da Vinci, among others. The possibility of using steam as a source of power was then mooted by Giambattista della Porta in his *Pneumaticorum libri tres* (1601), based on Heron's *Pneumatica*.[117] The first steam engine actually built in modern times seems to have been the one described in 1615 by Salomon de Caus; it operated an ornamental

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[114]Heron, *Pneumatica*, I, xliii, 204:16.

[115]Strabo, *Geography*, XIV, v §3; XIV, v §5.

[116]Heron, *Pneumatica*, II, xi; I, xxxviii.

[117]Della Porta's work was soon translated into Italian under the title *I tre libri de' spiritali* (1606). With the end of ancient science, the meaning given to *pneuma* by Hellenistic scientists had been forgotten, and the word was generally translated "spiritus" or its reflex in modern languages, in a sense similar to the one it had had in the classical period. Reversing in this way the path traveled by Ctesibius and Herophilus, we get the amusing result of the same adjective being used for spiritual beings and steam engines.

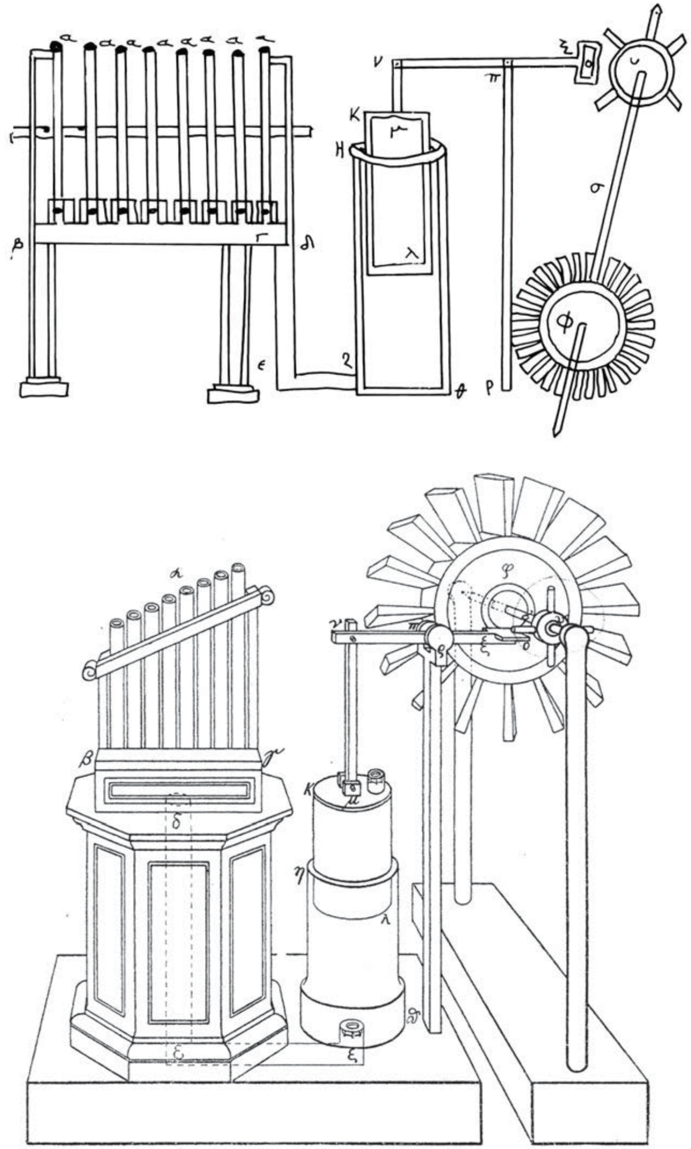

FIGURE 4.11. Wind organ described by Heron. Top: the figure as transmitted in the Codex Marcianus 516, Venice; taken from [Lewis: TH], p. 144. Bottom: reconstruction by W. Schmidt based on the manuscript's figure and the textual explanation; taken from [Heron: OO], vol. I, p. 205.

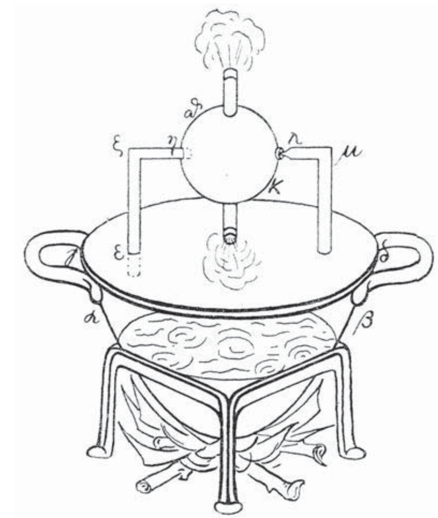

FIGURE 4.12. The eolipile (Schmidt's rendition). From [Heron: OO], vol. I, p. 230.

fountain intermittently. Thus the inheritance from Heron was so complete that it even concerned the end to which the machine was put. Heronian technology hung on for another century in various hands, until it became economically convenient to start building steam engines for industrial use — which is to say, when the rapidly growing energy needs of nascent industrialization no longer could be met by watermills alone.

4.8 The Antikythera Mechanism

In 1902, near the wreck of a ship that foundered by the islet of Antikythera, between the Peloponnesus and Crete, divers found some corroded bronze fragments that at first appeared to belong to some clock-like object with complicated gears. The find dates from the early first century B.C.,[118] but it appeared so qualitatively different from any known object from classical Antiquity that it gave rise to absurd speculations of all sorts.[119]

The partially readable inscriptions on the fragments make it clear that the mechanism has to do with the motions of the sun and the moon.

[118] In 1985, Jacques Cousteau found on the shipwreck coins issued in Pergamum in 86 B.C. (see [Casson: AM], p. 224), indicating that the wreck probably took place not long thereafter.

[119] Humorously recounted in [Price: Gears], p. 12. The early date of the find is secure, among other reasons because a paleographical study of the inscriptions shows they are contemporary with the shipwreck. For an accurate description of the finds and a reconstruction of the whole machine, see [Price: Gears]. The description in [Price: Instruments] was written before the author studied the fragments in person and is completely superseded.

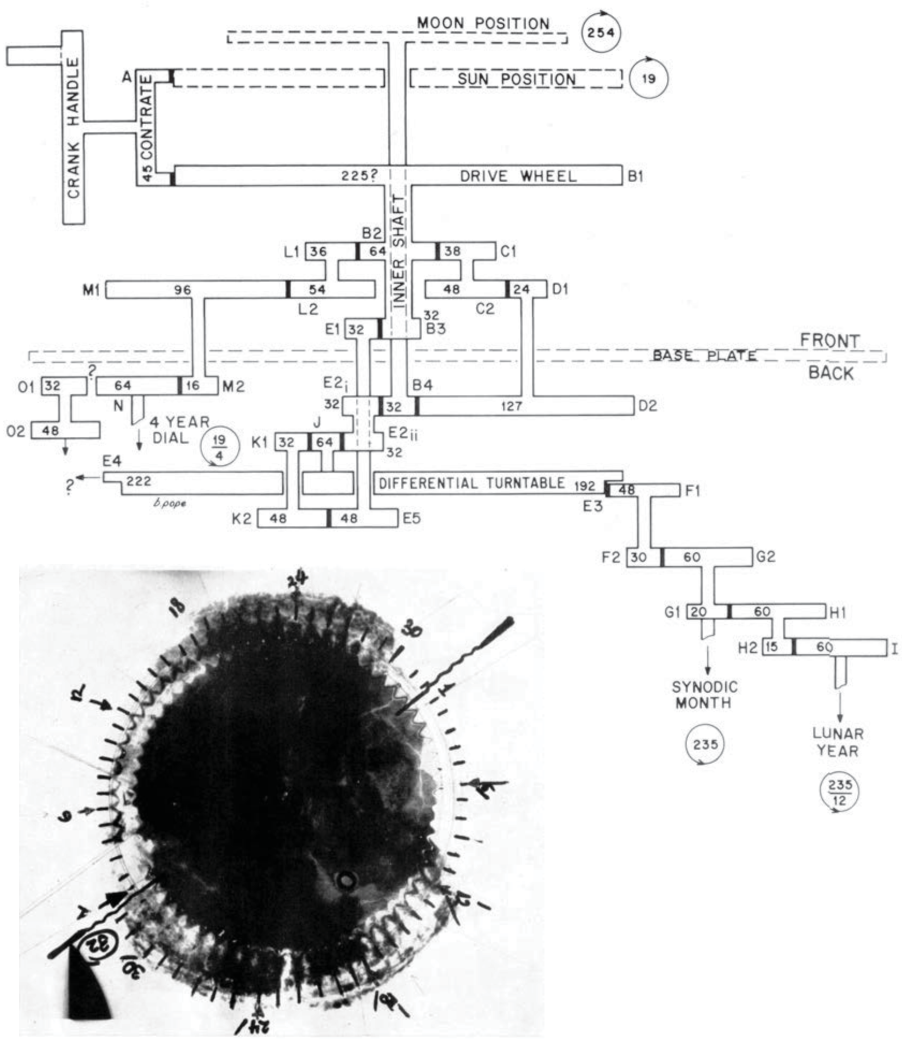

FIGURE 4.13. Top: Sectional diagram of gearing system found in Antikythera, as reconstructed by Price. Teeth counts are indicated. The remnants of the device lie in several fragments, each of which contains a number of gears or partial gears stuck together through corrosion. Radiography allowed the determination of the position and teeth count of most individual gears (see bottom inset with Price's working notes). From [Price: Gears], pp. 33, 28.

According to Price's reconstruction, it was a sort of perpetual calendar, allowing the calculation of the phases of the moon, past and future. To this end a gear train converted the motion of a wheel representing the solar cycle to another representing the sidereal revolution of the moon, according to the ratio of 254 lunar revolutions to 19 solar years.

Technologically speaking two features stand out. One is the complexity of the mechanism, which uses at least thirty gears. This intricacy is what makes one instinctively assign the machine to the category of clockwork. The second and most remarkable feature is the presence of a differential turntable, a mechanism that allows the addition or subtraction of angular velocities. The differential was used to compute the synodic lunar cycle (moon phase cycle), by subtracting the effects of the sun's movement from those of the sidereal lunar movement.

Price's verdict about the significance of the Antikythera mechanism is revealing: "We must suppose that from both Heron and Vitruvius we underestimate what was available in gearing technology in their times."[120] Indeed, he felt that the existence of this single object of "high technology" is enough to radically alter our ideas about classical civilization and lay to rest once and for all old clichés to the effect that the Greeks scorned technology and that the easy availability of slave labor led to an unsurmountable gap between theory and experimental and applied sciences.[121]

4.9 Heron's Role

The most famous written documentation on Hellenistic technology that has survived consists of the works of Heron of Alexandria. Heron, who lived most likely in the first century of our era,[122] described a great many "marvelous engines", especially in two of his works: the *Pneumatica* and the *Automata*. An example is a vending machine that, upon the introduction of a five-drachma coin, would dispense a fixed amount of liquid.[123]

[120] [Price: Gears], p. 54.

[121] [Price: SSB], p. 42; [Price: Gears], p. 51. Price's comments seem to be informed by a personal experience; he held very different views on Greek technology in [Price: Instruments], written twenty years earlier.

[122] The most useful datum for dating Heron is the lunar eclipse used as an example in the *Dioptra*, which was unequivocally identified as that of March 13, 62 A.D. in [Neugebauer: Heron]. Heron might conceivably be transmitting an account of an earlier author, but it is likely that he used a recent eclipse. In any case 62 A.D. is a *terminus post quem*. One piece of evidence against the dating once proposed by Heiberg and Heath as the most probable, namely the third century A.D., is that Sextus Empiricus, around 200 A.D., seems to refer to Heron, though not by name (see pages 322–323).

[123] Heron, *Pneumatica*, I, xxi.

Heron knows and uses precision screws, rack gears, reduction gears, transmission chains, camshafts,[124] pistons, valves of various types, and more. He puts to use many properties of fluids, the principle of jet propulsion and the sources of natural power already mentioned: water, wind and steam.

Conceptually, one of the most interesting features of Heron's machines is the pervasive presence of feedback mechanisms, capable of taking a system back to its initial state after being displaced from it, or of maintaining a device in a steady operation state (until the energy resource being used is exhausted).

As we shall see in this section, modern attitudes toward Heron's writings have generally been misguided by the often amusing uses to which he puts technology and by prejudices about classical civilization. A passage of Dijksterhuis may be quoted as a good example of the common opinion:

> He has as many physical and technical possibilities at his command as the eighteenth-century inventors who by their work made the industrial revolution possible. Why, one is continually inclined to ask, does he not accomplish anything comparable to their work, and why does he confine himself to the construction of instruments without any practical utility?[125]

To see things in perspective, one must first of all recognize that the technology described by Heron is too complex to be the creation of a single inventor. When dealing with theoretical arguments, too, he appears as more of a compiler and transmitter of information that as an innovator, and ever more so as our knowledge of Hellenistic science advances. For instance, he was once considered the inventor of "algebraic" methods, but the decipherment of cuneiform texts has shown that such methods had long been in use in Mesopotamia;[126] *Heron's formula* for the area of a triangle is attributed to Archimedes by the Arabic mathematician al-Bīrūnī;[127] Heron's *Definitions* are avowedly a compilation made for divulgation purposes; the principle of communicating vessels, though not discussed explicitly in the pre-Heronian texts that we possess,[128] is implicit in

[124]The cam (eccentric wheel) and camshaft convert circular motion into reciprocating (alternating linear) motion. They were used, for example, in the pipe-organ already discussed on page 126 (Heron, *Pneumatica*, I, xliii). They were long thought to have been invented in medieval Europe or in China.

[125][Dijksterhuis: MWP], p. 73.

[126]On this topic see, for example, [Neugebauer: HAMA], vol. II, p. 847.

[127][al-Biruni/Suter], p. 39.

[128]This absence has led to statements such as "the first attempt to explain their action [i.e., of syphons] was that of Hero" ([Forbes: HES], p. 669).

Archimedes' postulate and was the basis for the Hellenistic syphons built centuries before Heron.[129] Other examples could be added.

Heron's personal contributions concern at best some of the applications he describes, not the underlying technology. Thus we should inquire how far back the technology goes. We list here a few relevant facts.

Heron's use of mechanical technology and fluid technology is based on mechanics, hydrostatics and pneumatics, scientific theories that all date from the third century B.C. The same century witnessed the invention of foundational pieces of technology such as precision screws, gears and valves, and an extraordinary technological development demonstrated by, among other things, the already discussed testimonia on shipbuilding and war engines.

The link between Heron and his sources has been carefully analyzed in the case of the treatises on artillery by Marsden,[130] who concludes that "in spite of his own [Heron's] date, the technical content of his work belongs in the third century B.C."[131] Marsden cites as evidence, in particular, that details of weapons described by Heron (following Ctesibius' work) had already been critiqued by Philo of Byzantium,[132] and that Heron starts his *Belopoeica* by stressing the importance of artillery in safeguarding cities: a comment obviously lifted from sources that predated the establishment of the Pax Romana.

The building of automata also goes back to the third century B.C., more precisely to the first half. According to Vitruvius (who cites as his source the *Commentaries* of Ctesibius), the clocks built by Ctesibius could activate automata at preset times.[133] Callixenus, writing about a famous parade organized by Ptolemy II Philadelphus, mentions a statue of Nysa that would stand up by itself from a sitting position, pour libations of milk and sit down again.[134] Heron, in his book on the automatic theater (see page 139), mentions several times a work of Philo of Byzantium on the same subject, criticizing details and boasting that he could improve on Philo in some particulars, such as the number of simultaneous movements of each automaton. Evidently, the technology in this area was not very different in early Hellenistic times from what Heron describes. In particular, Lewis points out that many automata of the third century B.C. could not

[129]See pages 118–118.

[130]In [Marsden: HD]; see also [Marsden: TT], pp. 1–2.

[131][Marsden: HD], p. 3.

[132]This fact was once thought to imply that Heron preceded Philo (see [Heath: HGM], vol. II, p. 302, for example). The fallacy of this logic hinges, as it does in many analogous cases, on the falsehood of a premise implicitly taken as obvious: that the interval between the two scientists was one of technological progress and not regression.

[133]Vitruvius, *De architectura*, IX, viii §§4–5.

[134]Athenaeus, *Deipnosophistae*, V, 198f.

have worked without a mechanism for converting rotational motion into reciprocating motion, like the ones used by Heron.[135]

Ctesibius and Philo of Byzantium had of course known and described many pneumatic devices centuries before Heron. Indeed, sources for certain parts of Heron's *Pneumatics* have been identified and date from the third century: besides the works of Philo of Byzantium and of Ctesibius (the latter for the water-organ, for instance), Heron probably also relied on the writings of Strato of Lampsacus (especially for the introduction, as Diels was the first to propose).

The feedback mechanisms used by Heron also go back to the Hellenistic period; they are used systematically in Philo of Byzantium's *Pneumatica*.

In part, the problem of dating Heron remained open for centuries precisely because in his work he often mentions early Hellenistic scientists — Archimedes, Ctesibius, Philo of Byzantium — but never any contemporaries that we know about. Obviously Heron, contrary to what is often said, does not belong to a school that had stayed alive since Ctesibian times; he derives his knowledge from reading ancient works. Indeed, as already noted,[136] Alexandrian scientific tradition was traumatically interrupted in 145 B.C., and the main element of continuity during the dark centuries separating those tragic events from imperial-age Alexandria was the Library.

In sum, there is every evidence that most of the technology described by Heron goes back to the third and second centuries B.C. More than that, certain hints suggest that the technology had already been lost to some extent by the time of Heron's writing.[137] Namely:

The dioptra and other Heronian devices include small metal screws, but when Heron deals with the manufacture of screws, in the *Mechanics*, he states only the two methods mentioned on page 97, which are easy to implement (especially the first), but only work for big wooden screws. There are additional reasons to think that Heron's *Dioptra* is based on Hellenistic-period sources.[138]

In the *Mechanics*, in a theoretical context, Heron describes reduction gears, but in the *Automata*, when he must repeatedly transfer movement from one wheel to another, he never uses gears, only friction devices. This is easily explained if we assume that in his time it had become difficult to procure not just precision metal screws but even gears.

[135] [Lewis: MH], pp. 84–88.

[136] See Sections 1.2 and 3.2, especially note 60 on page 69.

[137] In other words, it may be that Price (see page 100) is right in saying that Heron's dioptra was an "invention whose complexity exceeded the technical resources of its time" — but not because it was "premature" in Heron's time, rather for the opposite reason.

[138] See note 24 on page 272.

In the few cases where direct documentation is available, the technology of earlier centuries appears more sophisticated than Heron's; for example, from his work we would never suspect the existence of differential gears of the type used in the Antikythera machine, which predates Heron by about 150 years.[139]

We see that Heron's writings provide precious, but late and incomplete, documentation about the level of Hellenistic technology, and cannot be used unless with great caution to evaluate the motivations that led to that technology's appearance and development in a completely different cultural and political climate, centuries earlier. Some relevant observations toward such an evaluation:

Mechanics and pneumatics arose in close connection with technology and, as we have seen, allowed the creation of many economically useful devices as early as the third century B.C.

Heron himself describes many devices that are not at all just amusements: artillery weapons, various types of press, machines to lift weights, the dioptra, the screw maker.

A growing number of aspects of Heronian technology are now known to have had serious uses in Hellenistic times. For example:

- We knew that some of Heron's models were moved by water power. Now we know that those models reflected real-life installations based on efficient vertical water wheels, which had been in use long before.
- We knew about the use of fluid pressure in Heron's "toys". From twentieth-century archeological excavations we have learned that the same principles were used to build pumps in widespread use and pressure pipes that supplied cities with water.
- We knew about the use of hydrostatic principles in Heron's works. The same principles, as we have seen, were very likely used in the third century B.C. in naval technology.

Sometimes Heron obtains the spectacular effects he seeks by altering simple experimental devices. Take for instance the thermoscope described by Philo in chapter vii of his *Pneumatics*, a simple setup to demonstrate that air expands when heated. It has a syphon that dips below water level in two partly filled containers, one airtight and the other open. When the air in the closed container is heated in the sun or by a fire, it pushes some

[139] See Section 4.8 above. Also Archimedes' planetarium must have reproduced the correct ratio of angular velocities not only of sun and moon, but also of the planets. Since we know that various gears were already in use in Archimedes' time and that science was already on the wane at the time of the Antikythera machine, we may imagine that Archimedes' planetarium was technologically even more sophisticated.

water through the syphon to the other container, and the opposite happens when the temperature drops. Essentially the same object appears in Heron's *Pneumatica*, II, viii, where it is disguised as a fountain having the amazing property of spouting only when it is exposed to the sun. The same setup with a fire as the source of heat appears in I, xii, as a trick to make automata pour libations onto an altar when a fire is lit. The construction is repeated with sound effects — a hissing snake — in II, xxi. Likewise the principle of jet propulsion behind the demonstration device called the eolipile (Figure 4.12) is used by Heron in II, iii for ornamental purposes (Figure 4.14).

At other times Heron refers to a preexisting technology with practical uses when introducing a derived entertainment-related use. For example, in the *Pneumatica*, I, x, he explains one of his famous fountains, which relied on pressure to create an astonishing gush of water. A key component is the flap valve, which he introduces with the words "called an assarium by the Romans". He proceeds to describe the valve in detail in I, xi (though it was obviously not a novelty), and uses it again in I, xxviii, in a pump that serves as a fire extinguisher. The two constructions are very similar: in either case a water jet is driven by air that is compressed thanks to pistons and valves, and the basic designs largely coincide. Since we know that

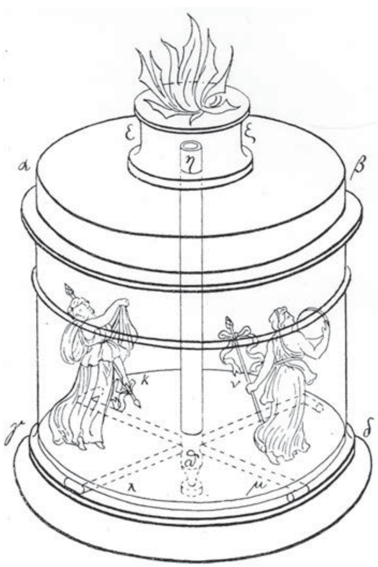

FIGURE 4.14. Rotating altar moved by hot air, in Schmidt's rendition; the figures in the glass case would start to move when the fire was lit. From [Heron: OO], vol. I, p. 215.

Ctesibius invented this pump, we conclude that valves had been playing a useful role for centuries, and that the purely ornamental fountain was an offspring of the pump. Elsewhere Heron or one of his predecessors adapts the mechanism of water clocks to light-hearted ends. Heron's use of a paddle-wheel "like an anemourion" to operate a pipe-organ (page 126) may be a further example of the same pattern.

Automata, which had been of interest since the third century B.C., are precisely mechanisms able to transform a simple rotation into complex motions, similar to those needed in human tasks. Mechanisms of this type had been used from early Hellenistic times for military purposes and to save labor, as we saw in the case of the repetition catapult and the sakiyeh. We shall see in Section 9.3 examples of automation in agriculture during the same period.

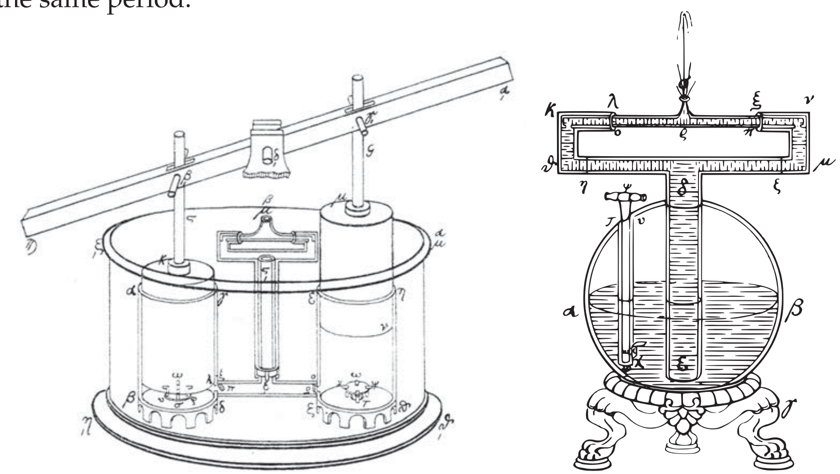

FIGURE 4.15. Two applications of the valve in delivering a jet of water under pressure (Schmidt's redition). Left: The Ctesibian pump configured for use as a fire hydrant, in Heron, *Pneumatica*, I, xxviii ([Heron: OO], vol. I, p. 133). Right: A Heronian fountain, *Pneumatica*, I, x ([Heron: OO], vol. I, p. 73).

Now consider that eighteenth-century inventors did not just have, in Dijksterhuis' phrase, *as many* physical and technical possibilities as Heron: they had *the same* possibilities. Because technology, like scientific theories, is not predetermined by our genes, being rather a cultural product, this coincidence should give us pause. It can only be due to the fact that at the source of eighteenth-century technology lay the Hellenistic works, studied since the twelfth century thanks above all to manuscripts available through the Arabs in Spain, and later, more intensively, by Europeans in general from the fifteenth century on. Thus it is that, in the early modern era, knowledge of many technological items such as syphons, differential gears and steam engines preceded any practical application for them. This would seem very peculiar, except in the light of the preceding observation, which explains it outright.

As Dijksterhuis implicitly observed, the Industrial Revolution in Europe is based in an essential way on devices described in Hellenistic works, and Heron's in particular. This shows how important the knowledge that Heron recorded may have been for production technology in Antiquity. Only wisps of information about Hellenistic production technology have come down to us, but this should not cause surprise, since so little remains of Hellenistic literature at all. And it is safe to say that the exceptional transmission of Heron's works through prescientific societies of low technological level owes a lot to the amazing and entertaining character of the devices described.

In conclusion, many of Heron's devices could be seen as byproducts of Hellenistic technologies originally created for other purposes, having managed to survive and thrive in the new conditions of the imperial age precisely, likewise, because of their amusing nature. If instead we are to accept the common opinion about Heron, it would be necessary to draw the depressing conclusion that modern European civilization, to develop its own technology, could do no better for centuries than to continue to draw ideas form the isolated work of an ancient toy builder.

4.10 The Lost Technology

Technology has always been treated as a sensitive subject. For centuries the Romans imported via intermediaries the coveted "ferrum sericum" produced in far regions of the Orient,[140] without being able to learn even its exact provenance, let alone how to produce it. Silk was imported from China for over a thousand years without its origin being known. Concerning the Hellenistic period in particular, we have mentioned (page 114) Strabo's remark that at Rhodes, Cyzicus and Massalia mechanical technology was shrouded in secrecy, and we will see (page 165) that chemical lore in Egypt was equally secret. In Hellenistic kingdoms, confidentiality about technological procedures was encouraged by the control that rulers held over the main industries[141] and, in the case of Egypt and Mesopotamia, by the ancient tradition of priestly control over areas of production reserved for temples: an approach maintained by the Ptolemies toward indigenous industries, but supplemented by other control systems in Greek commu-

[140] Pliny, *Naturalis historia*, XXXIV §145; Orosius, *Adversus paganos*, VI, xiii §2.

[141] Chapter 6 will discuss the interest of rulers in production technology.

nities. Thus it is not surprising that we know virtually nothing about, say, kiln techniques, weaving techniques, or methods used for producing perfumes or particular types of glass.

Obviously, then, it cannot be claimed that direct traces of all relevant Hellenistic technologies should be retrievable from the few sources that have reached us.

The idea that the "Ancients" had very powerful technology lived on throughout the Middle Ages. It is reasonable to think that the origin of this tradition lay in memories of ancient knowledge, since technology really had been in many cases superior to what it became in medieval Europe, as we have seen, and also because this admiration for ancient technology tended to be rekindled in times and places were ancient works were being recovered and read.[142] That some medieval authors may have had access to works no longer available today (see page 333) adds interest to their testimony, although the pervasive contamination of elements grounded in truth with other types of elements, often magical,[143] makes the use of such testimonies problematic. Consider, for example, what Roger Bacon wrote in the thirteenth century:

> It is possible to make sailing devices without rowers, so that great ships can move by river and sea with a single man in control, faster than if they were full of men. Likewise it is possible to make cars not pulled by any animal, which move with incalculable speed; we think the *currus falcati* which the Ancients used in combat were of this kind. It is possible to make flying machines where a man sits in the middle turning some device by means of which artificial wings flap through the air, like a bird in flight. It is also possible to make instruments small in themselves, but able to raise and lower almost infinite weights, whose usefulness on occasion cannot be surpassed. ... One might also easily make a device with which a single man can drag to himself a thousand, against their will, and attract other things as well. And devices can be made for walking in the sea or in rivers, going down to the bottom without bodily harm: Alexander the Great used them to look at the secrets of the sea, as Ethicus the astronomer tells us. And these things were made in Antiquity and have been made in our times, this much being certain — except for the flying machine, which I have not seen, nor do I know any who

[142] We will return to this point in Sections 11.1 and 11.2.

[143] Two examples of this phenomenon in the area of optics: the belief in the magic powers of crystal balls (probably arising from knowledge of the magnifying properties of spherical lenses) and the name of "magic lantern" given to a simple projector that goes back at least to the Arabs.

have seen it; but I know a scholar who has contrived to flesh out the design.[144]

At fist sight this may seem like a list of wishful fantasies. But first of all some objects mentioned are real: weight-lifting machines had certainly been built in Antiquity with greater efficiency that in Bacon's Europe. For other objects it is easy to identify a literary origin: the diver used by the Macedonian ruler appears in the *Romance of Alexander*,[145]a Greek source which Bacon seems to be indirectly acquainted with, and rich in legendary elements that would not be easily recognized as such in the thirteenth century. In other cases it is not hard to separate the distortions from a probable kernel of preexisting truth: the machine with which a single man could *drag* a thousand was of course unfeasible, but replacing "drag" by "balance" we have a machine of high mechanical advantage, such as the ones used for lifting weights. The brief description of the machine that flies by flapping its wings (which Bacon himself expresses doubts about) belongs to a tradition whose origin is not easy to trace, but some of whose developments are easily recognized: the idea was taken up again in very like terms by Leonardo da Vinci, after whom Rome's airport is named. One wonders whether the self-propelled cars and the rowerless ships, admittedly exaggerated in Bacon's rendition, are related to real objects; quite likely they originate in ancient sources, perhaps less fantastic than the *Romance of Alexander*, because Polybius records a self-moving engine displayed in a procession[146] and rowerless ships are illustrated in the *De rebus bellicis*.[147]

In certain cases the technology described in extant works may have been misunderstood. Consider Heron's *Automata*, a work in two parts, each devoted to one type of mini-theater played out by automata. The automata of the first part are called moving, those of the second static or standing (στατός). It is generally assumed that both kinds of show involved dummies operated by intricate machinery, an example of which appears in Figure 4.16. But intriguingly, Heron says the inner works of the *standing* automata were both safer and able to allow a greater variety of scenes than the other kind; they caused such amazement that "the Ancients used to

[144]Roger Bacon, *Epistola de secretis operibus*, IV (a very loose translation is given in [Bacon/Davis], pp. 26–27).

[145]*Historia Alexandri Magni*, II §38.

[146]Polybius, *Historiae*, XII, xiii §11. His description is taken from Demochares. The machine was built in Athens by Demetrius Phalerius (who later became one of the main inspirers of the cultural politics of the early Ptolemies) at the close of the fourth century B.C.

[147]*De rebus bellicis*, xvii and preceding illustration = [RB/Ireland], pp. 10–11 and Tabula XI. These are paddleboats, which the anonymous author of the fourth century A.D. thought were moved by oxen, through a mechanism similar to the sakiyeh.

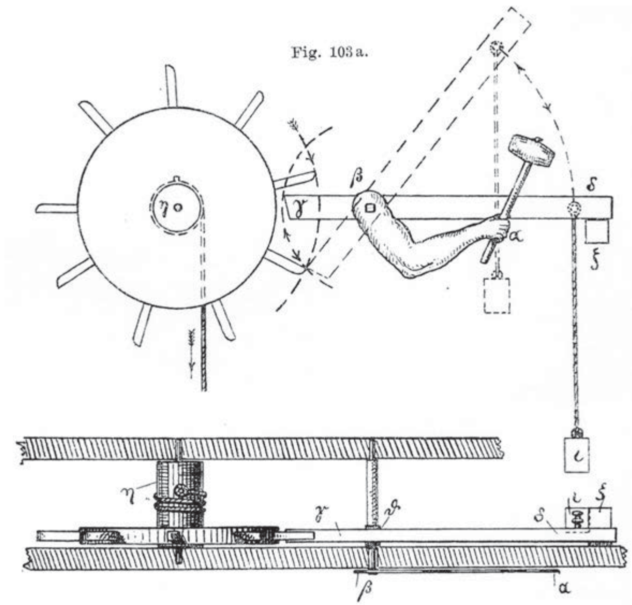

FIGURE 4.16. Detail of an automaton, in Schmidt's rendition. From [Heron: OO], vol. I, p. 425.

call the creators of such things wonder-makers" (θαυματουργοί).[148] Among the features of these mysterious standing automata shows were:

- The characters were figures (ζῷδια) painted (γεγραμμένα) on a board (πίναξ), yet they were capable of *appearing to be in motion* (ἐν κινήσει φαίνεσθαι).[149]
- These figures made up images that were displayed in rapid succession. In the extremely short time (ὀλίγον παντελῶς χρόνον)[150] between the display of two consecutive images, the stage was covered by special automatic doors.
- A knotted rope, pulled by a weight, moved an intricate mechanism that coordinated the stage coverings and uncoverings and the succession of images. The passage of each knot through a neck in the mechanism triggered a corresponding action.[151]

Conventionally, each interval between coverings of the stage has been thought of as a scene, and it has been assumed that mechanical motion

[148]Heron, *Automata*, i §7, 340:23 – 342:4 (ed. Schmidt, in [Heron: OO], vol. I).

[149]Heron, *Automata*, i §5, 340:13–15.

[150]Heron, *Automata*, xxi §2, 410:15.

[151]Heron, *Automata*, xxiii, 416–420; xxv–xxvi, 426–436.

told the story within each scene. But if this were all, it is hard to see why the automata should be drawings on a board and not three-dimensional objects, or why so much of the treatise should be devoted to the moving doors and the substitution of the scenes — or indeed, why the scenes should follow each other so fast that they can be controlled by a rope pulled by a weight. The preceding elements of the description suggest a different possibility: that these automata were called *still* because the component figures of the images were always the same, the illusion of movement coming from a quick run of images.[152]

It is agreed that Heron's source for the *Automata* is Philo of Byzantium. It is not easy to figure out through the filter of Heron's pen what exactly went on in Philo's automatic plays. It is true that Heron describes mechanisms to move various parts of the automata; but some elements of the exposition are of his own creation, and he does not always seem to understand his source fully. My conjecture is that the mechanism described by Philo was already obscure to Heron, and was subsequently forgotten for many centuries.

There are other ancient passages that might refer to optical tricks whose nature is no longer understood by the reporter.[153]

[152]This is consistent with Heron's remark that an early automatic playlet merely showed, by way of motion, a face with blinking eyes (Heron, *Automata*, xxii §1, 412:3–6) — something that is of course easy to accomplish with an alternation of just two images. Heron also says that with still automata one can either show a character in motion, or a character appearing or disappearing (ibid., i SS5–6, 340:13–21).

[153]In this direction one passage that deserves attention is Athenaeus, *Deipnosophistae*, IV, 130a. A discussion of it will appear in a forthcoming work.

5
Medicine and Other Empirical Sciences

5.1 The Birth of Anatomy and Physiology

Among the medical schools of classical Greece the most famous was that founded in the fifth century B.C. by Hippocrates of Cos. It played an important role in freeing medicine from magic and religious practices and in founding medical ethics. But Hippocratic thinking remained in the realm of *techne*, or professional medical practice, which it essentially founded; it did not generate autonomous sciences (in our sense). The key novelty of Hellenistic medicine was the creation, in the first half of the third century B.C., of anatomy and physiology based on the dissection of the human body. This was done by Herophilus of Chalcedon, active in Alexandria, and by Erasistratus of Ceos.

Again in this field, tradition favored the works still understandable in the Middle Ages. Thus we have the works of the Hippocratic corpus and those of Galen, from the imperial period, but all Hellenistic writings have been lost; no treatise by Herophilus or Erasistratus exists. But from fragments and testimonies it is possible to recover a certain fraction of their results, enough for a qualitative evaluation. We are indebted to Heinrich von Staden for a keen effort in reconstructing the results of Herophilus and his school, based on a critical analysis of all relevant testimonies.[1]

An impressive picture emerges. Human anatomy and physiology under Herophilus appear "modern" in many ways. A lot of anatomical concepts and terms still used today are directly traceable to him. For instance, it was

[1][von Staden: H].

Herophilus who first described the liver and the digestive system, distinguishing the intestine's various tracts and giving them some of the names used today (often in Latin translation), such as duodenum and jejunum.[2]

His most interesting discoveries were probably about the nervous system. Before Herophilus the role of the brain had not been clearly identified; some thinkers had intuited it correctly, but Aristotle thought that it consisted in cooling the blood.[3] Herophilus was the first to describe the anatomy of the brain; most importantly, he discovered the nerves,[4] whose existence was previously unknown, and, having understood their function, he distinguished between sensory and motor nerves. Among the pairs of cranial nerves described by Herophilus were the optic, oculomotor, trigeminal, motor root of trigeminal, facial, auditory and hypoglossal.[5]

Herophilus co-founded the anatomy of the circulatory system (which also owes much to Erasistratus). He described the heart's cavities and valves[6] and was the first to identify and describe the anatomical differences between arteries and veins, which had been first distinguished by his teacher Praxagoras of Cos. The introduction of specific terms such as *calamus scriptorius* ("writing quill") for the narrow lower end of the floor of the fourth ventricle of the brain and *torcular Herophili* (see page 150) for the confluence of the four cranial venous sinuses gives an idea of how detailed his description of the vascular system was. He made equally significant contributions to respiratory and reproductive anatomy; it was Herophilus, for example, who discovered the ovaries and the so-called *Fallopian tubes*, and gave an accurate description of the spermatic ducts (including the epididymis, which he discovered and named with the term still used).

Herophilus paid particular attention to the eye, the only organ to which it is said he devoted a specific treatise. His was the first description of the retina, which he named *arachnoides* ("like a spider's web"), and of three other membranes, probably to be identified with the sclera (and cornea), the iris and the choroid.

One gets the feeling that, if a person versed in anatomy and physiology could read Herophilus' treatises, he would have the same impression that

[2]The duodenum was so called because of its length of twelve finger-widths, and the jejunum because it was normally found empty upon dissection.

[3]Aristotle, *De partibus animalium*, II, vii, 652a:24 – 653a:36.

[4]This is demonstrated in [Solmsen].

[5]Galen, *De anatomicis administrationibus*, IX, ix, 8–9 (ed. Simon, from the surviving Arabic translation) = [von Staden: H], text 82.

[6]Apparently Erasistratus described cardiac valves more accurately than Herophilus (Galen, *De placitis Hippocratis et Platonis*, I, x §§3–4, 96 (ed. De Lacy) = [von Staden: H], text 119). The descriptions themselves are lost and surviving references involve general terms such as "membranes at the [heart's] openings" (πέρατα επὶ τοῖς στόμασι), so it is not certain that either researcher knew their role as valves, but this is likely in view of the considerations on pages 146–148.

a mathematician has in reading Euclid or Archimedes: getting past the differences between ancient and modern knowledge, he would recognize these treatises as works in his own field. This impression would certainly not be conveyed by the Hippocratic corpus, nor by Aristotle's works, nor by any other earlier text.

Herophilus also dealt with medicine proper: pathology, diagnosis and therapy. He introduced what became for two thousand years one of the main instruments of diagnosis: the measurement of the pulse. He noticed the relationship between heart rate and body temperature, as well as the variation of average heart rate with age. To measure the heart rate of his patients, according to Marcellinus, he had a water "stopwatch" built that could be adjusted for the age of the patient.[7] Since in those years, as we saw on page 102, Ctesibius was building water clocks in Alexandria that could be adjusted for the length of the day, there is no reason to doubt Marcellinus' report.

Without listing the many areas of Herophilean interest in pathology, we mention only that he was the first physician to describe the symptoms of mental illness.[8]

As for therapeutics, Herophilus on the one hand declares the importance of prevention, stressing for example the benefits of physical exercise, and on the other prescribes treatments of various types: diets (which, he says, are also important in prevention), simple medicines of plant, animal or mineral origin, as well as complex formulations involving a dozen or more ingredients in stated amounts.[8a] In an obvious allusion to the age-old habit of entrusting healing to the gods, Herophilus says that "medicaments are the hands of the gods".[9] For some diseases, such as the cholera, it is recorded that Herophilus handed down no treatment: this is perhaps the best proof of how serious he was in his medicine.[10]

5.2 Relationship Between Medicine and Exact Sciences

The birth of exact science is contemporaneous with the qualitative leap taken by medicine at the hands of Herophilus and his school, a leap so vast that one is inclined to call it the birth of scientific medicine. This simultaneity forcefully suggests two questions:

[7]Marcellinus, *De pulsibus*, xi, 463 (ed. Schöne) = [von Staden: H], text 182.

[8]Caelius Aurelianus, *Celeres vel acutae passiones*, I, pref. §§4–5 = [von Staden: H], text 211.

[8a]See testimonies in [von Staden: H], p. 422–423.

[9][von Staden: H], p. 417.

[10]See testimonies in [von Staden: H], pp. 413–414.

1. What features of Herophilus' work make it appear so much more scientific than pre-Hellenistic medicine? Put another way, is it the case, and if so in what sense, that Herophilus' anatomy, physiology and medicine are sciences?

2. What is the relation between the (possible) birth of scientific medicine and that of the exact sciences?

We start with some observations about the second question. Today, by and large, the history of medicine enjoys few points of contact with the history of the exact sciences and the history of technology (which themselves interact but little with each other). At the same time, none of these disciplines can by itself give an idea of the unity of third-century Hellenistic culture, which has never been recovered since. It is easy to imagine the interest with which Herophilus and his disciples followed advances in exact science and technology. As we shall see, this interest is reflected even in the choice of anatomical terms. Collaboration with other scientists must have been fecund in both directions. It is likely, in particular, that the physicians had dealings with Ctesibius. We know that the latter built a water clock with a variable scale that compensated for the variable length of the day, and we now see Herophilus using a timepiece of the same type for a different purpose. We know that Ctesibius first introduced valves, and we see Erasistratus and Herophilus describing heart valves. In the pseudo-Galenic *De historia philosopha* we read:

> Herophilus admits a motor capacity for bodies in the nerves, arteries, and muscles. He accordingly thinks the lung has an additional tendency to dilate and contract. The natural activity of the lung, he says, is, then, the drawing in of pneuma from the outside. . . [11]

That the lungs draw something in because they expand may seem banal today, but it probably first became clear around that time, in Alexandria, and precisely because of the studies of Ctesibius.[12] Thus the descriptions

[11]Pseudo-Galen, *De historia philosopha*, ciii, 317–318 (ed. Kühn) = [DG], 639:4–16 = [von Staden: H], text 143c. For *pneuma* see page 77; the meaning here may be simply "aspirated air" (cf. Heron's definition of the term), particularly in its physiological role, which would distinguish it from *aer*, air outside the body. Herophilus maintains that pneuma, besides being breathed in, is also present in the arteries. The surviving testimonia unfortunately say nothing about the passage of pneuma from lungs to arteries. It has been said that Herophilus thought that arteries contained only pneuma and no blood, but a closer analysis of the sources reveals that most likely he distinguished between the contents of veins (blood alone) and that of the arteries (blood and pneuma). For discussion see [von Staden: H], pp. 264–267.

[12]Heron also describes a medical device (syringe) based on suction due to expansion (*Pneumatica*, II, xviii). It does not seem that the idea of suction depending on expansion was clear to Aristotle: he does say that when the lung rises the air comes in, but he also says that when the air comes in the lung rises, so it's not clear whether and in what direction he postulates a causal link. It might be that, following his teleological scheme, he attributed the influx of air to the lung's porousness, which he stresses (*De partibus animalium*, III, vi, 669a). It may seem that the very early use of bellows should have been enough to suggest the "modern" explanation for suction, but Aristotle's passage shows that this is not so. This exemplifies the contrast between a product of prescientific technology and an object designed scientifically: the construction of the former does not have to (as we see) lead to a theoretical clarification; the construction of the latter by definition presupposes such a clarification.

of the circulatory and respiratory systems seem connected to contemporary progress in the new science of pneumatics.

The relationship between medicine and fluid mechanics suggests that physicians may have contributed indirectly to pneumatics. Couldn't it have been the descriptions of the physiology of the heart and heart valves that gave Ctesibius the idea of the valve-based pump? Extant testimonia don't allow us to assert with certainty that blood circulation was understood by Herophilus and Erasistratus, but neither do they allow one to conclude the opposite. According to Galen, Erasistratus believed that the pneuma is communicated from the heart to all parts of the body through the arteries, which are filled with blood.[12a] We also know that Herophilus believed that the pulsation of arteries served to spread nourishment.[12b] A mechanical action of contraction and dilation could hardly move pneuma and nourishment while the blood remains stationary.

Herophilus called *arteries* the vessels that contain what we call arterial (oxygenated) blood, and *veins* those that contain venous blood. Thus his terms differ from ours in the case of pulmonary circulation: the vessel that carries stale blood from the heart to the lungs was to him a vein. But recognizing its mechanical similarity to arteries, he called it the *arterial vein* (ἀρτηριώδης φλέψ). This name became canonical and, though sometimes considered a poor choice and a proof that Herophilus did not grasp the role of arteries and veins, it appears in the correct description of pulmonary circulation given by the thirteenth-century Arab physician al-Nafīs:

> The blood from the right chamber must flow through the arterial vein to the lungs, spread through [their] substance, be mingled with air, pass through the venal artery to reach the left chamber of the heart. . . [13]

The possibility of interaction between Herophilus and Ctesibius, suggested above all by the fact that in the case of the adjustable clock and of

[12a]Galen, *An in arteriis natura sanguis contineatur* viii, 18 (ed. Albrecht) = [von Staden: H], text 145a.

[12b]P. Londinensis 137 (Anonymus Londinensis, *Iatrica Menonia*), col. 28:46 – 29:15 = [von Staden: H], text 146.

[13]Ibn al-Nafīs, *Sharḥ tashrīḥ al-Qānūn*; see [Chehade]. In Europe, discussions of blood circulation appeared before Harvey in the Renaissance, at the same time as the first modern descriptions of

the valve it is plausible to postulate influences in opposite directions, is compatible with what little we know about Herophilus' dates: his *floruit* seems to have been the first half of the third century,[13a] while we know that Ctesibius was active in the time of Ptolemy II Philadelphus (283–246).

Herophilus' interest and proficiency in what we now call "physics" are demonstrated also by his work on the pulse. Significantly, his heart rate measurements are in all probability the very first measurements of intervals of time of the order of one second, ever. There were no terms in Greek to indicate such short intervals and he had to invent an appropriate unit; he chose the average period of the heartbeat of a newborn. It is particularly interesting that he used mathematical concepts in his theory of heartbeats: he studied the ratio between the times spent in systole and in diastole and distinguished rational from irrational ratios. Unfortunately our only source for this is a very obscure passage by the imperial-age physician Rufus Ephesius.[14] The way Rufus puts things is clearly self-contradictory: supposedly Herophilus found the systole and diastole of a newborn to have equal duration, but the ratio between the two to be irrational. Perhaps what Herophilus meant is that in a newborn the ratio is very close to 1 but not equal to 1 or to any other simple rational number.[15] In any case, Herophilus' familiarity with the terminology of the more advanced material in Euclid's *Elements* is impressive. Herophilus also applied to the theory of heartbeats musical and metrical terms, with which he names various cardiac rhythms. This interest in music, too, was shared by Ctesibius, who invented the water organ. Probably it was not just results and instruments of exact science that found use in medicine, but also the other way around.

The only bodily organ about which Herophilus wrote a specific book was the eye. Since all his anatomical interests seem to be directed toward an understanding of physiology, he was probably interested in the fundamentals of physiological optics. This interest would be contemporaneous with the development of optical science in Alexandria. The assumption of a close connection between Alexandrian physicians and the founders of optics may help understand certain parts of Euclid's *Optics* that have

the heart valves (see page 342). The terminology used by al-Nafīs is a further reason to suspect that the source of this knowledge lay in Antiquity.

[13a][von Staden: H], p. 50.

[14]Rufus Ephesius, *Synopsis de pulsibus*, iv, 223–225 (eds. Daremberg, Ruelle) = [von Staden: H], text 177. The passage is discussed in [von Staden: H], pp. 280–282.

[15]That is, the ratio would not be expressible as a fraction having a small enough denominator. It makes sense to call a ratio a/b of two homogeneous measured values rational only if it can be expressed as a fraction n/m with m small enough to make $1/m$ large with respect to measurement errors. What Herophilus says becomes comprehensible if we assume that he was aware of the point just made.

remained obscure since the imperial age.[16] We have seen, for example, that Herophilus knew the "weblike" structure of the retina, as implied by the name he gave it. This knowledge, plus the knowledge of the function of sensory nerves, could easily have suggested the existence of a discrete set of photoreceptors. To construct a mathematical model of vision, then, it is natural to consider a discrete set of "visual rays", one for each sensory element of the retina, and that is exactly what Euclid does. The resulting theory can explain quantitatively the resolving power of the human eye. In real life, distant objects appear not only smaller but also fuzzier, because the amount of information provided by the nerve endings decreases with the portion of the retina involved. This loss of detail is not easily explained within a continuous theory of vision; but in Euclid's model visual rays form a discrete set and are separated by at least some minimum angle,[17] so the loss derives from fewer visual rays intercepting the object. Modern scholars, no longer recognizing in classical optics a mathematical model of the physiological act of vision — in part because of the total absence, for the next two thousand years, of any mathematical models of physiological processes — have regarded Euclid's choice as a "false hypothesis".[18]

One may ask whether ancient optics incorporated other anatomical and physiological knowledge about the eye. The preface attached to Euclid's *Optics* in the redaction attributed to Theon contains some interesting observations, such as that, on reading, one's gaze moves so that the words read in succession are always centered in the field of vision, and that, analogously, when scanning for a small object one catches sight of it only when looking directly at it, at the center of the visual field.[19] Although the text does not say so explicitly, it is clear that to explain such phenomena in the

[16]The close connection between optics and ophthalmology is also suggested by the passage in the *Arenarius* where Archimedes deals with the measurement of the apparent size of the sun: one preliminary step is the measurement of the pupil's diameter (Archimedes, *Arenarius*, 139, ed. Mugler, vol. II).

[17]Euclid, *Optics*. This is already clear at the beginning of the work, in "definitions" 1 and 7 and in proposition 1. (In the *Optics*, "definitions" (ὅροι) is the term used for what are in fact the assumptions or postulates underlying the theory.) Definition 7 says that the sharpness with which an object is perceived depends on the number of incident visual rays, showing that the choice of a discrete model is due to the need to explain the eye's limited resolution power. Definition 7 can be regarded as a consequence of definition 3, which says that a thing is seen if and only if it is reached by visual rays. It follows that what is reached by more visual rays is seen in greater detail, and what is far enough is not seen at all (proposition 3) because it falls between adjacent visual rays. According to Euclid, the impression that we see a continuous image arises from rapid eye movements.

[18]These are Heath's words (see page 384). Today the term "false" is out of fashion, but Euclid's hypothesis continues to appear strange. More recent articles have recognized that Euclidean optics is in part a theory of visual perception (see [Jones] and references therein) but probably we have not yet exhausted what can be deduced from Hellenistic ophthalmology. See [Medaglia, Russo], pp. 46–54 for a fuller discussion.

[19][Euclid: OO], vol. VII, 146–148.

framework of Euclid's optical theory one must suppose that visual rays are not equidistributed, but more concentrated near the center of the cone of sight: an assumption compatible with, though not explicitly contained in, the text of the *Optics*.[20] The name "weblike" given by Herophilus to the retina seems to allude to a web's finer mesh size near the center, so the idea of a nonuniform distribution of visual rays may have come from anatomy.

5.3 Anatomical Terminology and the Screw Press

The anatomical terminology introduced by Herophilus can be a precious source of insights, probably not fully mined yet. For example, he gave the name *pharoid* (Pharos-like) to a certain elongated structure, the styloid apophysis of the temporal bone. Pharos had always been the name of an Alexandrian islet, but Herophilus had in mind, of course, the new meaning of the name — the lighthouse recently built on the island. This shows a willingness to borrow terms from the new technological reality.

The term *torcular Herophili*, adopted for the confluence of the cranial venous sinuses, is particularly interesting in this regard. The Latin word *torcular*, from the verb *torquere* (twist), suggests a screw shape, and this has always been considered the reason for the name. Because the screw shape of this structure is characteristic in oxen but not in humans, some have thought that Herophilus in this cased relied not on human autopsies but on bovine anatomy. However, as von Staden observes, the torcular shape does occur in humans, though more rarely.[21]

Now, the Greek term that Herophilus actually used is *lenos* (ληνός).[22] Its etymology is dubious, but in any case not originally related to screws; the early meanings were a large vat or trough, and a press of any sort. When screw presses were invented, the term acquired that meaning as well. Conceivably, Herophilus might have chosen the term before that invention took place, in which case he would have had in mind "an object that holds a certain amount of liquid"; but it would be a strange coincidence if, among the dozens of nouns available for containers, he had selected exactly the one that later came to mean the screw shape associated with

[20]The first postulate of the *Optics* (concerning the structure of visual rays) is certainly corrupt in the version that has come down to us. In trying to reconstruct it, it may be useful to take into account the portions of the preface that try to illustrate the postulate's meaning. The preface is admittedly late, but some of the notions it contains may predate the corruption of the text. For a discussion see [Medaglia, Russo].

[21][von Staden: H], p. 158.

[22]This is stated in two passages of Galen (*De anatomicis administrationibus*, IX, i, 712 (ed. Kühn, vol. II) = [von Staden: H], text 122a; *De usu partium*, IX, vi, 19 (ed. Helmreich, vol. II) = [von Staden: H], text 123).

the structure being named (sometimes in man and always in the ox — and Herophilus was probably even more familiar with bovine than with human anatomy.[23]) Thus it seems probable that when Herophilus chose the term *lenos* it already had the sense of "screw press".

We must conclude that probably at the time of Herophilus, in the first half of the third century B.C., screw presses already existed. The absence of references to this object (and to any other type of screw-nut combination) in classical-age literature, together with Herophilus' particular attention to technological progress, suggests that the invention took place during this time. The earliest surviving description of screw presses is from centuries later, written by Heron of Alexandria,[24] while Pliny mentions them as a novelty of his time.[25]

All of this tends to confirm on the one hand that the technology Heron describes actually dates from the third century B.C.,[26] and on the other Pliny's unreliability: he presents as a recent invention every Hellenistic product that had started being imported to Rome in the recent past.

5.4 The Scientific Method in Medicine

Turning now to the problem of the relationship between exact science and medicine, it goes without saying that Herophilus' intelligence, scholarship and instrumental use of results from science do not in themselves warrant labeling his work scientific. For that we must ask whether and to what extent he shared with the luminaries of Hellenistic exact science not only technical instruments and terminology, but also methodological elements.

Consider first that the dissection of cadavers is a complete novelty as compared to earlier medicine: for the first time a human body is handled not for healing, or for embalming, or for other immediate practical ends, but purely for knowledge's sake. We also know from Celsus that Herophilus was supplied by the king with condemned men for experimental vivisections.[27] Thus Herophilus' work certainly presents two of

[23]Almost all anatomical knowledge before Herophilus was based on the ox. Bovine anatomy was also well-known to priests, who sacrificed them. The religiously imposed bar to human dissection had forced "physicians" to rely primarily on analogies between man and ox.

[24]Heron, *Mechanica*, III §§19–20.

[25]Pliny, *Naturalis historia*, XVIII §317.

[26]See pages 132–134.

[27]Celsus, *De medicina* I, proem, §§23–26 = [von Staden: H], text 63a. Von Staden, who discusses at length all the available evidence (pp. 142–153), regards this statement as credible; in any case, as he notes, it's not possible to determine whether a nerve is sensory or motor, as Herophilus did systematically, without experimenting *in vivo*. The ethical problems raised by vivisection are certainly very serious; at the same time they have such a modern ring to them as to confirm, in a sense, Herophilus' proximity to modern "scientific method".

the features typical of empirical sciences: research as distinct from professional activities, and the experimental method.

For anatomy to be founded by Herophilus, two taboos of classical Greek culture had to be overcome. One was religious in nature and obvious: the ban on cutting up human bodies. The second was intellectual and subtler.

Fifth-century medicine developed without introducing neologisms.[28] In all of classical culture, discussions about concepts had been inseparable from discussions about the terms used to name them. The ancient doctrine that things have "natural names", still partly present in Plato,[29] was already contradicted by Aristotle,[30] but the latter considered that humans were free to choose only names as strings of sounds, not the demarcation of the world into individual nameable objects. For example, the *De partibus animalium* (*Parts of animals*) takes for granted that the parts of which one can talk always have a Greek name. The implicit assumption, in other words, is that there exists a finite, static set of all knowable objects, corresponding to the vocabulary of Greek or any other language. Aristotle does seem to have introduced new terms in zoological taxonomy, such as *entoma* ("segmented") for insects and *coleoptera* ("sheath-winged") for beetles.[31] But these terms were not being used conventionally; their meanings either preexisted or were obvious from the component parts. The words simply indicate all creatures that share the characteristic in question: all segmented creatures or all those having sheath-like wings. This is therefore only a preliminary step toward the introduction of a conventional terminology: the novelty in Aristotle's new names — an important one from the perspective of zoology, of course — has to do not with the meanings of the terms (which are understandable in the intended sense even by someone who has not been told about them), but with their systematic use in classification.

Even Democritus, though having offered several arguments for the conventional origin of names, does not seem to have fully overcome the traditional view,[32] which in the modern age only started to lose ground in the seventeenth century. Tullio De Mauro writes:

> Starting with the seventeenth century, the experimental method and [zoological and botanical] classifications revealed to the avant-garde of European culture that there are scientifically knowable things ... for which nonetheless there had never been a name either in the "perfect" Latin language or in any other.[33]

The freedom with which Herophilus introduces his anatomical names is analogous to that with which Hellenistic mathematicians create new mathematical terms.[34] This freedom would have been inconceivable not only in classical Greece but also after the decline of Hellenistic civilization, all the way to the seventeenth century. And note that the avant-garde of European culture of which De Mauro speaks had been studying Hellenistic works intensively for centuries — works containing on the one hand ideas underlying the classifications and the experimental method, and on the other the memory of scientists like Herophilus, who identified new objects of study by giving them a name for the first time.

Thus the notion of a conventional terminology is not at all trivial. In anatomy it is even less so than, say, in systematic zoology: new animal species are fairly easy to identify as such, but there is ample freedom in the choice of what anatomic structures deserve a name. When Herophilus picks from the continuous and enormously complex structure that is the circulatory system those particular morphological features that warrant a specific name (such as *calamus* or *torcular*) in view of his physiological and pathological purposes, he is creating new concepts. He is in fact inaugurating a new discipline in which not only the words but even the corresponding concepts are conscious creations.

This is, in my opinion, the source of the impression of "scientificness" that one has in reading Herophilus' anatomical excerpts, so different in texture from Aristotle's "anatomical" discussions.

The use of specific and consciously created theoretical concepts is in fact one of the essential features that characterize scientific theories in our sense. But the scientific theories of "exact science" described in Chapter 3, which were developing in the time of Herophilus, were also characterized by being:

– based on empirical data, without being uniquely determined by them;
– internally certain, thanks to a rigorously deductive structure;

[28]See [Irigoin], where it is said, for example, that in the Hippocratic *De locis in homine*, two-thirds of the anatomical terms had already been used in Homer.

[29]Plato, *Cratylus*. Though recognizing that words are a human creation, Plato insists on the objective similarity between a *good* name and the named object. Further, he denies the possibility that a run-of-the-mill contemporary of his might introduce new words; in his view, all names had been chosen by the original *legislators* who created the various languages.

[30]Aristotle, *De interpretatione*.

[31]Aristotle, *Historia animalium*, I, 487a:32 and 490a:14–15.

[32]These arguments are reported by Proclus (*In Platonis Cratylum*, §16, 5:25, ed. Pasquali = [FV], II, 148:3–26, Democritus B26). They are based on the existence of homonyms and synonyms and on the possibility of changing names, but the only example proposed for the latter possibility was of a proper noun. It's hard to see why a conventionalist like Democritus would fail to mention in this context the creation of conventional terms if that practice already existed in his time.

[33][De Mauro], chapter II, section 3.

[34]Archimedes systematically defines mathematical concepts by introducing new and conventional names for them (for example, geometric terms in the *On conoids and spheroids* and arithmetic ones in the *Arenarius*); so does Apollonius of Perga, to whom we owe the terms *ellipse*, *parabola* and *hyperbola*, among others.

– applicable to concrete problems, via "correspondence rules" lacking absolute validity.

Do these characteristics have analogues in Herophilean anatomy and medicine? Any attempt to answer this question must start from the existing testimonia about the scientific methodology of Herophilus. Unfortunately, in these passages, dating from the imperial period, the authors — Galen, in particular — do not seem to be in a position to appreciate the conceptual depth of their source.

The testimonia leave no doubt as to the fact that Herophilus considered it essential to found knowledge on empirical bases. Galen, for example, writes:

> We find, however, that this Herophilus concedes no small importance to experience, nay indeed, to speak the truth (and it is the fittest to be spoken), he makes experience all-important.[35]

And regarding the formation of the fetus:

> For he considers that anatomical descriptions do not produce any presupposition of knowledge on the basis of which [one might] say "this part arose from this other part", as some, misunderstanding, believe; [he thinks] that the faculties that govern us should be discovered on the basis of other phenomena and not simply by observing the parts.[36]

The *phainomena* ("appearances", "things seen") that interest Herophilus are therefore not just morphological data and they do not determine the theory in a mechanistic way; they include things that we would wholeheartedly call experimental data, as in this passage from Galen:

> ... [heart] rhythms, about which Herophilus discoursed at length, surveying observations and experiments rather than teaching a rational method.[37]

For Galen, the experimental method contrasts with rationality, which he evidently considers to be the hallmark of purely deductive expositions.[38] It is amusing that Polybius criticizes Herophilus for the opposite reason:

[35]Galen, *De experientia medica*, xiii §6, 109–110 (ed. Walzer, from the surviving Arabic translation) = [von Staden: H], text 52.

[36]Galen, *De foetuum formatione*, v, 678–679 (ed. Kühn, vol. IV) = [von Staden: H], text 57:4–9.

[37]Galen, *De praesagitione ex pulsibus*, II, iii, 278 (ed. Kühn, vol. IX) = [von Staden: H], text 53.

[38]This opinion of the person who is generally considered the greatest ancient physician, together with the fact that we have Galen's works but none by Herophilus, helps explain why in the few extant scientific works from Antiquity examples of use of the experimental method are rare.

> The rational part [of medicine] arose especially in Alexandria, in the hands of the so-called Herophileans and Callimacheans ... but if you lead [these rationalists] back to reality and put a patient in their hands, you will find that they are as useless as he who has not read a single medical treatise.[39]

If Herophilus, in spite of the opinion expressed by Galen, is considered a founder of "rational medicine" (which Polybius counterposes to "empirical medicine"), it is clear that the deductive aspect of the theory was not absent from his work. Moreover Herophilus knew that theories do not have an absolute truth value, as can be seen from other passages from Galen:

> Some say that there are no causes of anything; some, like the Empiricists, are in doubt between yes and no; others yet, like Herophilus, accept it hypothetically.[40]
>
> Although Herophilus casts doubt on every cause with many arguments and strong, he himself is later found making use of them.[41]

This last attitude seems contradictory to Galen, who, making a distinction only between true and false, is unable to understand the Hellenistic scientists' conscious use of theoretical models, that is, theories based on hypotheses. We will return to this point later.

Yet another Galenic passage:

> Herophilus and his followers state that prognosis [πρόγνωσις, foreknowledge] possesses firm certainty, whereas foretelling [πρόρρησις] does not. For, they say, many of the things that have been foretold do not come about — as if anyone is able to foretell without foreknowing [προγνῶναι].[42]

Galen (who returns to this question in two other passages) considers the Herophilean distinction absurd. But one can argue otherwise; Herophilus may be saying that the patient's observable condition, together with accepted medical doctrine, determine a prognosis that is *certain* in the sense of being in principle shared by all physicians — but one to which is associated an uncertain course of the disease. In other words, the prognosis is a concept internal to the theory, and therefore determinable with certainty,

[39]Polybius, *Historiae*, XII, xxv-d = [von Staden: H], text 56.

[40]Galen, *De causis procatarcticis*, xiii §162 = [von Staden: H], text 58:1–3. The words here translated as "hypothetically" are "ex suppositione."

[41]Galen, *De causis procatarcticis*, xvi §197 = [von Staden: H], text 59:2–4.

[42]Galen, *In Hippocratis prognosticum*, I, comment. I.4, 204–205 (ed. Heeg) = [von Staden: H], text 264, 1–4.

whereas the correspondence rules that allow one to apply the theory to a concrete case, associating to the prognosis a prediction, have no absolute guarantee of validity.

In conclusion, the impression of scientificity conveyed by Herophilus' results is fully confirmed at the methodological level. Herophilus emerges from the extant testimonial evidence as one of the founders of the scientific method, for his introduction of the experimental method to medicine, for his contribution to a new conception of language, and above all for his methodological awareness.

5.5 Development and End of Scientific Medicine

Although so far we have discussed Herophilus as the dominant personality in the birth of scientific medicine, he was not at all an isolated case. The development of medicine in the third century B.C. was not limited to the work of a few exceptional individuals: it had remarkable repercussions on common professional practice. The manifold increase in knowledge led to the specialization of medicine: in Alexandria there were not only *physicians*, but also dentists, gynecologists, and so on. Also, scientific medicine was not a phenomenon limited to Alexandria; one of Herophilus' contemporaries was Erasistratus of Ceos, whose medical activities probably took place in Antioch, in the court of Seleucus I.[43] Almost all of Herophilus' main scientific interests, from anatomic dissection to neuroanatomy, from the pulse to ophthalmology, seem to have been shared by Erasistratus, and to him, too, were attributed by some ancient authors the discovery of the nerves, the distinction between motor and sensory nerves, and the practice of vivisection. The fragmentary nature of the extant testimonial evidence makes it difficult to compare the contributions of the two scientists.[44]

We know from a papyrus of the second century A.D. that Erasistratus carried out at least one quantitative experiment in physiology. To prove that animals give off matter in some invisible form, he locked an animal in a container without food and compared its initial weight with the later weight of its body together with its excreta.[45] Similar experiments were conducted in the seventeenth century and are considered a sign of the appearance of the modern experimental method.

[43]Compare [von Staden: H], p. 47.

[44]For the fragments of Erasistratus, see [Erasistratus/Garofalo].

[45]P. Londinensis 137 (Anonymus Londinensis, *Iatrica Menonia*), col. 33:44–51 = [Erasistratus/Garofalo], 86.

Herophilus founded a school that remained active until the first century A.D. According to Hyginus, one of his immediate disciples was Agnodice, the first woman who dared challenge the exclusion of her sex from the medical profession.[46] In view of Agnodice's professional success the ban against female physicians was lifted — an example of the role played by women in Hellenistic civilization.[47]

One of the earliest and most significant representatives of the Herophilean school was Andreas, personal physician to Ptolemy IV Philopator and perhaps an immediate disciple of Herophilus. Like the latter, Andreas had wide-ranging interests, which included for sure pharmacology, surgery and physiology. A passage of Caelius Aurelianus on a case of *pantophobia* suggests that he shared Herophilus' interest in mental illness.[48] A machine built to order for Andreas to reduce dislocations of the limbs remained famous in later centuries and attests to the interactions between Alexandrian physicians and mechanicians. But already there is no clear indication that Andreas engaged in what had been Herophilus' main scientific activity: anatomic dissection. It is certain that among later members of the school, almost none practiced it.[49]

An obvious symptom of decadence of the Herophilean school in later centuries is the increasing importance given to the exegesis of Hippocratic texts. Nonetheless throughout its existence the school produced scientists who made important contributions to the development of knowledge in their fields of specialization. For example, Demetrius of Apamea studied the sexual organs, shifting in this area the focus of interest, which under Herophilus had been the description of reproductive physiology, to the treatment of ailments. Mantias, another important representative of the school, was probably the greatest pharmacologist of Antiquity: it seems he was the first person to prepare, describe and classify medicines obtained by combining several ingredients.[50]

The continuing vitality of the Herophilean school in the first century A.D. is attested by the work of one of its last representatives, Demosthenes Philalethes. This physician, though having written a work on the theory

[46]Hyginus, *Fabulae*, §274 = [von Staden: H], text 8.

[47]Among the Hellenistic painters mentioned by Pliny there are five women (Pliny, *Naturalis historia*, XXXV). The late examples of Mary the Jewess (page 166) and Hypatia indicate that scientific activities, too, were not out of the reach of women.

[48]Caelius Aurelianus, *Celeres vel acutae passiones*, III, xii §108. Further information about Andreas, including a list of testimonia, can be found in [von Staden: H], pp. 472–477.

[49]The only exception seems to have been Hegetor.

[50]However, Plutarch states that medicines obtained by combining plant, animal and mineral ingredients had been made by Erasistratus, who called them "the hands of the gods" (*Quaestionum convivalium libri vi*, 663C). This is an example of how traditions about the Herophilean and Erasistratan schools merged.

of heartbeats, devoted himself primarily to ophthalmology. The forty passages in which he is mentioned ascribe to him the study and cure of more than forty infirmities of the eye, from sties to glaucoma, many of which still maintain in all likelihood the names he gave them. The written works of Demosthenes Philalethes remained the foundation of knowledge about the eye throughout the Middle Ages.

After the first century A.D. the Herophilean school dies out. The ensuing methodological decadence, already mentioned in connection with Galen, is even more obvious in another of the best physicians of the imperial period, Rufus Ephesius. In his treatise *Names of the parts of the human body*, a source of invaluable testimonia on Herophilus, he scrupulously reports all the anatomical terms he knows, together with their origins. But Rufus' terminology is particularly exuberant regarding inessential features, such as facial hair;[51] this is a clear consequence of a passive attitude toward the terminology under discussion, which becomes very rich precisely in the case of parts of the body that, like the beard, are mentioned every day. Rufus not only makes no attempt to standardize this nomenclature by narrowing down the use of ambiguous terms; he even criticizes some terms coined by Alexandrian scientists as being the creation of "Egyptian physicians" with an inadequate mastery of Greek.[52] Other imperial-age physicians, too, often quibble with Herophilus' language, whose creativity they can no longer grasp. Caelius Aurelianus, for instance, in the same passage where he reports the Herophilean description of a mental case, has *deliratio* and *alienatio* as Latin counterparts of two words used by Herophilus in his pioneer work on psychiatry, but because he regards the words as synonymous, he reproaches Herophilus for unwisely juxtaposing them as if they had distinct meanings.[53]

To these men language had again become an external body of information which they could not influence except unconsciously. This is an important aspect of the death of the scientific method.

5.6 Botany and Zoology

Zoology and botany gained great impetus from Alexander the Great's conquests, which made possible the systematic study of many animal and plant species unknown or little known to classical Greece. Alexander himself had ensured that specimens of flora and fauna from the regions he

[51]Rufus Ephesius, *De nominatione partium hominum* (eds. Daremberg, Ruelle), §49, 139:8–10
[52]Rufus Ephesius, *De nominatione partium hominum* (eds. Daremberg, Ruelle), §133, 151:1–2.
[53]Caelius Aurelianus, *Celeres vel acutae passiones*, I, pref. §§4–5 = [von Staden: H], text 211.

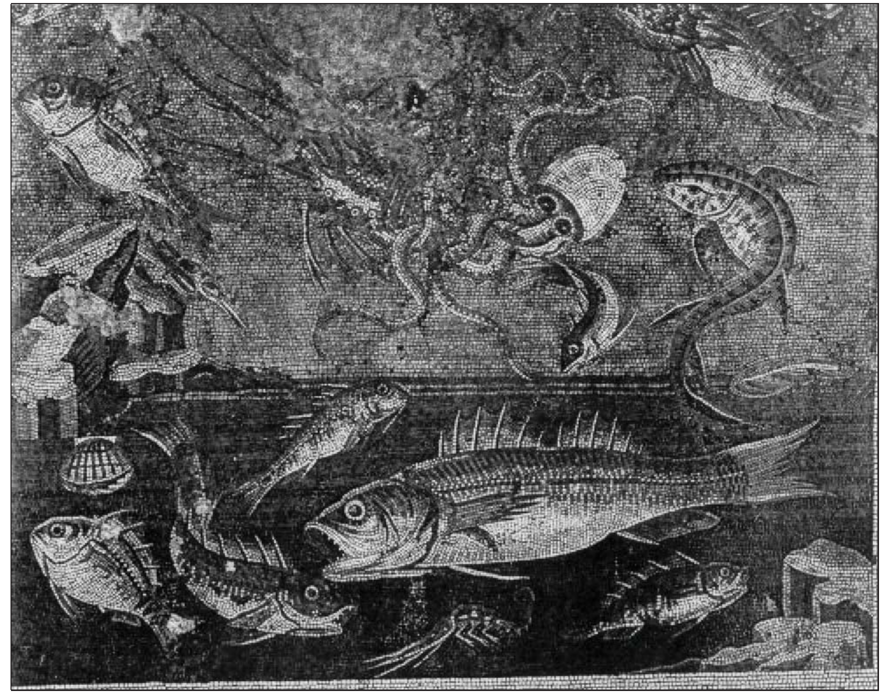

FIGURE 5.1. Detail of mosaic from the late second century B.C. found in a house in Pompeii, known as the House of the Faun. Other mosaics in the building show a variety of naturalistic birds, reptiles, beasts and plants. Museo Archeologico Nazionale, Naples.

crossed, especially in his Indian campaign, were sent home so they could be studied.

Hellenistic botany and zoology had an important precursor: Aristotle's research. The philosopher's teleological world view did not, in this area, prevent the collection of interesting results. Whereas, as already noticed, his anatomy was obstructed by a static concept of language, this was not a problem in zoology, where Greek names for animals provided a fairly suitable conceptual scheme. Aristotle not only described about five hundred animal species based on observations ranging from morphological to behavioral,[54] but he also recognized that animal life varies along a continuum, and so introduced a "natural classification scheme" for zoology.[55]

[54]Aristotle, *Historia animalium*. Many observations can be found in his other zoological works as well.
[55]On the subject of classification criteria, the salient texts are the *Historia animalium* and the first book of the *De partibus animalium*.

Theophrastus, who was for decades Aristotle's favorite disciple and succeeded the master at the head of the Lyceum, devoted himself to many subjects, including meteorology, mineralogy, and above all botany. His two surviving botanical treatises, *Historia plantarum* and *De causis plantarum*, contain the elements of a theory (in the empirical-science sense) of botanical physiology, built both on the gathering of ancient empirical lore and on recent observations and experiments.

Theophrastus discusses at length the modifications that living beings can undergo between generations.[56] Morphological variations of plants due to changes in soil or climate are clearly distinguished from spontaneous changes; Theophrastus stresses that mutations of the second type, which occur in plants and animals alike, do not happen to the individual already formed, but to the seed,[57] that they are hereditary, and that they can build up to gradual but extensive changes after many generations.[58] Of particular interest is Theophrastus' critique of the Aristotelian theory of final causes.[59] Actually on this subject the most gripping passage is not in Theophrastus, but, surprisingly enough, in Aristotle himself, in book II of the *Physics*: it examines with great clarity the possibility of replacing the teleological framework by a principle of natural selection:

> But a difficulty comes up: what prevents nature from acting not with a purpose or for the best but rather as Zeus makes rain — not in order that the grain may grow but by necessity (for the rising [vapor] must be chilled and, being chilled, turn into water and come down)? ... Likewise nothing prevents the parts [of livings beings] from behaving in the same way, so that front teeth by necessity arise sharp and adapted to cutting, and molars flat and useful for crushing the food, all of this happening not on purpose, but by chance; and nothing prevents this from being so also for other parts that seem to have arisen for a certain purpose.
>
> And so in the cases where everything happened by chance, but as if with a purpose, those beings survived because they were put together by chance in a suitable way; those for whom that was not so have perished and do perish.[60]

Aristotle then overcomes the "difficulty" by noting that the features mentioned (the shapes of incisors and molars, etc.) occur regularly and

[56] Theophrastus, *Historia plantarum*, II, iii.
[57] Theophrastus, *De causis plantarum*, IV, iv §11.
[58] Theophrastus, *De causis plantarum*, II, xiii §3.
[59] Theophrastus, *Metaphysica*, 10a:22 – 11b:26 (ed. Ross and Fobes).
[60] Aristotle, *Physica*, II, viii, 198b:16–31.

not sporadically, as they would if mere chance caused them.[61]

The denial of the existence of true final causes, and the lucid explanation of the correspondence between structure and function of animal organs based on natural selection, are very far from Aristotelian thinking, and in a matter that is central to it. Thus this passage may reflect a substantial divergence in opinion between Aristotle and someone who had raised the difficulty. The passage was well known to modern scientists[62] and its influence has probably been underestimated by historians of science.

Strabo (who on this subject reports also fragments of Posidonius[63]) says that Eratosthenes studied the transformations of the earth's crust and that he offered as proof that coastlines had moved in the deep past the existence of marine fossil deposits on land.[64] The argument actually goes back to Xenophanes (sixth century B.C.), who noticed the presence of shells and fish imprints in areas that lay inland in his time.[65] Among marine fossils known in ancient Egypt were the ammonites, so named in Antiquity on account of their spiral form, similar to the ram horns sported by the Theban god Ammon. Historians of paleontology have long maintained that the Ancients, though occasionally stumbling on large vertebrate fossils, ignored them because of a prejudice rooted in the belief in the fixity of species. But Adrienne Mayor has shown that it is rather a matter of historiographical prejudice preventing modern scholars from taking seriously the ample available textual evidence on ancient fossil finds.[66] She used the sources to demonstrate that:

- From the archaic period on there was a lot of interest in the frequent finds of fossils of large vertebrates, which were often kept in temples as precious relics because they were interpreted as remainders of giants or heroes. Several authors of classical or late Antiquity associated the buried finds of enormous bones with the events of the gigantomachia: a myth toward which the fossils themselves probably contributed.

[61] Aristotle, *Physica*, II, viii, 198b:32 ff. The argument is obscure, in that a sporadic appearance of organs adapted to their functions can become regular precisely through the mechanism of natural selection that Aristotle presents and contradicts. His solution for the difficulty would be very convincing if it had come immediately after the observation about the shapes of the teeth, that is, if he had not included the second paragraph of the excerpt just quoted, containing the idea of natural selection. One may therefore suspect that this paragraph was a later insertion, by a redactor of the *Physics*, the better to illustrate the "difficulty".
[62] For example, Darwin cites it in the preface to *The origin of species* (from the third edition on), adding: "We here see the principle of natural selection shadowed forth..." (footnote 1).
[63] Strabo, *Geography*, II, iii §6.
[64] Strabo, *Geography*, I, iii §4.
[65] Hippolytus, *Refutatio contra omnes haereses*, I, xiv §5 = [DG], 566:1–6.
[66] [Mayor]. There is no lack of texts espousing the point of view Mayor contradicts; having the luxury of the choice, she mentions [Rudwick] and [Sarjeant], among others.

FIGURE 5.2. Detail of the Monster of Troy vase (Museum of Fine Arts, Boston). Mayor identifies the monster's prototype as a fossil mammalian skull, calling attention to the articulated jaw, the broken premaxilla and the extended occiput. Extraneous features such as the tongue and the ring around the eye socket can be explained by conflation with other familiar skulls and by the artist's desire to endow the creature with life. Photograph by John Boardman.

– Starting from at least the fifth century B.C. some fossils were identified as belonging to animal species no longer extant.
– George Cuvier, the founder of modern paleontology, unlike some of his successors, knew the ancient prodromes of his discipline: he collected testimonia on ancient discoveries and descriptions of fossils dating from the fifth century B.C. to the fifth century A.D.[67]

One very striking depiction of a fossil in Greek art is the Corinthian krater that shows the "Monster of Troy" (Figure 5.2). Heracles and the princess Hesione, who was to be a sacrificial victim, are seen shooting arrows and rocks at the monster, whose features have long seemed strange. As Mayor points out,[67a] the beast is actually modeled on a fossil skull of a large mammalian from the Tertiary era, eroding out of an outcrop: a sight that occurs at many sites around the Mediterranean.

[67][Cuvier].
[67a]See [Mayor], pp. 157–165. The myth is briefly mentioned by Homer (*Iliad*, XX:146) and recounted by Apollodorus (*Bibliotheca*, II, v §9) and Diodorus Siculus (*Bibliotheca historica*, IV, xlii).

Unfortunately, as usual, we have not a single Hellenistic work on such subjects. But Theophrastus wrote two books *On petrification* (Περὶ τῶν λιθουμένων), which very likely dealt with fossilization.[68] The same interest in fossils that can be documented today on the part of Theophrastus and Eratosthenes was presumably shared by other authors.

We have seen, then, that the bases of modern evolutionism, namely the notions of mutation and natural selection, were both present in Hellenistic thought.[69] Did these notions lead to full-fledged evolutionary theories? The almost complete loss of zoological and botanical works precludes a definite answer, but it should be noted that general ideas about evolution in nature predate ideas about natural selection and mutations; they are repeatedly brought up (and criticized) by Aristotle, who attributes them once to Speusippus and to "some Pythagoreans" and again, more vaguely, to his own contemporaries.[70] Moreover, a rationalist explanation for animal evolution seems to be present already in Anaxagoras, who remarked that man owes his intelligence to his hands.[71] Prescientific versions of biological evolution were expounded by various pre-Socratic thinkers, from Anaximander[72] to Empedocles; the latter imagined, for example, that once there had lived on earth monstrous beings formed by the random combination of disjoint animal limbs.[73] Lucretius seems to be referring scientific objections to Empedocles' idea when he writes:

> That there were many seeds of things on the ground
> at the time when the earth first produced animals

[68]We know the title of the work from Diogenes Laertius, *Vitae philosophorum*, V §42. That it dealt with fossilization is suggested by Theophrastus' mention of "fossil ivory" (ἐλέφας ὀρυκτός; *De lapidibus*, §37) and "petrified Indian cane" (ἰνδικός κάλαμος ἀπολελιθωμένος; *De lapidibus*, §38), as well as by references to petrification made by Pliny and others.
[69]Regarding selection mechanisms, we see already in Plutarch the important observation that in the struggle for survival a species is placed in opposition not to its predators but to its competitors, those that would eat the same food. Plutarch also makes an interesting analogy with economic competition (*De fraterno amore*, 486B).
[70]Aristotle, *Metaphysica*, XII, vii, 1072b:30 – 1073a:3; XIV, iv, 1091a:31–36; XIV, v, 1092a:11–17. See also [Popper: OSE], note 11 to chapter 11.
[71]This important observation of Anaxagoras is reported by Aristotle (*De partibus animalium*, IV, x, 687a:8–10 = [FV], II, 30:5–9, Anaxagoras A102), Plutarch (*De fraterno amore*, 478D–E) and Galen (*De usu partium*, I, iii, 4, ed. Helmreich, vol. I). Aristotle, who just before this passage had noted the relationship between the use of hands and an erect posture, rejects Anaxagoras' thesis and states that, contrariwise, nature gave hands to humans because, being intelligent, they would be able to use them. Plutarch and Galen crib Aristotle's opinion.
[72]We know through Aetius that Anaximander maintained, in particular, that life started in the water (pseudo-Plutarch, *De placitis philosophorum* V, xix §4 = [DG], 430a:15–20).
[73]Fragments 6–7 in [Empedocles/Gallavotti] = frs. B35 + B57–61 in [FV], I, 326–328 + 333–334. (Gallavotti's edition contains a conjectural partial reconstruction of Empedocles' *Physical poem* assembled from fragments that were transmitted in isolation.) The concept of chance seems clearly expressed in the verse ταῦτα δὲ συμπίπτεσκον ὅπῃ συνέκυρσεν ἕκαστα (fr. 59 in Diels).

does not at all mean that beasts could have arisen
mixed among themselves, nor combined animal limbs,
because the species that even now populate the land
—of herbs, of fruit, of luxuriant plants—
cannot be created in confusion with each other,
but each springs forth in its way and preserves
its distinctive features, by a firm law of nature.[74]

We see that the notion of a species as a pool of individuals capable of interbreeding, often considered modern, must have been clear in Lucretius' source, for he transmits it in an intelligible form. Elsewhere he expresses the ideas about natural selection that we saw in the Aristotelian passage,[75] and he even says

For time changes the nature of all in the world
and everything must go from one state to another,
nor does anything remain like itself: all moves,
all is changed by nature, which imposes transformations.
One thing rots with time and languishes, weakened;
another, once insignificant, gathers strength. . . .
Many races of animals must have perished, must not
have been able to propagate by procreation.
Those you see now enjoying the life-giving air
Owe to their wiles or their strength or swiftness
Their preservation since the beginning of time.[76]

What Lucretius knew about biology is not terribly clear: shortly before this passage he discusses spontaneous generation (as having been once very common, now characteristic of animal organisms that arise from putrefaction)[77] while elsewhere he seems to maintain that specific seeds are needed for the generation of any living being.[78] The passage about extinct

[74]Lucretius, *De rerum natura*, V:916–924.

[75]Lucretius, *De rerum natura*, IV:823–842.

[76]Lucretius, *De rerum natura*, V:828–833 + 855–859. In the intervening lines (omitted), Lucretius talks about monstrous beasts of a distant past.

[77]A belief in the spontaneous generation of animals was espoused by Aristotle (*De generatione animalium*, I, 715a–b) and others.

[78]Lucretius, *De rerum natura*, I:159–207. The idea that animals cannot arise spontaneously but must be born from other animals is generally considered modern and attributed to the seventeenth-century physician Francesco Redi. But Alexander Polyhistor ascribes the same belief to the Pythagoreans (Diogenes Laertius, *Vitae philosophorum*, VIII §28). Redi, who devotes the first pages of his *Esperienze intorno alla generazione degl'insetti* to an examination of the opinions held by the "Ancients" on the subject, does not mention the Polyhistor passage reported by Diogenes; he prefers to cite Diogenes when the latter quotes the opposite opinion that in bygone eras humans were spontaneously generated ([Redi], p. 76; the English translation by Bigelow is incomplete).

animals that were not able to procreate might seem to imply abiogenesis, but Lucretius is talking of extinct *species*, not individuals, and then he states that the extant races (*saecla*) were preserved by their own fitness for survival. Thus he does not have abiogenesis in mind in this case. That in Lucretius' sources natural selection operated by the gradual change of specific characteristics is suggested by his mention of primitive men who had a distinct bone structure and no language.[79]

As we shall see, the early Hellenistic period witnessed the first direct contributions from learned men to agricultural techniques. The study of plants was also important to pharmacology, which became the main application of botany in the imperial age. The best source on this matter is the *De materia medica*, compiled in the first century A.D. by the physician Dioscorides. Down to the modern era it continued to be the best surviving treatise on the medicinal properties of plants.

5.7 Chemistry

Chemical studies started in the Hellenistic period. Such early studies are usually thought of as "alchemical", but what is properly called alchemy—a syncretism of Greek natural philosophy, Egyptian magic, allusions to Judaism and Christianity, craftsmen's recipes and empirical chemistry—is first documented in the writings of Zosimus of Panopolis, from the early fourth century of our era:[80] a time when all areas of Hellenistic science had already been overrun by irrationalist currents.

Very little remains of early chemical works. One reason is hinted at by Zosimus himself, who insists on the arcane character of the knowledge he is passing on.[81] He and his successors call theirs *the sacred art* (ἱερὰ τέχνη) and refer to ancient Egyptian religious centers, above all Memphis, as the birthplace of chemical lore. One imagines that from its beginnings Egyptian chemistry was controlled by the priestly class,[82] which as late as the Ptolemaic era was in charge of many economic activities, carried out in temples.[83] Our attempts at reconstructing Hellenistic empirical chemistry are thus thwarted by its confluence with alchemy in later centuries,

[79]Lucretius, *De rerum natura*, V:925–928 + 1028–1032.

[80][Zosimus/Mertens] is a recent critical edition of this author's *Authentic memoirs*, of which thirteen fragments exist. For his other writings one must use [CAAG], dating from 1888 and unsatisfactory in various respects.

[81]Zosimus, *Authentic memoirs*, IV, i, 17:30–34; VII, ii, 23:8–10; X, vii, 41:135–137 (ed. Mertens).

[82]On the relationship between ancient "alchemical" knowledge and the Egyptian priesthood (especially the Memphis sanctuary), see Mertens' complementary note 9 in [Zosimus/Mertens], pp. 187–189 and references therein.

[83]See page 262.

and we must be content with glimpses of it caught through the alchemists who, deciding what to transmit and how it would be combined with ingredients from other sources, filtered that knowledge.

Zosimus and other alchemists describe chemical devices such as stills and sublimation chambers, but although they attribute some to Mary the Jewess, "sister of Moses",[84] the device's names are all Greek.[85] In alchemical works we almost always encounter these three components: Greek names, elements of Egyptian magic and references to Judaism. This tripartite melange points to the trilingual city of Alexandria as the place from which alchemical knowledge radiated.

The fact that Zosimus' work contains, apart from the religious elements and the allegorical symbolism that ever more characterized later works, information about a variety of chemical compounds and reactions shows that much knowledge had accumulated in the preceding centuries on this subject.

It is also significant that the oldest "alchemical" work we have notice of, the treatise *Physica et mystica* attributed to Bolos Democritus of Mendes (a city on the Nile delta), probably from the beginning of our era, contains nothing of later alchemy. From the small portion of it that has survived and from many references we can deduce that this work dealt with the preparation of imitation gold, silver, precious stones and purple, describing traditional procedures used by craftsmen (painters, glassmakers, metalsmiths) and analyzing possible modifications thereto. Likewise the Leyden and Stockholm papyri,[86] dating probably from the late third or early fourth centuries A.D. and usually classified as alchemical works, have no reference to magic at all: they simply list recipes for preparing various substances. Their favorite subjects, as in the case of Bolos' treatise (extracts of which probably form the substance of these papyri) are imitation gems and precious metals. What most seems to interest the authors of these works is the color of the substances produced; this suggests that the main application of Alexandrian chemistry may have been the manufacture of dyes and other colorants.

Pliny explicitly distinguishes between natural and artificial pigments[87] and contrasts classical-age painters, who made do with only four colors, with Hellenistic ones, who used a great many different hues.[88] Procedures for preparing artificial pigments are given also by Vitruvius, who

[84] This woman actually lived in Alexandria, probably in the first century A.D., and wrote under the pseudonym *Miriam the Prophetess, sister of Moses*.
[85] See Mertens' *Introduction technique* in [Zosimus/Mertens].
[86] P. Leidensis X and P. Holmiensis. For a recent edition of these papyri see [AG].
[87] Pliny, *Naturalis historia*, XXXV §30.
[88] Pliny, *Naturalis historia*, XXXV §§49–50.

attributes some to the Alexandrians.[89]

We shall see that in Hellenistic times there developed also other industries involving the transformation of matter, from metal extraction and refining to the manufacture of cosmetics, fragrances and medicines.[90]

Already the Stoics, and particularly Chrysippus, in the third century B.C., had clearly in mind the distinction between heterogeneous materials, homogeneous mixtures and single compounds. We know this from various sources. Stobaeus, for instance, writes:

> Stoics like to distinguish between juxtaposition (παράθεσις), mixture (μῖξις), blend (κρᾶσις) and composition (σύγχυσις) … A mixture is the complete interpenetration of two or more bodies, preserving the properties of each … A blend, according to them, is the complete interpenetration of two or more liquids, preserving the properties of each. A blend displays simultaneously the properties of each of the blended liquids, such as wine, honey, water, vinegar and so on. That in such blends the properties of the constituents are preserved is clearly shown by the fact that one can generally separate them again with the right trick. If a sponge dipped in oil is introduced in a blend of water and wine, the water will be attracted to the sponge and separated from the wine. The composition (σύγχυσις) of two or more qualities is instead a transformation (μεταβολή) of the bodies that gives rise to qualities diverse from the original ones, as happens in the synthesis (σύνθεσις) of perfumes or medicines.[91]

We note that the noun here translated as composition never appears in Aristotle's works,[92] and that the concept it designates is illustrated here with examples from the new matter-transformation industries. The noun derives from the same root (of the verb χέω) that probably lies behind the word *chymeia* (χημεία or χυμεία), attested at a later date and from which we ultimately got *chemistry*.[93]

The modern concept of a molecule has an interesting forerunner in the

[89] Vitruvius, *De architectura*, VII, xi–xiv.
[90] See pages 253–255.
[91] Stobaeus, *Eclogae* I, xvii, 154:8 – 155:14 (ed. Wachsmuth) = [SVF], II, 471. Other passages on this point are Philo of Alexandria, *De confusione linguarum*, II, 264 (ed. Wendland) = [SVF], II, 472, and Alexander of Aphrodisias, *De mixtione*, 216:14–218:6 + 221:16–18 (ed. Bruns) = [SVF], II, 473, 474.
[92] The most intimate combination of substances considered by Aristotle is the homogeneous mixture, which he calls by the terms μῖξις and κρᾶσις (*De generatione et corruptione*, I, 328a).
[93] In Arabic the adjunction of the article transformed the Greek word into *alchimia*, which coexisted in Latin for centuries with *chimia*. In the modern age, chemistry ennobled itself by appropriating the Greek name, but usually when the Greek science is referred to, the Arabic word is scornfully used. This strange situation illustrates the complex and contradictory way in which modern scientists have regarded the relationship between themselves and the classical tradition.

Hellenistic notion of *oncos* (ὄγκος).[94] *Oncos* is conceived as the ultimate component of substances, but, unlike atoms (of which it seems to be made), it is capable of transformation, through reorganization of its parts, thus accounting for qualitative changes in substances.[95]

In the Leyden and Stockholm papyri, when amounts of ingredients are indicated (which is often not the case), the information is given in *parts*. The occasional use of weight units indicates that the parts are to be measured with a balance. There is evidence from several sources that this use of the balance led to the principle of conservation of mass[96] attributed to Lavoisier and regarded as one of the greatest achievements of eighteenth-century chemistry. In Lucretius the principle of conservation of mass is not only clearly stated (in the poetical form that enabled the work to survive, of course),[97] but even justified on the grounds that atoms are indestructible.

Another attestation is in Lucian's *Life of Demonax*: when someone asks Demonax "how many minae of smoke do you get burning a thousand minae of wood?", he gets the answer "weigh the ashes; the remainder is smoke".[98] Obviously it matters little that from our vantage point the proposed method is incorrect (because atmospheric oxygen also takes part in the combustion). More interesting than the answer is the question. Why on earth would one ask about the weight of a certain amount of smoke? The only sense the question makes is as an attempt to ridicule an existing scientific theory to the effect that all objects have a "mass" (or weight) and that the mass is preserved. Though such a theory has been regarded

[94]Sextus Empiricus (*Adversus physicos* II (= *Adv. dogmaticos* IV = *Adv. mathematicos* VIII), §318) attributes the use of this concept to Heraclides of Pontus (fourth century B.C.) and to Asclepiades of Prusa, a Greek physician who worked in Rome in the first century B.C. and maintained, for instance, that fevers can propagate through the emissions of corpuscles from the body (compare Sextus Empiricus, *Adversus geometras* (= *Adv. math.*, III), §5).

[95]Compare Sextus Empiricus (*Adversus physicos* II §§42–44). The original meaning of *oncos* is volume, mass, bulk. It would be interesting to investigate in detail what contribution the memory of the ancient concept of *oncos* made to the formation of the modern concept of molecules. Here we just make two observations in this direction. First, the term *oncos* in scientific texts was systematically translated in Latin as *moles* (bulk, large mass), even when the meaning is that of volume, as is clear from the Latin translations of Archimedes' *On floating bodies* made by William of Moerbeke, I. Barrow and G. Torelli. Second, the passage in which Robert Boyle introduces the modern idea (*Chymista scepticus*, London, 1661, chapter 1, prop. 2) is reminiscent of the Sextus Empiricus passage, and in fact the whole work is pointedly set against the backdrop of ancient Skepticism, a doctrine for which Sextus is almost our only source.

[96]An important prescientific precursor of this principle was already present in the statement of conservation of matter made by Empedocles (fr. 4 in [Empedocles/Gallavotti] = frs. 17, 14, 13, 17, 22, 20 in [FV], I, 314–321; see particularly verses 30–32 of fr. 17 = Simplicius, *In Aristotelis physicorum libros commentaria*, IX, 158:29 – 159:1).

[97]Lucretius, *De rerum natura*, II:294–296.

[98]Lucian, *Vita Demonactis*, 39:2–6.

by some as absent in Antiquity,[99] it is implicitly used by Heron of Alexandria, in connection with the same phenomenon, but in a more technical context. After observing that coal, after burning, undergoes a small change in volume but a large decrease in weight, Heron attributes the drop to the transformation of coal into particles of different nature (fire, air and earth), which partly go away as smoke and partly get absorbed by the ground.[100]

Conservation of mass appears to be contradicted in the *Pneumatics* of Philo of Byzantium: in the candle experiment (see page 77 and note 86 thereon) the explanation given is that the air "perishes". But a reading of the homonymous work by Heron, which was preserved in the original Greek (unlike Philo's: see page 76), suggests that this may reflect a simplification undergone by Philo's text in the late Arabic and Latin versions that have reached us. Indeed, Heron, who certainly uses Philo as a source, repeatedly uses forms of the verb φθείρω (destroy, spoil, corrupt; probably the same verb used in Philo's original) to describe what happens to air and other substances during combustion, but he always clarifies that the corruption consists in a *transformation* into other substances. He says, in particular, that the air enclosed in a glass container, if consumed by the fire, is in fact leaving through pores in the glass, leaving behind empty space that attracts other matter.[101] Thus conservation of matter was regarded by Heron as so certain that its apparent violations were explained by means of invisible processes. It is highly improbable that this view was a novelty of Heron's times, if we consider that Erasistratus, in the third century B.C., also seems to have been so confident in the conservation of matter as to deduce from the change in weight of an animal left in isolation the emission of invisible matter (for this experiment see page 156).

Some additional information about ancient chemical knowledge can be gleaned from papyri. An especially interesting case is that of the word *oxos* (ὄξος). It is usually translated as vinegar, that being its original meaning. But recipe 14 of the Leyden papyrus talks about *oxos from the purification of gold* (ὄξος ἀπὸ καθάρσεως χρυσίου). Since the methods given for purifying gold may have released hydrochloric or sulfuric acid but certainly not wine vinegar, it is clear that *oxos* is being used here in a sense similar to *acid*, a notion that must have been partly worked out by Alexandrian empirical chemists. The Latin term corresponding to *oxos* is *acetum*, also translated as vinegar (the related adjective is *acidus*). Now, Livy says that Hannibal used *acetum* to dissolve a blockage in a gorge.[102] This is unlikely

[99]Max Jammer, after quoting Demonax's answer, writes: "Such ideas, however, remained isolated statements.. . . And never did such ideas give rise to the formation of the concept of 'quantity of matter' in a technical sense" ([Jammer: CM], p. 27).

[100]Heron, *Pneumatica*, I, proem, 10:13–24.

[101]Heron, *Pneumatica*, I, proem, 16:10–14.

(and Polybius says nothing of the sort in his account), but it is very suggestive: Livy may have heard of an *acetum* that, much like the *oxos* from the Leyden papyrus, was a lot stronger than anything obtainable from wine.[103] Vitruvius tells us that pearls, lead, copper and pebbles can be dissolved in *acetum* — again, not ordinary vinegar.[104]

We conclude that the Hellenistic period saw the rise of chemistry as an empirical science, although we cannot determine the level of knowledge it achieved. Alchemy appeared only in the imperial age; it borrowed from earlier empirical chemistry instruments and certain procedures, but it had other goals and a different conceptual framework. It was also then that scientific astronomy was reduced to a handmaiden of ancient astrology.

[102]Livy, *Ab urbe condita libri*, XXI, xxxvii §2.
[103]This possibility is also raised by Halleux in [AG], p. 31.
[104]Vitruvius, *De architectura*, VIII, iii §§18–19.

6
The Hellenistic Scientific Method

6.1 Origins of Scientific Demonstration

One essential characteristic of scientific theories, as we have defined them and as we have encountered them in the Hellenistic works considered, is the use of *demonstrations*, that is, deductions of certain statements from others, following a chain of logical steps that makes these deductions, in principle, irrefutable: someone who accepts the premises cannot reject the conclusions, except by finding an error in the deduction.

The English word "demonstrate" is a calque, through Latin, of the Greek verb ἀποδείκνυμι, which initially meant "show, display, expound" (and was interchangeable with the unprefixed δείκνυμι). The original meaning of the corresponding noun, *apodeixis* (ἀπόδειξις), was "a showing, display, exposition" of an object or subject: Herodotus, for example, presents his work as an *apodeixis* (exposition) of his findings.[1] The evolution from this wider meaning, still present in the English "demonstration",[2] to the scientific meaning that interests us went hand-in-hand with the establishment and consolidation of what is called the *hypothetico-deductive method*. This evolution went through at least two intermediate stages, which we can exemplify through the use of *apodeixis* in Plato and Aristotle.

[1]Herodotus, *Historiae*, I §1.
[2]This English word — also a calque, through Latin, of *apodeixis* — means showings of various kinds, and it has even spawned the clipped version "demo", which applies to some of these senses. In the scientific meaning that is the subject of this chapter, it has been losing ground: it is now more common to hear "proof" than "demonstration". In this chapter we will generally write "demonstration" to underscore the semantic origins of the term.

In Plato the word is used in the sense of a rational argument capable of convincing someone else. In the *Hippias minor*, for example, Hippias proposes to demonstrate that Homer portrays Achilles in a better light than Ulysses.[3] The *Republic* gives various demonstrations of the possibility of realizing the proposed state model. Plato seldom uses the term in connection with geometry.[4] We have already remarked that Plato's work contains very interesting demonstrations (in the later, technical sense of the word),[5] but the method used in such cases is not distinguished by a specific term from convincing arguments of another nature.

In Aristotle's works on logic, *apodeixis* is associated with the feature of absolute irrefutability considered today to be necessary in a mathematical proof. This new type of demonstration stands out as the object of Aristotle's *Prior analytics*, where he describes and analyzes syllogisms.[6] He defines a demonstration as a *true syllogism* (one whose premises are true).[7]

A survey of the evolution of *apodeixis* from general argumentation to what we can call Aristotelian "syllogistic demonstration" would require a reexamination of a good part of the history of Greek philosophy, with special attention to the Eleatic school. But it would not be a story limited to philosophy in the narrow sense that this word usually has now, for the evolution owes much to the development of deliberative and judicial rhetoric — the art of arguing convincingly in assemblies and courts — that evolved especially in the Greek democracies of the fifth century. There was a crucial link between the existence of certain forms of democracy and the development of the argumentation skills that led to the hypothetico-deductive method. The relationship between demonstrations and public speaking is clearest in the Aristotelian *Rhetoric*, where the author stresses that the so-called *enthymemes* are none other than syllogisms, and identifies twenty-eight distinct types of rhetorical lines of argument.[8] Aristotle presents rhetoric, in large measure, as an application of the instruments he elaborated in his works on logic, but the historical order was clearly the reverse. A century before his day there were already treatises on rhetoric (now lost), so we can imagine that the theory of syllogisms arose, at least to some extent, from consideration of the rhetoricians' enthymemes.[9]

[3] Plato, *Hippias minor*, 369c.

[4] One exception is in the *Theaetetus* (162e–163a), where a contrast is drawn between methods that do not provide "true demonstrations" and the method used by Theodorus and other geometers.

[5] See page 37 and note 21 thereon.

[6] Aristotle, *Analytica priora*, I, i, 24a:11–15.

[7] Aristotle, *Analytica posteriora*, I, ii, 71b:18–25.

[8] Aristotle, *Ars rhetorica*, 1355a + 1397a ff.

[9] It seems that Aristotle was the first to use the title *The art of rhetoric*. Earlier works on the subject had probably been called *The art of discourse* (τέχνη τοῦ λόγου), revealing in the very name the genealogy of later works on logic (from *logos*). Mathematics too had certainly been food for thought for the founders of logic. But it should be noted that Egyptians and Babylonians, who for centuries had cultivated mathematics, but not democracy or rhetoric, never did reach the demonstration stage.

For some deductive schemes we can recognize an origin in fifth-century rhetoric and sophistics: the scheme called "consequentia mirabilis" in the Middle Ages (a variant of the proof by contradiction, consisting in proving *A* by proving that non-*A* implies *A*) was used by Protagoras and Gorgias.[10]

The link between *apodeixis* and rhetoric was still discernible during the imperial age, when rhetoric was used only in law. Quintilian writes, illustrating the usefulness of the study of geometry in the training of orators:

> Geometry proves consequences from premises and unestablished things from established ones; do we [orators] not do the same when we make a speech? Doesn't the solution of the proposed questions rely almost entirely on syllogisms? . . . So also [the orator] will use, if necessary, syllogisms and of course enthymemes, which are rhetorical syllogisms. Finally, of all lines of proof the most powerful are those called γραμμικαὶ ἀποδείξεις [linear demonstrations]; and what could be more desirable in an oration than good lines of proof?[11]
>
> . . . so that an orator cannot ever be ignorant of geometry.[12]

Syllogistic demonstrations were an important element in the scientific method, but one that gave it life only in combination with other elements, which, through the creation of scientific theories, profoundly altered the very role of demonstrations.

A Hellenistic scientific theory is something very different from a set of syllogisms. To begin with, the statements of the theory make up a single network, being all provable from a small number of premises. They also involve *theoretical* terms, that is, terms specific to the theory, in contrast with the syllogisms considered by Aristotle. To build a scientific theory, then, is is not enough to be able to deduce one statement from another; one must choose appropriately the premises and terms of discourse. Also essential was the use of elements other than verbal argumentation, drawn from observation and from technical activities; one important example is the role played by constructions in geometric demonstrations.

The next few sections will be devoted to these aspects of the Hellenistic scientific method.

[10] For Protagoras see Sextus Empiricus, *Adversus logicos* I (= *Adv. dogmaticos* I = *Adv. math.* VII), §§389–390 = [FV], II, 258:36 – 259:3, Protagoras B15. The history of *consequentia mirabilis* is discussed in [Bellissima, Pagli].

[11] Quintilian, *Institutio oratoria*, I, x §§37–38.

[12] Quintilian, *Institutio oratoria*, I, x §49.

6.2 Postulates or Hypotheses

One important aspect of scientific method was that it marked explicitly and unambiguously the premises that were to be accepted within a given theory. These premises were called αἰτήματα (*postulates*, which is to say demands), or λαμβανόμενα (*assumptions*), or yet ὑποθέσεις (“*hypotheses*”).

This last term merits a digression. Its original meaning, “foundation, base”, never disappeared in Greek: Aristotle uses the expression ὑποθέσεις τῆς πολιτείας for the foundations of government, and Theophrastus says that the trunk is the ὑπόθεσις of the tree.[13] In either case there is nothing “hypothetical” in the sense familiar to us. In philosophy the term was used for the logical foundation of a chain of deductions, and in scientific theories for more or less what we call principles.[14] When Archimedes, in describing the heliocentrism of Aristarchus of Samos, writes that the latter ὑποθεσιών τινων ἐξέδωκεν γραφάς (published the text of some “hypotheses”),[15] he means that the immobility of the sun and the rotation and revolution of the earth were the ground assumptions of the Aristarchan theory (though oddly, many commentators have taken the word in the modern meaning of “hypotheses”).

What criterion was used in choosing the initial assumptions (postulates or “hypotheses”) of a theory? The first that may come to mind, namely picking the simplest and most easily checked statements, has been put forth by many authors, ancient and modern, but it does not agree at all with the facts.

First of all, the statements that seem simplest may turn out to be useless for deriving interesting consequences. In astronomy, for instance, an assumption that the earth is fixed may look like an obvious choice, but does not offer a particularly useful basis for the description of planetary motion. In geometry one might think that the simplest entities are points, but the attempts of the Pythagorean school to build up geometry on statements dealing with points alone ended with failure (as we saw in Section 2.1) and with the acknowledgement that one may not even recover the properties of the straight line by starting from statements about points alone.[16] The road chosen by Euclid consists in *not* starting from absolutely “elementary” theoretical entities, such as points appear to be, but directly

[13]Theophrastus, *Historia plantarum*, IV, xiii §4.

[14]For example, Sextus Empiricus calls an ὑπόθεσις each of the postulates of geometry (*Adversus geometras* (= *Adv. math.* III), §§1–4). We will return later to the question of the somewhat different meaning of “principles” in modern physics.

[15]Archimedes, *Arenarius*, 135:8–9 (ed. Mugler, vol. II).

[16]An interesting work dealing with this problem has survived: the pseudo-Aristotelian *De lineis insecabilibus*, dating probably form the late fourth century B.C. It argues for the impossibility of constructing geometric magnitudes (and lines, in particular) by putting points together.

from statements on lines and circles.

In the second place, were the postulates or “hypotheses” of Hellenistic theories verifiable statements?

In almost every case, verifiability is excluded by the universal character of the statement in question: how can one check that something holds “for every pair of points” or “for every straight-line segment”? We can only check (approximately) particular statements implied by the postulates.

And generality is not the only problem. Consider, for example, the postulate on which the Archimedean treatment of hydrostatics is based (see page 73). This statement cannot be verified even in particular cases. How can one check, in non-equilibrium conditions (for instance, immediately after opening a dam), the action that a particular small portion of liquid, immersed in the whole, exerts on a neighboring portion? It is clear that Archimedes is making an assumption that is not directly verifiable, and that its interest for him lies in the possibility of deducing from it many verifiable statements about what happens in equilibrium conditions.

In sum, it is clear that the initial assumptions of Hellenistic scientific theories were neither obvious nor verifiable. What, then, could have been the criterion with which they were chosen? The next sections are devoted to this problem.

6.3 Saving the Phainomena

In his *Outlines of Pyrrhonism*, the Skeptic Sextus Empiricus writes:

> We [Skeptics] do not contest that which compels us to involuntary assent [συγκατάθεσις] to a sensorial impression, which is to say the *phainomena*.[17]

He is reporting here the Stoic conception of phainomena or “appearances” (φαινόμενα). According to the Stoics, a phainomenon does not involve just a (passive) sensorial impression; also essential is the assent (συγκατάθεσις) of the subject, at once active and involuntary.[18] Strato of Lampsacus, too, wrote that every sensation is obtained through an active participation of the intellectual faculties, even if we’re not conscious of it.[19]

It is important to notice the distance in meaning between the Greek

[17]Sextus Empiricus, *Pyrrhoneae hypotyposes*, I, x §19.

[18]The Stoics considered also voluntary assent, but we will be interested primarily in the involuntary kind, a typical example of which is provided by the recognition of a known person, which happens through a comparison between visual impression and memory. The Stoics discussed at length the possibility of identifying an erroneously given assent; see [Gould], pp. 56–62, as well as [Frede] and the references therein. See also Figure 6.1.

[19]Plutarch, *De solertia animalium*, 961A.

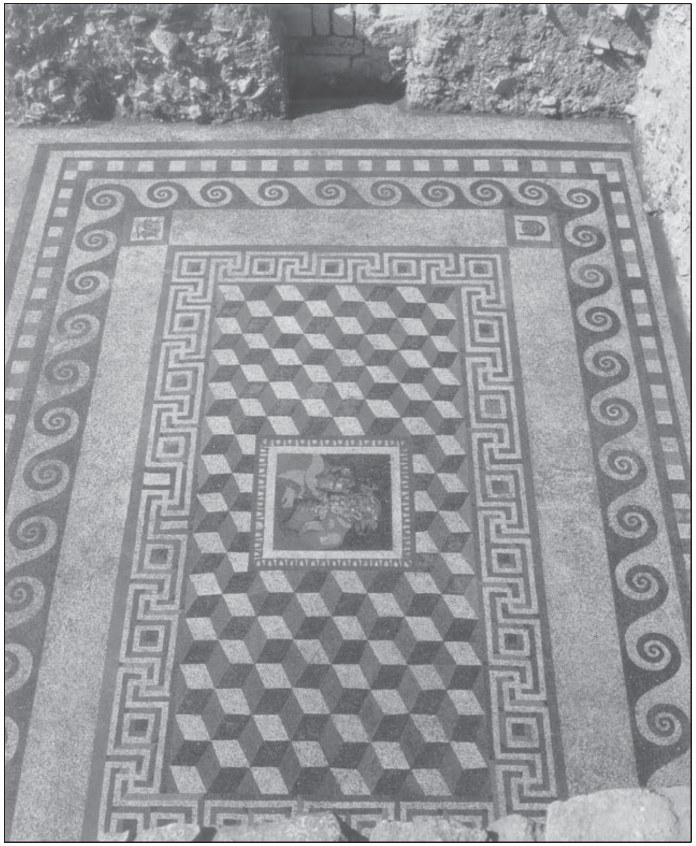

FIGURE 6.1. Hexagonal tessellations shaded to represent cubes in perspective are a recurring motif in Hellenistic mosaics. The corners can be perceived alternately as sticking out or in. Possibly this phainomenon was explained through the possibility of assenting to different interpretations of the same impression. The photo shows a mosaic from Pergamum (with Silenus and child Dionysus at the center). Courtesy of Deutsches Archäologisches Institut, Istanbul, negative Perg. 91/80,6, photo by E. Steiner. See also [Dunbabin], pp. 223–224.

word *phainomenon*, which refers to the interaction between subject and object constituted by perception, and our derived word *phenomenon*. In modern times phenomena have long been considered simply as facts that take place independently of, and can be known directly by, the observer, through a mechanism that need not be looked into. Now the point for us is that Sextus Empiricus identifies for us the only epistemological element about which there was no doubt at all (even among Skeptics!) in

Hellenistic times.[20] Not long before the passage cited, he gives a more specific example:

> For a Skeptic gives his assent to impressions caused necessarily by sensation: for example, being hot or cold, he would not say, "I don't think I'm hot or cold".[21]

Sextus Empiricus was a physician. It is possible that his example was not independent of the analogous observation made centuries earlier by Herophilus and reported by Galen:

> So what does [Herophilus] say? "It is by nature impossible to find out whether there is a cause or not; but I can assess whether I'm cold, or hot, or satiated with food or drink."[22]

If *phainomena*, because of their immediate evidence, are singled out as the only indubitable epistemological data, they are the best candidate for a departure point in the construction of scientific theories. But they can only be the departure point in a heuristic, not a logical, sense; they must then be explained by a theory logically based on "hypotheses" that are not directly verifiable, and in this theory they play the role of effects. On this point there is an important testimonium in the medical work of the Anonymus Londinensis:

> . . . as Herophilus observes, saying "let the phainomena be described first, even if they do not come first."[23]

In the case of optics — the theory of sight — the only certain data from which one can start are visual perceptions. One of the propositions that Euclid demonstrates is:

[20]Here we are interested more in the method actually used by scientists than in the epistemological musings of Hellenistic philosophers (who, in the surviving fragments, rarely deal with scientific theories). But it is nonetheless significant that to several Hellenistic philosophical schools, various aspects and forms of perception lie at the base of knowledge: to the Cyrenaics, *feelings* (πάθη: Eusebius, *Praeparatio evangelica*, XIV, xix §1); to the Epicureans, *perception* (αἴσθησις: Epicurus, *Letter to Herodotus*, lines 49–53, in Diogenes Laertius, *Vitae philosophorum*, X, §24); to the Stoics, *impression* (τύπωσις: Diogenes Laertius, *Vitae philosophorum*, VII §50 and Sextus Empiricus, *Adversus logicos* I (= *Adv. dogmaticos* I = *Adv. math.* VII), §§250–251). We will discuss later the notion of κατάληψις (also translated as *perception*).

[21]Sextus Empiricus, *Pyrrhoneae hypotyposes*, I, vii §13.

[22]"Quid igitur ait [Herophilus]? 'causa vero, utrum sit vel non, natura quidem non est invenibile, existimatione autem puto infrigidari, estuari, cibo et potibus repleri' " (Galen, *De causis procatarcticis*, xvi §198 = [von Staden: H], text 59a:7–9). Von Staden interprets it differently, translating "… it is through a supposition that I think I am chilled…". Galen, who is criticizing Herophilus, may in fact be using "existimatio" to some such effect. But even not taking into account the preceding passage by Sextus, it seems to me that the original word translated "existimatio" in the Latin, referring as it does to the perception of sensations such as cold or satiety and counterposed with the impossibility of verifying causes, was probably not being used by Herophilus in a limiting function.

[23]P. Londinensis 137 (Anonymus Londinensis, *Iatrica Menonia*), col. 21:20–23 = [von Staden: H], text 50a:3–4.

> If several magnitudes move [in the same direction] each at its own speed, and the eye also moves in that direction, those magnitudes that move at the same velocity as the eye appear stationary, those that move more slowly appear to fall behind, and those that move faster appear to advance.[24]

The phainomena, which heuristically speaking represent the starting point, are here deduced from (not directly verifiable) statements about the state of motion of the observer and the objects observed. Euclid's proposition has a great deal of methodological interest, recognizing as it does that, at least in the case of visual perception, the phainomena do not speak about the object directly, but only about a relationship between object and observer.

A connection between Euclid's proposition and the astronomical problems of his time is transparent: retrogressing planets were very soon to be seen as a case of bodies that seem to move backward because their motion is slower than the observer's.

We now see clearly what essential requirement the "hypotheses" of a theory must satisfy: they need not be directly verifiable, they may even be surprising at first sight, but the important thing is that they must allow the logical deduction of the phainomena: in the case of astronomy, the observed motion of heavenly bodies. The Aristarchan "hypothesis" that the sun was stationary and the earth had movements of rotation and revolution certainly appeared strange and remote from intuition, but (and this is the crux!) it allowed their inventor to "save the phainomena" (φαινόμενα σῴζειν), as Archimedes says, by deducing from them the planetary motions actually observed.

The passage in Aristotle's *Physics* that mentions natural selection[25]is apparently similar in spirit to Aristarchus'. The "hypothesis" that animal organs may in the beginning have had accidental shapes, like the Aristarchan "hypothesis", is not directly verifiable and appears at first to flagrantly contradict observations — here the complex and functionally well-adapted structures seen in animals. And yet, by drawing all the consequences of that hypothesis, because only the more suitable forms would have allowed survival and reproduction, one manages to explain much more than through an appeal to final causes. Indeed, one can explain not only the shapes of the organs based on their functions, but also *why* organs are adapted to their functions, just as Aristarchus succeeded not only in accounting for the apparent motion of fixed stars and the sun, but even in explaining planetary retrogression.

[24]Euclid, *Optics*, proposition 51.
[25]See pages 160–161.

6.4 Definitions, Scientific Terms and Theoretical Entities

Anyone who has used the term *trapezium* (or *trapezoid*) in school was never in doubt that it describes a geometric figure rather than a concrete object. Euclid's students, to designate the same geometric figure, used the term τραπέζιον, from which we derive ours. But to them, the word was also part of the ordinary language: it meant a stool or small table. The abstraction process through which it came to designate a theoretical entity, therefore, was necessarily more conscious and explicit. As Bruno Snell wrote:

> The relation between language and the formation of scientific concepts ... can, strictly speaking, be observed only in Greek, because only here did the concepts grow organically from the language. Only in Greece ... was there a native formation of scientific terms — all other tongues fed on Greek, borrowed from it, translated from it or depend on it in some less direct way.[26]

To create a "scientific term" the Greeks resorted to one of two methods. The first and more obvious to us was through a *definition* (ὅρος).

In the history of thought, two profoundly diverse notions of definition have alternated. According to the first, which we will call *essentialist* or *Platonist* because we find it in Plato (though also in Aristotle), the purpose of a definition is to identify the essence of the thing defined.[27] Thus, for example, the many attempts to define "good" and "justice" in Plato's Socratic dialogues. In the Platonist view, essentialist definitions apply just as well to mathematical entities, which are regarded as having an objective reality, the mathematician's function being solely to describe and use them.[28] This view prevailed in the imperial age, in the Middle Ages and in the early modern age.

Karl Popper wrote:

> The development of thought since Aristotle could, I think, be summed up by saying that every discipline, as long as it used the Aristotelian

[26][Snell], p. 199.
[27]For Aristotle's opinion that defining something means to identify its essence, see for example *Topica*, I, v, 101b:36; *Metaphysica*, VII, v, 1031a:13 and VIII, i, 1042a:17. For the substantial agreement on this subject between Aristotle and Plato see [Popper: OSE], Chapter 11, §2, following note 31.
[28]Plato's conception of mathematical entities (put forth, for example, in the *Republic*, VI, 509c–511a) was, it is true, criticized at length by Aristotle, who maintained that these entities were not immanent in objects and did not possess a separate reality (see in particular *Metaphysica*, XI, iv; XIII; XIV). Aristotle's position can be summarized approximately by saying that mathematical beings have a particular type of existence: they exist only as properties of perceivable objects. But although his view has different philosophical bases than Plato's, the difference is not such that the attitude toward the mathematician's work is significantly changed. In this respect the essential point is that, for Aristotle as for Plato, humans do not construct mathematical entities: they somehow preexist.

method of definition, has remained arrested in a state of empty verbiage and barren scholasticism, and that the degree to which the various sciences have been able to make any progress depended on the degree to which they have been able to get rid of this essentialist method.[29]

One may take issue with Popper's opinion about the "empty verbiage" of the Aristotelian method. In fact, the Aristotelian method of definition, which consists in pinpointing the essence of what is being defined through a series of dichotomies, is useful and applicable for singling out an existing object among a finite set of possibilities; this is the case of animal species, which Aristotle was particularly interested in. One can, for instance, define the swallow by saying that it is a bird and by listing enough features to allow it to be distinguished from all other known bird species. But it was not this method that led to the creation of the scientific terminology that concerns us. In exact science, indeed, a definition is not meant to identify a concrete object among a finite set of possibilities, but to characterize uniquely a theoretical entity among infinitely many possibilities.[30]

Typical definitions fit for creating new scientific terms are the following, made by Archimedes:

> . . . we suppose the following: if an ellipse, its major axis staying still, rotates so as to get back to its initial position, the figure enveloped by the ellipse will be called a lengthened ball-shape [παραμᾶκες σφαιροειδές]. If an ellipse, its minor axis staying still, rotates so as to get back to its initial position, the figure enveloped by the ellipse will be called a flattened ball-shape [ἐπιπλατὺ σφαιροειδές]. For either ball-shape. . .[31]

What we have translated as "lengthened", "ball-shape" and "flattened" are everyday Greek words, which acquire a new and precise meaning after this point in the text, becoming abbreviations or tags for certain long expressions made up of other terms already known. Note the contrast with the essentialist mode of definition: Archimedes is not at all troubled by the fact that his ball-shape may look nothing like a ball, but rather like a needle or a lentil.[32] We will call definitions of this type *nominalist*; they

[29][Popper: OSE], Chapter 11, §2, following note 26.

[30]If in order to define an object one must take into account its differences vis-à-vis all others, any definition has as a prerequisite the knowledge of all reality. This difficulty (raised, according to scholiasts, by Speusippus) had already been faced by Aristotle in other contexts (*Analytica posteriora*, II, xiii, 97a:6–10), but it becomes insurmountable in the case of mathematics.

[31]Archimedes, *De conoidibus et sphaeroidibus*, 155:4–13 (ed. Mugler, vol. I).

[32]By contrast, for Plato the same term really meant round like a ball (Plato, *Timaeus*, 33b). The Greek σφαιροειδές is of course the etymon of our "spheroid", the word traditionally used to translate this term in Archimedes; the technical expression used nowadays for the same notion is *ellipsoid of revolution* (*prolate* for lengthened and *oblate* for flattened).

are common in the writings of Hellenistic scientists.[33] There is clearly a close connection between a nominalist notion of definition and linguistic conventionalism, which, as already seen, arose at that time.[34] The use of nominalist definitions in mathematics was accompanied in Hellenistic times by a new concept of mathematical entities. For example, we know from Proclus that Apollonius of Perga described the origin of fundamental geometric concepts from everyday experience, saying for instance that the notion of a line arises from considering things such as roads about which one can say "Measure its length" without fear of misunderstanding.[35]

The notion of a point (στιγμή) was analyzed at length in pre-Hellenistic times in the framework that we have called Platonist. The discussions of the notion in Aristotle were of this type.[36] Euclid avoids the word στιγμή in the *Elements*, using instead σημεῖον, which originally meant "sign".[37] This replacement suggests that Euclid may have wished to cut himself off from the tradition of Platonic speculations on the true nature of the point, sticking to a conventionalist notion of language and a new notion of mathematics. This new way of seeing mathematical entities is in evidence in some of Euclid's definitions, in particular the definition of proportion. If one thinks of the "ratio between magnitudes" as something that exists in and of itself, the equality of two ratios seems like an obvious notion (as it did to Galileo[38]), whereas Euclid's definition, as we saw in Section 2.5, is tantamount to a subtle and complex implicit definition of the notion of the ratio between magnitudes.

Nominalist definitions are certainly very valuable for enriching scientific terminology, but they cannot create it from scratch. Any definition

[33]Like the one just quoted, many definitions, particularly in Archimedes and Apollonius, consisted of a long expression that was identified with a new term by means of some form of the verb καλέω (to call). In Euclid's *Elements* there are also some definitions of Platonist type, for instance for point, line and plane. We will return in Section 10.14 to the problems offered by these latter definitions.

[34]See page 153 and especially footnote 34 thereon. A hint of nominalist definitions in mathematics is perhaps present already in Plato (*Theaetetus*, 184a–b). The passages in the *Theaetetus* dealing with the practices of the mathematician Theodorus and his school seem to look forward to later scientific elements that are not otherwise present in Plato (whose notion of language, as laid down in the *Cratylus*, is miles away from conventionalism).

[35]Proclus, *In primum Euclidis Elementorum librum commentarii*, 100:4–8 (ed. Friedlein).

[36]Sample passages are listed in note 245 on page 326; see also the surrounding discussion.

[37]The occasional presence in some works of the Aristotelian corpus of this word σημεῖον in the sense of "point" is neither here nor there, because these works were revised in post-Euclidean times. The term recurs, for example, in some geometric constructions contained in the *Meteorologica* (see especially III, iii, 373a and III, iv–v, 375b–377a), whose redaction into the form that has come down to us is probably due to a student of Theophrastus.

[38]We will return to this on page 350.

of this type can only reduce the meaning of a new term to that of terms already assumed known. Just as the hypothetico-deductive method requires demonstrationless statements on which to build, so the nominalist definition procedure requires definitionless terms from which to start.

The awareness of the need to avoid infinite regress, which must be clear to a person who shares a nominalist view of definitions, is documented already in pre-Hellenistic philosophy. Thus, Aristotle reports that, according to the school of Antisthenes, since every definition requires a reference to something else, it is only possible to define what is composite (whether materially or conceptually), not what is simple.[39] This observation of Antisthenes is not an isolated one, because around 200 A.D. Sextus Empiricus wrote:

> And given that, if we want to define everything we define nothing at all, because of regression to infinity, whereas if we admit that some things can be understood without definition we are declaring that definitions are not necessary for understanding, ... we must either define nothing at all or declare that definitions are not necessary.[40]

The conclusion drawn by Sextus Empiricus reflects his own Skeptic ideas. What concerns us is that the possibility of "admitting that some things can be understood without definition" — of presupposing, that is, the existence of some definitionless entities — was still taken into consideration in his time.

How were the first Greek purely scientific terms, and therefore the first theoretical entities of science, created? It is not hard to realize that the essential tools were provided by the postulates of the various theories and the hypothetico-deductive method. Take the first postulate of the *Elements*, for example. In Euclid's text it reads, literally,

> Let it be demanded that a straight line be drawn [i.e., drawable] from every sign to every sign.[41]

This statement contains words from ordinary Greek, signifying concrete objects: "straight lines" are originally traces drawn or carved (the Greek word γραμμή indicates this explicitly), the "signs" are equally concrete in nature, and the whole can be read as a sentence of ordinary language, with a clear meaning relating to the concrete activity of a draftsman. Naturally the draftsman can draw, for instance, a green or a red line, thicker or thinner, and make signs of different types. But now suppose that we take this

[39] Aristotle, *Metaphysica*, VIII, iii, 1043b:23–32.
[40] Sextus Empiricus, *Pyrrhoneae hypotyposes*, II, xvi §§207–208.
[41] Ἠιτήσθω ἀπὸ παντὸς σημείου ἐπὶ πᾶν σημεῖον εὐθεῖαν γραμμὴν ἀγαγεῖν.

sentence, together with the other four that Euclid wrote, as postulates in his theory based on the hypothetico-deductive method. Because none of the postulates mentions color, clearly no proposition deducible from them can say anything about color. The "lines" of the theory are thus automatically colorless. The same can be said of their thickness or the shape of the "signs". In other words, the use of the hypothetico-deductive method automatically restricts the semantic extension of the terms used in the postulate, generating new entities that are "theoretical" in the sense that the only statements one can make about them are those deducible from the postulates of the theory.

Another example: in the *Optics*, again of Euclid, the initial assumptions of the theory contain an essential term, *opsis*, which in Section 3.1 we translated as "visual ray". The meanings of the word in Greek are manifold: view, aspect, image, spectacle; or yet, in an active sense, sight, look, organs of sight; it can even mean the "evil eye".[42] In natural philosophy there were various doctrines of sight based on the idea that an *opsis* was something actually emitted from the eye. In the Euclidean theory all of that no longer matters; all these possible meanings of the term, since they do not play a role in the theory's assumptions, are automatically eliminated from the theory itself. The visual rays of Euclidean optics are completely characterized as entities that associate (according to precise rules) visual perceptions to half-lines originating in the eye. In particular, since in none of the assumptions is there any reference to propagation along the ray one way or the other, the direction of propagation remains outside the theory, just like color in Euclidean geometry.

Of course, theoretical terms, being formed from ordinary words by a process of semantic pruning, maintain some relation with their ordinary meanings. It is this relation that gives rise, in Hellenistic scientific theories, to what we have called correspondence rules between theoretical entities and concrete objects. It is clear, moreover, that this correspondence will never be perfect, since actual observable phenomena depend also on those properties of the concrete objects in question that were pruned away in the process of abstraction. For example, if we wish to check a theorem of Euclidean geometry on a drawing, the thickness of the lines, though absent from the theory, will play in the drawing a role that will prevent the verification of the theorem beyond a certain precision.

The abstraction process just described enjoys a very important feature: because ties are never completely broken with common sense and daily language, even theoretically incorrect results (those not following from the hypotheses) tend to remain applicable within the framework in which the

[42] Plutarch, *Quaestionum convivialium libri vi*, 681A.

theory was devised, if they are grounded in experience. Thus, for example, Euclid states that an exterior angle of a triangle must be greater than either of the nonadjacent interior angles. His proof is wrong: this statement does not in fact follow from the assumptions previously made.[43] But it is true in the particular model of plane geometry that Euclid had in mind and to which he intended to apply his theorems.

Analogously, the goal of research on logic (which meant the theory of *logos*, or discourse) was to reach forms of deduction at once universally acceptable yet expressible in the terms of ordinary language. Such studies always remained closely connected with studies about language. "Logical" paradoxes, or paradoxes of discourse, were of special interest in part because they highlighted the need to sharpen the tool that is language, and helped do so. Chrysippus devoted many books to them. As in the case of mathematical theories, there was no break with everyday experience, no formal language divorced from everyday language.

We can summarize the matter so far by stating that the fundamental entities of a theory were defined implicitly by the postulates of the theory itself, so long as we keep in mind that this implicit definition process was not the same as the modern formalist reduction of the "meaning" of terms to the logical rules that must be followed in their use in discourse (à la Hilbert with his tables, chairs and beer mugs). Instead, it is a consequence of the choice of assuming as postulates certain statements formulated in ordinary language and regarding them as unambiguous. Of course, the postulates, once the theory was developed, also took on a new face (following the semantic pruning) as statements in the theory, but they also kept their original naïve meaning, thus playing the role of a bridge between concrete reality and what we would now call the theoretical model.

It may be asked whether and to what extent Greek scientists were aware of the procedure we have described. There are many indications that they were. An awareness of the model nature of scientific theories is unavoidable in cases where alternative models were used simultaneously; we have discussed one such case (Archimedean hydrostatics) and will see others in the next sections. Likewise, an awareness about the mode of formation of new scientific terms is attested by surviving references to Stoic semantics, a framework in which the meaning of words boils down to whatever is intended by their user (see Section 7.5 for further discussion). Regarding the implicit definition of entities by means of postulates, the procedure

[43] Euclid, *Elements*, I, proposition 16. The non sequitur comes from a hidden assumption: that a certain segment arising in the construction-proof lies within the exterior angle. In spherical geometry the proposition is false whereas the assumptions made in Euclid's theory up to that point are valid.

appears to be conscious whenever a fundamental entity of the theory is introduced without definition, as in the case of the notion of visual rays in Euclid's *Optics* or that of barycenter in the Archimedean treatise *On the equilibrium of plane figures* or that of the length of a class of curves in the work *On the sphere and cylinder* by the same author.

6.5 Episteme and Techne

Humans interact with the external world by observing it and acting on it. These two fundamental modes provide the heuristic bases of ancient science: phainomena or perceptions on the one hand, technical activities on the other. Geometry, in particular, kept a close link with the techniques of drawing; we have seen that, in the *Elements*, three of the postulates (dealing with the possible uses of ruler and compass) and many of the propositions are feasibility statements: in effect, problems, whose solution ends not with the formula "as was to be shown" but with "as was to be done".[44] In the demonstration of problems and theorems alike, many logical steps are not verbal, but consist in drawing lines or carrying out other, more complex, operations whose feasibility had been demonstrated earlier.

Geometric drawings were not used simply as accessories to abstract reasoning: in geometry it was rather abstract thinking that was conceived as a functional instrument for drawing. Drawing is a technical activity that played a privileged but not isolated role in the development of Greek science. Among the elements of a problem Proclus includes the "construction" (κατασκευή).[45] Though he is referring to geometry here, the word also meant much more concrete things than geometric constructions — although the latter, too, were meant to be really carried out and not just imagined. Consider the following two propositions:

> Construct an equilateral triangle on a given segment.
>
> With a given force move a given weight by means of gears.

The first is taken from the *Elements*,[46] the second from Heron's *Mechanics*.[47] From the point of view of Hellenistic science these two statements (or "problems") are strictly analogous: both are followed by an exposition

[44] See note 30 on page 40.

[45] Proclus, *In primum Euclidis Elementorum librum commentarii* (ed. Friedlein), 203:10–12.

[46] Euclid, *Elements*, I, proposition 1.

[47] Heron, *Dioptra*, xxxv, 306:22–23, in [Heron: OO], vol. III. The *Mechanics* has survived only in Arabic translation, but the quoted proposition, together with its proof, is known in the original because it appears also in the *Dioptra*.

of the necessary construction and then the demonstration that, based on propositions already known, the construction does satisfy the statement's conditions. A similar scheme is followed in the pneumatics works of Philo of Byzantium and Heron. The "demonstrations" there are less rigorous, but the scheme problem–construction–demonstration is still present, and the exposition is reminiscent of works on geometry (or mechanics or optics) even in its formal features, such as the use of letters to indicate simple elements (which in this case might be valves or tubes).

Hellenistic mathematics was certainly constructive (every new figure introduced by Euclid comes with a description of its construction), but in a sense much stronger than that of modern constructivism, because the construction was not a just a metaphor used for providing a demonstration of existence, but the actual goal of the theory, just as the machine described by Heron was constructed to lift weights and not just to prove a "theorem of existence" about the machine.[48]

At bottom, the criterion of value for a scientific theory was based on the relationship between the theory and concrete objects, and this was twofold: on the one hand, the ability of the theory to account for the phainomena; on the other, its ability to allow the design of viable, functioning objects.

Whereas with ordinary language one can talk reasonably only about existing objects and observable phenomena, the hypothetico-deductive method allows one to deduce from the "hypotheses" properties of objects and phenomena that are virtually possible, but not yet in existence. Science thus provides a formidable tool for technical design. Obviously empirical tests remain essential, because one can never be sure of having included among the "hypotheses" all the data relevant to one's purpose.

A discussion about the relationship in Greek civilization between *episteme* (ἐπιστήμη, meaning "science") and *techne* (τέχνη, meaning "art, craft, technique") would require many pages, if nothing else because of the amount of ink that Plato and Aristotle devoted to the subject. But the close ties between the two in Hellenistic times are completely obvious. Several of the subjects that for us are sciences are named after a Greek techne: thus mechanics was originally ἡ μηχανικὴ τέχνη, the art of building machines. Even a very specific techne, such as making mirrors or theater backdrops, could gave rise to a science: Geminus calls catoptrics and scenography subdisciplines of optics.[49]

In some cases (optics, for instance) the name of a science arises in Greek

[48]Zeuthen was the first to stress the essential nature of geometric constructions in Euclid, but he backdates the modern preoccupation with proofs of existence. See [Zeuthen], for example.

[49]Geminus, in [Heron: OO], vol. IV, 104:9–12.

as an adjective that can be construed with either noun, *episteme* or *techne*. This usage of course does not efface the difference in meaning between the two nouns but it underscores the close relationship between the scientific organization of certain disciplines and their origin in and application to a particular techne.

Sextus Empiricus, in the part of his work called *Against the rhetoricians*, refutes the widespread opinion that rhetoric is the art (techne) or science (episteme) of discourse. For this purpose he transmits the following definition of techne:

> Every techne is a system of acquisitions of knowledge exercised together in connection with some useful purpose in life.[50]

This Stoic definition is reported with little variation by several authors[51] (and gave Lucian a pretext for writing an amusing satirical piece, making the point that, on this basis, being a moocher or sponger is also a techne[52]).

Another passage of Sextus Empiricus is also worth quoting:

> Every existing techne and episteme is mastered through the works (ἔργα) produced by that techne or episteme and associated with it.[53]

It is not by accident that the already quoted theoretical reflection of Philo of Byzantium on the experimental method[54] appears in a work on the construction of catapults and mentions the experiments needed to determine the optimal design features of the weapon. Likewise the science of pneumatics, in which the experimental aspect is more evident, is closely linked to the construction of objects such as pumps or water supply systems.[55] An analogous link can be glimpsed between acoustics and the design of theaters and musical instruments.

6.6 Postulates and the Meaning of "Mathematics" and "Physics"

The criterion we have identified for the choice of postulates, namely the ability to "save the phainomena", is certainly essential, but it does not lead to a unique choice of postulates. Consider again the example of motion: as Euclid observes,[56] the same phainomena can be saved by different sets

[50]Sextus Empiricus, *Adversus rhetores* (= *Adv. math.* II), §10.

[51]The versions that have come down to us are cataloged in [SVF], II, testimonia 93 to 97.

[52]Lucian, *De parasito, sive artem esse parasiticam*.

[53]Sextus Empiricus, *Adversus ethicos* (= *Adv. dogmaticos* V = *Adv. mathematicos* XI), §188.

[54]See page 111.

[55]See Section 4.6.

[56]Euclid, *Optics*, proposition 51. Compare page 178 above.

of postulates saying which objects are really stationary and which are in motion, inasmuch as the same visual impressions are deducible from both sets. This eventually led to the idea of a free choice of a reference system with respect to which motion is to be measured, but it predates the notion of a reference system. Because they were stated using verbs from ordinary language, postulates about states of motion necessarily had to involve "absolute" motion at first, so what we regard as a free choice of reference systems was then seen as an equivalence of mutually contradictory postulates. The epistemological importance of this example was therefore much greater than it might appear today.

Sextus Empiricus writes that one must suspend judgement about the question of what objects are truly stationary.[57] Here as elsewhere, he is probably echoing a thought that goes back at least to Herophilus, as seems to be implied by the following passage of Galen:

> Again, he [Herophilus] will express doubt in another manner by justly making the following distinction . . . : "That which sees produces sense-perception of that which is seen, either because what sees is stationary, and so is what is seen, or because what sees is in motion and the seen object stationary, or because both are in motion, or because what sees is stationary and the object is in motion." Then, showing that it is not plausible that sense-perception takes place according to any of the aforementioned, he annuls the fact that we see anything at all.[58]

Galen evidently misunderstood his source, and ascribed to Herophilus an absurd statement.

If two distinct and apparently incompatible statements, such as the sun is stationary versus the sun moves, can both represent good starting points for distinct but equivalent theories, it is clear that one cannot apply to such statements the ordinary idea of "truth". The only criterion for judging postulates, namely the verifiability of their consequences, allows one to welcome distinct postulates as equally valid. Of course, throughout Antiquity many "more traditional" notions of scientific truth were held (and are amply documented), but in the Hellenistic period the notion just discussed was put forth as well, and it is attested in several works.

Diogenes Laertius, for example, presenting two of the five modes in which the Skeptic Agrippa reached a suspension of judgement, wrote:

[57] Sextus Empiricus, *Pyrrhoneae hypotyposes*, I, xiv §107.

[58] Galen, *De causis procatarcticis*, xvi §§203–204 = [von Staden: H], text 59a:40–45 (von Staden translation).

> The mode "regarding something" says that nothing can be understood in itself, but [only] regarding something else. All things, therefore, are unknowable.
>
> Another mode results from "hypotheses": some say that it is necessary to assume as true without qualification certain initial assertions, and not to postulate[59] them; this is senseless, for someone else will assume the opposite.[60]

What interests us here is not Agrippa's sceptic opinion, but the fact that he considered it a generally accepted idea that postulating something does not necessarily means claiming its truth.

Epicurus seems to already display the same methodological attitude when he writes that anyone who takes one causal explanation over another that is equally compatible with the phainomena is wallowing in mysticism.[61]

The scientific method as described so far — and in particular its twin foundations, postulates capable of saving the phainomena being one and demonstrations/constructions the other — was used to some extent even in medicine, but in subjects such as geometry, optics, hydrostatics and astronomy, it reigned uniformly. The modern distinction between physical and mathematical sciences was alien to Hellenistic science, which was unitary. This point cannot be stressed too often, given our well-nigh unavoidable tendency to think in terms of modern categories: the theories developed in Archimedes' *On floating bodies* and *On the equilibrium of plane figures* or in Euclid's *Optics* are homogeneous with Euclid's more famous work, the *Elements*, not merely in their instrumental use of geometric notions and results, but in that they are made of theorems based on postulates of hydrostatics, statics and optics — just as the *Elements* is made of theorems based on geometric postulates. Conversely, just as works on statics and optics bear a clear relation to concrete activities such as the use of balances and optical instruments (dioptra, astrolabe, and so on), the exact same relation, as we have seen, obtains between Euclidean geometry and drawing with ruler and compass.

It is essentially correct to say that the original name of the unitary science that we have been discussing was *mathematike* (ἡ μαθηματική, also in the neuter plural, τὰ μαθηματικά). Substituting "mathematics" for the Greek term requires the use of quotation marks and the awareness that the meaning of the word changed profoundly in the modern age. In fact it

[59] The verb here, αἰτέω, is the same used by Euclid to introduce his postulates.

[60] Diogenes Laertius, *Vitae philosophorum*, IX §89.

[61] Epicurus, *Letter to Pythocles*, in Diogenes Laertius, *Vitae philosophorum*, X §87. See also Lucretius, *De rerum natura*, V:526–533.

changed twice: the original meaning was "all that is studied", coming from the verb μανθάνω (learn) and the noun μάθημα (object of study, subject of learning): Plato uses the term in this sense at least twice,[62] and in the Pythagorean school "mathematicians" were the disciples who shared in the more profound teachings. Later came the sense, attested in Aristotle and systematically in Hellenistic authors, of a body of knowledge characterized by a certain methodological outlook: much broader than what we understand today as mathematics, but narrower than the etymological meaning.

Anatolius, in the late third century A.D., recorded an explanation for the origin of the Hellenistic use of the word:

> Why does mathematics have this name? The Peripatetics say that, whereas rhetoric, poetry and popular music can be practiced even without being studied, no one can understand what's called mathematics without first having studied it. Thus they explain why the theory of these things is called mathematics.[63]

From this we must conclude that, at least according to the interesting Peripatetic opinion reported by Anatolius, mathematics is so called because one must study it.[64]

Several writings give us an idea of the Hellenistic use of the term. The Plutarchan dialog *De facie quae in orbe lunae apparet*, an important source on Hellenistic science, has no less than nine occurrences of "mathematicians" and "mathematics", and one of its characters, Menelaus, is presented as a mathematician. Everything that is mentioned in the dialog as a typical subject of concern for mathematicians belongs to optics or astronomy. Recall also (see note 3.6 on page 79) that Ptolemy's astronomical work now best known as the *Almagest* was originally titled *Mathematical treatise*.

Around 200 A.D., Sextus Empiricus wrote a treatise *Against the mathematicians* (Πρὸς μαθηματικούς) in six parts: *Against the grammarians*, *Against the rhetoricians*, *Against the geometers*, *Against the arithmeticians*, *Against the astronomers*, and *Against the music theorists*.[65] "Mathematics" was therefore a very broad subject for him; moreover, he refers to certain statements in

[62]In the neuter singular: τὸ μαθηματικόν (*Sophista*, 219c:2; *Timaeus*, 88c:1). Plato did use, in a sense close to the modern one, narrower terms such as "geometry".

[63][Heron: OO], vol. IV, 160:17–24. The passage was preserved with "*Heron's Definitions*" (compare note 8 on page 58).

[64]Proclus proposes an alternative, and much less convincing, origin for the term, stemming from the Platonic theory of reminiscence (*In primum Euclidis Elementorum librum commentarii*, ed. Friedlein), 44:25 – 45:21.

[65]The books *Against the logicians*, *Against the physicists*, and *Against the ethicists* are sometimes also included under the title *Against the mathematicians*, but another tradition collects them under the name *Against the dogmatics*.

grammar and music as *theorems* (θεωρήματα).[66]

Proclus reports two classifications of "mathematical sciences": the old "Pythagorean" one, which went back to Archytas and divided the subject into arithmetic, geometry, music and astronomy,[67] and Geminus' classification, which recognized as mathematical subjects not only arithmetic and geometry (though he emphasized these two) but also mechanics, astronomy, optics, geodesy, music theory and the art of calculation.[68] In any case there is no doubt that ancient mathematics included even works on mathematical geography and hydrostatics. Thus the term "mathematics", faithful to its etymological origin, does not indicate a specific discipline of study but the unitary method we have described.

"Physics", too, is a Greek word: what did it mean? The verb φύω (generate, grow) gave rise to the noun *physis* (φύσις), meaning everything that lives, grows, or (by extension) comes into existence; this was rendered by the Latin *natura*. Therefore the corresponding adjective, *physikos* (φυσικός), means "natural". These terms appear systematically already in the pre-Socratic philosophers, many of whom wrote poems *About nature* (Περὶ φύσεως), and who by virtue of their interests were called φυσιολόγοι[69] or φύσικοι[70] — terms that can be partly transliterated as "physiologists" or "physicists" but which meant simply "students of nature". Aristotle, whose works had a deep influence on medieval and modern terminology, speaks specifically of physical science (φυσικὴ ἐπιστήμη).[71] He wrote a work on this argument whose Latinized title, *Physica*, is the ancestor of our contemporary word "physics". But Aristotelian physics differs profoundly from the homonymous modern science, both in subject matter (it encompassed not only plants and animals, but even the "prime mover") and in method.

Diogenes Laertius takes as generally accepted a division of philosophy into three sectors: physical (which is to say natural), ethical and logical.[72] According to him, the division is due to the early Stoics.[73] Thus the term "physics" corresponds to what came to be called "natural philosophy" in the modern age.

[66]Sextus Empiricus, *Adversus grammaticos* (= *Adv. math.* I), §§132–133; *Adversus musicos* (= *Adv. math.* VI), §30.

[67]Proclus, *In primum Euclidis Elementorum librum commentarii* (ed. Friedlein), 35–36.

[68]Ibid., 38.

[69]For instance, Diogenes Laertius says that Aristippus of Cyrene and Chrysippus called their respective works on natural philosophers Περὶ φυσιολόγων.

[70]This term is often used by Aristotle, for example in the quotation of a statement of Anaxagoras (*Metaphysica*, XII, vi, 1017b:27).

[71]See, for example, *Metaphysica*, VI, i, 1025b:19.

[72]Diogenes Laertius, *Vitae philosophorum*, I §18.

[73]Diogenes Laertius, *Vitae philosophorum*, VII §39.

Now, given that optical and astronomical phenomena, which were studied by mathematicians, are themselves natural and so within the scope of physicists or natural philosophers, in what way are the latter different from mathematicians? This question, already considered by Aristotle, was taken up again several times in Antiquity.

Geminus, for example, explains that it is not the business of optics (to him a part of mathematics) to inquire into the actual direction of propagation of rays, or the role played by air or ether in the transmission of light: these matters are evidently within the realm of natural philosophy.[74]

The most interesting passage available today about the relationship between mathematical sciences and "physics" (natural philosophy) is also by Geminus and we are able to read it because it made its way secondhand into Simplicius' commentary on Aristotle:

> Alexander [of Aphrodisias] quotes ... a passage from Geminus' epitome of his commentary on the *Meteorologica* of Posidonius. Geminus, drawing from ideas of Aristotle, says: "It is characteristic of physical science to consider what has to do with the substance of heavens and celestial bodies, their powers and quality, their generation and corruption ... Astronomy, however, does not concern itself with all that.... In many cases astronomers and physicists will set out to demonstrate the same topics, for example the size of the sun or the roundness of the earth, but they don't follow the same route. The latter will deduce whatever it may be from substance [οὐσία] or powers [δύναμις], or from optimality arguments [τοῦ ἄμεινον οὕτως ἔξειν], or from generation or transformation, whereas the former will deduce it from appropriate figures or magnitudes or the measurement of motion and corresponding times. The physicist, with an eye towards productive power, often touches on causes, whereas the astronomer, when he is constructing proofs based on what comes from outside, is a poor observer of causes[.]
>
> Sometimes [an astronomer] through a "hypothesis" [ὑπόθεσις] finds a way to save the phainomena. For example, why do the sun, the moon and the planets appear to move irregularly? If we suppose that their round orbits are eccentric or that these bodies move on epicycles, the apparent irregularities will be saved. One must investigate in how many different ways the phainomena can be represented...[75]

Note that in Simplicius' time (early sixth century), "mathematician" and "astronomer" were often used synonymously, in part through the influence of the title of Ptolemy's work; just before the passage cited, Simplicius contrasts physics with "mathematics and astronomy".[76]

The astronomical example reported by Simplicius, namely the possibility of explaining the same observable motions through an eccentric or an epicycle, alludes to a theorem demonstrated by Apollonius of Perga and then later by Ptolemy in the *Almagest*.[77] The result is the following. If a point B has uniform circular motion around a point A and a third point C has uniform circular motion around B (following a so-called *epicycle*), then in the particular case that the two angular velocities are the same, the resulting motion can still be uniform and circular, but around a center distinct from A. Thus the motion of C can be described in two different ways: by saying that C goes around a circular orbit eccentric relative to A, or that it moves on an epicycle based on an orbit around A. From our point of view these are two descriptions of the *same* motion, but for Posidonius and Geminus they are two hypotheses about real motions.

The passage quoted on the previous page pinpoints the distinguishing feature of the "mathematician" or "astronomer": limiting oneself to finding "hypotheses" capable of saving the phainomena, and not aspiring to know the absolute truth, whose pursuit is left to the "physicist", or natural philosopher. This situation is, as we have already seen, a necessary consequence of the scientific method (or mathematical method, as it would be called then). Indeed, if two theories, based on different hypotheses, are both consistent and explain equally well what is observed, the choice between them is not within the purview of the scientist as such. This does not mean that scientists necessarily reject the existence of ultimate truths or global explanations; they may accept such theses characteristic of natural philosophy. But the scientific method, shared by scientists regardless of philosophical outlook, has a different goal: to develop theoretical frameworks useful in describing the phainomena and in advancing technology.

This was still quite clear to Thomas Aquinas, who, taking up again Simplicius' contrast between "physics" and "astronomy" and the example about eccentrics and epicycles, writes:

> Reason may be employed in two ways to establish a point: firstly, for the purpose of furnishing sufficient proof of some principle, as in natural science [*physica*], where sufficient proof can be brought to show that the movement of the heavens is always of uniform velocity. Reason is employed in another way, not as furnishing a sufficient proof of a principle, but as confirming an already established principle, by showing the congruity of its results, as in astronomy the

[74]Geminus, in [Heron: OO], vol. IV, 102:19 – 104:8.
[75]Simplicius, *In Aristotelis physicorum libros commentaria*, in [CAG], vol. IX, 291:21 – 292:19.

[76]Ibid., 291:19–20.
[77]Ptolemy, *Almagest*, XII, i, 451–544 (ed. Heiberg).

> theory of eccentrics and epicycles is considered as established, because thereby the sensible appearances of the heavenly movements can be explained; not, however, as if this proof were sufficient, forasmuch as some other theory might explain them.[78]

Thomas Aquinas is obviously using the word *physica* still in the classical sense of philosophy of nature. He still knows the ancient scientific method. That ancient science renounces all claim to an unambigous identification of true first principles establishes, in his eyes, its inferiority with respect to natural philosophy and theology.

6.7 Hellenistic Science and Experimental Method

That Greek science knew the experimental method has often been maintained and more often denied. As a representative of the former view, we quote Neugebauer:

> But if modern scholars had devoted as much attention to Galen or Ptolemy as they did to Plato and his followers, they would have come to quite different results and they would not have invented the myth about the remarkable quality of the so-called Greek mind to develop scientific theories without resorting to experiments or empirical tests.[79]

This opinion seems to have remained minoritarian, but if we extend our sights beyond the imperial-era Galen and Ptolemy and consider Hellenistic scientists such as Herophilus and Hipparchus, we reach conclusions even stronger that Neugebauer's.

Obviously, a verdict on whether Hellenistic science knew the experimental method will depend on the definition of "experimental method". If we take the expression to mean simply the systematic collection of empirical data obtained through the investigator's direct intervention, the appearance of the experimental method is discernible, despite the meagerness of our sources, not only in the physical and mathematical sciences, but also in anatomy, in physiology,[80] and in other empirical sciences like zoology and botany — areas where the knowledge amassed through husbandry and agriculture was being complemented by that derived from

[78]Thomas Aquinas, *Summa theologica*, part I, question 32, article 1, reply to objection 2. The translation is by the Fathers of the English Dominican Province (Benziger Brothers, 1947), except that "astrology" has been replaced by "astronomy".

[79][Neugebauer: ESA], §63, p. 152.

[80]As we have seen, one source of scientific physiological knowledge was experimentation *in vivo* on humans. These experiments, however repugnant, in themselves belie the widespread notion that Hellenistic science was speculative and cared little for experimental checks.

experiments performed in dedicated venues, such as the Ptolemies' zoo and the gardens devoted to this purpose by the Pergamene dynasty.[81]

If an essential characteristic of the experimental method lies in making quantitative measurements, the systematic use of such measurements had been present for many centuries in astronomy (and if we exclude observational astronomy from the experimental sciences, Newtonian mechanics itself risks being denied an experimental basis). In the early Hellenistic period quantitative measurements were extended not only to fields such as mechanics and optics,[82] but to the medical and biological sciences, as shown by the systematic use of water clocks in Herophilus' studies on the pulse (page 148) and the use of a balance in Erasistratus' physiological experiment (page 156).

If by experimental method we understand the practice of observation under artificially created conditions, the most significant examples are perhaps in pneumatics, where we see the systematic construction of experimental gadgets for demonstrations, but examples are documented in other areas as well.[83]

Von Staden, examining five physiological experiments performed in the third century B.C., finds in each of them one or more of the features seen by modern philosophers of science as characteristic of the experimental method. Shunning generalizations about "ancient science", he stresses the sudden emergence of the experimental method in the third century and its equally rapid decline in the second.[84] Among the deniers of the experimental method in Antiquity there are those who recognize the existence of well-documented ancient experiments such as the ones considered by von Staden, but claim they were sporadic events that did not add up to a method.[85] But making true experiments in the absence of an experimental method would be a bit like casually writing a few sentences before writing was invented. The notion of an experiment implies a qualitative methodological leap that cannot occur at random.

Because no one doubts that the "experimental method" was fully operational in eighteenth-century European physics — that it was in fact an essential feature of it — the question of what is meant by the expression can be illuminated by a study of the paraphernalia in use at the time. Consider, for example, a 1794 inventory of the contents of the "physical theater" of the University of Rome (then called "Archigimnasio"). It

[81]See Chapter 9, particularly pages 247 and 250.

[82]For instance, measurements of refraction (see page 64) and rate of water flow (page 103).

[83]For pneumatics, see page 77. For an example of an experimental device in optics, see note 2 on page 270.

[84][von Staden: EEHM].

[85]Thus M. D. Grmek on Erasistratus' physiology experiment; see [Grmek], Chap. V, for example.

comprises (besides modern devices such as electrostatic machines and microscopes) objects such as a pneumatic pump, glassware for pneumatic experiments, devices for experiments on the "elasticity of air", hydrostatic balances, inclined planes, barycenter finders, levers, beam balances, pulleys, winches, screws, Archimedean screws and a Heronian fountain.[86] Thus, experimental physics developed in part thanks to the reintroduction of devices whose Hellenistic origin is clear even from their names. Only after the recovery was consolidated did it become possible to believe that there was no experimental method in Antiquity.

Of course, there are important differences between the experimental method of Hellenistic science and the contemporary one. In comparison with the early modern age, Hellenistic exact science was project-oriented rather than experimental: technology was more important than experimentation in driving the interplay between theory and practice. And a concept long considered essential to modern science was absent in Hellenistic science, namely the "crucial experiment". If by that expression we understand an experiment designed for choosing between two alternative hypotheses on a particular phenomenon, then crucial experiments are present in ancient science: the nerve section practiced by Herophilus to decide if it was a motor or a sensorial nerve is an example of this type. But crucial experiments in the sense of something decisive for establishing the truth of a whole theory are certainly absent.

Notwithstanding these differences, if we want to regard as users of the experimental method not only twentieth-century physicists and biologists but also Galileo, Francesco Redi and Robert Grosseteste, it seems perverse to exclude Ctesibius, Herophilus and Philo of Byzantium.

6.8 Science and Orality

The importance of oral culture in the Greek world was for a time underestimated, but in the last few decades it has been the subject of much literature.[87] In the fifth century and to some extent down to Plato's time, writing had a subordinate role relative to oral culture, in the sense that books were written and bought not for readers, but as professional instruments for those who performed the contents in song, theater or declamation.

The genetic link between rhetoric and the hypothetico-deductive method suggests that perhaps the scientific method, too, had its roots in oral culture, thus going back to long before the Hellenistic period.[88]

[86] *Inventario delle machine esistenti nel Teatro Fisico dell'Archigimnasio Della Sapienza. Adì 26 dicembre 1794.* Preserved in the museum of the Physics Department of the University of Rome *La Sapienza*. A summary can be found at http://www.phys.uniroma1.it/DOCS/MUSEO/catalogo1794.html.

[87] For a bibliography one can consult [Harris].

Naturally, it is difficult to demonstrate fully the existence or nonexistence of particular practices in an oral culture of which, by definition, we lack direct documentation. But while, as we have seen, the connection with rhetoric (a verbal art by definition) implies that the origins of the hypothetico-deductive method go back to oral culture, the diffusion of books certainly caused important changes, above all in uniformizing the choice of postulates.[89] Indeed, a fundamentally oral culture is no bar to the development of even very sophisticated forms of deductive reasoning, such as arose in classical Greece; but one would expect in this context variations in the choice of assumptions, to suit the needs of the moment.[90]

A second important effect of the diffusion of books was the fostering of a conventional terminology. Anything like Archimedes' definition of "ball-shape", which associates a new meaning to an old word, is likely meant to be written, not just spoken. Only those who, like Archimedes or Herophilus, know that their work will last in the written medium and remain available to specialists in the field can change the meaning of a term without causing confusion. Thus, it seems not to be a coincidence that linguistic conventionalism and nominalist definitions arose around the time when written culture emancipated itself from oral culture.

Because a fully scientific methodology (in our sense of "scientific") requires the formation of vast unified theories, based on shared premises and precisely defined terms, what we have called the scientific revolution depends on the diffusion of written culture,[91] and so would not have been possible before the fourth century. Of course, the diffusion of writing came at a cost (which Plato thought too high). In particular, the homogeneity of methods and premises that allowed any student to solve as an exercise a problem internal to a full-fledged scientific theory is paid for by the giving up of many ideas which, although present in the earlier culture, for various reasons did not make it into the victorious systematization.

6.9 Where Do Clichés about "Ancient Science" Come From?

This book espouses theses openly opposed to certain widespread opinions about "ancient science", summarizable in three interconnected statements:

– The Ancients did not know the experimental method.

[88] This belief is expressed in [Cerri].

[89] On this point see [Cambiano].

[90] See page 37 and also Plato, *Meno*, 86e–87b.

[91] Of course this prerequisite is not sufficient. Egyptian culture had used writing for millennia, but its evolution led in other directions.

– Ancient science was a speculative form of knowledge, unconcerned with applications.
– The Greeks created mathematics but not physics.

We discussed the experimental method in Section 6.7. This section tries to explain the origin of the other two assertions.

A first cause of misunderstandings is the idea that there was such a thing as the "Ancients". Talk of ancient science, supposedly spanning the millennium and more from Thales to Simplicius and represented by such diverse people as Parmenides, Archimedes, the elder Cato, Plutarch and Seneca, makes as much sense as talk of a "second millennium science" cultivated by Thomas Aquinas, Nostradamus, Galileo, Lavoisier, Freud and Dr. Mengele.

Lack of interest in applied science is of course documented among many classical-era Greek thinkers (who lived before the full blossoming of the scientific method) and among imperial-era Roman intellectuals (to whom the scientific method remained alien). These two groups of intellectuals share with Hellenistic scientists the label "Ancient"; if one believes in a homogeneous attitude of the "Ancients" regarding science, one can be tempted to reconstruct it by dismissing as unrepresentative all the true scientists we have notice of. This misunderstanding is further compounded by the fact that most of what we know about Hellenistic scientists comes to us through the sieve of imperial-era writers.

The best counterexample to the idea that Hellenistic science was unconcerned with applications is provided by Archimedes, who wrote a treatise on mirrors and founded the science of machines; who wrote the first theoretical treatise on hydrostatics and followed the construction of the biggest ship of his time; who devised new machines for lifting water and for waging war; who showed (according to tradition by means of public demonstrations, and at any rate in his writings) how natural philosophy could be surpassed by creating a science that, through theoretical design, was closely linked to technology. And yet many scholars have sworn that Archimedes was not interested in technology! Since this lack of interest is attested neither in his remaining works nor by documented facts, it is cast in terms of an inner feeling or philosophical attitude that supposedly caused Archimedes to carry out his numerous achievements willy-nilly. Fraser, for instance, writes:

> Archimedes had a profound contempt for applied mechanics.[92]

We might as well say that developers of today's cutting-edge military technology keep their work under wraps because they're ashamed of it!

[92][Fraser], vol. I, p. 425.

Where does this very widely shared opinion about Archimedes' private sentiments come from? It is essentially pulled out of the hat of a single sentence in Plutarch's *Parallel lives*.[93] Archimedes' feelings are deduced from the testimony of a writer who, three centuries after the scientist's death, gratuitously ascribes to him his own Platonist tendencies.[94] It was actually authors like Plutarch (who, being Greek in origin, had a shining carrer in the service of the Romans) that created the myth of a homogeneous "Greco-Roman" civilization, by writing works such as the *Parallel lives*. It is this extraordinarily persistent myth that has led so many people to believe that Archimedes' feelings on technology can be gleaned from imperial-age works.

Stobaeus tells us that so-and-so started to study geometry with Euclid, and after having learned the first theorem he asked the master: "But what will I get out of it once I've learned all of this?" Euclid called his slave and told him, "Give him three obols, the man must profit from what he's learning."[95] Some historians of science have deduced from this anecdote that Euclid did not care for concrete applications of mathematics.[96]

In fact, the very breadth of the applications that were starting to be derived from mathematics in Euclid's time made imperative a division of labor, in which the mathematician had a certain role in which he took pride and which was very different from the role of, say, the engineer, who applied mathematical procedures invented by others. While allowing the resolution of concrete problems with newly found efficacy, the rise of scientific theories, which is to say theoretical models of parts of the concrete world, led to the equally new circumstance of certain persons working within the theory itself. In other words, the birth of science was closely connected with the appearance of scientists. In the eyes of writers such as Stobaeus, who belonged to the subsequent (prescientific) society to whom we owe the record of this new class of professionals, its members, immersed as they were in theoretical work, seemed uninterested in the practical aspects of life.

Moreover it was precisely the division of labor between scientists and technicians that demanded extreme rigor from those who worked on the theory, and so gave impulse to the new scientific method. Indeed, if the

[93]The *Parallel lives* is a series of biographies where each Greek character has a Roman "parallel". No scientists are represented, which is not surprising given that Plutarch would not have been able to find a single Roman parallel. The statement about Archimedes that motivates Fraser's remark appears in the *Life of Marcellus* (xvii §§3–4) — that is to say, in the biography of the very Roman general under whose watch Syracuse was sacked and its greatest scientist killed.

[94]An even later source reports the same claim: Pappus, *Collectio*, VIII, 1026:9–12, ed. Hultsch.

[95]Stobaeus, *Eclogae*, II, xxxi, 228:25–29 (ed. Wachsmuth).

[96]See, for example, [Boyer], p. 111 (1st ed.), p. 101 (2nd ed.).

person who obtains a mathematical result also knows its only possible application, it doesn't matter if the result is exact: a reasonable approximation is enough, as was generally the case in the mathematics of Pharaonic Egypt or Old Babylonia, which did not distinguish, for example, between exact and approximate area formulas. But when the result obtained is consciously kept internal to the theory, that is, when it must be applied, often indirectly, to a variety of problems not known a priori, a mathematician's rigor becomes essential.

Thus, in the absence of direct information, a good way to get an idea of the breadth of applications of mathematics in a particular historical moment is to inspect its level of rigor.

In the preface to Book IV of his fundamental treatise on conic sections, Apollonius writes:

> Moreover, apart from these uses, they [some theorems of Conon of Samos] are worthy of being accepted for the demonstrations' sake alone, in the same way that we accept many other things in mathematics for this reason and no other.[97]

There is no question that Apollonius is sincere (and that he is right!), but the need to justify the value of pure science is so characteristic of civilizations where science is the locomotive of technology that this quote by itself would be enough to document the existence of applied mathematics in Hellenistic times. Had there been no applied mathematics, Euclid's quip and Apollonius' apologetics would be unthinkable: no one would defend in such stark terms the value of "pure" mathematics unless to distinguish it proudly from existing, and well-known, applied mathematics. In fact, the same contrast is seen in modern times. While Galileo had to rack his brains to invent practical applications that might persuade the Venetian government to increase his stipend, once physics took on a prime role in technological development it won the luxury of producing "theoretical physicists" apparently uninterested in potential applications of their own research.

Apollonius himself is generally thought of as the quintessential pure mathematician. But note that this impression comes primarily from the sifting of his works carried out by later generations. It is known that he wrote books on astronomy and one on catoptrics, but all these have been lost, and of the work on conics, the only one that was partially preserved in Greek,[98] the treatise on conics, we have lost the eighth and last book,

[97]Apollonius, *Conics*, preface to Book IV.

[98]One other work by Apollonius has come down to us, in Arabic translation. It studies the problem of finding a line whose intersections with two fixed half-lines form segments having a fixed ratio. See [Apollonius/Macierowski, Schmidt].

which likely was devoted to applications of the theory.[99]

One other source of the myth that Hellenistic science had no applications was the introduction of new computational tools in the modern age. In the three centuries that preceded the invention of digital computers, calculations were perfomed using:

- arithmetic operations on numbers written in decimal notation;
- numerical tables of logarithms and of certain other (e.g., trigonometric) functions;
- operations of analysis, such as differentiation and integration, on functions expressible in terms of "elementary functions" (which is to say those whose values had been tabulated).

Ancient geometric methods, which from the beginning of the modern age had started to become less useful given the systematic use of positional notation, were definitely surpassed as computational tools at least as early as 1614, when the first tables of logarithms were published. By contrast, Euclidean mathematics remained a peerless model of rigor until 1872, when a rigorous theory of real numbers was founded (see page 47). Between these two dates, mathematicians used Euclidean geometry as a framework and prime example of the hypothetico-deductive method, and decimal numbers and tables of logarithms for the calculations needed in the solution of concrete problems. Also, certain ancient problems inherited unsolved from Hellenistic mathematicians, including the trisection of the angle, the doubling of the cube and the even more famous quadrature of the circle, continued to fascinate, and the demand that they be solved with ruler and compass, in spite of having lost its original motivation,[100] was accepted as a "rule of the game" that characterized what became known as the classical problems.[101]

It was this, then, that nourished the belief that "classical mathematics" was good only for theory, and so strengthened the prejudices in this direction that had arisen from the loss of records on ancient technology and from the fact that Hellenistic mathematics was part of "Greek thought", a

[99]This is the implication of a remark in the preface to Book VII of the *Conics*, where Apollonius says that the theorems contained therein (on diameters of conics) were applicable to "problems of many types" and that examples of such applications would be given in Book VIII.

[100]See the discussion on page 41.

[101]In the nineteenth century these three problems were proved to be unsolvable with ruler and compass. Note that all three had some practical interest in antiquity. The trisection of the angle was probably suggested by the need to draw divisions corresponding to the hours in sundials (compare [Neugebauer: ESA], p. 265). The quadrature of the circle was tied to the need to compute trigonometric functions, essential in topography and astronomy. The extraction of cube roots was useful, for example, in the design of catapults (see page 111).

term usually targeting primarily the literary and philosophical works of the classical period.

The remaining cliché, that the Greeks developed mathematics but were incapable of creating physics, is closely connected to the one just discussed.

Even Sambursky, one of a handful of authors who have taken an interest in the physics of the Greeks, believed that they had in fact no real physics.[102] I fear that his belief, like other clichés we have been discussing, was fostered in part by terminological naïveté. Many historians of science, taking as eternal the current boundaries between subjects, have felt that they could reach conclusions about the existence of Greek physics by concentrating on the writings of those whom the Greeks themselves labeled "physicists", ignoring what was called "mathematics". As a result, they basically looked at ancient natural philosophy (which lacked, among other things, the experimental method), and failed to notice the birth of the first quantitatively and experimentally based scientific theories about nature. How else to explain Sambursky's decision to base his analysis on the writings of various pre-Socratic philosophers, of Plato and Aristotle, and to devote a whole book to the "physics" of the Stoics, while showing very little interest in scientists such as Euclid, Ctesibius, Philo, Archimedes and Hipparchus?

Of course, the idea that Hellenistic theories such as hydrostatics and geometric optics, which today belong in physics textbooks, were at the time pure mathematics is due not only to the semantic shift undergone by the name "mathematics", but to the hypothetico-deductive nature of the expository texts that have come down to us. Yet note that in the modern age it was only after a few centuries of scientific development that a similar structure was constructed for mechanics, thermodynamics and classical electromagnetism.

[102]See [Sambursky: PWG] and [Sambursky: PS]. For example, Chapter 10 of the former book argues that the Greeks lacked altogether the ability to make experiments.

7 Some Other Aspects of the Scientific Revolution

7.1 Urban Planning

The influence of the scientific revolution on everyday life is particularly evident in the changes undergone by cities in the early Hellenistic period.

Hellenistic architecture revels in experimentation, a feature connected with an increasing interest in complex structures. It does not limit itself to designing buildings, but, thanks to new engineering possibilities, it intervenes actively in the landscape, urban or otherwise: roads built on causeways transform islands into peninsulas, canals create new islands, hills are terraced, artificial hills are made.[1] As a project worthy of the ruler's fame, the architect Dinocrates even proposed to Alexander that Mount Athos be carved into human shape, with a city placed in one hand and a lake in another. The plan was not viable, but Alexander so liked this intention of moulding nature, grandiose to the point of folly, that he put Dinocrates in charge of designing Alexandria.[2]

The new city soon became the most populous on earth. It was a cosmopolitan metropolis: its inhabitants were mainly Greeks, Egyptians and Jews, but there were immigrants from all parts of the known world. Dio Chrysostom mentions Syrians, Persians, Romans, Libyans, Cilicians, Ethiopians, Arabs, Bactrians, Scythians and Indians. [3]

The two main roads, over thirty meters wide, were flanked by porticos lit all night by torches (Figure 7.1). An underground network of canals

[1]See [Lauter], Sections C.I and D.I.
[2]Vitruvius, *De architectura*, II, preface, §§1–4.
[3]Dio Chrysostom, *Orationes*, xxxii, 40:1–5.

FIGURE 7.1. Watercolor by Jean-Claude Golvin, showing Ptolemaic Alexandria as it can be imagined from surviving descriptions. The view is along the Canopic Road toward the west. On the foreground we see the city's other main road; in the distance, starting near the far walls, is the causeway to Pharos Island.

FIGURE 7.2. Nineteenth-century visitors explore an ancient cistern for storage and purification of water, in Alexandria. Cisterns such as this ("subterranean cathedrals", in the words of an eighteenth-century traveler) and the canals that fed them were a feature of the city from the beginning. More cisterns were built under the Arabs, and some still functioned in the modern age. All but forgotten in the late twentieth century, this underground network is now being studied and recovered by the Centre d'Études Alexandrines.

distributed water from the Nile to private homes, after it had been made fit to drink by sedimentation of suspended impurities in large cisterns, a system that survived until the modern age (Figure 7.2).[4]

The city boasted parks, theaters, stadia, gyms, the great Hippodrome and temples of several religions. The most stunning building, according to Strabo, was the Gymnasion.[5] At the center of the city — which was so crowded that a law from the third century B.C. set a minimum distance of a few feet between buildings[6] — were the sacred groves and the Paneion, a conical artificial hill from whose top one could admire the view all around. The Pharos was reached by a road along a causeway that lay between the two main harbors, joining the islet of the same name to the mainland.

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[4] See Pseudo-Caesar, *De bello alexandrino*, v, where the unhealthy water of the Nile, drunk by those who did not live in homes served by city water, is contrasted with the clear water delivered through the underground network. Sedimentation tanks made for this purpose are documented at several Hellenistic sites: see, for example [Tölle-Kastenbein], under index entry *Absetzbecken*. For photos and discussion of Alexandria's cisterns and other historical features, see the beautifully illustrated [Empereur].

[5] Our main source about Hellenistic Alexandria is a long description by Strabo (*Geographia*, XVII, i §§6–10), who visited the city at the beginning of the Roman occupation.

[6] P. Halle 1, lines 84 ff.

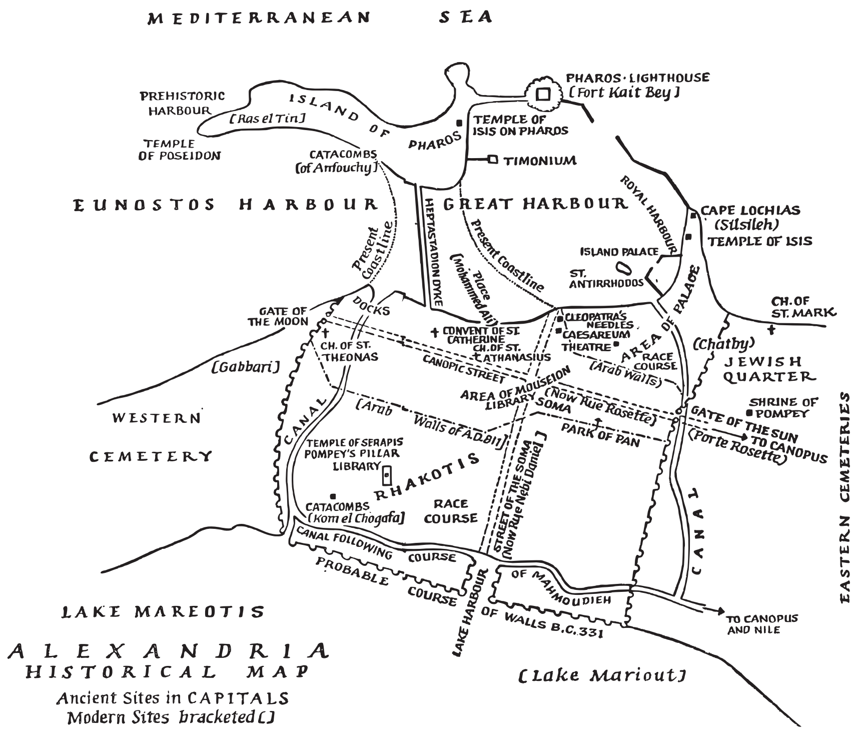


FIGURE 7.3. Plan of Ptolemaic Alexandria, from [Forster]. A more detailed map, from [Botti], can be found on the back endpaper of this book.

Poëte draws the picture vividly:

> In Alexandria the urban organism, more complex, gives an inkling of the nascent modern city... We see it with its mass of buildings, its numerous population, its concentration of riches, its masterful application of technology to community life[.]

Referring more generally to Hellenistic cities, he writes:

> We feel here for the first time what a city really is—its vast area occupied by buildings, its fast pace, its imposing organization, its conveniences for body and spirit, its splendid luxury and stark misery, its amusements and its vices, its sprawl into a more or less extensive suburban area. A breath of modernity seems to reach us from this distant world. We have the impression that we would not be too out-of-place in a city such as Alexandria or Antioch[.][7]

[7] [Poëte], pp. 280 and 344.

The influence of science on the fabric of the city is not limited to the use of scientific technology in building certain works such as, in Alexandria, the water supply system, the Pharos or the causeway. More momentous is the change undergone by the very notion of a city during the period of the scientific revolution. The classical city was understood as an organism whose natural dimensions were fixed and whose various parts existed in a natural equilibrium.[8] Overpopulation was mitigated through birth control and the foundation of colonies. Now, instead, urban development is rationalized and directed, rather than combated. Martin writes:

> [A Hellenistic city] is no longer just a city of citizens, the harmonious framework for the exercising of political activity: it becomes a city with a growing population, a business center, where artisanal and commercial activities win over political aspects. . . . The phenomenon of modern urbanization has its roots and beginnings in the urbanization of the Hellenistic world.[9]

The new urban centers existed in a dynamic equilibrium that foresaw open-ended development and regulated it. The existence of an open and infinitely extensible structure led Benevolo to compare Hellenistic cities with American cities of the eighteenth and nineteenth centuries.[10]

Other aspects of Hellenistic, and more specifically Alexandrian, society may bring to mind modern America: an immigration hub that quickly becomes the biggest city in the world; the claiming of great expanses of virgin land for agriculture; interest in technology; a taste for the colossal;[11] the diffusion of the culture industry; the relationship of Greek emigrés with Greece, which in some ways parallels that of Americans with Europe.

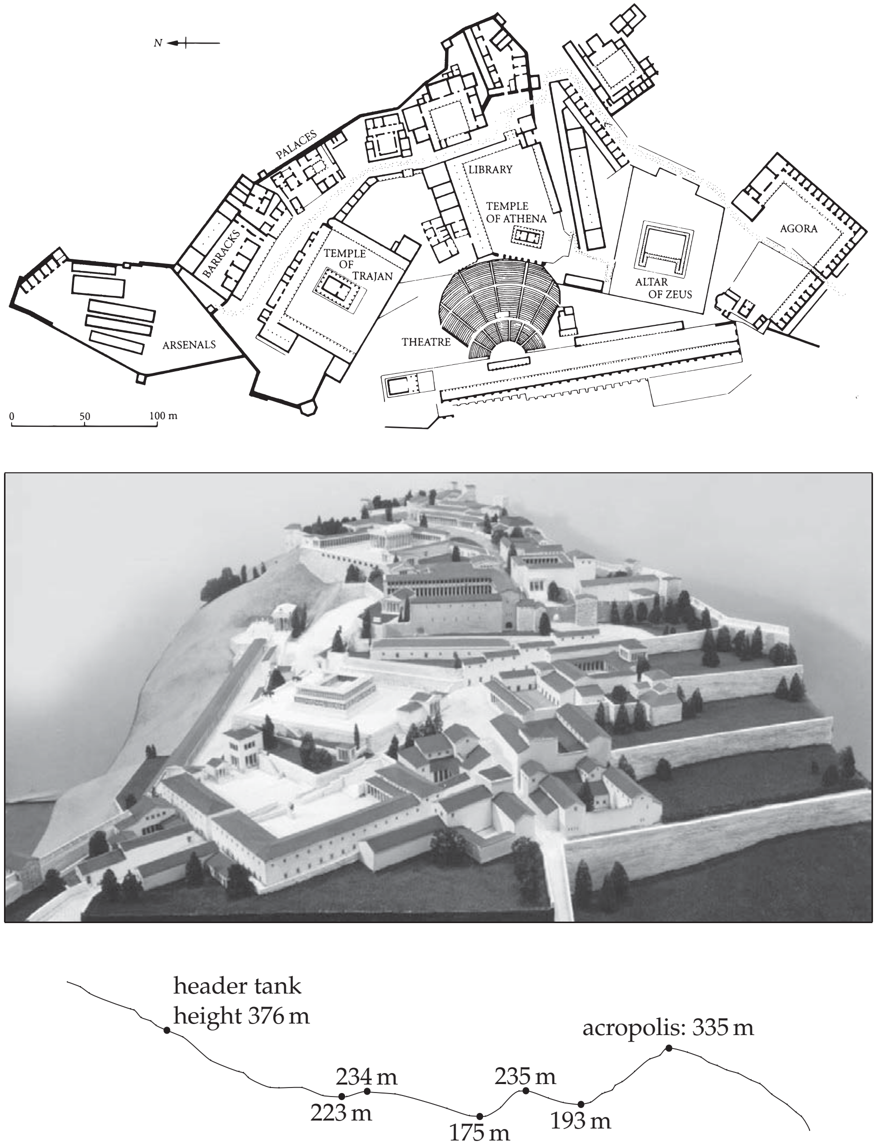
Among the many differences between the two historical situations, a major one is the radically different relationship with native populations: whereas the modern immigrants to North America encountered nomadic hunters, the Greeks who went to Egypt and Mesopotamia came into contact with the oldest civilizations in history. The Greeks acquired the upper hand not because of superior technology—in fact, as we have seen, they were initially technologically less developed than the indigenous popu-

[8] See, for example, Aristotle, *Politica*, VII, 1326a–b.

[9] [Martin], p. 573. We will return in Section 9.5 to the economic role played by Hellenistic cities, discussing also opinions quite different from Martin's.

[10] [Benevolo], p. 37.

[11] As far as I know, the United States is the first country in modern times where mountains were carved and giant statues put up. When the French sculptor Bartholdi designed the Statue of Liberty, much was borrowed from the Colossus of Rhodes, as tradition had it: not only the general idea, the harbor location and the torch, but even the crown and the rays spreading therefrom; this was originally an attribute of the Sun, to which the Colossus was dedicated, but for us it has become a familiar detail to which we cannot assign a meaning.



Length of aqueduct: 3.2 km. Vertical scale extended.

FIGURE 7.4. Top and middle: the Pergamum acropolis (reconstruction model in the Pergamonmuseum, Berlin). Bottom: Profile of the syphon aqueduct discussed on page 118, which supplied the acropolis in Hellenistic times. After [Hodge: HAWS], p. 43, and [Garbrecht], p. 23.

lations — but thanks to their cultural instruments. This was probably one of the causes of the great attention paid by the Greeks in the Hellenistic states to cultural and educational institutions.

7.2 Conscious and Unconscious Cultural Evolution

For millennia, technological innovations were rare and most of the time unintentional. Craftsmen tried to follow tradition faithfully, but did not always succeed; an automatic selection mechanism, similar to the one operating in biology, let through useful variations, thus causing a slow evolution. The fruits of this unconscious progress included not only most prehistoric technological innovations but also many that took place in the ancient empires.

An example is the long process, reconstructed (to some extent conjecturally) by Denise Schmandt-Besserat, whereby writing arose from an accounting system.[12] First came tokens or counters of various shapes, some geometric and some evocative of people or objects. Then the tokens started being "archived" in clay containers, which were closed and sealed. Later, to allow the contents of the archive to be known from the outside, people started imprinting the shapes of the content tokens on the containers, prior to archiving, one imprint per token. This rendered the actual tokens useless, but they did not disappear suddenly and deliberately, only through a slow process of natural selection. Indeed, the passage from the complex system just described to that of simple tablets with incisions happened through intermediate steps, represented first by the use of empty containers and then of curved tablets, reminiscent of the shape of the ancient containers.

The positional number system arose along similar lines. In the Old Babylonian number system, symbols had multiple meanings distinguished by size and differing by a power of the base: for example, the symbol for 4, when enlarged appropriately, also meant 40 or 400 (this system was used in base 10 and 60 alike). The habit of writing the component symbols of a number in order of size made the size superfluous, since the position by itself was enough to determine the value. But the superfluous size distinctions lingered on for centuries. Eventually they disappeared, and the result (apart from the ambiguities arising from the lack of the zero) was the positional system, which therefore no-one "invented".[13]

[12] For details see [Schmandt-Besserat].

[13] See [Neugebauer: ESA], chapter 1, §14. It seems that the absence-of-zero ambiguities were not fully overcome until the Hellenistic period.

As a last example, recall the growth of the dimensions of the temple in Southern Mesopotamia, which characterized early urbanization. The growth can be considered "rapid" on a multimillennial scale, but even its fastest phase required many centuries.[14]

Scientific technology altered this situation profoundly. Technological innovations became intentional and, because many appeared within a single generation, their usefulness became obvious to everyone.

One sign of the importance given to new technological achievements in the Hellenistic period is in the Seven Wonders of the World. Since it includes the Pharos at Alexandria and the Colossus of Rhodes, the canonical list must have been compiled after the construction of the former (about 280 B.C.) and before the destruction of the latter (in 226 B.C., as a result of an earthquake) — in other words, during the golden age of the scientific revolution. This list of the most remarkable human works in history, whose popularity is proved by its preservation down the centuries, thus provides an interesting testimony about the prevailing ideas in the third century B.C. The Seven Wonders encapsulate the three main elements of Hellenistic culture: the ancient empires' traditions are represented by two wonders (the Great Pyramid and the Hanging Gardens of Babylon), classical Greece by three (the statue of Zeus at Olympia, the temple of Artemis at Ephesus and the Mausoleum at Halicarnassus), and the present by the two already mentioned, made possible by recent technological developments. Casting colossal bronze statues and shining light thirty miles out into the sea — the ability to do such things was a lofty accomplishment in the eyes of the Greeks of the third century B.C., on a par with the best of the past.

The new scientific culture transformed many aspects of everyday life. The flourishing of treatise writing, an offshoot of professional specialization, shows clearly the importance that the formation of new and specific theoretical knowledge acquired in every area. In many fields of endeavor the work was now being either carried out or directed by specialists who relied on know-how connected to scientific developments: physicians, engineers, navy and army technicians, and many others. We see a veritable explosion of treatises — on agronomy, beekeeping,[15] fishing,[16] veterinary medicine, perfumery,[17] and so on.

[14]See the graph in [Liverani], p. 32.

[15]See page 251.

[16]Some authors of books on fishing are cited by Athenaeus (*Deipnosophistae*, I, 13b–c). The only one that has come down us is by Oppian of Cilicia.

[17]Theophrastus had already written a treatise *On smells*, and several others followed suit. A physician from the Herophilean school, Apollonius Mys, wrote a work *On perfumes*, from which Athenaeus quotes (*Deipnosophistae*, XV, 688e–689b). Athenaeus also cites a work on perfumes by Philonides (*Deipnosophistae*, XV, 691f).

Figure 7.5. The Pharos, an object of enduring admiration, is the prototype of this ancient terracotta lamp, where we see the three levels with their characteristic cross-sections: square, hexagonal and round. By the kind permission of Prof. Jean-Yves Empereur, Centre d'Études Alexandrines, Alexandria.

Conscious technological innovations cannot but be accompanied by the notion of progress. The classical word to indicate growth in knowledge, ἐπίδοσις — literally, "addition" — was replaced by προκοπή, "headway", from which our word "progress" is a literal translation through Latin.[18] As a matter of fact, already in the late fifth century Thucydides wrote:

> It is inevitable that [in politics], as in a techne, novelties always prevail.[19]

Seneca wrote, evidently drawing from the same Hellenistic source he used for the astronomical discussion that comes right before:

> Why do we marvel if the comets, which are such a rare astronomical phenomenon, have not yet been described by firm laws . . . ? The day will come when all this knowledge that is now hidden will be brought to light . . . and our descendents will marvel that we might have been ignorant about such obvious facts.[20]

Despite this, most scholars have held that the "Ancients" totally lacked the notion of progress. I think this belief has two sources. First, it was true

[18][Edelstein], p. 146. The surrounding pages analyze the conceptual shift in some detail.

[19]Thucydides, *Historiae*, I, lxxi §3.

[20]Seneca, *Naturales quaestiones*, VII, xxv §§3–5. This passage is often cited, but usually (and incongruously) as if reflecting Seneca's own thoughts on science.

for almost all of Antiquity. When, for example, Ptolemy talks about Hipparchus or Heron about Archimedes, their tone is not what we would use for scientists of three centuries ago, but the tone we use for our illustrious contemporaries. Clearly (and for good reason!), imperial age scientists see no scientific progress between the Hellenistic period and their own.

The second source is a profound difference between the Hellenistic notion of progress and ours. In Hellenistic writers we see an awareness of achievements made and the confidence that others will come, but not the idea, to which we are now accustomed, of an automatic and effortless progress which necessarily follows the course of time and which we can cash in just as if it were interest earned by money in the bank.

A fragment of Chrysippus illustrates well the new climate of confidence in the possibilities of mankind. Aristotle, analyzing in the *Categories* the contrast between possession and deprivation of natural faculties, exemplified by sight and blindness, wrote that one could go from possession to deprivation and not the other way round: the seeing can lose their sight, but the blind cannot reacquire theirs.[21] Chrysippus remarks that this example is belied by the existence of people whose sight was restored by cataract removal operations.[22] The importance of this critique is enormous. Aristotle, believing he lived in an essentially static world (as probably everyone who lived before him had believed), regarded as impossible anything that had never happened before. Chrysippus, on the other hand, saw so many things happen for the first time: not only did pneumatics and mechanics allow the construction of machines to perform tasks that Aristotle thought impossible, but physicians were able to restore light to the blind![23] It became necessary to revise the very notion of impossibility, denying the validity of judgments about impossibility formulated on empirical bases.

This was one example of an interaction between philosophy and medical science; such interactions seem to have been pervasive and fecund in both directions. We have already noted the epistemological interests of Herophilus. Chrysippus' epistemology too, and especially his theory of perception (κατάληψις),[24] clearly seem (from what we can deduce from the sources[25]) associated with new knowledge about the nervous system. Particularly interesting is his idea that perception does not consist only

[21] Aristotle, *Categoriae*, x, 13a:35–36.

[22] Cited in Simplicius, *In Aristotelis Categorias commentarium* ([CAG], vol. VIII), 401:7ff.

[23] It is well-known that Demosthenes Philalethes, the first century A.D. follower of Herophilus whom we mentioned on page 157, removed cataracts; but Chrysippus' comment shows that the procedure was already practiced in the third century B.C.

[24] Cicero translated this Chrysippean term with the Latin *perceptio*, whence our "perception".

[25] The extant sources on Chrysippus are collected in volumes II and III of [SVF]. See also [Gould], pp. 48–65, and [Solmsen].

in a modification of the state of the *psyche* (the latter being pretty much identified with the nervous system) caused by messages originating in the sensory organs, but above all in the act of *assent* (συγκατάθεσις) that may (or may not) follow.[26] The recognition of the subject's active function in perception was forgotten for many centuries, but at the time it probably had important consequences not only in psychology but elsewhere, including esthetics.[27]

The ideas discussed in this section are aspects of the same intellectual revolution described in earlier chapters. The scientific revolution's key novelty lay in that people became aware, for the first time, of being able to consciously create their own cultural categories. This is the common foundation of all the conceptual upheavals we have discussed in connection with the birth of science: the overcoming of natural philosophy thanks to the experimental method (or "project-driven method") of Ctesibius and Archimedes; the transition from a Platonist to a constructivist view in mathematics; the novel creation of conventional terminology in the empirical sciences.[28]

Even political and religious ideas were influenced by the new cultural climate. In the political sphere we encounter the (originally Sophistic) idea that law and state stem from an agreement among humans, grounded on mutual benefit.[29] Euhemerus attempted a rationalist explanation of religions, maintaining that cults originated in the deification of exceptional humans.[30] It is telling that early Ptolemaic Alexandria was the theater for the only known episode of intentional creation of a divinity: it seems that the persona of the god Serapis was simply made up by Ptolemy I Soter and his advisors. Fraser remarks: "The operation of creating a new deity, bizarre though it may seem to us, probably did not appear so at the time".[31]

[26] See page 175.

[27] An echo of this can be found in a beautiful passage of Philostratus concerning the active part of the observer of a work of art (*Life of Apollonius of Tyana*, II, xxii).

[28] Bruno Snell has masterfully described the increasing awareness in Greek culture from the archaic to the classical period ([Snell]). I think, however, that he and later classicists have missed the importance of the subsequent cultural leap, which led to the appearance of exact science. See, for example, [Snell], p. 214, where the conclusion that the Greeks "lacked a genuine concept of motion" is drawn from considerations on Zeno's and Aristotle's thought.

[29] This idea, which would not come back until the eighteenth century, is clear in the *Principal doctrines* of Epicurus, in particular numbers 33, 36, 37 and 38. For precedents among the Sophists we mention Lycophron's statement (cited in Aristotle, *Politica*, III, 1280b) that the law is an agreement meant to guarantee men's just rights toward one another.

[30] Rationalist explanations of the origin of religions, though best known from Lucretius, also go back to the ancient Sophists. Particularly important in this connection is a fragment of Critias quoted by Sextus Empiricus (*Adversus physicos* I (= *Adv. dogmaticos* III = *Adv. mathematicos* IX), §54).

[31] [Fraser], vol. I, p. 252 (quote), pp. 246–259 (discussion of episode).

7.3 The Theory of Dreams

It would be fascinating to reconstruct what was thought in Hellenistic times about the analysis of the *psyche*. Here we will treat only one aspect of the problem, albeit an important one: the analysis of dreams.

One work about the subject that has enjoyed great popularity ever since the Renaissance is the *Interpretation of dreams* (᾿Ονειροκριτικά) of Artemidorus of Daldis, written in the second half of the second century A.D. A recent reprint of a venerable Italian translation has an introduction by the psychoanalyst Cesare Musatti, from which we quote to give an idea of the content of Artemidorus' work relative to modern psychoanalysis. Musatti first stresses the profound difference between Artemidorus' conceptual framework and ours:

> Artemidorus accepts the idea, widespread in the ancient world and down to modern times, that dreams have a premonitory value. . . .
>
> Freud maintains as a general thesis that dreams are hallucinatory realizations of desires. . . .
>
> Even for Freud, dreams may be prognostic. . . .
>
> Admittedly [they foretell] a future that will be manufactured by the dreamer, rather than facts that will unfold in the outside world and that he would run into unawares. For Artemidorus the future tracked in dreams is of the latter type; for Freud it may be just a future constructed by the subject himself.[32]

On more technical matters, however, Musatti finds much in common between the two. For example, discussing the distinction Artemidorus makes between two types of dreams, ὄνειρος and ἐνύπνιον, he writes:

> One is tempted to consider these images (the ἐνύπνιον of Artemidorus) similar to, if not in exact correspondence with, those we now call hypnagogic.[33]

Artemidorus subdivides his other class, "true dreams" (ὄνειροι), into *contemplative* (θεωρηματικοί) and *allegoric* (ἀλληγορικοί). Musatti remarks:

> Artemidorus says that the former are realized immediately, with no delay, whereas for allegoric dreams the realization happens only after a while. This notion that contemplative dreams have a very short deadline has a curious match in what Freud says about the "residues of the day" (images corresponding to elements really lived during

[32][Artemidorus/Musatti], pp. 7, 10, 11, 12.
[33][Artemidorus/Musatti], p. 9.

> wakefulness and inserted without change in the dreams): they will appear in dreams no more than forty-eight hours later. . . .
>
> [Among allegoric dreams] an interesting category is that of κοσμικοὶ ὄνειροι (cosmic dreams). They appear fairly often in today's psychoanalysis, and we too call them cosmic dreams. . . .
>
> But let's consider in its totality the category of allegoric dreams, those in which things may be represented not by their own images but by other images that signify them. "He who loves a woman will not see the beloved woman but a horse, or a mirror, or a ship, or the sea, or a female animal, or a woman's dress, or something else that stands for a woman."[34] This is the principle of dream symbolism, which is accepted by the Freudian theory as well. . . . And let it be said right away that, in Artemidorus' passage, the symbols listed for a woman are, so to speak, all correct and exact from the point of view of psychoanalytic experience. . . .[35]

Musatti finds other correspondences between Artemidorus and modern psychoanalysis. A last quote can serve to summarize his thought:

> It is certainly fascinating to find in this author from another era, for whom dreams had a completely different meaning which we do not accept, familiarity with types of ideas that scientific psychology only through great pains has managed to uncover in the last century.[36]

Musatti is struck by the apparent contradiction between the presence in Artemidorus' thinking of so many elements present in modern scientific psychology and his faith in the divinatory potential of dreams. But this apparent contradiction may be explainable by the historical context: Artemidorus belongs to the same late Hellenistic period represented by Galen, Ptolemy and Heron, when scientific theories had been increasingly contaminated by cultural elements of indigenous origin, often magical (think of Ptolemy's interest in astrology). Scientific results acquired in the preceding centuries were being used for individual and immediately practical ends. We have seen how Galen, in spite of the anatomical and physiological knowledge he inherited from the school of Herophilus (and put to use not least in the service of his extraordinary personal career) was no longer able to fully understand the methodology underlying that knowledge; we have seen Heron using elements from Alexandrian technology to design wonderful toys. One may suspect that, likewise, Artemidorus uses for divination purposes the remnants of an ancient "scientific

[34]Musatti is quoting Artemidorus.
[35][Artemidorus/Musatti], pp. 14–15.
[36][Artemidorus/Musatti], p. 16.

theory of dreams". In fact it is not hard to track down such a theory; we have already met the first author to whom it is attributed — Herophilus of Chalcedon.[37] In the pseudo-Galenic *De historia philosopha* we read:

> Herophilus says that some dreams are sent by a god[38] and arise by necessity, while others are natural and arise when the soul makes for itself an image [εἴδωλον] of what is to its advantage and of what will undoubtedly happen; but the "compound" dreams [arise] spontaneously, according to the impact of the images whenever we see what we wish, as happens in the case of men who harbour affection, when in their dreams they make love to the women they love.[39]

Several elements stand out clearly: the relationship between dreams and the future, based on a knowledge of what must happen; the importance of sexual dreams; the relation between dreams and wishes. Alas, we have none of Herophilus' writings on dreams, but given the author's known intellectual level and his status as the founder of psychiatry, so to speak, they must have been very interesting indeed. His thinking was picked up and modified by the Stoics, and it influenced the Christian literature of the first centuries A.D., with the replacement of sexual dreams by "demonic dreams".[40] A fragment of Posidonius quoted by Cicero attests to an intermediate stage between Herophilus and Artemidorus: it relates a Stoic classification of dreams that is still reminiscent of Herophilus', but now inserted in the context of divination.[41]

Thus the scientific psychology elements present in Artemidorus' work seem to stem from an old and distinguished tradition. But it was the part that makes modern scientists smirk — the use of the theory for divination — that allowed the work to survive down to our days, preserved by the same ancestors of ours who decreed the annihilation of all works by Herophilus.[42]

[37] Among earlier authors of works on the interpretation of dreams can be counted the Sophist Antiphon (Suda, under Antiphon = [FV], II, 334:9–10, Antiphon A1), to whom was attributed the ability to suppress suffering through persuasion (Plutarch, *Vitae decem oratorum*, 833C = [FV], II, 337:2–4, Antiphon A6). We have no information about Antiphon's ideas about dreams.

[38] "Sent by a god" is the literal meaning of the single original word θεόπεμπτος, which can also mean simply "extraordinary". The use of this word by our anonymous author, therefore, does not imply that Herophilus thought that such dreams really had a divine origin.

[39] Pseudo-Galen, *De historia philosopha*, 106 = [DG], 640 = [von Staden: H], text 226c, von Staden translation.

[40] See [von Staden: H], pp. 309–310.

[41] Cicero, *De divinatione*, I, xxx §64; see also [von Staden: H], p. 308.

[42] The same fate befell other works subsequent to Artemidorus and probably more "scientific" than his. For instance, the mathematician Pappus is known to have written an Ὀνειροκριτικά, again according to the Suda (under *Pappus*).

At this point you might be thinking that Freud's precursor in Antiquity happened to be not so much Artemidorus but Herophilus and his school. But there is more to it. In his *Interpretation of dreams*, Freud writes:

> I am far from wishing to assert that no previous writer has ever thought of tracing a dream to a wish. . . . Those who put store by such hints will find that even in antiquity the physician Herophilos, who lived under the First Ptolemy, distinguished between three kinds of dreams. . .[43]

Freud then continues with the trichotomy found in the already quoted pseudo-Galenic passage. Earlier in the same book he writes:

> The unscientific world, therefore, has always endeavoured to interpret dreams, and by applying one or the other of two essentially different methods. The first . . . is symbolic dream-interpretation. . . . The second . . . might be described as the cipher method. . . . An interesting variant of this cipher procedure . . . is presented in the work on dream-interpretation by Artemidoros of Daldis.
>
> The worthlessness of both these popular methods of interpretation does not admit of discussion. . . . So that one might be tempted to grant the contention of the philosophers and psychiatrists, and to dismiss the problem of dream-interpretation as altogether fanciful.
>
> I have, however, come to think differently. I have been forced to perceive that here, once more, we have one of those not infrequent cases where an ancient and stubbornly retained popular belief seems to have come nearer to the truth of the matter than the opinion of modern science.[44]

Freud cited Artemidorus often, and with the highest regard. For example:

> Artemidorus of Daldis . . . left us the fullest and most careful work of the Greco-Roman world on the interpretation of dreams.[45]

Thus, to judge from Freud's authoritative testimony, the modern psychoanalytic theory of dreams had at its origin the elements "near to the

[43] Sigmund Freud, *The interpretation of dreams*, adapted from the translation by A. A. Brill, 1913. The original is *Die Traumdeutung*, third edition, 1911 (and subsequent editions). The passage is in a footnote at the end of Chapter 3 ("The dream as wish-fulfilment").

[44] Ibid., Chapter 2 (at 4% and 18%), Brill translation.

[45] Sigmund Freud, ibid., Chapter 2, third footnote. In his *Introductory lectures on psycho-analysis* he likewise wrote: "Of the literature dealing with dreams we fortunately have the main work, by Artemidorus of Daldis" (*Vorlesungen zur Einführung in die Psychoanalyse*, 1915–1917, fifth lecture). It is not clear whether these statements are based on (a priori) faith in the workings of chance, or on a belief that Artemidorus could not be surpassed.

truth" found in Artemidorus' work (which Freud must have studied carefully, if he thought it was the most important on the subject) and based on the "foreshadowings" of Herophilus, to whom Freud correctly, if unintentionally,[46] attributes the paternity of the theory of wish-incited dreams. If it has been possible to reconstruct in this way at least part of the thinking of the Herophilean school, it is no wonder that Musatti finds so many "curious matches" for Artemidorus in Freud, nor that they use the same terms, such as "cosmic dreams" and "hypnagogic images",[47] nor that all the symbols listed by Artemidorus are generously approved as "correct and exact" by the modern heirs of his thinking.

There remains to explain why on earth it took "great pains" (to use Musatti's words) for scientific psychology in the last century to uncover the "types of ideas" that Artemidorus showed "familiarity" with. One might suspect that the effort lay in translating the ideas into a scientific language conforming to the current canon. But if, in the case of trigonometry, halving the chord was enough to make the theory unrecognizable to generations of historians of science,[48] the effort shouldn't have been so great after all.

We have talked of dreams only, but in Artemidorus there are interesting "foreshadowings" of other psychoanalytic elements. For instance, Musatti writes:

> Regarding the dream of incest between son and mother, Artemidorus talks as if he knew of and agreed with Freud's ideas about the Oedipus complex.[49]

7.4 Propositional Logic

Aristotle devoted much attention to logic, and syllogisms in particular. But in his analysis of the various forms of syllogisms their validity was justified only through the evidence provided by examples. In other words, he described the use of logic, but he did not formulate a *theory* thereof, in our sense of the word.

[46] The facetious tone in which Freud cites Herophilus only for the sake of those who "put store by such hints" ("Wer auf solche Andeutungen Wert legt") suggests that he too was subject to the well-known phenomenon of "Freudian repression", which makes it seem hilarious that one may have been foreshadowed by a Hellenistic author.

[47] Musatti is *tempted* to consider these images similar to the concept found in Artemidorus, and this not only because of the similarity in meaning but also of the word itself: the modern word "hypnagogic" is a direct borrowing of Artemidorus' word.

[48] See Section 2.8.

[49] [Artemidorus/Musatti], p. 17.

The first steps toward a scientific theory of logic seem to have been taken around 300 B.C. by Diodorus Cronus and Philo of Dialectician. The latter is known to have defined the conditional proposition "if p then q" as "not (p and not q)".[50] But although his awareness of the need for a crisp, unambiguous definition of the conditional seems to herald proofs of theorems in logic, on the whole the existing testimonia point to Chrysippus as the founder of the scientific theory of propositional logic. While Aristotle adopted variables to represent generic terms in propositions, Chrysippus used them to stand for the propositions themselves, and constructed a theory of logical inference based on five postulates.[51] If we use p and q to denote generic propositions, the five inference rules assumed by Chrysippus as *undemonstrated* correspond to the five lines below; the first two entries of each line are premises and the last says what can be deduced from them.

1.	if p then q	p	q
2.	if p then q	not q	not p
3.	not (p and q)	p	not q
4.	p or[52] q	p	not q
5.	p or[52] q	not p	q

From these five postulates an unlimited number of inference schemes could be deduced as theorems. But none of Chrysippus' theorems in logic has been preserved.

Chrysippus also wrote several books devoted to the study of antinomies and logical paradoxes.

Apparently no historian of philosophy had grasped the substance of Chrysippus' work before Benson Mates did so around 1950.[53] Indeed, an understanding of Chrysippus had to wait for the development of modern propositional logic and for someone to study both propositional logic and ancient philosophy — two subjects then regarded as far apart. And yet Mates and others have been criticized for using modern propositional logic in interpreting Chrysippus. Long, a historian of philosophy, wrote:

[50] Sextus Empiricus, *Adversus logicos* II (= *Adv. dogmaticos* II = *Adv. math.* VIII), §113; *Pyrrhoneae hypotyposes*, II, xi §110.

[51] These five postulates are listed by Diogenes Laertius (*Vitae philosophorum*, VII §§79–81) and by Sextus Empiricus (*Pyrrhoneae hypotyposes*, II, xiii §§157–158). Another passage of Sextus (*Adv. logicos* II §§224–227) reports only the first three postulates but gives more information about theorems in logic.

[52] Here "or" is understood in the exclusive sense: either p or q is true but not both.

[53] [Mates]. For another very interesting study about ancient logic and its relationship to modern logic, see [Bocheński] (which was partly based on Mates' work, then still unpublished).

> [S]ome scholars have recently argued that modern logic may not be the best key to understanding this or any other ancient logical theory. The debate continues[.][54]

It is important for our purposes to dissect the objections mentioned by Long. They are based on one reasonable consideration and one implicit assumption that is often applied unconsciously in reflecting about the past. The reasonable consideration is that the key to understanding an author is seldom found in a culture distant from that author: this is simply an expression of culture's historical relativity (which, by the way, was first recognized in Hellenistic times). The implicit assumption lies in the measurement of the distance between two cultures. In talking about "modern logic" and "any ancient logical theory", one is visibly judging this distance primarily according to a dichotomy between ancient and modern. But so many things become clearer if one manages to abandon this ancient/modern opposition that has taken root so deeply since the Renaissance! Truly the distance between cultures cannot be judged according to this split, nor can it be measured in centuries or miles, but only by identifying the discontinuities due to cultural revolutions and by following the often subtle threads of deep influence.

Chrysippus' logic is very different from any other ancient logical theory in that it is a product of the scientific revolution we have been discussing. It is the transposition to the realm of logic of the same conceptual scheme used in Hellenistic geometry, which Chrysippus must have studied carefully.[55] This makes his new logic radically different from, say, Aristotelian logic. Thus, if one wishes to find the key to understanding Chrysippus in a culture "close" to his, one can try looking for a culture in which the Euclidean hypothetico-deductive scheme is essential.

But trying to axiomatize logic following the scheme used by Euclid would not make sense for someone who does not even manage to follow the Euclidean method in the case of mathematics. To apply the Euclidean scheme to logic and thus to found propositional logic, as Chrysippus did, it was first necessary to recover Euclid completely. Useful instruments for understanding Chrysippus therefore must be sought in the mathematical circles of the late nineteenth century, particularly in the German school, to which we owe the recovery of Euclid (especially in the work of Weierstrass and Dedekind in 1872, as mentioned on page 47). Thus it is not surprising that the first steps toward modern propositional logic (including the

[54][Long], p. 139.

[55]Proclus, quoting Geminus, says that Chrysippus used a theorem of geometry to illustrate with an analogy the role of ideas (Proclus, *In primum Euclidis Elementorum librum commentarii* (ed. Friedlein), 395:13–18).

restatement of Philo's definition of the conditional) were taken in 1879 by the German mathematician Frege.[56]

7.5 Philological and Linguistic Studies

Literary history and philology came into existence in the early Hellenistic period and throve especially in Alexandria, at the hands of Callimachus, Eratosthenes, Aristophanes of Byzantium, Aristarchus of Samothrace, and others, until the tragic interruption of 145 B.C.[57]

Modern philology, not only in its humanistic phase but also in its more technical developments begun in the eighteenth century, was born from the rediscovery of Alexandrian philology. An important impetus was the renewal of Homeric philology that took place in the late eighteenth century after the finding of certain ancient scholia on the *Iliad*, containing a storehouse of information on Aristarchan textual criticism.[58]

The birth of philology and literary history has often been put forth as evidence of Hellenistic cultural decadence, the view being that such studies represented the musty erudition of the effete Alexandrian scholars, instead of the fresh and robust inspiration of the classical age. Such opinions, coming from philologists and historians of literature, may perhaps give an indication of the origin of the erasure phenomenon (or Freudian repression) that we have observed starting from Chapter 1.

We have seen how the scientific revolution generated a new concept of language, involving the manifestly useful possibility of conscious lexical enrichment through the introduction of new terms for novel conceptual constructs. This new possibility, which probably underlay the increasing awareness of the historical evolution of languages observed around that time,[59] was soon applied within the framework of the study of language itself. Thus was born Greek grammar, with its distinction between the parts of speech, the five cases of nominal and adjectival declension, verb tenses, and so on. In other words, a theory (in the sense of empirical science) was formulated, and new and conventional terminology was introduced to de-

[56]See [Frege].

[57]See page 11 and note 60 on page 69. Even Aristarchus of Samothrace had to leave Alexandria. For the history of Hellenistic philology see [Pfeiffer], for example.

[58]The scholia were on a tenth-century codex preserved in Venice and were published in 1788. Seven years later, F. A. Wolf re-posed the ancient Alexandrian questions in his famous *Prolegomena ad Homerum*, thus founding modern Homeric philology.

[59]Whereas Aristotle still has a static notion of language, Sextus Empiricus stresses both the infinity of possible meanings and signs, and the evolution of the lexicon (*Adversus gramaticos* (= *Adv. math.* I), §82). The second point was taken up also by Horace (*Ars poetica*, 60 ff.).

note the concepts internal to it.[60] The earliest work on Greek grammar that we have adequate information about, thanks to a Byzantine compendium, is the *Art of grammar* of Dionysus Thrax, from the second century B.C., which already uses the current canonical terminology. However it is clear from other testimonia that the beginnings of Greek grammar date from no later than the third century B.C.[61] The subject was developed in particular by eminent Stoic thinkers, including Chrysippus himself;[62] main results such as the description of nominal and verbal inflection were already in place at the time of Aristophanes of Byzantium, Dionysus' teacher.[63]

We have plenty of information about grammar because it was coopted by the Romans.[64] We know less about semantics, which was developed in the Stoic school in close connection with the theory of knowledge and with logic. The extant citations are late and do not allow a reconstruction of the results obtained in this field, but they are enough to give an idea of their level. Long writes:

> One of the most interesting features of Stoic semantic theory is the fact that it allows a distinction to be drawn between sense and reference. This distinction, which was first formulated in a technical sense by the German logician Gottlob Frege, has been extremely fruitful[.][65]

Next Long illustrates the Stoic distinction through passages from Diogenes Laertius.[66] After all that was said in the preceding section, it should not come as a great surprise that Frege was the first modern thinker to revive Hellenistic ideas on semantics. But Long's qualifying phrase "in

[60]An important forerunner of Hellenistic grammatical studies can be found in Aristotle, *Poetica*, xx, 1456b–1457a. But the passage only makes a distinction between noun and verb and makes a general remark about (substantival and verbal) inflection, without describing it. In any case Hellenistic grammarians made essential use of the possibility, extraneous to Aristotle's thought, of introducing conventional terminology.

[61]Its relationship with Sanskrit grammar, developed in particular by Pāṇini, is not clear. However, the continuity between Stoic epistemology, Stoic semantics, the creation of a conventional terminology in other fields, and Hellenistic grammatical studies seem to preclude the possibility that Greek grammar appeared as a simple transposition of a theory imported from India.

[62]Two passages, one by Diocles of Magnesia (in Diogenes Laertius, *Vitae philosophorum*, VII §57 = [SVF], II, text 147) and one by Galen (*De placitis Hippocratis et Platonis*, VIII, iii = [SVF], II, text 148) report Chrysippus' classification of words into parts of speech. Proper nouns, verbs, conjunctions and articles appear on both lists; Diocles' list also includes common nouns and Galen's prepositions. A later stage of grammatical studies is attested in Plutarch: in discussing the functions of the various parts of speech he also includes adverbs and pronouns (*Platonicae quaestiones*, 1009B–1011E).

[63][Pfeiffer] is a useful reference for Hellenistic linguistic studies. For grammar, see in particular pp. 202–203 and 266 ff.

[64]Latin grammar started with Remmius Palaemon, who, during the reign of Tiberius, adapted the work of Dionysus Thrax to Latin.

[65][Long], p. 137.

[66]Diogenes Laertius, *Vitae philosophorum*, VII §§94–102.

a technical sense" brings to mind similar remarks already discussed, for instance in the case of trigonometry. A few lines later Long concludes:

> But, returning to Frege, we should beware of assimilating his theory of meaning and the Stoics'. The Stoics have no term which corresponds clearly to Frege's use of *Bedeutung*, 'reference'. Its place in Stoicism is taken by 'bodies' (the thing referred to) or the 'grammatical subject' (*ptôsis*).[66a]

One milestone of modern linguistics is the work of Ferdinand de Saussure. In even more recent times the contributions to the theory of meaning made by Roland Barthes, the founder of modern semiotics, have had great resonance. Let's see how Barthes, in his *Elements of semiology*, expounds Saussure's theory of meaning and his own:

> Saussure himself has clearly marked the mental nature of the signified by calling it a *concept*: the signified of the word *ox* is not the animal *ox*, but its mental image ... These discussions, however, still bear the stamp of psychologism, so the analysis of the Stoics will perhaps be thought preferable. They carefully distinguished between φαντασία λογική (the mental representation), the τυγχανόν (the real thing) and the λεκτόν (the utterable). The signified is neither the φαντασία nor the τυγχανόν, but the λεκτόν; being neither an act of consciousness, nor a real thing, it can be defined only within the signifying process, in a quasi-tautological way: it is this 'something' which is meant by the person who uses the sign. In this way we are back again to a purely functional definition[.][67]

The Hellenistic linguistic notions appropriated by Barthes constitute an important aspect of the scientific revolution. In the third century B.C. there appeared in all fields new theoretical constructs, based on the use of new concepts, consciously elaborated and denoted by conventional terms. To reduce the meaning of words to the *lekton* (λεκτόν), that is, to "what one wishes to signify", is to use words as "signs" not for natural objects but for freely created notions, and this practice can only arise from reflection on the creation of intellectual notions. Thus, Stoic semantics is none other than an aspect of the same revolution in thought that led also to science and to Chrysippus' theory of active perception.[68]

[66a][Long], p. 138.

[67][Barthes], p. 43 (section II.2.1).

[68]The main sources on Stoic semantics are, first and foremost, Sextus Empiricus, and then comments by Stobaeus, Diogenes Laertius and others. The terms used by Barthes in the preceding quotation are explained in Sextus Empiricus, *Adversus logicos* II (= *Adv. dogmaticos* II = *Adv. math.* VIII). (The tripartite distinction made by the Stoics between sign, meaning and object is in II §§11–12, and the Stoic notion of lekton is illustrated in II §70.)

It is certainly true, as Long says, that one should not shoehorn Stoic semantics and Frege's into the same mold. But one reason may be that, at least to judge from Barthes' passage, in Frege's time the reacquisition of Stoic semantics was still far from complete.

7.6 The Figurative Arts, Literature and Music

It has often been remarked that Hellenistic figurative art works display surprisingly modern features. As in early modern Europe (in seventeenth-century Holland, say) painting got the upper hand over sculpture, and subject matter changed radically compared to the preceding period. We see the birth of different styles, including those that have been named the *ancient baroque*, *naturalism*, *classicism* and *impressionism* — terms that do not merely refer to vague analogies with modern styles, but on the contrary seem very apposite.[69] The ease with which Hellenistic artists can be classified using modern terminology may appear surprising. But on further reflection, in the case of the baroque it is not surprising that the term may be applicable without exaggeration to ancient works whose study was certainly connected with the rise of the baroque style. One wonders whether Hellenistic artists also succeeded in "foreshadowing" much of later art?[70]

In literature, too, styles and genres multiplied. Some literary genres maintained their old names, but had little in common with their classical incarnations. The classical epigram, for example, was a commissioned inscription, whereas its Hellenistic counterpart, of which Callimachus was the greatest virtuoso, is a pretend inscription. Thus a cultural product of social exigencies is transformed into an occasion for free and deliberate invention. Also born then were bucolic poetry, the comedy of manners and the luckiest of the new literary genres: the novel.[71]

Was there a connection between science and the new character of art and literature? A positive answer is suggested by the simultaneity not only of their first appearance but of their reappearance in modern times: the

[69]For example, Bianchi Bandinelli writes that a mosaic found in Palermo "echoes a painting performed wholly without outlines and relying completely on light effects, in a way that is, in effect, almost impressionistic" ([Bianchi Bandinelli], p. 477).

[70]The evolution of the modern styles homonymous with those cited happened during centuries when a great many Hellenistic-inspired ancient works were brought to light and studied, at the hands of artists who regarded the study of ancient art as part of the very foundation of their own culture.

[71]The Hellenistic origin of the novel was long obscured. It was thought that Greek-language novels first appeared in the late imperial age; this changed in 1945 when a papyrus was found in Oxyrynchus that dates from the first century B.C. and contains fragments of the *Novel of Ninus*. Now many scholars think that the novel originated in the second century B.C.

FIGURE 7.6. Detail of floor mosaic from the second century B.C., found in the 1990s during the excavation of the site of the new Library of Alexandria. Courtesy Prof. Jean-Yves Empereur, Centre d'Études Alexandrines.

same features in art were reacquired in the modern age together with the scientific method.

Science of course lent art technical and conceptual tools, as we have seen concerning the link between optics, scenography and painting, and as we shall see in music.

But there is also a deeper relationship between science and art. The main aspect of the scientific revolution lies in that the creation of culture became a conscious act. It is this awareness that gives rise to artistic experimentation, causing styles and genres to multiply. At the same time, the focus of interest shifts from cultural categories, "mythicized" (in different ways) in both the archaic and classical ages, to the individual, who becomes recognized as the creator of such categories, and to the individual's concrete life. This endows figurative art with a new realism, a novel interest in the depiction of emotions and psychological states (see photos on page 226), and a wealth of fresh subject matter from everyday life — whether in sculpture (page 227), mosaics (page 159 and this page), or painting, where personal portraits, landscapes and still lifes become common (page 62).[72] The same

[72]Particularly significant, in this respect, are pictorial representations of the painter's atelier or some other craftman's shop. Paintings with such subject matter are known to have been made by

FIGURE 7.7. Bronze from the second century B.C., found in pieces in a shipwreck off Cape Artemisium. The young rider, nicknamed the Jockey of Artemisium, has African features and rode a galloping horse: he held the reins in his left hand and a whip in the right hand. Hirmer Photoarchiv.

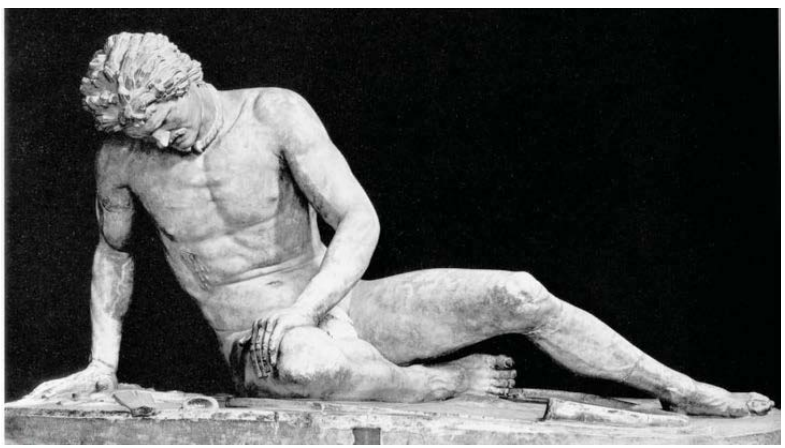


FIGURE 7.8. The Dying Gaul (Capitoline Museums, Rome). Roman copy of a lost original that belonged to a group commissioned by the Attalids for a monument built in the Pergamum acropolis around 275 B.C. to commemorate their victory against the Gauls.

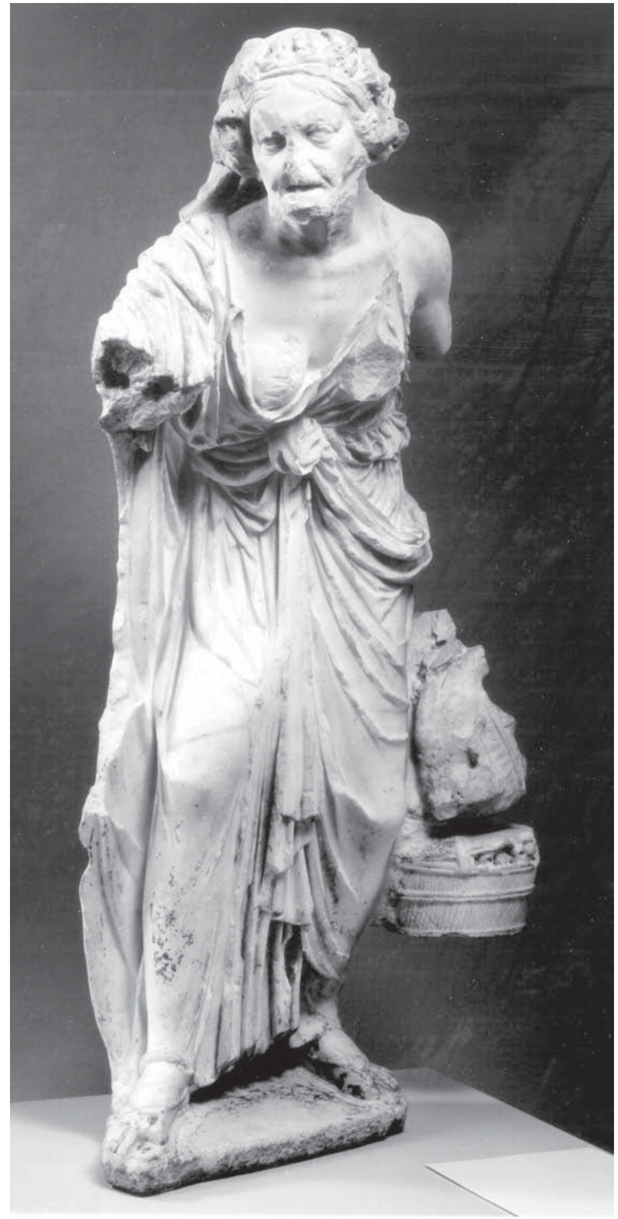


FIGURE 7.9. Old woman in market, Roman sculpture after Hellenistic original. Courtesy Metropolitan Museum of Art. All rights reserved.

trend is observed in comedy, which acquires characters and situations that mirror the spectator's world.

The novel's appearance is linked both to the new idea of cultural production as a deliberate invention, and to the new type of target audience (individual readers) and mode of reading (private).[73] As in modern Europe, many aspects of Hellenistic art depend on the existence of a public of readers and buyers of art works.

In classical Greece the word "music" (μουσική) meant music, singing and dance, thought of as an indivisible whole. Our notions of "music" and "musician" date from the early Hellenistic period.[74] The second half of the fourth century B.C. also witnessed the development of the first true theory of music, by Aristoxenus of Tarentum. We read in Franchi's contribution to an authoritative collection about Greek civilization:

> These doctrines still constitute today the basis for any study about sound systems, and their accuracy is often astonishing, as when Aristoxenus actually foreshadows equal temperament, which was not achieved until the late seventeenth century.[75]

As in all cases examined, from semantics to shipbuilding, from dream theory to propositional logic, any specialist who in the course of studies in his own field turns his attention to the period of the scientific revolution is invariably *astonished* to discover that modern knowledge was *foreshadowed* at the time. The idea of *foreshadowing* of later theories is a bit like the premonitions in which Artemidorus of Daldis believed. Now that we have succeeded, in the case of oneirology, in reconstructing a scientific theory, should we not try to do the same for the history of culture? Should we not replace these *foreshadowings* by the study of the influences of Hellenistic thought on modern thought?

The main Hellenistic innovation in the area of musical instruments was the introduction of the first keyboard instrument: the water organ,[76] which seemingly was also the first scientifically designed musical instrument (Figure 7.10). Its invention, attributed to Ctesibius,[77] was clearly linked to the new science of pneumatics created by the same man. In the Ctesibian

Antiphilus, active in the court of Ptolemy I Soter (see, for example, Pliny, *Naturalis historia*, XXXV §§114 + 138) and by several others, such as Philiscus and Piraeicus (Pliny, *Naturalis historia*, XXXV §§112 + 143).

[73]For an analysis of the relationship between Greek literary works and their modes of enjoyment, see [Gentili: PP].

[74]See, for example, [Gentili: MR].

[75][Franchi], p. 624.

[76]The instrument is described in Vitruvius, *De architectura*, X, viii.

[77]Sources for this attribution include Athenaeus (*Deipnosophistae*, IV, 174b+d–e), who adds that Aristoxenus (born around 370 B.C.) did not know the instrument (IV, 174c).

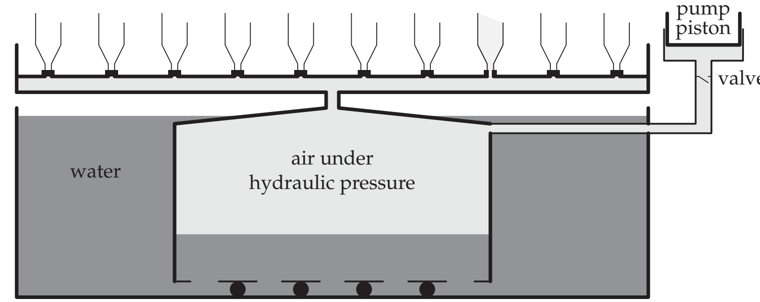


FIGURE 7.10. The Ctesibius water organ as described by Vitruvius and Heron. Based on a drawing of James A. Diamond.

organ, the air is forced into the pipes because it is kept under pressure in an underwater chest. The air spent in the tubes or otherwise lost is replenished by means of a pump. The water, so long as its level does not vary too much, keeps the pressure in the air chest approximately constant. Remnants of an exemplar dating from the imperial period and having four registers, each with thirteen tubes, have been found at Aquincum (now Buda) and are on display at the local museum. Remnants of an older water organ were found in 1994 in Dion, near Mount Olympus. Medieval organs are the direct descendents of Alexandrian ones, through imperial and Byzantine inheritance.[78] This is a typical case of the pervasive role played by Byzantine culture in the transmission of Hellenistic musical heritage.[79]

Musical theory had close relations with acoustics, whose theoretical developments are poorly documented;[80] as to applications, we know, for example, that theaters were equipped with acoustic chambers that dampened echoes and served to amplify certain frequencies, thanks to bronze resonators.[81]

[78]The first organ in medieval Europe was received in 757 by the Frankish king Pepin the Short as a present from the Byzantine emperor Constantine V. It was a water organ.

[79]A lack of interest in Byzantine music was for a long time a factor in hiding the continuity between ancient and medieval music. See [Touliatos].

[80]Certain qualitative observations have come down to us: about the speed of sound (for example, in Sextus Empiricus, *Adversus astrologos* (= *Adv. mathematicos*, V), §69), the analogy between spherical sound waves propagating in air and plane waves on the surface of the water (for example, in Diogenes Laertius, *Vitae philosophorum*, VII §158), the relation between pitch and frequency of vibration (for example, in Plutarch, *Platonicae quaestiones*, 1006A–B, and more extensively in Porphyry, *In Harmonica Ptolemaei commentarius* 56, ed. Düring = [FV], I, 431–435 (Archytas B1). On the other hand the elementary results traditionally attributed to Pythagoras show, if nothing else, that quantitative developments also took place.

[81]Vitruvius, *De architectura*, V, v. The passage mentions the existence of a quantitative theory and that the Greek systems he describes were not yet in use in Rome. Recent restorations have revealed such acoustic chambers in the theater at Scythopolis (now Beit Shean in Israel).

Again in the case of music, modern scholars generally pass stern judgement on the Hellenistic period. The *Oxford History of Music* says:

> Certainly, however, sophisticated poets were growing incapable of making music, and musicians of writing sophisticated verse. When the classical unity of Music was broken, the 'music' (in our narrow sense of the term) was supplied by a professional engaged in the performance.[82]

Classical Music versus 'music': broken unity: what an odd way to think of the beginnings of what we understand by music! Such is the stigma of Hellenistic culture. Elsewhere in the same article we read:

> The forms of rural music were afterwards collected by Alexandrian scholars, more from literary references than from life. They catalogued over fifty generic or regional types of dance, with innumerable songs of shepherd loves or of rustic labours — the 'practical songs' sung at work by spinners, millers, reapers, and water-drawers.[83]

Despite the (unsubstantiated) jab at library-bound work — perhaps the author fears competition in the realm of second-hand musicology? — we cannot help but see developments that "foreshadow" modern attitudes: scholarly interest in popular musical motifs and in the preservation of musical heritage in general, to which we may add a frequently cited emphasis on professionalization and technical excelence.[84]

[82][Henderson], p. 400.

[83][Henderson], p. 391. For popular music and work songs see Athenaeus, *Deipnosophistae*, XIV, 618e–620a.

[84][Henderson], p. 400.

8 The Decadence and End of Science

8.1 The Crisis in Hellenistic Science

Hellenistic science and indeed Hellenistic civilization, after their extraordinary development in the third century B.C., ran into a crisis during the next century.

The resumption of studies in the imperial period secured a revival of ancient knowledge, but did not yield any new scientific theories; even the scientific method itself was rejected. The rejection of theoretical concepts, which as we have seen is implicit in Galen and will be seen explicitly in Plutarch, was theorized on a philosophical level by the Skeptics. The best extant exposition of Skepticism is the work of Sextus Empiricus *Against the mathematicians*, written around 200 A.D. (see page 190).

Sextus writes against the possibility of constructing theoretical models. His arguments are well represented in the following passages:

> If there is such a thing as *mathema* and it is attainable by humankind, it presupposes agreement on four things: the thing which is taught, the teacher, the learner, and the method of learning. However, the thing which is taught does not exist, nor does the teacher, the learner, nor the method of learning, as we shall demonstrate. Therefore, there is no *mathema*.[1]

[1]Sextus Empiricus, *Adversus grammaticos* (= *Adv. mathematicos* I), §9. We use D. L. Blank's translation (*Against the grammarians*, Clarendon, Oxford, 1998) except in leaving the word *mathema* untranslated; its meaning is learning, study, or an object thereof (see page 190).

Sextus then gives an argument for the nonexistence of the thing being taught:

> Furthermore, since some somethings are bodies and others are incorporeal, the things which are taught will, as somethings, have to be either bodies or incorporeal.... Now body would not be a teachable thing, and especially not according to the Stoics, for things that are taught must be *lekta*, but bodies are not *lekta* and hence not taught.... Nor indeed can the incorporeal [be taught, for] ... every [incorporeal] is obviously under investigation..., some insisting that they exist, others that they do not, and others undecided.[2]

The *lekta* (literally "utterables", which is to say meanings) were for the Stoics the only possible object of teaching; they are of course conceptual constructs. In the imperial age, when the notion of theoretical models had been lost, such entities were conceivable only as real objects: the alternative between "bodies" and "incorporeal beings" thus became ineluctable. Some such entities were indeed made corporeal — witness the crystalline celestial spheres which replaced the spheres of Eudoxus of Cnidus and the epicycles of Apollonius of Perga. Likewise, the "visual rays" of optics reacquired the character of physical objects emitted by the eyes, which was not present in Euclid's theory. This new interpretation appears already in a preface to Euclid's *Optics*[3] dating perhaps from the fourth century A.D. and prepended to the work in manuscripts that contain the recension generally attributed to Theon.[4] Other entities, such as those of geometry, were given an incorporeal reality.[5] This placed geometry in the realm of Platonic thought, a position that Hellenistic mathematics had left behind.

(The memory of the function played by theoretical entities in Hellenistic science did not disappear completely. As late as the fifth century A.D., Proclus muses, probably harking back to an ancient epistemological debate:

> What shall we say about eccentrics, which are still the subject of talk, and about epicycles? Are they mere inventions or do they really exist in the sphere to which they are attached?[6]

[2] Sextus Empiricus, *Adversus grammaticos* §§19–20, 28, Blank translation (leaving *lekta* untranslated: see subsequent text and also page 223, where the singular, *lekton*, is discussed).

[3] See [Euclid: OO], vol. VII, p. 150, where it is said that if the eyes were to be receivers of something rather than emitters they would have to be hollow, like the nose and ears.

[4] This attribution was made by Heiberg, who also discovered a different version of the work, which he thinks is original, and which was transmitted by certain other codices. Both versions are published in [Euclid: OO], vol. VII.

[5] For an explicit statement of this position, see for example Iamblichus, *De communi mathematica scientia*, xxviii.

[6] Proclus, *Hypotyposes astronomicarum positionum*, 236:15–17.

But he criticizes both possibilities.)

What were the causes of the crisis in science?

Many seem to share C. Préaux's opinion that the development of ancient science was hindered by Aristotle's excessive authority.[7]

But note that Aristotle, for whom the brain had a cooling function, could not possibly have enjoyed excessive authority in the eyes of Herophilus and his disciples, the founders of neurophysiology, nor in those of Archimedes and Ctesibius, who had designed machines that could perform operations whose impossibility Aristotle had "demonstrated". We have seen an analogous supersession of Aristotle in Aristarchus' heliocentric theory, in mathematical, linguistic and logical concepts, and even in the criticism to finalism advanced by Theophrastus, the favorite disciple. It is clear that science, to whose birth Aristotle had contributed, had since developed without consideration to authority of any sort.

In the third and second centuries B.C. the main philosophical school, the Stoa, found its main opponents to be the Epicureans and Skeptics; it did not take issue with Aristotle except marginally. It is clear that the "excessive authority of Aristotle" applies only to later ages and is often backdated; therefore it may perhaps have been an effect, but certainly not a cause, of the crisis in science.

A more serious hindrance to scientific activity probably lay in the long wars between Rome and the Hellenistic states. We have already recalled the plunder of Syracuse in 212 B.C. and the killing of Archimedes.[8] In several cities the whole population were reduced to slavery.[9] The decisive phase of the wars ended in 146 B.C., the year in which Carthage and Corinth were razed to the ground. In the following year it was King Ptolemy VIII who took it upon himself to eliminate the Greek community of Alexandria.[10]

The Roman civilization of the third and second centuries B.C. was of course not at all that of Virgil and Horace. The refined culture acquired by the Roman intelligentsia was a result of continuing contact with the Hellenistic civilization, through Greeks deported as slaves and through looted books and works of art. But this took several generations.

[7] [Préaux].

[8] One always reads that this killing was a tragic mistake that greatly saddened the commanding general Marcellus, whose express orders were that Archimedes should be spared. This may be a revisionist version of events, attested first in Livy and later embellished by Plutarch; there is no confirmation in Polybius, who already in Livy's time was probably the sole trustworthy source on the siege of Syracuse. The notion that a Roman general of the third century B.C. was in awe of a scientist is probably anachronistic.

[9] Thus Anticyra in 211, Oreus in 208, Dyme in 207 B.C.

[10] See pages 11 and 69.

The circumstances attending the arrival of the first Hellenistic painting in Rome are recounted by Pliny. With the destruction of Corinth in 146 B.C., the Romans found themselves in possession of numerous art works, which they auctioned off. For a painting of Aristides the Pergamene king offered such an exorbitant bid that the Roman general Lucius Mummius carted it away with him to Rome, in the belief that it had magic powers.[11] By the turn of the century the civilizing of Rome had progressed to the point where the Senate decreed an end to human sacrifices.[12]

From the middle of the second century B.C. there were practically no Hellenistic cultural centers left. For a while Rhodes was an important holdout, but its economic role was drastically rescaled by the Romans, who eventually sacked the island in 43 B.C. The wars finally ended in 30 B.C. with the conquest of Alexandria, rounding off the submission of the Mediterranean world to Rome's sway.

It was in the first century B.C. that interest in Aristotle started to wax. Until then, he had been ignored by philosophers; this we have for example on Cicero's authority, and he was in a position to know.[13]

The Library at Alexandria survived the Roman conquest,[14] but other Hellenistic libraries ended up as war booty and ornaments for the villas of successful generals.[15] The end of libraries must have been a significant factor in the crisis of science; likewise the fashion among Roman aristocrats of acquiring cultured Greeks as slaves and using them as readers, tutors and copyists. Cicero, in a letter to his brother Quintus, laments the poor quality of Latin books, saying that in Rome one could not easily find books that weren't full of scribal errors, unless they were in Greek.[16] We must conclude that Rome had a lot more Greek-speaking copyists than native ones; this gives an idea of the scale of the deportations that had befallen the limited class of Greek intellectuals in the Hellenistic states.

[11]Pliny, *Naturalis historia*, XXXV §24. The same Mummius also carried home the bronze resonators that had enriched the acoustics of Corinth's theater (see page 229) — not through any interest in acoustics, of course, but to make a votive offering to the Moon (Vitruvius, *De architectura*, V, v §8).

[12]Pliny, *Naturalis historia*, XXX §12. In truth, ritual killings did not stop; they simply turned into gladiatorial games, losing part of their religious significance.

[13]Cicero, *Topica*, i, §3. Interest in the philosopher was reawakened by the arrival in Rome of a copy of his works in Silla's booty from the plunder of Athens in 86 B.C., and the subsequent publication of a Roman edition by the grammarian Tyrannion, who, taken to Rome as a slave of Lucullus, was later employed as a librarian and a tutor to Cicero.

[14]The conflagration of books that happened soon after Caesar's arrival in Egypt was for a while thought to have destroyed the Library, but in fact what burned down was a warehouse near the harbor; the Library lingered on until the war against Zenobia under Aurelianus (late third century A.D.), when it was destroyed for the first time. See [Canfora], pp. 68–70 and p. 195 (or pp. 76 ff. and p. 201 of the original).

[15]See, for example, [Fedeli], pp. 31 ff.

[16]Cicero, *Litterae ad Quintum fratrem*, III, v–vi.

8.2 Rome, Science and Scientific Technology

What was Rome's attitude toward science? To give an idea of the level of Roman interest in the scientific method, it may suffice to mention that, as far as is known, no one even attempted to translate Euclid's *Elements* into Latin until the sixth century A.D. The first complete translation seems to have been Adelard's: the year was around 1120 and Adelard was an Englishman (from Bath) translating from the Arabic.[17]

When Varro lists in his agricultural manual earlier treatises on the subject, he says that Theophrastus' writings are not so much for people who care to cultivate land but for those who want philosophical learning.[18] Why were the Greek scientist's books, which contained besides much else principles on which viticulture was reformed throughout the Hellenistic world,[19] labeled as philosophical texts with no practical utility? Evidently because Theophrastus talks of theories. Varro, probably the most erudite of Romans, is turned off by such things, which he does not understand. He classes their content with the only "theory" whose existence he's aware of: philosophy.

Varro represents a prescientific culture, to which science was utterly alien. By contrast, later Roman writers like Pliny or Seneca are fascinated by Hellenistic scientific works: they cannot follow the logic of the arguments, but nonetheless admire their conclusions, precisely because they appear unexpected and marvelous. These authors try to emulate their models while eliminating the logical connecting threads or replacing them with ones which, though arbitrary, are easier to visualize and so lead faster to the desired result, the wonderment of the reader. This contact with the results of a science whose methodology remains impenetrable then has the glaring effect of causing faith in common sense — a quality that earlier writers like Varro did not lack — to be jettisoned.

Pliny twists his sources to such an extent that it is difficult to recognize even known information. An example:

> Some beasts of burden suffer from eye disease when the moon waxes. But only man is freed from blindness upon the emission of fluids. After twenty years [of blindness] sight has been restored to many.... The great authorities say that the eyes are linked to the brain through

[17]A full list of medieval manuscripts containing even fragments of Euclid's work can be found in [Folkerts]. For the bigger picture, see [Stahl], an amusing book that makes patent the nonexistence of "Roman science".

[18]"... non tam idonei iis qui agrum colere volunt quam qui scholas philosophorum" (Varro, *De re rustica*, I, v §§1–2).

[19]See page 250.

> veins. I would think to the stomach too: certainly anyone who has his eyes ripped out is bound to vomit.[20]

If we did not know that cataract-removal operations had been practiced for centuries[21] and that Herophilus had described the optic and oculomotor nerves, it would be just about impossible to figure out what "emission of fluids" and what "veins" were meant in Pliny's sources.

Pliny devotes many pages of his work to the life of bees. Having read somewhere about the reason why beehives have hexagonal cells, he replaces the complex scientific arguments with the following explanation, which naturally seems simpler to him: "Every cell is six-sided because each side is the work of one leg."[22]

Pliny reports the measurement of the meridian, of 252,000 stadia, correctly attributing it to Eratosthenes and showing admiration for the result. How does he think this result was reached? He talks about the burial of a geometer, Dionysidorus, and recounts:

> It is said that they found in Dionysidorus' tomb a letter from him to the living, according to which he had traveled from the tomb to the lowermost point of the earth, and the distance was 42,000 stadia. And there were geometers who interpreted the letter to mean that it was sent from the center of earth, which is down as far as possible from the top, like the center of a ball. From which data a computation was carried out, and they announced that the length of the meridian was 252,000 stadia.[23]

Thus Pliny makes much of the calculation whereby the circumference was obtained from the radius given in the letter (using the value 3 for *ß*, to boot). About Eratosthenes' method, not a word. Pliny is only able to imagine as evidence a direct measurement. The problem is not stupidity, of course. Eratosthenes' procedure — using a scientific theory as a model for the concrete world — is absolutely incomprehensible to someone who belongs to a prescientific culture. Pliny is thus forced to replace the true intellectual voyage of Eratosthenes by the imaginary concrete voyage of Dionysidorus, though he prefaces it by saying that it is a prime example of "Greek boasting".[23a]

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[20] Pliny, *Naturalis historia*, XI §149.

[21] See page 212 and note 23 thereon.

[22] Pliny, *Naturalis historia*, XI §29. The scientific explanation was probably in Pliny's sources; see page 251.

[23] Pliny, *Naturalis historia*, II §248.

[23a] Pliny's incomprehension of science was matched by his scorn for Hellenistic figurative art. Books XXX to XVI of his *Natural history*, which form our main literary source on Greek art, stop at the beginning of the third century, when, according to Pliny, art entered into steep decadence.

Seneca says that wine struck by lightning becomes solid and stays that way for exactly three days, after which time it kills or maddens anyone who drinks it. He mentions his "research" on the reasons for such effects of lightning.[24] Regarding mirrors he offers some brief "scientific" remarks (leaving open the possibility that duplicates of the reflected objects exist within the mirror[25]), but the pièce de résistance is an account of a man's enjoyment of magnifying mirrors while copulating with partners of both sexes: a story that affords Seneca a brilliant conclusion to his discussion of mirrors, fulminating against their depraved uses.[26]

Writings such as Pliny's and Seneca's have for centuries been considered as masterpieces of ancient science and as concentrated extracts of all knowledge worthy of transmission, rendering inconsequential the loss of so many other scientific works. One famous propagator of this optimistic view was Gibbon, who wrote in his famous and influential *Decline and fall of the Roman Empire* (1776–1788):

> Yet we should gratefully remember, that the mischances of time and accident have spared the classic works to which the suffrage of antiquity had adjudged the first place of genius and glory: the teachers of ancient knowledge, who are still extant, had perused and compared the writings of their predecessors [such as Galen, Pliny, Aristotle]; nor can it fairly be presumed that any important truth, any useful discovery in art or nature, has been snatched away from the curiosity of modern ages. [27]

Here is what Seneca had to say about technological innovations:

> Also the question whether the hammer or the tongs were used first does not seem very interesting to me. Both were invented by a clever, sharp mind, not a great and lofty one. The same goes for anything that is to be sought by looking at the ground, with the body bent. . . .
>
> It is well-known that certain things date from our times, like the use of window panes that let daylight through the translucent glass, or the raised fixtures for baths and the pipes hidden in the wall that spread the heat uniformly up and down. . . . These are all inventions

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This judgement, shared by Vitruvius and Pausanias, though accepted uncritically until at least the nineteenth century, now appears remarkably prejudiced (see in particular Section 7.6); in any case it is at variance with the actual tastes of cultured Romans, whose appetite for Hellenistic art, original or in imitation, was if anything greater than for classical objects.

[24] Seneca, *Naturales quaestiones*, II, xxxi §1 + liii §1.

[25] Seneca, *Naturales quaestiones*, I, v §1.

[26] Seneca, *Naturales quaestiones*, I, xvi–xvii.

[27] Gibbon, *History of the decline and fall of the Roman Empire*, chapter LI, part VII.

> of base slaves. Wisdom has her throne higher up, and not the hands but the minds does it teach.[28]

Vitruvius is known to have been Rome's main writer on architecture. In his book on the subject he tries to give a complete picture of current technology, from building construction to the manufacture of automata, from clocks to organs to war engines. Some examples will illustrate his understanding of scientific technology. After having described water levels,[29] he says:

> Perhaps readers of Archimedes will say that a true level cannot be made with water, since he asserts that the surface of water is not even [*libratam*] but is the surface of a sphere centered at the center of the earth.[30]

Thus he is aware neither that the surface of the water can be at once horizontal and spherical, nor that the roundness of the earth can have no effect on objects the size of a water level. Later in the paragraph he "overcomes" the "difficulty" with this remark:

> It is necessary that the place where the water is poured in should have a bulge or curvature in the middle, yet the heads of the left and right water columns should be level against one another.

We should not chalk this lack of understanding up to Vitruvius' personal limitations, of course: it is an inevitable consequence of the absence of the notion of a theoretical model. And it should be said, to his credit, that unlike other authors Vitruvius was quite aware of how difficult it was to understand and translate Greek sources. He says, for example, that acoustics is treated in works "obscure and difficult, particularly for those who don't know Greek".[31]

In Vitruvius' work, the hydrostatics of Archimedes boils down to the observation that, if you immerse something into a full container, the liquid overflows in an amount equal to the volume of the object. After recounting this "discovery" as one of the scientist's most dazzling ideas, Vitruvius closes his discussion of hydrostatics with the vignette of the hollering Archimedes running home naked.[32]

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[28] Seneca, *Epistulae ad Lucilium*, xc, §13.

[29] These were essentially communicating columns of water; cf. page 100.

[30] Vitruvius, *De architectura*, VIII, v §3.

[31] Vitruvius, *De architectura*, V, iv §1. Likewise Lucretius, the Roman intellectual who came closest to understanding Hellenistic science, stressed at the beginning of his poem how hard it was to cast into Latin the "obscure discoveries of the Greeks" (*De rerum natura*, I:136–139). Of course it's even harder for us to translate the other way around, that is, to reconstruct the contents of lost sources based on writers who at best found them obscure.

[32] Vitruvius, *De architectura*, IX, preface, §§9–12. For over two thousand years Vitruvius has been the favored source on Archimedean hydrostatics, over Archimedes himself, and this although *On*

Vitruvius' regard for the role of applied science is the greatest of any Latin author. His enumerates the fields of knowledge required by a good architect (a term encompassing those who build all the sorts of objects he discusses). They are writing, drawing, geometry, arithmetic, history, philosophy, music, medicine, law and astronomy.[33] But consider the ensuing explanation of the uses of this knowledge: astronomy is regarded as necessary, in essence, for determining the four points of the compass,[34] and geometry for understanding the use of squares and levels. The computation of the total cost of a building heads the short statement of the applications of arithmetic; much more space is devoted in this introduction to "history" (understood as a collection of anecdotes that the architect should know in order to be familiar with the subjects of ornamental statues) and to law, needed for contract drafting and for the long and varied lawsuits that accompany the building process.

Vitruvius' work represents the highest level achieved by a Roman technical treatise. As for the rest, Frontinus, the author of the main Latin work on aqueducts, systematically mixes up the flow rate of a pipe with its cross section, thus ignoring, in particular, the role of the slope.[35] The high technological level of Roman aqueducts[36] seems hard to reconcile with such incompetence, but we should not forget that Frontinus was not an engineer but the bureaucrat in charge of Rome's water supply (the powerful *curator aquarum*), whereas the actual designers, builders and maintainers of aqueducts were slaves[37] who of course were not in a position to write books.

In the same way we find that, for all productive activities with technological content, Rome had to import either finished goods or workers from the East.

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floating bodies has survived. Thus, for example, [Geymonat], vol. I, p. 298, and [Boyer], p. 137 (1st ed.), p. 122–123 (2nd ed.). Discussing the anecdote of Hiero's crown, Boyer takes Vitruvius at face value, discarding as an idle complication the idea — which at least is based on Archimedean principles! — of determining density by measuring buoyancy. ("It is also possible, although less likely, that the principle [of buoyancy] aided [Archimedes] in checking [the crown]. Such fraud could easily have been detected [in the manner told by Vitruvius]". In fact, as anyone who has attempted the experiment knows, the spilled-water method is grossly inadequate for the determination and comparison of irregular volumes.) The buoyancy method is explained and attributed to Archimedes in the didactic poem *De ponderibus et mensuris*, written around 400 A.D. (*Anthologia Latina*, I.2, 32:125 – 37:185, ed. Riese).

[33] Vitruvius, *De architectura*, I, i §§3 ff.

[34] Further astronomical knowledge is deemed necessary only to builders of clocks, who must take into account the seasonal variation in day length.

[35] Frontinus, *De aquis urbis Romae*, I §§25–63.

[36] [Hodge: RAWS].

[37] This we know from Frontinus himself (*De aquis urbis Romae*, II §§96 + 118); see also [Finley: AE], p. 75.

8.3 The End of Ancient Science

The level of science in the first two centuries of our era, though low if compared with the early Hellenistic period, is still very high relative to later centuries. This can be seen from the literature. In some cases, as in Pappus' *Collection*, dating from the fourth century A.D., the quality of the results contained in a late work is still very high; however, the work is not original, but, as the name implies, a collection of earlier results, which acquired importance with the loss of the sources. Pappus' scientific proficiency can be gauged when his contribution stands next to the source, as in the case of his commentary on the *Almagest*: it then becomes clear that he was no scientist, but a compiler with scarcely any intellectual autonomy.[38]

Sextus Empiricus is a bitter critic of the scientific method, in which he does not believe. But he still represents the culture that nurtured the scientists and philosophers he polemizes against: he can read their work and use against them arguments couched in the rational language of ancient philosophy. It is not by accident that his writings are for us a major lode that can mined for information about, for instance, propositional logic and semantics.

As time went by, the climate in what had once been the great Hellenistic intellectual centers got overrun by irrationalist winds. Chemical knowledge, contaminated by magic and religious elements, gave rise to alchemy; astronomical lore dwindled and turned into no more than a lingo for the casting of horoscopes. Thus was science smothered, while the ever-present human tendency toward superstition gained new and fertile channels of expression. And never since then did pseudoscience — the combination of irrational beliefs with a language borrowed from science but devoid of scientific methodology — yield its position of supremacy, at least as far as popular attention is concerned.

Hellenistic philosophy became incomprehensible and attention turned to authors ever more distant in time: to the interest in Aristotle and Plato, which started to grow in the first century B.C., was added an interest in Pythagoras. Thanks to the return of numerology, spearheaded by the neo-Pythagoreans, even mathematics was plunged into an atmosphere of irrationalism.[39]

Hipparchus of Nicaea was one of the authors who were retroactively lumped with the Pythagorean tradition. A *Letter from Lysis to Hipparchus* was concocted in which Lysis censures Hipparchus for having spread

[38]See, for example, [Neugebauer: HAMA], vol. II, p. 968.

[39]Among the attestations of this tendency we recall the works of Nicomachus and Iamblichus, as well as the *Theologoumena arithmeticae* that have been attributed to the latter.

Pythagorean knowledge outside the school, in violation of secrecy vows.[40] The origin of this anachronism (Lysis was a Pythagorean of the fifth century B.C. and Hipparchus lived three centuries later) is surely a confusion between Hipparchus and Hippasus of Metapontus.[41] Thanks to this apocryphal letter, Hipparchus was regarded by many moderns as a representative of the Pythagorean school; for example Copernicus, thinking the *Letter from Lysis to Hipparchus* to be authentic, included a translation of it in his *De revolutionibus orbium caelestium*.[42]

Even the amalgams of ancient traditions with scientific elements ran into hard times, and eventually all remnants of ancient scientific culture was destroyed. The Serapeum, which had been the first public library, was demolished by Theophilus, patriarch of Alexandria, in 391. In 415, as we have mentioned, Hypatia was lynched. She was the last commentator of scientific books in Alexandria.

[40]The letter was published by Marcus Musurus, ᾿Επιστολαὶ διαφόρων φιλοσόφων ῥητόρων σοφιστῶν (*Letters of various philosophers, orators and wise men*), Venice, 1499. A shorter version is reported by Iamblichus (*Vita pythagorica*, xvii, §§75–78).

[41]Diogenes Laertius (*Vitae philosophorum*, VIII §42) quotes from a certain *Letter from Lysis to Hippasus*, which probably served as the model for the letter to Hipparchus. The contents seem to be much the same — Lysis scolds Hippasus for having publicly taught men from outside the school — but this letter, though probably just as apocryphal, is at least plausible, since Hippasus was according to tradition the Pythagorean who divulged the secrets of the school. Hippasus is called Hipparchus by Clement of Alexandria (*Stromata*, V, ix §58). A scholium to Plato's *Phaedo*, reported in [FV] under Philolaus, A1a (vol. I, p. 398), yokes Hipparchus and Philolaus as the only two Pythagoreans to have been saved from Cylon's persecution (thus placing Hipparchus in the role that the tradition assigns to Lysis). The confusion between Hipparchus and Hippasus was first remarked by Diels in a passage of the *Placita*; see [DG], prolegomena, p. 213.

[42]This translation was in the manuscript of the first book, but was suppressed in the 1543 edition and subsequent ones. It can be found in Koyré's French edition of the *De revolutionibus* (*Des revolutions des orbes celestes*, Paris, F. Alcan, 1934).

9
Science, Technology and Economy

9.1 Modernism and Primitivism

In the mid-century *Social & economic history of the Hellenistic world* we read:

> We may accept, for instance, the view of many modern scholars, that the brilliant development of exact science in the Hellenistic period contributed largely to the improvement of methods of production and exchange, by the invention of new technical devices in the economic spheres in question.[1]

Leaving exchange aside for the moment, we can analyze the opinion that Rostovtzeff accepts and summarizes in this passage by splitting it into four theses:

1. During the Hellenistic period there was a brilliant development of exact science.
2. Hellenistic science had nonmarginal technological applications.
3. Potentially important applications of scientific technology to production methods were devised.
4. Some of these applications were in fact used to a significant extent, leading to appreciable economic progress.

Rostovtzeff's modernist theses contrast with the views of primitivists who have denied all four points above.

Thesis 1 might seem downright obvious. But for a long time a majority of scholars maintained primitivist views even on science proper, engaging

[1][Rostovtzeff: SEHHW], vol. II, p. 1180.

in a debate on the causes of the putative scientific backwardness of Hellenistic culture in articles such as C. Préaux's "Stagnation de la pensée scientifique à l'époque hellénistique".[2] Of course, any change can be seen as indicative of stagnation or regression by those who choose to place elsewhere the apex of the process under study (say by regarding Aristotelian thought as the unsurpassed high tide of scientific thinking).

In any case, the other three theses have had a hard life and few historians of technology or economics have subscribed to them.

Among scholars who have denied any relationship between science and technology in Antiquity many have wondered how on earth ancient scientists were so oddly unable to find technological applications for their science. One answer that enjoyed popularity for some time was that they were all afflicted by "mental block".[3]

Others, less charitable, think that technology was channeled toward useless goals through a conscious and perverse decision. Here, for example, is the opinion of two intellectuals of very high caliber:

> Alexandrian technology was directed almost entirely toward games and amusements, ever more costly and elaborate, in which a constellation of rich parasites sought relief for their ennui.[4]

Certain historians of science, such as Dijksterhuis, having a profound acquaintance with Hellenistic technology, have indeed accepted thesis 2 of the preceding page, realizing that Hellenistic scientists created a good part of the technology that underlay Europe's Industrial Revolution; but these same historians then wonder why ever this technology was created only "for amusement", with no awareness of its usefulness.[5]

For several decades the contest between *modernist* or *maximalist* views on the one hand and *primitivist* or *minimalist* views on the other leant decisively toward the latter, thanks in no small part to M. Finley's influential writings.

Finley, who once boiled all the technological innovations of "the Greeks and Romans" down to a "fairly exhaustive" list of thirteen items,[6] also wrote:

[2][Préaux].
[3]The mental block theory was put forward in 1938 in [Schuhl].
[4][Enriques, de Santillana], p. 497.
[5]See Dijksterhuis' quote on Heron, on page 131, and the subsequent discussion.
[6][Finley: TIEP], first paragraph. The entries are: gear, screw, rotary and water mills, screw-press, fore-and-aft sail, glass-blowing, hollow bronze casting, concrete, dioptra, torsion catapult, water clock, water organ, automata driven by water and wind and steam. Astoundingly, he concludes: "it adds up to not very much for a great civilization over fifteen hundred years" (as if the rate of innovation had been constant throughout that period).

> [The Ptolemies] reclaimed great quantities of land, they improved and extended the irrigation system, they introduced new crops, they moved Egypt belatedly from the bronze age into the iron age . . . all in the interest of the royal revenue, and all amounting to nothing more than giving Egypt the advantages of already existing Greek technology and Greek processes. Simultaneously, the Ptolemies founded and financed the Museum at Alexandria, for two centuries the main western centre of scientific research and instruction. Great things emerged from the Museum, in military technology and in ingenious mechanical toys. But no one . . . thought to turn the energy and inventiveness of a Ctesibius to agricultural and industrial technology. The contrast with the Royal Society in England is inescapable.[7]

Finley warns us against the danger of mapping onto ancient societies conceptual and economical structures characteristic of the modern world, and denies that the ancient world ever witnessed in a significant way nonagricultural economic activities, capital investments in production or real economic growth.

He also maintained — and this is a key point in primitivist thought — that in Antiquity the city was a center of consumption but not of production. Admittedly, the argument goes, a certain number of city dwellers engaged in craftsmanship (primarily to supply their own city), but on the whole cities and towns lived at the expense of their surroundings.

Leaving that aside until Section 9.5, we have seen ample evidence that the first two theses that summarize Rostovtzeff's position (page 243) are correct, and we will devote the rest of this chapter to an analysis of the role played by scientific technology in the production of goods and in the economy. Since the primitivist views refer to all of Antiquity and since we have much more information about the imperial period, we will not be able to limit ourselves to the Hellenistic period.

9.2 Scientific and Technological Policy

One argument of primitivists is that interest in technological progress was altogether absent in Antiquity. An oft-cited anecdote in this regard is about the Emperor Vespasian, who supposedly vetoed the installation of some device capable of move heavy columns at a low cost, in order "that he might be able to feed the mob" with labor-intensive projects.[8] Another well-known story goes like this: an artisan, who had invented a method

[7][Finley: AE], p. 148.

[8]Suetonius, *De vita Caesarum*, *Vespasianus* §18.

to make glass unbreakable, sought to see Tiberius in the hope of a reward. The emperor, after ascertaining that the man had not shared his secret, put him to death, fearing that the spread of infrangible glass would debase the value of gold.[9] Of course a lack of interest in technology on the part of the Roman ruling class cannot be denied — and we have seen examples of it before[10] — but it can hardly be thought representative of the "Ancients" as a whole.

In the Hellenistic states the attitude of authorities toward science and technology was markedly different.

An Egyptian papyrus of the third century B.C. (Figure 9.1) contains a letter to the king in which one Philotas, otherwise unknown, urges the adoption of a new water-lifting machine he had invented.[11] We don't have the response, but from the progress documented in this sector (Section 4.6) it seems unlikely that the story ended badly for Philotas.

The Ptolemies founded the Mouseion (Museum) at Alexandria, the first public research institution we have notice of.[12] Strabo tells us that meals were served in a common room.[13] This shared life must have favored cultural exchanges: if we keep in mind that Herophilus may have regularly sat at table with Euclid and Ctesibius, it is easy to imagine the extent of their mutual influences.

In the Mouseion, for the use of its scholars, was the famous Library. Ptolemy II Philadelphus stocked it through purchases in every market, through requests to other states with which he had dealings, and through his famous "book toll": every ship that put in at Alexandria had to declare all books on board and donate them to the Library, receiving a copy in exchange. At the same time he fostered the publication of many new books, particularly translations of foreign works. In a few decades the collection grew to about half a million books. A separate section of the Library, called the Serapeum, was open to the public: in Callimachus' time (third century B.C.) it contained 42,800 books.

Science and technology had pride of place among the studies promoted by the Ptolemies: not only did Alexandria's scholars include such names

[9]Told by Petronius (*Satyricon*, ix) and various other authors.

[10]See pages 237–239.

[11]Edfou papyrus 8. The author of the letter asks to be assigned to the strategus Ariston, who presumably was in charge of waterworks. It may have been the same Ariston to whom Philo of Byzantium devoted his *Pneumatics* (for the various forms in which the name appears in the manuscripts see [Philo/Prager], p. 48).

[12]The Academy and the Lyceum in Athens had many features in common with it, but were private institutions.

[13]Strabo, *Geography*, XVII, i §8. We have no account of the Mouseion dating from the third or second centuries B.C., but it seems likely that the practice in question held also in the early Ptolemaic period, if only because it was adopted at the Lyceum at well.

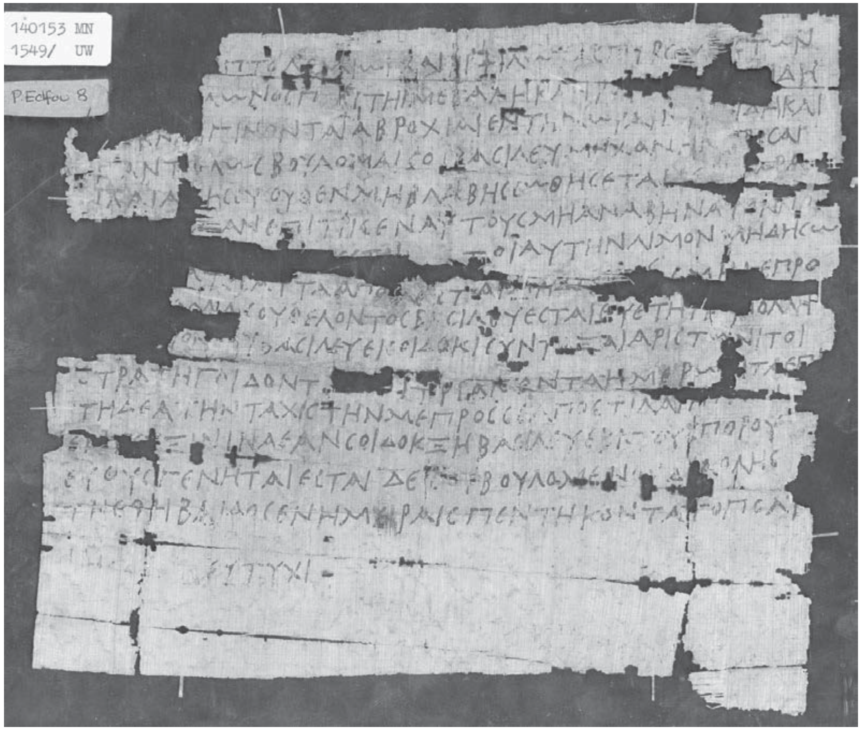


FIGURE 9.1. Papyrus Edfou 8 is a letter by the inventor of a water-lifting machine, urging the authorities to adopt his creation. (Courtesy University of Warsaw, Institute of Archaeology.)

as Euclid, Ctesibius, Eratosthenes and Apollonius of Perga, but all scientists of the time maintained contacts with the city.

How about Hellenistic states other than Egypt? One case where the little we know is enough to show the existence of an energetic scientific policy is the statelet of Pergamum. The Attalids created there a library second only to that of Alexandria. Attalus III wrote a treatise on agriculture, and he and other rulers of the same dynasty sponsored experimental studies in botany, even devoting themselves and the royal gardens to them. The Attalids were particularly eager to develop civil and military engineering. We have already mentioned the Pergamene syphon aqueduct,[14] part of an extensive water distribution complex. Biton's book on military technology was dedicated to Attalus I. The protection extended by these rulers to the greatest experts in military construction bore good fruit, to judge from

[14] See page 118.

the fortifications they built, whose innovative characteristics include the extensive use of arches and vaults as well as new types of towers designed to accommodate the latest forms of artillery.[15]

About the Seleucid state we have little information, but we know that naval technology in the third century vied with that of the Ptolemies; we also know about the development of mathematics and mathematical astronomy and we have seen that the rise in Alexandria of the Herophilean school of medicine was matched by the contemporaneous activities of Erasistratus, probably in Antioch. We have notice of libraries in other states, such as Macedonia. We have seen that the first water mill on record was built at Cabeira by the king of Pontus; his palace also boasted a zoo.[16] Strabo mentions astronomical observatories in Caria and in Libya, going back to Eudoxus.[17] And it is well-known that Syracuse's ruler, Hiero II, used naval and military technology based on Archimedes' scientific achievements.

These examples suggest that the attention paid to science and technology by the rulers of Egypt and Pergamum was shared, to a greater or lesser degree, by the whole Hellenistic world.

Cultural policy absorbed considerable economic resources. Many of the books published in the Alexandrine Library were translated into Greek by teams of bilingual experts brought in from their countries of origin;[18] the funding for such editions, given their very great number, must have been a sizable budget item even for the rich Ptolemaic dynasty. The nature of the Hellenistic rulers' interest in culture is well illustrated by the "paper wars": Ptolemy II Philadelphus, in order to stunt the growth of the Pergamene library, stopped exporting papyrus out of Egypt.[19] This can be interpreted not so much as a bibliophile's act of jealousy towards a rival but as an attempt to prevent other states from acquiring a product of perceived strategic importance. Clearly, Hellenistic rulers supported culture not out of intrinsic high-mindedness but because they saw in knowledge an essential source of power.

State-funded cultural initiatives, including book publishing, often had political ends. The translation of the Hebrew Bible into Greek known as the *Septuagint* was at once an instrument and an effect of the Ptolemies' policy of assimilation of the important Jewish community in Alexandria.

[15] [McNicoll], pp. 118–156.

[16] Strabo, *Geography*, XII, iii §30.

[17] Strabo, *Geography*, II, v §14; XVII, i §30.

[18] According to Pliny (*Naturalis historia*, XXX §4), just the Zoroastrian corpus involved the translation into Greek of two million verses, a task requiring extensive organization and coordination.

[19] As is well-known, Pergamum's reaction was not to stop making books but to develop the writing medium that we now know as parchment (from the Greek *pergamene*).

In the first half of the third century B.C. the Egyptian priest Manetho compiled an *Aegyptiaca* and the Babylonian priest Berossus wrote a *Babylonian history* dedicated to Antiochus I Soter. The goal of these Greek-language works was to give the ruling class in those states a better understanding of the local culture. Hellenistic kings realized that knowing one's subjects is a way of strengthening one's power.

Interest in science also seems to have had economic motivations. This is most obvious for the small Attalid kingdom, whose rulers, as we just saw, sponsored research in areas that promised an immediate practical return, such as agriculture and engineering. That this interest in agriculture was not a hobby but aimed at acquiring knowledge useful in improve crop yields may be seen from the fact that most of the authors of treatises on agriculture mentioned by Varro[20] are connected in some way or another with Pergamum.[21]

9.3 Economic Growth and Innovation in Agriculture

Whereas opinions differ on whether there were economically important nonagricultural activities during certain periods of Antiquity, there is no doubt that agriculture did remain the mainstay of the economy throughout the ancient world. In investigating whether scientific development had economic consequences, therefore, we must start by asking if science was applied to agriculture.

That theoretical knowledge was applied to agriculture is indicated by the flourishing of treatises on the subject. Varro writes that in Greek there were fifty such works, and he names forty-nine of them.[22] Not a single one has been preserved, nor do we have reliable quotes from them. It is indisputable that they were the ultimate origin of all Roman knowledge about the subject, but there is very little hope of reconstructing any significant fraction of their content based on the writings of learned Romans. Indeed, it seems that the chief source used by Varro and other Roman writers on agriculture was Diophanes of Bithynia, who summarized the translation made by Cassius Dionysius of the large handbook written by the Carthaginian Mago, itself a compilation of various Hellenistic treatises on agronomy.[23]

Many plants indigenous to other lands were acclimated for the first time in the Hellenistic kingdoms. Preexisting plants were improved thanks to

[20] Varro, *De re rustica*, I, i §§8 ff.
[21] [Rostovtzeff: Pergamum], p. 694.
[22] Varro, *De re rustica*, I, i §§8 ff.
[23] [Rostovtzeff: SEHHW], vol. II, p. 1183.

seeds imported from different countries. Progress in husbandry was likewise impressive: animals from elsewhere were acclimated, breeds were improved through crosses, and wild animals such as hares, dormice and boars began being raised,[24] as did fish species. Innovations of this type had of course been happening since neolithic times, but on a wholly different time scale. There are clues that in early Ptolemaic Egypt the acclimation and hybridization of species were carried out under the supervision of Mouseion scholars: for example, in a passage by Athenaeus about the zoo founded by Philadelphus in the royal district, the subject is brought up in connection with new bird breeds obtained there,[25] and the close link between the study of botany and the development of cultivation techniques is clear in the botanical works of Theophrastus.

We also know that viticulture was reformed following directions laid down by Theophrastus.[26] In this regard R. J. Forbes writes:

> Theophrastus believes that plants derive their vital spirit (*pneuma*) from the soil and draw it up through the pith, together with water. From this theory he deduces the correct way of striking cuttings from good vines, the conditions under which they should be planted, the porosity and moisture of the soil, and the care of the cuttings. Grafting he rejects, but he discusses the uses and methods of pruning. Despite the defects in his knowledge of plant physiology, his advice is generally good and often so much in accordance with modern views that we may reflect how little the practical experience of vine-growers has advanced during the last 2200 years. It shows that Greek genius raised viticulture to a very high level of achievement.[27]

If, as is likely, Theophrastus judged the value of a theory by its usefulness and simplicity, he would be amazed to read that his theories are considered "defective" today by those who still follow his advice in spite of having acquired vastly more complex knowledge.

Some of the advances lay simply in the adoption and spread throughout the Hellenistic world of the best techniques practiced in the various parts of the ancient empires. For example, the seeder, already used in Mesopotamia, was introduced to Egypt, while egg incubators,[28] long traditional

[24] Varro says that the first Romans who tried breeding these species learned about the practice in the books by Mago and Cassius Dionysius (*De re rustica*, III, ii §§13–14). In the same context he mentions the introduction of fish-farming.
[25] Athenaeus mentions hybrids of pheasants with guinea fowl (*Deipnosophistae*, XIV, 654b–c).
[26] Theophrastus, *De causis plantarum*, III, xi §1 – xvi §4.
[27] [Forbes: FD], pp. 131-132.
[28] Artificial incubation in a temperature-controlled environment is mentioned by Pliny (*Naturalis historia*, X §154).

in Egypt, became known to the Greeks.[29] (In the early sixteenth century Thomas More wrote admiringly that in Utopia "vast numbers of eggs are laid in a gentle and equal heat, in order to be hatched",[30] but incubators would remain a mere literary memory still long after that.)

In Hellenistic times, as in classical Greece, honey and wax had multifarious and extensive uses, and apiculture enjoyed considerable economic importance. The growth of knowledge in this field (and, for that matter, in entomology in general[31]), is demonstrated by the existence of treatises on apiculture. Pliny mentions two: one by Aristomachus of Soli, who "did nothing else" in his whole life but study bees, and one by Philiscus of Thasus.[32] We do not have these works, but a lovely page of Pappus gives us a taste of the scientific level of these studies and an example of the interaction between exact and empirical sciences. Pappus, in the introduction to the book where he deals with minima, passes on the observation that bees, in building hexagonal hives, have solved an optimization problem, because among regular polygons that tile the plane the hexagon is the one with the least perimeter for a given area, and therefore the one that allows the use of the least amount of wax to hold a given amount of honey.[33] This remark was made again many times in the modern age, becoming a cliché of sorts.

About advances in agricultural machinery we know little. Rostovtzeff, after mentioning the use of technological innovations to extend the area under cultivation, says:

> The question arises how far in the same period Egyptian agriculture benefited by similar technical inventions. Our information on this point is meagre. There is hardly any literary evidence; the papyri occasionally mention agricultural implements, but the references are not easy to interpret; and the agricultural implements themselves, though found in large numbers, have never, as I previously remarked, been collected, described, and analysed from the technical and historical standpoint.[34]

One big step forward in agriculture, reflecting probable breakthroughs in ore extraction and metallurgy, was the diffusion of iron tools and ma-

[29] Diodorus Siculus, *Bibliotheca historica*, I, lxxiv §§4–5; Aristotle, *Historia animalium*, VI, 559b:1–5.
[30] Thomas More, *Utopia*, book II, at 3%.
[31] Incidentally, Plutarch offers us a glimpse of the introduction and subsequent abandonment of the experimental method in entomology when he berates certain scientists (whom he nonetheless uses as sources!) for having systematically cut up anthills to study their internal structure (Plutarch, *De sollertia animalium*, 968A–B).
[32] Pliny, *Naturalis historia*, XI §19.
[33] Pappus, *Collectio*, V, 304–306.
[34] [Rostovtzeff: SEHHW], vol. I, p. 362.

chines with iron parts, which were at first extremely rare both in Egypt and in Greece. There was automation too. Pliny mentions animal-powered automatic harvesters with teeth and blades.[35] Similar implements were still in use in Gaul in the fourth century A.D. and were described in greater detail by Palladius in his work on agriculture.[36]

The Ptolemies put to use much virgin land, draining marshes and irrigating the edge of the desert; in both cases essential use was made of water-lifting machinery, and likewise in regulating the floods of the Nile (that is, watering fertile land in years when the flood was weak and draining it in the opposite case).[37] The source of lifting power was sometimes the stream itself, thanks to a wheel that combined paddles and water-holding compartments.[38]

All of this shows that the Hellenistic scientific revolution made possible very significant changes in agriculture, but we do not have enough direct information to figure out to what extent the new possibilities were effectively put to use. Still, what demographic data we have speak eloquently: whereas in the late Pharaonic period the population of Egypt is estimated to have been about three million, at the beginning of our era it was eight million: half a million in Alexandria[39] and seven and a half in the rest of the country.[40] An interesting comparison can be drawn with an estimate of Egypt's production capacity, published in 1836, to the effect that eight million was the maximum population that could be fed if all land capable of cultivation were sown.[41] The same source put the actual population at the time at less than half that number. By 1882, after economic reforms and a half-century of growth, it had reached 6.8 million.[42] One might add that Hellenistic and Roman Egypt was a major exporter of agricultural produce and grain in particular.

We note that while it is easy to document the use of new agricultural know-how in the Hellenistic period but difficult to quantify its impact, most scholars who recognize the existence of growth in the agricultural

[35] Pliny, *Naturalis historia*, XVIII §296.
[36] Palladius, *De re rustica*, VII §§5–7. The machine was very simple, but still beyond the ken of medieval and early modern Europe. In the 1830s an English translation of Palladius came into the hands of an Australian farmer, who derived "Ridley's stripper" from it; see for example [Thompson], pp. 80–81.
[37] See, for example, [Oleson: WL], p. 247.
[38] Vitruvius, *De architectura*, X, v §1.
[39] This number is based primarily on a statement of Diodorus Siculus (*Bibliotheca historica*, XVII, lii §6) that the city had about 300,000 free residents. For discussion see [Fraser], vol. II, pp. 171–172, note 358.
[40] Flavius Josephus, *Bellum iudaicum*, II, xvi, 385. His number is compiled from tax rolls, so we may presume it is, if anything, an underestimate. Josephus wrote around 75 A.D.
[41] [Lane], last three pages of introduction.
[42] [Walek-Czernecki]; cited in [Bowman], p. 17.

economy in Antiquity place that growth in the early imperial period, on the basis of a variety of evidence.[43] We will return to this point.

9.4 Nonagricultural Technology and Production

Many scholars continue to maintain that technical innovations were irrelevant to economic output in the classical world, but admit an increasing number of "exceptions".

In several places in northern Africa there is archeological evidence of big plants for the production of olive oil: their study has shown that the output was probably meant for export and that production was certainly based on up-to-date technology.[44] This then is one of the "exceptions". But keep in mind that screws were a technological novelty of the Hellenistic period and that their introduction was connected with science, as attested by the fact that one of our few (and reticent) sources about presses, in particular olive-presses, is Heron of Alexandria.[45]

In Hellenistic times new types of glass were produced and the new technique of glass blowing was introduced, which in Syria gave rise to blowing in molds. Today there remain many objects with the trademark of Ennius, the most famous imperial-age producer of blown-glass objects. His business was headquartered in Syria and had a branch at Rome. The magnitude of his and similar firms was such that some have suggested that glass blowing was the only exception to the economic negligibility of technological innovations in Antiquity.[46]

Returning to water lifting, we now know from archeological evidence[47] that an advanced technology, developed in close connection with science, was employed on a vast scale and had significant economic consequences. Water lifting was essential not only in agriculture, but also in draining mines and ships. The most powerful drainage installation known from the imperial age is that of the gold mines at Rio Tinto, in Andalusia, Spain. At least eight pairs of water wheels, placed at different levels and working in series, allowed water to be raised by about thirty meters (Figure 9.2).

Among nonagricultural economic activities, mining was particularly significant and had been since classical Greece. The lead and silver mines of Laurium, near Athens, had over ten thousand workers. Metal yields

[43]See [Millett], for example. But he warns: "I am only too aware that my discussion ignores the possibilities for growth through the Hellenistic world; in particular, Ptolemaic (and later Roman) Egypt, where materials for such a study may come closest to existing" (p. 41, note 46).

[44][Mattingly: ORW]; [Mattingly: OPRA].

[45]Heron, *Mechanica*, III §§13–20.

[46]See [Manning], for example.

[47]Much interpretive work has been done by J. P. Oleson; see, in particular, [Oleson: GRWL].

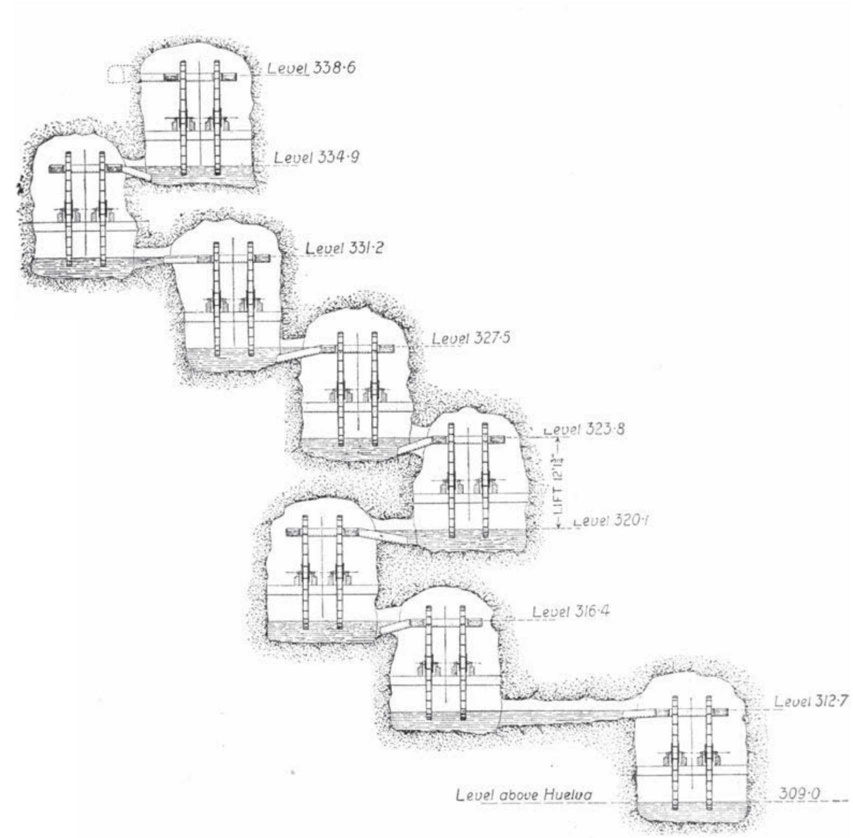


FIGURE 9.2. General arrangement of Roman water wheels uncovered during the years 1919 to 1921 at Rio Tinto mine. From [Palmer], p. 303.

benefited from progress both in metal refining and in tunnel digging. The remarks in Pliny and Livy about the use of "vinegar" to eat through rock have generally been taken skeptically, but to pass judgement one would need to know what "vinegar" they meant.[48]

To what extent did metal production change by virtue of progress in excavation, drainage, extraction and refining techniques? A partial response is provided by the fact that in early Hellenistic times iron, which until then had been scarce and was saved for weapons and knives, came into common use for tools and machinery of every kind.

[48]See footnote 102 on page 170, and Pliny, *Naturalis historia*, XXIII §57; XXXIII §71.

About the question of mining output we do have some potentially quantitative evidence of a global nature. Investigations of the northern polar ice cap have shown a peak in lead and copper pollution in the atmosphere corresponding to the early imperial age.[49]

From the little information we have about metallurgical procedures we can glean certain technological innovations in the area of metal refining. For instance, Polybius tells us about new blacksmith's bellows, perhaps fed by the Ctesibian pump.[50]

Among matter-transformation industries, some of the most important in the Hellenistic period were the manufacture of medicines, unguents, perfumes and dyes. These industries made essential use of new medical, botanical and chemical knowledge in their processing of plant, animal and mineral substances. (Some of the medicines mentioned by Dioscorides are of mineral origin, such as verdigris and the copper mineral called *chalkitis*.)

One hint about both the availability of new products and the increasing importance of materials of mineral origin is in a passage by Plutarch that mentions nonflammable fabrics made from asbestos fibers. Admittedly this was not an economically important use, but neither was it an isolated curiosity, since the fibers were mined until the vein was exhausted.[51]

The most important of the ancient technologies long ignored by historians is probably the utilization of water power. Thus Marc Bloch, in spite of having been the first to recognize the Greek origin of water mills, wrote:

> For one should make no mistake: the water mill, though an ancient invention, is medieval from the point of view of its effective diffusion.[52]

His opinion was based primarily on the absence of clear literary references to water mills in the first three centuries A.D. and was long shared by all historians, who felt that the intensive utilization of water power was a major mark of technological progress in the Middle Ages.

Of the ancient sites where the use of water power is archeologically attested, the most important is the flour plant at Barbegal, near Arles, where imposing remains have been found: grain was milled by 32 stones turned by sixteen vertical water wheels arranged in pairs on eight levels. Though many researchers think that the flour was exported, R. Sellin believes that the plant was designed to serve local needs.[53] The plant was originally

[49] [Hong et al.: Lead]; [Hong et al.: Copper].
[50] Polybius, *Historiae*, XXI, xxviii §15.
[51] Plutarch, *De defectu oraculorum*, 434A–B.
[52] [Bloch: Moulin], p. 545.
[53] [Sellin]. He reaches this conclusion by supposing, inter alia, that the wheels ran idle half the time and that the population of Arles consumed 350 grams per capita per day of mill-produced

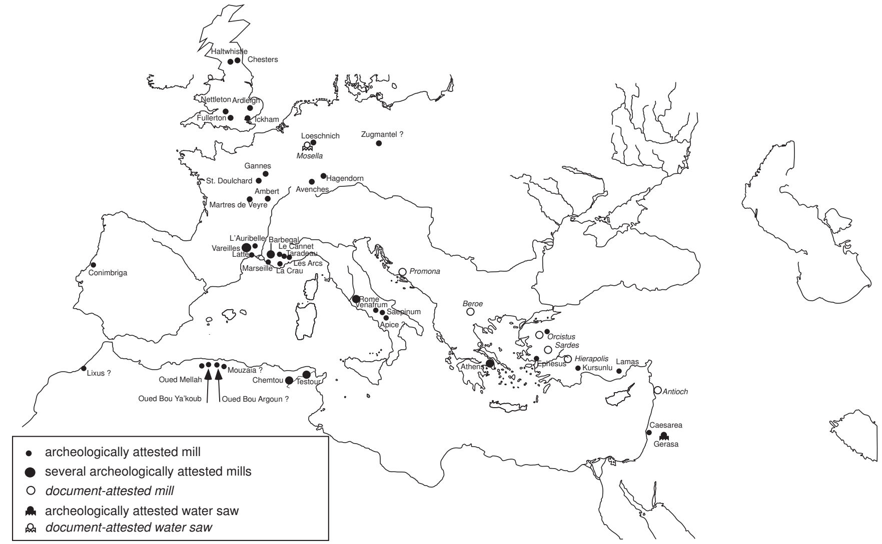


FIGURE 9.3. Sites where Roman-era water mills and saws have been attested. Map by Jean-Pierre Brun. Used with permission.

thought to date from late Antiquity, but recent studies have proved that present remains go back to the early second century A.D.[54] Archeological discoveries of smaller mills going back to the early imperial age continue to multiply (Figure 9.3). They have been made throughout the western extension of the Empire; in the eastern part, we know from epigraphical evidence that in the Phrygian city of Hierapolis around 200 A.D. there were enough water mills to justify the foundation of a millers' league. It is now accepted that, in spite of the almost complete silence of our written sources, water mills were widespread[55] — which certainly belies the traditional contention that the "Ancients" were not interested in saving labor.[56]

Cereal grinding and water lifting[57] were not the only uses of water power in Antiquity. In the poem *Mosella*, of about 370 A.D., Ausonius

flour. He also estimates that the power of each wheel did not exceed that of a 250 cc motorcycle. This mordant observation highlights how technologically uncivilized we ourselves are, in destroying the environment by wasting on a few motorcycles enough energy to produce food for a whole town. But after all is said and done, our current system, based on the exploitation of nonrenewable energy sources, will last but a moment on a historical scale.
[54] [Leveau].
[55] [Wikander: WM], in particular p. 398 for the water millers' league (συντεχνία τῶν ὑδραλετῶν).
[56] [Wikander: EWPTS].
[57] See page 252.

records the use of water-powered saws for marble.[58] Michael Lewis has argued on the basis of indirect evidence that the Ancients built pounding pestles and other reciprocating machines moved by water.[59]

9.5 The Role of the City in the Ancient World

Finley's model of the consumer city is true for Rome, with a vengeance. Documentation abounds. Rome was indeed, economically speaking, "the complete parasite-city".[60] It was the destination point for an enormous flow of riches from the whole Empire: taxes from the provinces, war booty, rents from provincial land and mines owned by the Emperor and others. Part of this bounty made its way to the bulk of the population through public and private channels, helping ensure its subsistence. Caesar started distributions of free grain; from Augustus onwards the plebs periodically received other foodstuffs also, as well as clothing, money and other benefits, such as free admission to public shows and baths. At the same time, every rich man had his own slaves and supported his *clientes* in various ways. Other income distribution channels are easily imaginable.

The case of Rome is unique, but Finley contends that the uniqueness was only in scale, and that (almost) all ancient cities were consumption centers[61] that lived essentially on local agricultural resources, taxes, land rents, commerce and tourism, the contribution of manufacturing being negligible.[62]

This thesis has been supported by several analytical studies,[63] but more recently it has been contested as well, particularly in view of new and reevaluated archeological evidence from the early imperial period.[64] For

[58] Ausonius, *Mosella*, lines 362–364. In the 1960s the theory that the poem was an apocryphal tenth-century invention was put forth, with no serious support, and it was not until the 1980s, when doubts started to be cast on the primitivist case, that it was definitively abandoned.

[59] See [Lewis: MH], in particular p. 114 for the marble saw, pp. 93–105 for pestles and trip-hammers, and p. 8 for an illustration of sixteenth century water-powered ore-crushing stamps. For an overview of the industrial uses of water power in Antiquity, see [Wikander: IAWP].

[60] The expression is from [Finley: AE], p. 130.

[61] [Finley: AE], p. 130 ("only in scale"), pp. 122–149 (economic role of town and city). Finley admits the existence of "exceptional cities" (ibid, p. 194) as a way of shielding his thesis against disproof by any given piece of contrary evidence. This is analogous to the already observed technique of maintaining the nonexistence of the experimental method in Antiquity in the face of documented experiments, on the grounds that these were performed only exceptionally.

[62] [Finley: AE], p. 139. But he includes among income sources such "local resources" as silver and other metals, premium wines and olive groves (all of which of course depend on transformation processes). Tourism he considers one of the main sources of income for Athens (p. 134).

[63] For example, [Jongman] for the city of Pompeii and [Whittaker] for the administrative centers of the northern provinces of the Empire.

[64] See [Mattingly, Salmon].

example, it appears that the town of Thamugadi, founded in 100 A.D. and now called Timgad (Algeria), can hardly fail to be thought of a production center, twenty-two small shops for fulling (and perhaps dying) wool having been found there.[65] This may have been an exceptional case, but it cannot be ruled out that it was instead a typical one, particularly in northern Africa. A statistical study of surface-level archeological evidence in Leptiminus (another Punic city, taken by Rome in 111 B.C., now Lamta in Tunisia) has shown that this was the site of a lively production industry — of metal, amphors, fish- and olive-derived goods — very likely geared toward exports. The authors of the study remark that over fifty sites along the Tunisian coast have been found to be associated with such manufacture, some operating at a very considerable scale, and conclude that, rather than being exceptional, Leptiminus' situation reflects a need to abandon Finley's single model in favor of a model allowing a variety of economic roles for ancient cities, such as production and commercial centers.[66]

If not all ancient cities were parasitical "consumer cities", we should ask what category the great Hellenistic centers belonged to.

Alexandria, founded by Alexander in 331 B.C., soon became the greatest city in the known world. Not long before it was conquered by Rome, in the mid-first century B.C., it was home to about half a million people.[67] Diodorus Siculus calls it

> . . . the first city of the civilized world . . . certainly far ahead of all the rest in elegance and extent and riches and luxury. The number of its inhabitants surpasses that of other cities.[68]

It is hard to see how such a city could have lived on local produce and tourism. The possibility that Alexandria, like Rome, was fed by the exploitation of subject territories can be excluded, since the Roman conquest of Egypt did not plunge its economy into a crisis. Dio Chrysostom, in an oration probably dating from the reign of Vespasian, says to the Alexandrians:

> Not only do you have a monopoly in sea traffic over the whole Mediterranean . . . but both the Red Sea and the Indian Ocean are in your reach. . . . Alexandria is at the crossroads of the world . . . as if the world were an immense market in the service of a single city.[69]

[65] [Wilson: Timgad].

[66] [Mattingly et al.].

[67] See note 39 on page 252.

[68] Diodorus Siculus, *Bibliotheca historica*, XVII, lii §5, based on Welles' translation.

[69] Dio Chrysostom, *Orationes*, xxxii, 36.

Here is how Alexandria is described as late as the fourth century A.D. in a Latin-language letter apocryphally attributed to Hadrian:

> Its inhabitants are very factious, arrogant and violent; the city is rich and prosperous, and no-one is idle. One blows glass, another makes papyrus into paper, yet another weaves flax: everyone seems to practice a craft. The gouty, the mutilated, the blind, all do something. Not even cripples live in idleness. Their only god is money, worshiped by Christians and Jews and all others. If only this city had better moral principles...[70]

This anonymous Roman author finds industriousness morally reproachable and a sign of a blasphemous yearning for wealth. He seems very irritated at the prosperity of Alexandria.

The city does not seem to have been significantly depopulated even in late Antiquity. At the time of the Arab conquest there were still four hundred theaters and other entertainment venues, as well as four thousand public baths.[71]

There seem to be only two possibilities, which are not mutually exclusive: Alexandria was either a trade center or a production center. We don't have enough information for a firm answer,[72] but the small amount of data we do have unanimously point to the conclusion that Alexandria's vast sea traffic consisted above all of imports of raw materials and exports of finished products. We know, in particular, that Alexandria imported gold, iron, tin, silver, copper and many pharmacological and perfumery ingredients, while it exported medicines, unguents, perfumes, dyes, fabrics, glassware, papyrus paper and metalware.[73]

The special relationship attested during the early Ptolemaic period between Alexandria and Rhodes, the main Hellenistic mercantile center, was apparently based on the complementarity of the two economies.[74]

Fortuitously, we have from Cicero one piece of information regarding commerce between Alexandria and Italy in the first century B.C. He talks of "many ships" that had put in at Puteoli from Alexandria, chock-full of

[70] *Historia Augusta: Firmus Saturninus Proculus et Bonosus*, VIII §§5–7 (in, e.g., Loeb Classical Library, vol. 263).

[71] These figures are in a letter sent by the Emir Amr ibn al-As, the conqueror of Alexandria, to the Caliph Omar. The letter is preserved in the *Annals* of Eutychius, a tenth-century patriarch of Alexandria; the passage in question is given in [Canfora], p. 83 (p. 92 in the original).

[72] Fraser leans toward the prevalence of trade, but regards the question as open. However he takes it as certain that Alexandria's industry was much more developed than can be directly documented today; see [Fraser], vol. I, p. 143.

[73] Much of the available information on Alexandrine trade is collected in [Fraser], vol. I, pp. 132–188.

[74] See [Fraser], vol. I, pp. 162–169; the complementarity, in particular, is stressed on p. 164.

paper, linen and glassware — all goods known to be produced in Alexandria.[75]

Strabo says that in his time Alexandria exported much more than it imported, as anyone could notice by comparing how deep-laden the ships were upon arrival and departure.[76]

One letter of 258 B.C. is particularly interesting. The author, a certain Demetrius, perhaps a functionary of the mint, writes to Apollonius, dioecete (chief financial officer) of the king Ptolemy II Philadelphus, about some difficulty (which we can no longer make out):

> The foreigners who arrive here by sea — merchants, adventurers, and so on — bring with them good coin of their countries.... They get furious because we don't supply the banks with coins ... so they can't send their agents to the countryside to buy merchandise; they say their gold is resting idle, causing heavy losses.[77]

The presence in Alexandria, in the time of Philadelphus, of foreign merchants who went out into the countryside to buy seems to indicate that at least at that time Egypt was more of a producer than a trading country.

Josephus says that in his time (first century A.D.) Rome was fed by two thirds of the crops sent as tribute from Egypt and from the province of Africa.[78] Now, when Rome's imperial role ran into a crisis, the severance of political ties with Egypt caused urban life to crumble in Rome, but not in Alexandria: the two cities had evidently followed divergent economic paths. One can hardly help seeing a link between Alexandria's economically active role and its leading part in the development of science and technology.

9.6 The Nature of the Ancient Economy

Modernist and primitivist theses are antagonistic also in what concerns the organization of the economy and of finances in the ancient world. In his *Social & economic history of the Hellenistic world*, Rostovtzeff discussed the subject using terms such as bourgeoisie, industry, capitalism, industrialization of agriculture, capital investments and credit transfer systems.

Finley, on the other hand, insists on the absence in Antiquity of many elements, including conceptual ones, that are essential to today's capitalist system. He argues for the primitive character of the ancient economy and

[75] Cicero, *Pro Rabirio Postumo*, xiv, §§39–40.

[76] Strabo, *Geography*, XVII, i §7.

[77] Papyrus Cairo Zenon 59021.

[78] Flavius Josephus, *Bellum iudaicum*, II, xvi, 383.

its lack of "economic rationality", stressing, for example, the complete absence of the notion of amortization,[79] the nonexistence of long-term loans for investment purposes[80] and even the impossibility of translating the word *broker* into Greek or Latin.[81] He writes:

> The absence of credit-creating instruments and institutions remains as an unshaken foundation of the ancient economy.[82]

While Finley's observations about the absence of stock markets and credit formation institutions are accurate and have played an important role in suppressing anachronisms, a growing number of scholars now casts doubt on the idea that one can characterize an economic system or a civilization by listing the ways in which it falls short of Wall Street. It is certainly true that in Antiquity the economic and especially the financial sphere had not acquired the commanding position they hold today. People saw money as one instrument for acquiring tangible goods and power — things they had fought over for thousand of years even in the absence of currency — and had not yet developed the idea (now apparently widely viewed as an inevitable result of rational thought) that goods and power are but instruments to make money.

Our knowledge of the economic and legal organization of Hellenistic states is sadly incomplete. For Ptolemaic Egypt it is relatively less so, and we know that in that country ancient institutions going back to the Pharaohs were preserved and merged into a new system, creating a structure both composite and organic. This is well illustrated by the legal status of agricultural land. Much of it was "royal land", cultivated on behalf of the king. Other areas were "sacred land": their utilization was controlled by the priestly class and provided for the maintenance of the temples. In addition to these two possibilities, inherited from the Pharaohs' time, there were two more: private land ownership, which may have been introduced during the Persian domination, and finally "revocable grants" of land to civil servants, which certainly were a novelty of the Ptolemies.

The production of vegetable oil, linen fabrics, salt and beer were royal monopolies. The state had a record of all the looms in private homes of weavers[83] and after their production quotas were met all looms and oil presses were kept under seal to prevent illegal use.[84] The production of

[79][Finley: AE], p. 116.
[80][Finley: AE], p. 117.
[81][Finley: AE], p. 118.
[82][Finley: AE], p. 198.
[83]See [Rostovtzeff: PtE], p. 136.
[84]See [Rostovtzeff: SEHHW], vol. I, p. 303 (looms) and p. 306 (presses).

paper, too, although not a monopoly, was concentrated to a significant extent in royal factories.

In Pharaonic times a good part of the economic activity was carried out in temples. An Egyptian temple was not just a place of worship; its interior contained factories and workshops of various kinds — producing cloth, oil and beer, for example — and small libraries; some temples also functioned as places of healing. Thus temples were places where all types of knowledge — religious, technical and medical — were concentrated under priestly control. The Ptolemies respected the age-old prerogatives of the temples. For example, when a monopoly in the manufacture of oil was introduced, temples were allowed to keep their presses, though only to make oil for their own use.[85]

Slavery in the Ptolemaic kingdom did not share at all the importance and features that it had in Classical Greece and in Rome.[86] Slaves were not heavily used either in agriculture or in mining and quarrying.[87] The workers who manned the government presses, too, were free. Nor were slaves used in the war fleet.[88] In sum, we do not know of a single economic activity that depended on slavery, an institution that was primarily domestic in Hellenistic Egypt. We know a fair amount about conditions in the household of Apollonius, the dioecete of Ptolemy II we have met before,[89] thanks to a trove of papyri associated with Apollonius' chief aide, Zenon. These documents show that the dioecete's private household staff included free men and women as well as slaves, both receiving compensation in cash. Some papyri mention complaints from the staff about payments in arrears.[90]

Privately owned factories, with wage-earning employees, are attested in Egypt, in many cases down to the Roman period. Also documented are wage negotiations between employers and workers[91] and strikes for salary raises.[92]

In Ptolemaic Egypt we encounter for the first time a central State Bank, whose business is to manage the state's finances and to invest public re-

[85]We have good information about the oil monopoly because the relevant law was found in its entirety (*Pap. Revenue Laws*, lines 38–56, ed. Grenfell).
[86]The extent and economic importance of slavery in Ptolemaic Egypt have long been a subject of debate. Much information can be found in [Bieżuńska-Małowist].
[87]See, for example, [Bieżuńska-Małowist], pp. 81, 111.
[88][Bieżuńska-Małowist], pp. 81–82; [Casson: SS], pp. 322–328.
[89]See page 260.
[90]See, for example, P. Cairo Zenon 59028, in which the cithar player Satyra bemoans the delays in receiving her earnings.
[91]Apprenticeship contracts specifying wages appear for example in Papyrus Oxyrhynchus 275 (A.D. 66), P. Oxy. 724 (A.D. 155), P. Oxy. 725 (A.D. 183), P. Oxy. 2977 (A.D. 239). Wage negotiations in the imperial age are documented in P. Oxy. 1668, for example.
[92]Papyrus Bremen 63; [Aubert], p. 107; [Minnen], pp. 62–63.

sources, even making loans to private individuals.[93] Bank deposits are very widespread, even among artisans and retailers. Among transactions we see for the first time papers transferring sums from one bank account to another.

To summarize, it seems that the use of terms such as bourgeoisie and capitalism in a situation like that of Hellenistic Egypt is at once reasonable — given the presence of elements such as privately owned factories with salaried employees — and dangerous — given the strong role of the state, the survival of Pharaoh-era structures, and the lack of certain features we tend to associate with capitalism, such as a capital market of the type that arose in modern Europe.

It should be stressed, too, that many of the features we have discussed lasted until Roman times in Egypt, but were never present in the Western provinces of the Roman Empire, where slavery, for instance, played a much greater economic role. Geography here seems to provide a better basis for a breakdown than time periods, and not only in terms of the economy but also regarding other institutions. For example, the possibility of dissolving a marriage through the wife's initiative, long inconceivable to a Roman, is recognized in Egyptian matrimonial contracts of the Hellenistic period.[94]

9.7 Ancient Science and Production

The considerations made so far allow certain conclusions and raise important problems.

I think there can be no doubt about the importance that ancient science and Hellenistic technology could *potentially* have had for production processes, but in assessing the extent of the applications actually deployed in Antiquity we must avoid certain traps that lurk in making comparisons, whether explicit or implicit, with our own age.

In Chaplin's movie *Modern Times*, the tokens of modernity are screws, gears, transmission belts, valves, steam engines, automata: a smorgasbord of inventions from ancient Alexandria. How can one say that these innovations were useless back then? Yet, though so much of the technology that made up the movie's factory goes back to the third century B.C., it is clear that in that century there were no factories like Chaplin's.

The Western world has experienced since the late seventeenth century a unique phenomenon in human history, characterized by an exponential

[93] [Rostovtzeff: SEHHW], vol. II, pp. 1283–1288.

[94] Some marriage, divorce and repudiation documents from the Hellenistic, imperial and late ancient periods are collected in [SP], vol. I, pp. 2–30.

increase in several technological and economic indicators, and the source of achievements and problems without parallel. (This growth certainly cannot continue for long at the same exponential pace.) The primitivists are right in warning us against the pitfalls of "modernizing" Antiquity by reading into it the accoutrements of modern life. There was certainly no Hellenistic Industrial Revolution, there were no stock brokers in Alexandria and the Mouseion was not the Royal Society. On the other hand, using today's Western world as a sort of universal standard, lumping all ages other than ours into an undifferentiated "underdeveloped" category, can be highly misleading.[95] If we think that biology has predetermined a unique possible path for the human race, culminating in the "economic rationalism" of today, it may be possible to define other civilizations by how far they are from ours; but human history is much more complex than that.

The application of scientific technology to production does not necessarily mark the beginning of the process in which we find ourselves now, where technology itself grows exponentially. Having made this clear, I think it must be agreed that scientific technology did have in Antiquity important applications to production. The Mouseion's economic role was not comparable to that of the Royal Society, but that does not mean this role was insignificant, nor does it imply lack of wisdom or foresight on the part of ancient scientists. The process of exponential development starting with what is usually called the Industrial Revolution was triggered by a plethora of economic, social, political, cultural and demographic factors that we have not yet understood in depth. It is more sensible to try to figure out what happened in Europe in the late seventeenth century than to ask why the same thing did not happen two thousand years earlier. Hellenistic scientific development was violently arrested by the Roman conquest. We may wish to speculate on what might have happened had this interruption not taken place. Nothing authorizes us to conclude that things would have gone the way they did in seventeenth century Europe; we do know, however, that the recovery of ancient knowledge on science and technology played a major role in the modern scientific take-off (more on this in Chapter 11).

The data examined so far limn two features that may appear contradictory. Almost all of the advanced technology used in Antiquity appeared in Hellenistic times and seems more or less directly connected with science. Even for technological elements attested only in Roman times, such as ball bearings (Figure 9.4), one may suspect a Hellenistic origin hidden by a scarcity of archeological evidence. Moreover the interest of Hellenistic

[95] This is stressed in [Greene: TIEP].

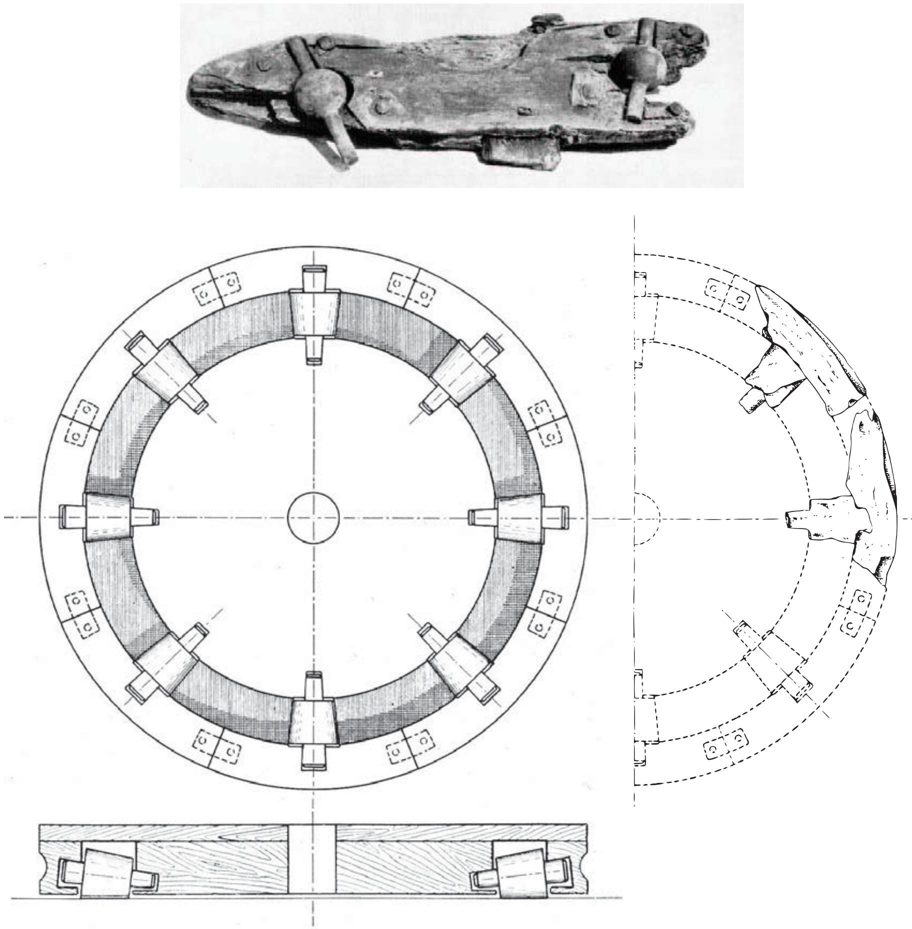


FIGURE 9.4. Brass rollers and remnants of rotating wooden trays encountered in a shipwreck in Lake Nemi and dating from Caligula's time.[96] Top: portion of the rim of a tray about 90cm in diameter. Middle and bottom: reconstruction of another tray with conic rollers. From [Ucelli], pp. 186, 189.

political rulers and the higher classes in science and technology is clear, whereas in imperial times an equally clear lack of interest is attested. Yet archeology appears to indicate that the quantitative development of technological applications, particularly in production, was concentrated in the early imperial period. How can this be?

[96]It is tempting to think that the rotating room of Nero's *domus aurea* used something similar.

There are at least two possible answers to this apparent contradiction. The first is that the archeological evidence may be biased toward the more recent. Thus we have imperial-age remains of water-drainage equipment from the mines at Rio Tinto, but we know that those mines were already being exploited by Carthage, and Diodorus Siculus mentions the use of Hellenistic technology by the Carthaginians in draining Spanish mines.[97] Strabo records both the traditional proficiency of Massalians with mechanical technology[98] and Massalian control over the Rhone estuary,[99] so it may be speculated that the Barbegal water mills likewise represent a pre-Roman technological tradition that the Romans found it convenient to keep. Similar examples can be multiplied: one could plausibly conjecture a Hellenistic precedent for almost every known Roman technological installation.

There is, however, a second possible answer: that there is no actual contradiction between the timing of inventions and that of their spread. Two successive civilizations are not independent, because the second has as its disposal the cultural instruments created by the first, if their record has not vanished. A logical link between two cultural phenomena, here scientific development and its technological consequences, does not have to imply synchronicity; one period may be consuming resources accumulated in an earlier one. The cultural policy of Hellenistic rulers clearly favored technological research and *innovation* more than the Roman one (indeed there are very few technological novelties in Roman times), but the *widespread diffusion* of technology already invented requires something different from research. A long period of peace accompanied by a heavy fiscal burden may have been more effective than the Mouseion ever was at causing an increase in production and the application of all available technological resources.

The Romans were not interested in science and favored the legal and military structures of dominion over technological and economical ones, but they certainly did not reject the benefits of technology and wealth: they simply thought that it behooved lowly folks to generate them and aristocrats to enjoy them. Rome found an effective solution for the problem of acquiring and controlling competent technicians, but not for that of forming them. It was a system based on the exploitation of provinces where, independently of the central power, a technological and scientific cultural tradition still survived. Together with the crumbling of Rome's political

[97]Diodorus Siculus, *Bibliotheca historica*, V, xxxvi–xxxvii. According to him, the Carthaginian introduced the use of multiple Archimedean screws in series, while the massive use of slaves dates from Roman times.

[98]Strabo, *Geography*, IV, i §5.

[99]Strabo, *Geography*, IV, i §8.

system came the cultural, economic and technological crumbling of the West. In the eight century the Venerable Bede was the greatest mathematician in what had been the Western empire. In his most exacting work he describes a method for representing numbers with hand gestures. Many could still do it up to ten: Bede, using a sort of sign language, manages to go a bit further. When this is what a "mathematician" can aspire to, urban life has already disappeared. In the Eastern empire, where the nexus of science, technology and economy subsisted, however precariously, things came out differently.

10
Lost Science

Besides being just a small part of the whole, extant Hellenistic scientific writings have often been altered in intervening centuries by editors, who adapted them to their own notions. Here we will try to reconstruct certain ideas of Hellenistic science, sometimes through plausible conjecture and sometimes in a less tentative way. Our evidence will include indirect testimonia present in literary works that are not often regarded as source material for history of science studies; in passing, it may be said that our knowledge of ancient science would greatly benefit from a systematic collection of fragments and testimonia on various scientific theories.

10.1 Lost Optics

Only two sizable works of ancient optics are extant. Authored by Euclid and Ptolemy, they stand almost five hundred years apart. Euclid's *Optics* represents an early stage of the science, which was then developed in lost treatises by some of the greatest scientists of Antiquity, including Archimedes, Apollonius and Hipparchus, and finally hit a crisis like all other scientific disciplines. We have several reasons to think that Ptolemy's book is more of a partial recovery of earlier stages than an improvement on them. Ptolemy discusses only plane and spherical mirrors, without ever applying to mirrors the theory of conics, but Diocles' short work *On burning mirrors* shows that such applications, later taken up by the Arabs, were very much older than Ptolemy. Ptolemy's *Optics* is the only surviving work that includes a theory of binocular vision, but certain testimonia imply that such a theory had already been developed by Hipparchus.[1]

[1] Plutarch mentions a theory of binocular vision (*Quaestionum convivalium libri vi*, 625E–626E). Aetius attributed such a theory to Hipparchus; cited in Stobaeus, *Eclogae*, II, lii, 483:19 – 484:2 (ed. Wachsmuth) = [DG], 404b:3–8.

The state of the sources leaves us in the dark about many areas of physical optics. For example, we know very little about research on colors[2] and on dispersion (the dependence of refraction on the color of light, which gives rise to the rainbow and allows the separation of colors through a prism). But interest in these subjects is demonstrated by many references found in the literature: for example, in Diogenes Laertius,[3] Plutarch,[4] Lucretius[5] and Seneca, who talks about glass objects "with many angles" that, when hit by sunlight, give back the colors of the rainbow.[6] Apuleius says that Archimedes too studied the rainbow phenomenon.[7]

It would be particularly valuable to know what the state of knowledge on refraction was. Among extant scientific works, the first to discuss the subject is Ptolemy's *Optics*, where we find an extensive, but unfortunately truncated, treatment of the phenomenon.[8] Yet refraction had important and early technological applications, first and foremost to lenses.

The use of lenses to focus sun rays is documented in Greek literature from at least the fifth century.[9] Theophrastus, among others, mentions this method of lighting a fire,[10] and Pliny talks of the medical use of converging lenses for cauterization.[11]

Because of a lack of clear literary references, it has often been denied that classical Antiquity used lenses to magnify images. (Such lenses must be made of better-quality glass, or else crystal, and require better grinding techniques, than those used to start fires.) But today it seems very likely from the archeological evidence that magnifying glasses were indeed in use. Archeologists have found lenticular objects at many sites, including

[2] A very interesting passage in Ptolemy's *Optics* (II §96, 60:11–19, ed. Lejeune) discusses spinning disks with sections of different colors — the kind later called Newton's disks. As Lejeune remarks, they appear here "as a true experimental instrument".

[3] Diogenes Laertius, *Vitae philosophorum*, VII §152.

[4] For example, *De facie quae in orbe lunae apparet*, 921A; *De Iside et Osiride*, 358F–359A.

[5] Lucretius, *De rerum natura*, II:799–800.

[6] Seneca, *Naturales quaestiones*, I, vii §1. The passage mentions a glass *virgula*, a term that is usually translated "little rod". But *virga*, besides rod, is also the name of the mini-rainbow that arises as a result of dispersion, and which Seneca discusses in the continuation of the passage cited. Thus *virgula vitrea* could have been, in Seneca's source, a small straight rainbow obtained with a glass object. As to the "many angles", it is possible that Seneca's source did not refer to objects with particularly many facets, but to the refraction angles of the various colors.

[7] Apuleius, *Apologia*, xvi.

[8] Ptolemy, *Optics*, V = 223–269 (ed. Lejeune). The work breaks off in the middle of this book V. See page 64 for a discussion of Ptolemy's tables of refraction angles.

[9] Aristophanes, *Clouds*, lines 766–772.

[10] Theophrastus, *De igne*, xiii §73, 20 (ed. Gercke). The fragment actually talks about lighting a fire by converging sun rays, and it's not totally clear whether a mirror or a lens is meant; but since both glass and metals (bronze and silver) are mentioned, Theophrastus probably has in mind both possibilities.

[11] Pliny, *Naturalis historia*, XXXVII §§28–29.

Tyre, Pompeii, Cnossos and the Fayyum.[12] Although the finds at Pompeii (which started coming up in the eighteenth century) and some other sites have often been interpreted as jewelry, and some others were probably lenses used for ignition, the especially high quality of some of the lenses found recently leaves little room for doubt that they were magnifiers. One of two plano-convex lenses found in Crete in 1983, for instance, magnifies well at least seven times and still has visible signs of polishing.[13] The Archeological Museum in Heraklion, Crete, has 23 lenses on display, some of prime quality, and has others in storage. Some of the finds, like the ones from Cnossos, go back to the bronze age and show that lens technology, though completely lost in early medieval Europe, is in fact very old.

To accept the idea that magnifying glasses were used in Antiquity we must account for the scarceness of literary references. We have seen that literary sources fail to mention other technological products that existed for sure. Also we can imagine that the use of magnifiers was confined to a few fortunate souls: probably some professionals to whom they would be of most help, such as fine engravers and jewelers,[14] and the very rich. Nero and his famous emerald monocle may be a case in point.[15] Moreover the lack of literary references to magnifiers may to some extent derive from the beliefs of modern scholars: a person convinced that something did not exist in Antiquity cannot but misunderstand any passage mentioning that thing. Thus, Alcaeus wrote that wine is a man's *dioptron*.[16] He probably meant that drink as it were magnifies a person's behavioral traits, putting them in evidence,[17] but the belief that lenses were unknown has caused *dioptron* to be translated as "mirror", sacrificing both the general sense of the passage and the natural meaning of the word, which on etymological grounds can be presumed to mean something that is *seen through* (whereas a mirror is *katoptron*). Strabo uses the same word to refer to chunks of a transparent mineral exported from a certain place in Asia Minor.[18]

[12] See [Beck] for the Cnossos find. The scant literary evidence is discussed in [Kisa], vol. II, pp. 357–359.

[13] [Sines, Sakellarakis]. This article also discusses other recent finds and some older ones.

[14] Already in the eighteenth century the jewelry carver Johann Natter, based on his study of ancient techniques, became convinced that his ancient colleagues could not have carried out all of their work with the naked eye; see [Natter]. The same opinion is held today by several scholars based on an analysis of gold craftmanship, particularly in the Roman period; see [Sines, Sakellarakis].

[15] Pliny, *Naturalis historia*, XXXVII §64. This continues a passage where Pliny mentions that engravers used emeralds to lessen eye-strain and provide bigger images; but Pliny believed that the engravers rested their eyes by contemplating the emeralds and that the enlarged images were those of the emeralds themselves.

[16] οἶνος γὰρ ἀνθρώπω δίοπτρον (Alcaeus, fr. 333 Voigt).

[17] I am indebted to Bruno Gentili for this observation.

[18] Strabo, *Geography*, XII, ii §10.

While the ancient use of lenses for lighting fires is certain and that of magnifying glasses seems at least very probable, few have been willing to entertain the possible existence of telescopes in Antiquity. But, if nothing else, this hypothesis would explain the many medieval pictorial and written references to an object that supposedly would not be invented for several centuries yet![19] As more direct evidence we may cite a passage in Strabo mentioning "reeds" or "tubes" by means of which images can be magnified thanks to the refraction of visual rays,[20] and perhaps also one in Geminus explaining that experts in geodesy, who used the dioptra, based some of their work on the phenomenon of refraction.[21]

The conjecture that dioptras provided with lenses — in essence, telescopes — were known in the Greek world may seem very daring, but the possibility would become less far-fetched if there were confirmation for the suggestion of Giovanni Pettinato that such instruments were already used by Mesopotamian astronomers in the Late Assyrian period.[22]

Hipparchus was an expert in dioptrics, in that he perfected and described dioptras[23] and probably already knew the instrument described by Heron.[24] It is not inconceivable that he found a way to apply knowledge about refraction to the dioptra.

We will return in Chapter 11 to the question of the possible existence of telescopes in Antiquity. [25]

[19]See page 347.

[20]Strabo, *Geography*, III, i §5. Many editors have adopted Voss's emendation of αὐλῶν (reeds) into ὑάλων (glass pieces), thus admitting the existence of magnifying glasses but perhaps missing the full import of the passage.

[21]In [Heron: OO], IV, 100:17–18. The Greek term ἀνάκλασις is usually rendered as "reflection", but there are several passages where "refraction" must be meant: for example, Sextus Empiricus, *Adversus astrologos* (= *Adv. math.* V), §82. Both this and the complementary word κατάκλασις seem to have been used for both phenomena (their root is κλάω, "break").

[22][Pettinato], p. 103. The idea is by no means new; we read in Hoppe's *History of optics* (1926) that "some astronomers are of the opinion that such precision [in Babylonian astronomical measurements] could not have been achieved without telescopes" ([Hoppe], p. 2).

[23]Ptolemy mentions the use of optical devices by Hipparchus and his written description of a dioptra (*Almagest*, V, v, 369; V, xiv, 417, ed. Heiberg). Pliny mentions Hipparchus' contribution to the perfecting of optical instruments for astronomical observations (*Naturalis historia*, II §95). The precision of the optical instruments used by Hipparchus and in particular the accuracy of his angular measurements can be gauged from the excellent approximation of his measurement of the distance to the moon (see note 96 on page 79).

[24]So Toomer in [Ptolemy/Toomer], p. 227, note 20. The suspicion that on this subject Heron may have used Hipparchus as his source is suggested by the fact that the method described by Heron in the *Dioptra* for determining the longitudinal difference between Alexandria and Rome, based on the difference in local time for the same lunar eclipse, had been proposed by Hipparchus, as we know from Strabo (*Geography*, I, i §12). But dioptras analogous to the one Heron describes may be even older than Hipparchus: the thesis that they were already in use in the early third century is expounded in [Goldstein, Bowen].

[25]See pages 343–348, particularly the text surrounding notes 53, 54, 65 and 66.

10.2 Eratosthenes' Measurement of the Meridian

Eratosthenes, with the method discussed in Section 3.2, obtained the value 252,000 stadia as the earth's circumference along a meridian. Estimating the accuracy of this measurement is not easy: there has been controversy on the value of a stadium in this context. The most likely reconstruction puts Eratosthenes' stadium in the range 155–160 m,[26] implying an error of at most 2.4% below or 0.8% above the true value. Such remarkable accuracy has often been seen with suspicion, especially because it is true only to a coarse approximation that Syene and Alexandria lie on the same meridian and that Syene lies on the tropic. Moreover, whereas modern measurements, first attempted by W. Snell in 1615,[27] involved triangulation over distances of a hundred kilometers or so,[28] it is generally held that the distance between Syene and Alexandria was estimated by counting days of travel. The conclusion ordinarily accepted is that Eratosthenes did get an excellent value, but only as the result of a very lucky cancellation of errors.[29]

The size of the degree is not the only distance measurement that Eratosthenes is reported to have made. He in fact compiled a map of the whole known world. One of the data transmitted by Strabo is the distance from Alexandria to Rhodes, which Eratosthenes found to be 3750 stadia.[30] This value, too, is generally regarded as the result of a rough estimate[31] that is

[26]A value of 157.5 m for the stadium used by Eratosthenes was determined by Hultsch in his thorough investigation of Greek measurements ([Hultsch: GRM], p. 61). Although different from the traditional value used in Greece, it has been accepted by most subsequent scholars as substantially correct. The argument is based primarily on a passage of Pliny (*Naturalis historia*, XII §53), where the ratio between the stadium and the schoenus is reported to have two alternative values, one of them being called "Eratosthenes' ratio" (*Eratosthenis ratione*). For the view that Eratosthenes used the traditional stadium, of about 185 m, see [Rawlins: ESNM].

[27]Snell — best known for the sine law of refraction, about which more on page 348 — explicitly designed his measurement as an attempt to duplicate Eratosthenes' feat. The work, carried out in the Dutch flatlands and described in Snell's *Eratosthenes Batavus* (1617), relied on his recovery of ancient methods of triangulation and of spherical geometry, which culminated in the book *Doctrina triangolorum* (Leiden, 1627). On p. 62 of the latter we find a tantalizing tidbit of terminological information, noted in [Carnevale]: according to Snell, Hipparchus and Menelaus used the term *tripleuron* for spherical triangles. Because we possess no ancient testimonium on Hipparchus in connection with spherical geometry, this may mean that Snell had some source no longer extant. (In the case of Menelaus the source may be Pappus.) Note that Hipparchus preceded Theodosius, the author of the oldest extant work on spherical geometry.

[28]After Snell's and other attempts involving distances too small to be effective, in 1669 the French Academy undertook two careful measurements of distances over 100 km, under Picard's direction, and so obtained the first reliable values for the degree of the meridian, namely 57064.5 and 57057 Paris fathoms (toises de Paris). Picard related this unit to a precisely defined pendulum, so we know it quite exactly (1949 mm); this gives an error of about 0.1%. See [Picard].

[29]See, for example, [Heath: HGM], vol. 2, p. 107; compare [Neugebauer: HAMA], p. 653.

[30]Strabo, *Geography*, II, v §24.

close to right by accident.

Do we know for sure that Eratosthenes assumed that Alexandria and Syene lay on the same meridian and that Syene was on the tropic? The primary source on the subject, Cleomedes, actually wrote:

> Eratosthenes' method, being geometric, seems more difficult [than the previously explained method of Posidonius]. What he says will become clearer if we allow ourselves to make two assumptions. We assume first that Alexandria and Syene are on the same meridian.[32]

Cleomedes does not give a detailed account of the method — that would be pointless, Eratosthenes' work still being available — but a pedagogical précis meant for readers scared of the complex geometric arguments of the original work. He attains his goal of explaining Eratosthenes' method by taking an ideal case obtained by eliminating all technical difficulties; how else could he compress into three pages a work that occupied two books? Cleomedes also rounds off the numbers, evidently so as not to bother the reader with calculations inessential to an understanding of the method.[33] We should not automatically attribute to Eratosthenes the simplifications adopted by his popularizer.

Elsewhere Cleomedes records a precious detail: that at noon on the day of the summer solstice sundials cast no shadow within a zone 300 stadia wide about the tropic line.[34] Clearly, many measurements with sundials had been made, over a wide area, and the tropic was fixed as the midline of the shadowless zone.[35] Thus it is reasonable to think that this line could be located precisely within a few tens of stadia, or a few minutes of arc.[36] We must conclude that Eratosthenes, desirous of measuring the distance from Alexandria to the tropic, first took the trouble to find it precisely and didn't just assume on someone's say-so that it went through Syene. Cleomedes and other authors probably name Syene because it was the

[31]See, for example, [Neugebauer: HAMA], p. 653.

[32]Cleomedes, *Caelestia*, I §7, 35:49–52 (ed. Todd).

[33]Cleomedes' value for the circumference (250,000 stadia, instead of the 252,000 reported by all other sources) and for the difference in latitude between Syene and Alexandria (1/50 of the whole circle) are clearly obtained by rounding, an understandable liberty taken by someone whose aim is avowedly just to illustrate the method.

[34]Cleomedes, *Caelestia*, I §7, 36:101 – 37:102 (ed. Todd).

[35]That this datum must have been determined by personnel sent on site for the purpose is said already in [Hultsch: PGES], p. 14. But J. Dutka objects that "it is questionable whether in that era royal surveyors would be used for a purely scientific purpose" ([Dutka], p. 61).

[36]We also know that Eratosthenes could detect astronomically differences in latitude between spots more than 400 stadia apart along the same meridian (Strabo, *Geography*, II, i §35). The accuracy with which one can locate the tropic is much better than this margin of error, since it is easier to distinguish precisely between no shadow and some shadow than it is to distinguish between two approximately equal nonzero magnitudes. The main source of subjective error in the shadow measurement is that the sun is not a point source.

Egyptian town closest to the tropic and the most convenient base for an expedition to the tropic.[37] As for the well whose bottom was lit by the sun at the solstice, Pliny says that it was dug out for a demonstration.[38]

Here a digression is warranted on an important aspect of experimental methodology. It is commonly thought that Hellenistic scientists were ignorant of the technique of averaging multiple measurements, because there is no direct documentary evidence for its use.[39] But the placement of the tropic at the center of a shadowless zone, as logically implied by Cleomedes' statement just discussed, seems to be a case where the literature preserves an indirect trace of the method in question. The lack of direct testimonia about experimental averaging is hardly surprising, since the manuscript tradition preserved neither the works where the technique might have been used (such as Herophilus' research on heartbeats) nor the theoretical treatise by Eratosthenes titled *On means*, which might perhaps have cast some light on the issue.[40]

Next, the determination of the tropic through multiple simultaneous observations affords an accuracy that would be pointless if the distance to Alexandria were then estimated using days of journey. Is it possible that the distance was actually measured? Eratosthenes was the first person to make a map of Egypt. The degree of precision with which he managed to measure the distance from Alexandria to the tropic — that is, to the southern border of the kingdom — is equivalent to that with which the chart was made.

A record of the work involved in this topographical survey can be found in the sources. Martianus Capella writes that the distance measurements on which Eratosthenes' estimate of the size of the earth relied were furnished by the royal surveyors (*mensores regii*),[41] and Strabo relays some data from Eratosthenes' map of Egypt.[42] We know that already in the Pharaohs' time a detailed measuring of the land (γεωμετρία) was made annually throughout Egypt. Under the Ptolemies the measurements were

[37]Strabo, Pliny and Arrian all say that Syene is on the tropic. The town lies near the first cataract of the Nile, which marked the boundary between Egypt and Ethiopia: therefore to get to the tropic one had to cross the border.

[38]Pliny, *Naturalis historia*, II §183.

[39]See, for instance, [Grasshoff], p. 203, where it is mentioned that the first documented instances of averaging of experimental results are due to ninth-century astronomers in Baghdad.

[40]We know the title of this work from Pappus, *Collectio*, VII, 636:24–25 (ed. Hultsch). The only other information we have is what we can deduce from another passage of Pappus (*Collectio*, VII, 662:15–18), where, apparently referring to the same work, the author states that Eratosthenes treated *geometric loci related to means* and that such loci, because of their particular definition, did not fall within the scope of Apollonius' classification.

[41]Martianus Capella, *De nuptiis Mercurii et Philologiae*, VI:598.

[42]Strabo, *Geography*, XVII, i §2. However, these data got corrupted on their way to us; a partial reconstruction can be found in [Rawlins: ESNM].

entrusted to technical staff and royal inspectors in every village; they were then collected and coordinated by the *toparchs* for each *topos* (a subdivision of the nome), and then further up by the *nomarchs* for each nome (province). The reports finally reached Alexandria, were they were used for the preparation of tax rolls.[43] By combining this tentacular bureaucratic organization with the new methods of scientific geodesy, good maps of Egypt would not have been out of reach.

To sum up, it is not out of the question that Eratosthenes really did measure the meridian within a margin of error of less than 1%.

In Section 3.2 we posed the question how come Marinus and Ptolemy, although aware of the method used by Eratosthenes, did not repeat the measurement and instead chose to rely on old and misunderstood data. What we have said so far points toward a partial answer: they could still read Eratosthenes' work and knew that it was based on sophisticated survey work that could no longer be carried out under prevailing political conditions. This also explains why for so many centuries, even as recently as Galileo's time, no one was able to improve on Eratosthenes' findings.

Later ages, no longer acquainted with so much as the possibility of a state-funded scientific project, passed on the tale of Eratosthenes' measurement as if it had been the isolated idea of a genius. Isn't that what happened also to Archimedes' hydrostatics, for that matter?

The French Academy's measurement of the earth in 1669 stood on three accomplishments of the human mind: mathematical geography, triangulation methods and survey instruments. All three go back to Hellenistic times. But in order to measure great distances accurately another ingredient is needed: the ability to organize and manage projects on a large scale. Is that an exclusive feature of modern civilization? In 1669 Europe was still far from being able to dig a canal through the Red Sea: this took another two hundred years and resources well beyond what the French Academy could have mustered. Yet the same task had been carried out by the Ptolemies decades before Eratosthenes' measurement.

Being necessary for the compilation of accurate maps, the measurement of the earth's circumference was probably a scientific enterprise financed by the state with the same largesse used to build other works useful to mariners, such as the Pharos and the canal. The general oversight of the work and the credit for it were assigned to Eratosthenes as the Head of the Library, which is to say the highest-ranking person in charge of the government's scientific policy.

It has been observed that the Eratosthenes' value of 252,000 stadia for

[43]For the organization of land registry and measurements under the Ptolemies, see [Rostovtzeff: SEHHW], vol. I, pp. 275–276.

the meridian is divisible by all numbers from 1 to 10 (their least common multiple is in fact 2520). This is a very useful property and it is unlikely that he came by it accidentally. He might have fudged the data in order to get this convenient result.[44] But Pliny's reference to the value of the stadium "according to Eratosthenes' ratio" (or scheme or reckoning) suggests another possibility, in line with Hellenistic conventionalism: that Eratosthenes introduced a new stadium equal to a convenient fraction of the meridian, just as with the meter's definition in the eighteenth century.

10.3 Determinism, Chance and Atoms

A passage from Laplace's *Essai philosophique sur les probabilités* (1825), very often cited as a nutshell statement of nineteenth-century determinism, is frequently considered as a "determinist manifesto":

> An intelligence who, for a given moment, knew all the forces that act in nature and the respective situation of its component beings, if it also were ample enough to analyze these data, would encompass under the same formula the motions of the largest bodies in the universe and those of the lightest atom. Nothing would be uncertain to that intelligence; the future, just like the past, would lie open before its eyes. The human mind, in the perfection to which it has been able to take astronomy, offers a pale reflection of such an intelligence.

By Laplace's time determinism already had a very old history. It goes back at least to Democritus[45] and underwent interesting developments at the hands of the Stoics.[46] Cicero, reporting Stoic ideas, writes:

> Moreover, since everything is caused by fate (as is shown elsewhere), if there could be a mortal able to grasp in his mind the chain of all causes, nothing at all would escape his knowledge: for he who knows the causes of future events must perforce know also what these events will be. And since this grasp is beyond any but the gods, it is left for man to foretell future consequences by means of certain declaratory signs. For events in the future don't come about suddenly: as the uncoiling of a rope is the passage of time, creating nothing new but instead unfolding the old.[47]

[44]This is the opinion put forth in [Rawlins: ESNM], where the remark about divisibility seems first to have been published.

[45]Democritus, A68 ff. in [FV], vol. II.

[46]One particularly interesting source about Stoic determinism is the *De fato* by Alexander of Aphrodisias. Fragments 915–951 and 959–964 in vol. II of [SVF] are devoted to this subject.

[47]Cicero, *De divinatione*, I, lvi §127.

One can speculate that in Antiquity too reflections on the predictability of astral motion — and the creation of a successful planetary theory[48] — were factors in articulating determinism.

The notion of chance, like determinism, has a long history in Greek thought, apparently going back to Democritus.[49] The random combination of limbs considered by Empedocles introduced chance in biology,[50] and Aristotle makes extensive reflections on chance.[51]

Plutarch, referring to Homer and Hesiod, says that men, at a time when the term "chance" (τύχη) was not in use yet, when in the presence of causal chains so irregular as to generate unpredictable events attributed those events to the gods.[52] This argument is heard in our days too.[53]

The existence of unpredictable and at least apparently random events such as the motion of a particle of dust is at first glance incompatible with determinism. Greek thought found two radically different ways to solve this puzzle.

The Epicurean solution consists in rejecting determinism altogether, ascribing to each atom the possibility of a minute and wholly unmotivated deviation (*clinamen*) from the trajectory that it would otherwise follow based on its weight and its collisions with other atoms.[54] (Naturally, the idea that atomic motion is subject to intrinsic stochastic fluctuations has gained renewed interest due to the establishment of quantum mechanics.)

In contrast, the Stoics reiterated the general validity of determinism, explaining the apparent randomness of many events through our inability to follow causal chains that are too complex.[55] Chrysippus, in particular, criticized the Epicurean hypothesis of unmotivated motion and explained that in every case, even that of a die that lands on a particular face, there is always a hidden cause.[56]

[48]See the end of Section 10.5 and the beginnings of Sections 10.7 and 10.8.

[49]Simplicius, *In Aristotelis Physicorum libros commentaria*, [CAG], vol. X, 330:14–18 = [FV], II, 101:11–16 (Democritus A68); Lactantius, *Divinae institutiones*, I, ii = [FV], II, 101:33–36 (Democritus A70).

[50]See page 163.

[51]Aristotle, *Physica*, II, iv–vi, 195b–198b.

[52]Plutarch, *Quomodo adolescens poetas audire debeat*, 24A.

[53]For example, [Rényi], p. 129: "Primitive people have a tendency to be very superstitious: if something goes wrong they try to attribute it to somebody's maliciousness. . . . The study of probability theory can help to erase these remains of magical thinking from the Stone Age[.]" Note that, whereas Plutarch talks of an important conceptual evolution that happened at a time already far removed from him, Rényi, an important probability theorist, makes a direct contrast between the specialized contents of his own field of study and Stone Age magic (being perhaps unaware that he is repeating a point made in classical times).

[54]This idea does not appear in the letters of Epicurus that have come down to us. The earliest author who reports it is Lucretius (*De rerum natura*, II:216–260).

[55]Plutarch, *De animae procreatione in Timaeo*, 1015B–C. See also [SVF], II, texts 965–973.

[56]Plutarch, *De stoicorum repugnantiis*, 1045B–F.

That atomic motion is essentially random and unpredictable may not have been an idea concocted by the Epicureans to account for the existence of random events (though this point stood out after the triumph of quantum mechanics); it may have played a role in theories of atomic motion created to save phainomena of various types.

Anaxagoras had considered the motion of dust lit by a beam of sunlight.[57] Lucretius revisited the same theme, explaining the phenomenon's "randomness" through the invisible and orderless movement of atoms.[58]

The path of an atom in a gas was thought of as being determined by a continuing succession of collisions.[59] (The idea that gases are characterized by chaotic atomic motion apparently fell into oblivion for centuries, but it is peculiar that the word "gas" itself was coined in the seventeenth century from the Greek term *chaos*,[60] which then, as in ancient times, had many meanings.)

Atomic motion was used to explain, besides the behavior of dust, other phenomena related to matter and heat. Unfortunately, only echoes of such explanations have survived, scattered in literary texts. For instance, some scientific ideas about thermal phenomena have come down to us through Plutarch, such as the remark that wool clothing makes us warm because of its insulating power rather than because of inherent heat, as demonstrated by the use of wool to keep snow from melting.[61] Plutarch also discusses — and exaggerates — the effects of temperature on the density of water.[62] Several of his passages hint that temperature differences may have been understood to reflect changes in atomic velocity: he reports (without attribution) the doctrine that coldness is merely the absence of heat and has the property of being stationary;[63] and the comment that "hotter" means "faster".[64]

Regarding states of matter, already Epicurus distinguished two types of atomic motion: the vibration of atoms around a fixed position (in solids) and the free movement of atoms separated by large distances (in gases).[65]

[57]Plutarch, *Quaestionum convivalium libri iii*, 722A–B = [FV], II, 24:10–15 (Anaxagoras A74).

[58]In the same lovely passage already mentioned in note 33 of page 23 (*De rerum natura*, II, 112-141).

[59]See for example Plutarch, *Adversus Colotem*, 1112B.

[60]The neologism is due to J. B. van Helmont (1577–1644), who used a Flemish phonetic equivalent for the letter χ. I am grateful to Federico Bonelli for bringing this etymology to my attention.

[61]Plutarch, *Quaestionum convivalium libri vi*, 691C–692A.

[62]Plutarch, *Quaestiones naturales*, 914A; see also note 27 on page 27.

[63]Plutarch, *De primo frigido*, 945F. See also *Quaestiones naturales*, 919A–B.

[64]Plutarch, *Quaestionum convivalium libri vi*, 677E.

[65]Epicurus, *Letter to Herodotus*, lines 43–44. The passage does not talk explicitly about solids and gases, but this correspondence (more or less obvious, in any case) is spelled out in Lucretius, *De rerum natura*, II:95–111.

The thermal dilation of gases was well-known. Philo of Byzantium, describing the thermoscope (page 134), observes that the thermal expansion of a gas is owed to the increase in interatomic distances, which cannot be observed directly.[66] Unfortunately, in his extant works Philo makes only a bare mention of these theoretical ideas, referring the reader to another work of his that has perished, titled *De mirabilibus arbitriis* in the Latin manuscripts of the *Pneumatica*.[67]

Without trying to speculate on the contents of this treatise, we mention a very interesting remark of Philo's on the possible origin of randomness, found in his artillery book:

> Many who have undertaken to build machines of equal size, using the same structure, the same type of wood and the same metal, not changing even the weight, have made some with long reach and great destructive power and others far inferior; and if asked why, they had no explanation. One might apply here the observation made by the sculptor Polycletus, who said that the good [creation] is obtained through many calculations, thanks to small differences. In the same way in this techne [artillery design] many calculations are needed, and someone who makes a small departure in the individual parts causes a large error in the result.[68]

In passing, we note that the use of mathematics to treat experimental data was so entrenched that it was not discarded in this case in spite of the unpredictable variability of concrete results. But the most interesting point for us here is how an apparently random result is explained by means of a chain of mathematical relations that magnify initially negligible variations: essentially a mathematized version of the Stoic argument for reconciling chance and determinism. This notion that "chance" can boil down to small causes generating large effects seems to have been forgotten for many centuries after Philo of Byzantium. It was taken up again, in quite another technical context, in modern theories of deterministic chaos.

[66]Philo of Byzantium, *Pneumatica*, vii. See also the chapters of Heron's *Pneumatics* mentioned on pages 134–136.

[67]Philo of Byzantium, *Pneumatica*, iii = [Philo/Prager], p. 129. Several authors have interpreted *arbitria* as an Arabic-mediated corruption of "automata" or "organs", assuming perhaps that these are the only wonder-evoking (*mirabilia*) entities in pneumatics. But, as noted by Prager (loc. cit., note 409), the context shows clearly that the reference is not to automata or organs, but to a discussion of the motion of atoms in the void.

[68]Philo of Bizantium, *Belopoeica*, 49:13 – 50:9 = [Marsden: TT], pp. 106–107.

10.4 Combinatorics and Logic

Cicero says that anyone who thinks that the ordered universe we know might have arisen accidentally, through the casual concourse of material particles, should also allow that, by shuffling and scattering on the ground a bag of letters of the alphabet, one might get Ennius' *Annals*, ready for reading.[69] The curious thing is that, in choosing an event whose practical occurrence can be excluded even though it is possible in theory — what we now call an event of extremely low probability — Cicero should use an example that seems to suggest an awareness of the enormous number of ways in which a set of letters can be combined. Plutarch not only reports a similar example,[70] but writes:

> Disorder, like Pindar's sand, "eludes numbering".... The facts allow only one true statement, but an unlimited number of falsehoods. Rhythms and harmonies follow precise ratios, but no one can comprehend all the musical slips that people make playing the lyre or singing or dancing.[71]

It has often been thought that combinatorial calculations were unknown to ancient science, but twice in Plutarch's dialogues we find this remark:

> Chrysippus said that the number of intertwinings obtainable from ten simple statements is over one million. Hipparchus contradicted him, showing that affirmatively there are 103,049 intertwinings[.][72]

This passage stumped commentators until 1994, when David Hough, then a graduate student in mathematics, noticed that 103,049 is the *tenth Schröder number*,[73] representing the number of ways in which a sequence of ten symbols can be bracketed (subdivided into hierarchically organized groups). Hough's discovery has shown that in combinatorial problems of considerable complexity had been approached and solved by the time of Hipparchus, and forced a reevaluation of our notions of what was known about combinatorics in Antiquity. (Fabio Acerbi, on the basis of the meager source material, has made some progress toward a reconstruction of

[69]Cicero, *De natura deorum*, II, xxxvii, 93.

[70]Plutarch, *De Pythiae oraculis*, 398B–399E.

[71]Plutarch, *Quaestionum convivalium libri iii*, 732E–F.

[72]Plutarch, *De Stoicorum repugnantiis*, 1047C–E, and *Quaestionum convivalium libri iii*, 732F (in the latter passage the number transmitted by the manuscripts, 101,049, was long ago emended to agree with the other passage, the corruption from "three thousand" to "[a] thousand" being by far the more likely one).

[73]This notion was introduced in 1870 in [Schröder]. The link between it and Plutarch's passage was published in [Stanley]; further remarks can be found in [Habsieger et al.].

that knowledge.[74]) Plutarch specialists up to Schröder's time did not even have a chance to explain this passage and could not have imagined that this was because combinatorics in their time was yet to recover a concept known to Hipparchus.

Plutarch also mentions simpler combinatorial problems; for example, he says that Xenocrates estimated the number of syllables that can be made with the letters of the alphabet as 1002 billion.[75]

One conclusion that can be drawn from this episode is worth stressing: at least sometimes, Plutarch faithfully recorded sophisticated scientific results of Hipparchus that are not otherwise documented. In subsequent sections we will see that this was probably not the only case. Needless to say, reconstructing scientific results from fragmentary allusions in literary sources is tricky at best: even the meaning of Hipparchus' calculation in terms of symbolic logic has not been fully clarified.[76]

10.5 Ptolemy and Hellenistic Astronomy

One often hears that the *Almagest* rendered earlier astronomical works obsolete.[77] This view is inconsistent with a crucial, if often overlooked, reality: whereas astronomy enjoyed an uninterrupted tradition down to Hipparchus (and especially in the period since Eudoxus), the subsequent period lasting almost until Ptolemy's generation witnessed no scientific

[74][Acerbi: SH]. Some of the sources are Pappus, *Collectio* VII, 833–837; Boethius, *De hypotheticis syllogismis*, I, viii §§1–7; a scholium to Euclid's *Data*, published in [Euclid: OO], vol. VI, p. 290.

[75]Plutarch, *Quaestionum convivalium libri iii*, 732F. This estimate is obscure in that we do not know how many letters are to be combined nor the rules to be followed in combining them; moreover the number "1002 billion" is certainly rounded off and so offers little in the way of clues to the original computation.

[76]Probably the ten simple statements are to be combined using logical implications. The Plutarchan term translated here literally as "intertwining" is συμπλοκή, which in Stoic logics ordinarily means what we call logical conjunction ("and"). But bracketing (grouping) an expression that involves only conjunctions is an idle exercise, since conjunction is associative. On the other hand, if by intertwining is meant a nonassociative operation (of which the most obvious example is implication), different groupings lead to essentially distinct logical compound statements, and counting such compound statements acquires interest. (More precisely, the conjecture is that the problem posed was to count chains of implications under different groupings, and that — as is perhaps natural when one uses everyday language instead of operator symbols — the ungrouped "intertwining" of A, B, C meant the chain of implications now represented by $(A \Rightarrow B) \wedge (B \Rightarrow C)$, as distinct from $(A \Rightarrow B) \Rightarrow C$ on the one hand and $A \Rightarrow (B \Rightarrow C)$ on the other; compare our $a > b > c$. The interest of ancient logicians (particularly Stoics) in long chains of logical implications is documented in several loci; see in particular Alexander of Aphrodisias, *In Aristotelis Analyticorum priorum librum I commentarium*, 283, 7ff. (ed. Wallies) = [SVF], II, 257, where this way of linking propositions is implied.) For a different view see [Acerbi: SH].

[77][Ptolemy/Toomer], p. 1: "…its success contributed to the loss of most of the work of Ptolemy's scientific predecessors, notably Hipparchus, by the end of Antiquity, because, being obsolete, they ceased to be copied." Likewise [Grasshoff], p. 1.

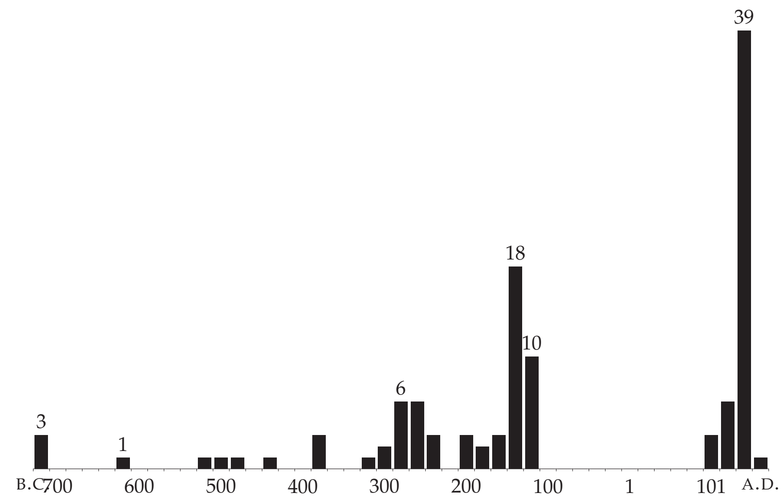


FIGURE 10.1. Histogram of astronomical observations referred to in Ptolemy's *Almagest*. Each bin is a 20-year period centered on January 1 of the year indicated. The number of observations is written on top of selected bars. Data gathered from the index in [Ptolemy/Toomer].

activity: there was a deep cultural discontinuity. This break, attested in different ways, is illustrated especially clearly by the astronomical observations mentioned in the *Almagest*. They are spread over a period of nine centuries, from 720 B.C. to 150 A.D., but leaving a major gap of 218 years: from 126 B.C., the date of the last observation attributed to Hipparchus, to 92 A.D., corresponding to a lunar observation by Agrippa (see Figure 10.1). It was during this hiatus that intellectual conquests such as the possibility of consciously creating new terms or new pictorial styles were abandoned, together with the ability to formulate new sets of postulates. We have already seen in several cases how the interruption of oral transmission made ancient works incomprehensible. (As a further example among many possible, consider that Epictetus, regarded at the beginning of the second century A.D. as the greatest luminary of Stoicism, freely confesses to being unable to understand the works of Chrysippus:[78] a fact that helps explain their disappearance.)

The loss of the scientific method led to a "realistic" interpretation of surviving scientific theses: they were no longer regarded as statements within a model but as absolute statements about nature. In astronomy, decompositions of planetary motions, which had been invented in the

[78]Epictetus, *Enchiridion*, xlix.

early Hellenistic period as mathematical models useful in calculations, were now in the imperial period regarded literally, each component motion having its physical reality. Even the concentric spheres of Eudoxus were interpreted as material.[79] That the theory of epicycles was originally a mathematical model is clear with Apollonius of Perga, who proposed two equivalent models for the same motion.[80] "Realistic" interpretations, thought not present in the *Almagest*, appear in another of Ptolemy's works, the *Planetary hypotheses*, which describes a complicated mechanical system whereby circular motions are supposed to take place on material spheres embedded in rotating spherical shells.[81] The idea of material planetary spheres was not abandoned again until the modern age.

Ptolemy wrote a work on astrology, something no astronomer from the golden period ever did.[82] In this work, the *Tetrabiblos*, planets are sorted according to two binary classifications: male versus female, beneficial versus harmful.[83]

The relationship between the *Almagest*'s star catalog and Hipparchus' lost star catalog has been the subject of much debate. From Tycho Brahe to the beginning of the twentieth century it was accepted that at least a large part of Ptolemy's star coordinates were deduced from Hipparchan data and not from independent observations. This thesis was later contested by Vogt and by Neugebauer,[84] but it has recently been proved by Grasshoff through a careful statistical analysis of all the available data.[85]

We get a hint of Ptolemy's attitude toward his sources from the fact that when he treats the same problem in different works, he changes his methods each time, and hardly ever makes self-references. Thus, as we have seen, in each of the three books that deal with planets (*Almagest*,

[79] After the Hellenistic period, the purely algorithmic role of the Eudoxan spheres only came to be understood again by Schiaparelli, in the nineteenth century.

[80] See page 192.

[81] The *Planetary hypotheses* survived in Arabic and partially in Greek. The standard edition is in [Ptolemy: OAM], 69–145. A translation from the Arabic of a part omitted from it is given in [Ptolemy/Goldstein].

[82] We do not know of any horoscope, either in Greek or in demotic, datable from before 80 B.C. See [Neugebauer: HAMA], vol. 3, p. 1371, for a histogram showing the number of horoscopes from each decade.

[83] Ptolemy, *Tetrabiblos*, I §§5–6. Some planets partake of opposing natures: Mercury, for example, is hermaphroditic.

[84] [Vogt]; [Neugebauer: HAMA], vol. 1, pp. 280–284.

[85] [Grasshoff], particularly p. 4 and pp. 178–197. Because of the loss of Hipparchus' catalog, the only term of comparison we have is the coordinates given in the *Commentary on the Phenomena of Aratus and Eudoxus* (see page 79). Grasshoff concludes that, although we of course cannot rule out that Ptolemy included in his catalog coordinates measured by himself, the information found in the *Commentary on the Phenomena* suffice to prove that at least half of Ptolemy's catalog is based on Hipparchan data. Even before such statistical investigations were made there had been proofs of the non-originality of Ptolemy's star catalog; see [Wilson: Ptolemy], p. 39.

Planetary hypotheses and *Tetrabiblos*), the treatment is completely different. The apparent increase in the size of heavenly bodies near the horizon is discussed in the *Almagest* and in the *Optics*,[86] but the two explanations offered are completely different — one based on refraction and the other psychological — and neither passage mentions the other.

One particular, about geography, will illustrate Ptolemy's methodology. Both Eratosthenes and Hipparchus chose Alexandria for defining zero longitude,[87] much as British astronomers adopted Greenwich. Ptolemy, in his *Geography*, prefers instead to reckon longitudes from the faraway and shadowy "Blessed Islands". Why on earth? Evidently in view of these islands' property of being on the leftmost edge of the map. This choice of a reference meridian is not particularly useful for compiling new maps, nor for navigation, but once established it is the most convenient for data transfer and armchair geography, since it avoids the need to specify east or west.

The *Almagest* presents a system for predicting the motion of the planets but no explanation about how the system was obtained. In other words, the book gives a recipe or algorithm, which depends on certain parameters also given, but it does not say how the parameters can be derived from experimental data. As in other fields, so in astronomy too: knowing how to build theories no longer matters, only how to use them.

For it to be true that the *Almagest* incorporated all the astronomical knowledge present in earlier works, it would have been necessary for Ptolemy to have known them and mastered their methods thoroughly. The considerations above cast doubt on whether this second condition was satisfied. As to the first, we have mentioned evidence of Ptolemy's incomplete knowledge of Hipparchus' works, regarding instrumentation (Ptolemy shows no knowledge of the dioptra described by Heron, which in all likelihood goes back to Hipparchus) and geography (Ptolemy is ignorant of the length of a degree of the meridian, which was measured by Eratosthenes and which Hipparchus knew well).[88] More direct evidence is provided by Ptolemy himself, when he writes:

> Hipparchus did not even begin to formulate theories for the planets, at least in the works that have reached me.[89]

The disclaimer, which may seem due to plain conscientiousness,[90] gains its full import in the light of Ptolemy's awareness of the *titles* of all works

[86] Ptolemy, *Almagest*, I, iii, 13 (ed. Heiberg); *Optics*, III §59, 115:16 – 116:8 (ed. Lejeune).

[87] We know this from Strabo (*Geography*, I, iv §1).

[88] See pages 69 and 276.

[89] Ptolemy, *Almagest*, IX, ii, 210 (ed. Heiberg).

[90] Toomer takes it so: [Ptolemy/Toomer], p. 421, note 10.

by Hipparchus; indeed, earlier in the *Almagest* he had cited verbatim a passage from a *Catalogue of my own works* by Hipparchus.[91] We conclude that Ptolemy knew that not all of Hipparchus' scientific writings were available to him.

The *Almagest* sentence just quoted appears in the chapter that introduces the study of planetary motions. Ptolemy claims there that he was the first to create a planetary theory, and his statement about Hipparchus (the only scientist mentioned at that point) is an essential part of his priority claim. In devoting a good chunk of prose to the non-existence of a Hipparchan planetary theory, Ptolemy must have been attacking an existing belief that Hipparchus had at least started to formulate such a theory. Otherwise, how to explain that a scientist about to treat a scientific topic should devote so much space to a predecessor of three centuries earlier that had not indeed studied the subject?

The idea that the *Almagest* included all earlier astronomical knowledge must be tautologically based on the fact that, because of the loss of older works, earlier astronomy was reconstructed based on the *Almagest*. And we do know that certain Hellenistic astronomical ideas do *not* appear in the *Almagest*: if nothing else, heliocentrism and the infinite universe. We have seen in Sections 3.6 and 3.7 that there is good reason to think that, contrary to common opinion, these were not isolated notions that were abandoned abruptly.

Thus a reasonably accurate picture of Hellenistic astronomy as it existed in Hipparchus' time will depend crucially on analyses of the literature preceding the *Almagest*.

10.6 The Moon, the Sling and Hipparchus

Toward the end of the first century A.D. Plutarch wrote a dialog on the appearance of the moon, titled *De facie quae in orbe lunae apparet*, where we read:

> And further, to help the moon, that it may not fall, there is its motion itself and the whizzing nature of its rotation, just as objects placed in a sling are prevented from falling by the circular motion. For each body is guided by motion according to nature, if it is not turned aside by something else. For this reason the moon does not follow its weight, which is cancelled by the counterweight of the rotation.

[91]Ptolemy, *Almagest*, III, i, 207 (ed. Heiberg). Rehm was the first person to correctly interpret this passage of the *Almagest*, and Toomer is confident of this interpretation: [Ptolemy/Toomer], p. 139, note 25.

> There would perhaps be much greater reason to marvel if it kept motionless and still like the earth.[92]
>
> One should not pay attention to philosophers if they want to ward off weirdnesses with weirdnesses and, to fight the astonishments of one doctrine, make up things even more weird and astonishing. Take the folks who introduced the thrust toward the center. What weirdness is missing there?... Not that multi-ton, white-hot boulders thrust through the depths of the earth, upon reaching the center, should stay still with nothing touching or supporting them; nor that if thrust down with impetus they should overshoot the center and turn back again and keep bobbing back and forth from these [turning points].... Not that a furious stream of water thrust down, when reaching the center point (which they themselves call incorporeal), should stand suspended, go around in circles, swinging with an incessant and perpetual swing?[93]

"Each body is guided by motion according to nature, if it is not turned aside by something else." To clarify the meaning of this we must first of all ask what is meant by "according to nature" (κατὰ φύσιν). To Aristotle, who discussed the question at length, in the *De caelo* in particular, the answer depended on the body's nature: for heavy bodies, motion according to nature is downward (toward the center of the earth); for light bodies it is upwards; whereas heavenly bodies move according to nature in circles.

For the pebble in the sling, Plutarch's passage might suggest that, as in Aristotle, "according to nature" means the downward movement, due to weight, that would occur in the absence of rotation. But the divergence between Plutarch's source and Aristotle is patent in the case of the moon, which is here made to parallel that of the pebble: in Plutarch's source, moon and pebble had the same motion according to nature, and obviously a uniform circular motion is not according to nature for a pebble. This a key point, because the idea that uniform circular motion is the natural motion of heavenly bodies is usually associated with all of Antiquity,[95] and was still shared by Galileo.

The Greek word we have translated as motion is *kinesis* (κίνησις). The effect of gravity is not described as a *kinesis* toward the center of the earth, but as a thrust (*fora*, φορά) toward the center. We cannot translate *fora* as motion, because the motions described by Plutarch as subjected to *fora*

[92]Plutarch, *De facie quae in orbe lunae apparet*, 923C–D. The ideas we're about to present about this passage first appeared in [Russo: Plutarco], where they are discussed at greater length. As fully explained there (p. 81), we adhere to the text as preserved in the manuscripts at some of the spots where editors and translators have emended it.

[93]Plutarch, *De facie...*, 923F–924C. The claim that it is absurd that an incorporeal "point" should influence material bodies is typical of the Skeptical critique of scientific theories. See, for exam-

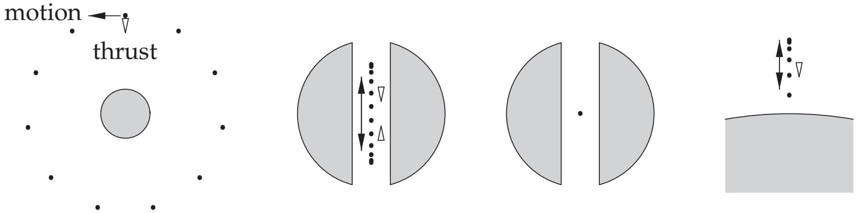


FIGURE 10.2. Several ways in which a body (black dot) subject to a "thrust toward the center" might move: a circular orbit, like that of the moon around the earth or the stone in a sling; an oscillation about the center; no movement if the body is already at the center; a single up-and-down trajectory of an ordinary body thrown up in the air. The first three possibilites are alluded to by Plutarch; see page 292 for an ancient testimonium on the last case. Arrows show direction of motion, white triangles direction of thrust.

toward the center are not in general themselves directed toward the center:[96] in the case of the multi-ton boulder that, arriving at a certain speed in the center of the earth, goes beyond it and starts to oscillate, the effect of gravity (the "thrust toward the center") is a decrease in speed when the boulder is moving away from the center and an increase when it is moving toward it; and the same can be said about the torrent of water in perpetual back-and-forth movement. The same thrust toward the center may have the effect of changing an object's velocity in direction alone, leading to a circular uniform motion, as the passage says regarding the water and the moon. In each case the movements described as subject to *fora* toward the center are those that, in modern terms, have an acceleration in that direction. This does not mean that the scientific treatises where the theory was presented necessarily used a mathematical concept coinciding with our acceleration.[96a] But it does seem clear that Plutarch's source regarded a motion "according to nature" to be a uniform linear motion, so their

ple, Diogenes Laertius, *Vitae philosophorum*, IX §99, or the passage of Sextus Empiricus cited on page 232: *Adversus grammaticos* (= *Adv. mathematicos* I), §28.

[95]For example, in [Koyré: EG], where the author maintains from the introduction on that the principle of inertia was unknown in Antiquity and that circular motion starts not being regarded as according to nature only in the modern age.

[96]Nevertheless, the word is usually translated as motion in this passage — for example, by H. Cherniss in the Loeb edition. This imprecision has led to confusion between the theory we're discussing ("thrust toward the center") and the older theory of "motion toward the center" (ἐπὶ τὸ μέσον κίνησις), attributed to Chrysippus and it, too, reported by Plutarch (*De stoicorum repugnantiis*, 1054B–1055C), among others. Obviously the idea of a tendency toward a central point is as old as the realization that the earth is round, but the theory under consideration here seems to be a new, "dynamical" version of the old idea and is of great interest for that reason.

[96a]As we shall see in Section 10.8, at least some ancient sources described the situation in terms of a combination of bits of unperturbed straight-line motion with displacements directed to the center.

dynamical theory was based on the principle of inertia in some form. This may seem out of place because, notwithstanding an interesting precedent in Democritus,[97] it is generally said that the principle of inertia and the notion of friction were unknown in Antiquity. But Plutarch's testimony is not wholly isolated. Notice that the notion of friction is indissolubly connected to the formulation of the principle of inertia. Common experience teaches us that to move a weight on a horizontal plane one must apply force. This observation is compatible with the principle of inertia if and only if friction is included among the possible forces. Several sources mention the resistance a body in motion faces from the medium in which it moves, vanishing only in the case of motion in the void.[98] Heron, in his *Mechanics*, unquestionably uses the notion of friction,[99] and he also says:

> We demonstrate that a weight in this situation [that is, on a horizontal, frictionless plane] can be moved by a force less than any given force.[100]

Heron proves this statement by taking arbitrarily close approximations to the horizontal plane with decreasing slope.

Heron's *Mechanics* has many overlaps with the homonymous pseudo-Aristotelian work. In particular, the exposition of the parallelogram rule for composing displacements is so similar in the two works that we may suppose a common source.[101] The pseudo-Aristotelian *Mechanics* follows this exposition with the remark that a point in uniform circular motion is subjected simultaneously to two motions: one "according to nature" (κατὰ φύσιν) along the tangent and one "against nature" (παρὰ φύσιν) directed toward the center.[102]This suggests that, as in Plutarch's dialogue, only linear motions were regarded as "according to nature". Unfortunately, the text of this work is corrupt and the quantitative analysis is not always clear.

Any theory based on a principle of inertia must allow as a consequence that the same gravity can generate different motions (depending on initial velocity). Plutarch, for the water near the center of the earth, does list three possible motions for the same body subjected to the same *fora* toward the

[97]Democritus postulated that atoms have a continual motion devoid of cause (transmitted by Cicero, *De finibus*, I, vi §17). The mid-nineteenth century historian of philosophy Eduard Zeller, not having assimilated the principle of inertia, misunderstood Democritus' idea, and his enormous influence on later historiography helped perpetuate the incomprehension. The matter is clearly explained in [Enriques, de Santillana], pp. 147–150.

[98]For example, Sextus Empiricus, *Adversus grammaticos* (= *Adv.~mathematicos* I), §156: "It is characteristic of the vacuum not to offer resistance" (κενοῦ γὰρ ἴδιον τὸ μὴ ἀντιτυπεῖν).

[99]Heron, *Mechanica*, I, §§20–21.

[100]Heron, *Mechanica*, I, §20.

[101]Heron, *Mechanica*, I §8; Pseudo-Aristotle, *Mechanica*, 848b:14–30.

[102]Pseudo-Aristotle, *Mechanica*, 849a:14–17.

center: rest, uniform circular motion around the center and unending oscillation about the center. As we know now, these are indeed three possible motions for a body in the conditions considered. In the case of a boulder, too, Plutarch mentions not one but two possibilities: rest and oscillation.

Another very interesting statement made by Plutarch is that the moon moves faster when closer to the earth.[103]

The whole passage suggests that Plutarch's source taught an inertial theory of dynamics where what we call force (and gravity in particular) does not determine motion uniquely, but only the variation in motion. Therefore it is valuable from a history of science perspective to investigate what that source may have been.

Lamprias, the *De facie* character who ridiculed the theory of the thrust toward the center, had earlier told Apollonides (who typifies "mathematicians" in the dialogue): "the deviations of visual rays are not in your purview nor in that of Hipparchus."[104] Thus Hipparchus is singled out as a polemical target in the course of the same antiscientific polemic where the supposed weirdnesses of our dynamical theory are highlighted. This suggests the possibility that he was Plutarch's source for the theory.

This conjecture gains plausibility in the light of several considerations. First, the *De facie* alludes to other results certainly due to Hipparchus, such as the observation that the moon has a measurable parallax[105] and numerical data from the Hipparchan lunar tables.[106]

Second, the conjecture that the theory was due to Hipparchus is consistent with the distribution and small number of the sources that hint at it. It seems not to have been known to Philo of Byzantium when he wrote the *Belopoeica*,[107] in the late third century B.C., so it probably belongs to the second century. It also seems to have been unknown to many Alexandrian scholars of the imperial period, which suggests that it had no time to sink in before the interruption of scientific activities at Alexandria in 145–144 B.C. (and in particular that it was not documented in the Library).

We have seen that Heron is one of the few authors whose writings contain ideas akin to those that appear in our Plutarchan passage. This is significant because there is other evidence that he may have known Hipparchan works that were not available to other Alexandrian scholars.[108] That may have been a result of his well-attested familiarity with the Mesopotamian scientific tradition: since we know of scholarly exchanges between Rhodes, where Hipparchus worked, and the neighboring Seleucid kingdom,[109] it may be suspected that the astronomer's works appearing after the pogrom of 145–144 were preserved in the East better than in Egypt.

The theory we are considering, by unifying the study of the motion of weights with that of heavenly bodies such as the moon and treating both as particular cases of motion subject to a *fora* toward the center, performs a synthesis of astronomy and ballistics. It is not surprising that such a synthesis should be performed by Hipparchus, the greatest astronomer of his time and a denizen of Rhodes, then the main center of ballistic studies.[110]

Other evidence, in my opinion conclusive, is provided by Simplicius. He tells us that Hipparchus wrote a work on gravity titled *On bodies thrust down because of gravity*.[111] This is the same terminology used several times by Plutarch: the boulder, the stream and so on are said to be "thrust down" because of gravity. And though Plutarch introduces the theory with the words "thrust toward the center" rather than "thrust down", the equivalence of the two is implied by the very fact that later in the dialogue there is an extensive critique of the identification of a single incorporeal point (the center) with "down" (κάτω).[112]

Before we analyze Simplicius' testimony, we note that already Aristotle recognized the acceleration of falling weights, and that from other lines of Simplicius we know that Strato of Lampsacus made decisive progress in understanding the effect of gravity.[113] He noticed that acceleration is easily provable (visualizable, even) in the case of a trickle of water in free fall: after going down for a while as a column, the water breaks up into drops. Simplicius, alas, does not tell us by what argument Strato deduced from this that the water is gaining speed. From the viewpoint of modern physics the separation into drops comes from the cross-section of the trickle shrinking below a certain critical value, and the decrease in the cross-section is what's equivalent to the increase in velocity (the rate of flow being of course constant). It is likely that Strato used just this

[103]Plutarch, *De facie...*, 933B.
[104]Plutarch, *De facie...*, 921D.
[105]Hipparchus measured the lunar parallax (Ptolemy, *Almagest*, V, v, 369). In the *De facie*, as Neugebauer notes, lunar parallax is mentioned just before Hipparchus is named (921D). The passage was mistranslated by Cherniss, but the meaning was put to right in [Neugebauer: HAMA], p. 661.
[106][Flacelière], p. 217; [Cherniss], p. 145.
[107]The book discusses the motion of weights, but there is no trace of a reference to the principle of inertia.

[108]One such work may have been a source for Heron's *Dioptra*; see note 24 on page 272.
[109]For example, Hipparchus used Mesopotamian astronomical data.
[110]Philo of Byzantium, *Belopoeica*, 51 = [Marsden: TT], p. 108.
[111]Περὶ τῶν διὰ βαρύτητα κάτω φερομένων (Simplicius, *In Aristotelis De caelo commentaria*, [CAG], vol. VII, 264:25–26). That this work of Hipparchus, unknown to many Alexandrian scholars, was familiar to Simplicius and (as we know from the latter) to Alexander of Aphrodisias, both coming from Asia Minor, corroborates the conjecture that some Hipparchan works fared better in the East than in Egypt.
[112]Plutarch, *De facie...*, 925E–926B.
[113]Simplicius, *In Aristotelis Physicorum libros commentaria*, [CAG], vol. X, 916:12–27.

argument to deduce the increase in velocity from the decrease in cross-section.[114] Simplicius mentions Hipparchus' theory in the case of an object thrown straight up. The sequence of events are clearly described: first an upward motion with decreasing velocity, then a downward motion with increasing velocity. At this point Simplicius adds:

> [Hipparchus] recognizes the same cause also for bodies let fall from above.[115]

Thus Hipparchus' theory treated identically the motion of a body thrown up and of one dropped from on high. This unification suggests that the astronomer, besides using the same terminology used by Plutarch, gave it the same meaning. Only a theory that recognizes as essential variables not just velocities but variations in velocity can unify the treatment of both motions.

Simplicius offers us another highly valuable insight:

> Hipparchus contradicts Aristotle regarding weight, as he says that the further something is, the heavier it is.[116]

Hipparchus' statement seems inexplicable if regarded as referring to small displacements from the surface of the earth: in normal experience, things don't get heavier the higher up they are. If anything, the fact that weights accelerate as they fall may suggest the opposite idea, which was indeed maintained by Aristotle. Therefore the weight change that Hipparchus

[114]That the cross-section shrinks cannot fail to be noticed by anyone who has observed with any attention a water trickle in free fall. The conjecture that Strato connected the formation of drops with the decrease in cross-section, coming up with an explanation similar to the "modern" one, seems very plausible in the light of three circumstances. First, the notion of flow rate, on which the explanation is based, is used in a completely analogous way by Heron in his *Pneumatics*, a work for which Strato was probably one of the main sources (see page 133). Next, the thirteenth century *Liber de ratione ponderis*, attributed to Jordanus Nemorarius and certainly based on classical sources, explicitly makes the link between acceleration and decrease in cross-section, concluding with "and so it breaks up [into drops]" (Jordanus Nemorarius, *Liber de ratione ponderis*, proposition R4.16, in [Moody, Clagett], pp. 224–227). Finally, the relationship established by Strato between drop formation and acceleration seems hard to explain in the absence of the theoretical notions just discussed, as shown by the modern historians of philosophy who have not succeeded in grasping Strato's argument (see, for example, [Rodier], p. 64, note 2).

[115]τὴν αὐτὴν δὲ αἰτίαν ἀποδίδωσι καὶ τῶν ἄνωθεν ἀφιεμένων (Simplicius, *In Aristotelis De caelo commentaria*, [CAG], vol. VII, 265:3–4).

[116]περὶ δὲ τοῦ βάρους τὰ ἐναντία τῷ Ἀριστοτέλει φησὶν ὁ Ἵππαρχος· βαρύτερα γάρ φησι τὰ πλέον ἀφεστῶτα (ibid., 265, 9–11). This passage has been cited many times (in [Clagett: SM], for example) in the translation given in [Cohen, Drabkin], which reads "… bodies are heavier the further removed they are from their natural places" (p. 210). The last four words, added without bracketing, have no correspondence in the original. It is true that specifying what the bodies are further from makes the sentence more readable. It is also true that the reference point intended by Simplicius is the center of the earth, which for him is indeed the natural place of a weight. None of this justifies putting in Hipparchus' mouth the Aristotelian notion of natural place.

had in mind becomes manifest only when the distance to the center of the earth changes appreciably. But if we think of bodies far away from the earth, the statement is even odder. The only way to make it comprehensible is to suppose that Hipparchus meant the weight of bodies inside the earth, recognizing that it decreases as the body nears the center. (It can be seen from simple symmetry considerations that the weight vanishes at the center, and it is natural to think that a small displacement from the center will affect the weight but little.[117])

We must conclude that Hipparchus' work dealt also with the motion of weights inside the earth, covering distances not negligible with respect to the distance to the center. This is precisely the situation of the boulder and the water in Plutarch's passage. The other example Plutarch gives is the moon, and Hipparchus was certainly the main source on the moon for this dialogue;[118] his name is mentioned explicitly (in connection with optics). All this makes it very probably that the "weirdnesses" ridiculed by Lamprias come from Hipparchus' work, possibly through intermediates.[119]

10.7 A Passage of Seneca

In 62 or 63 A.D., about 90 years before the *Almagest*, Seneca wrote his *Natural questions*, where we read:

> Of these five stars, which display themselves to us and which pique our curiosity by appearing now here now there, we have recently started to understand what their morning and evening risings are, where they stop, when they move on a straight line, why they move backward; we learned a few years ago whether Jupiter will rise or sink or retrograde (for this is the name given to its backward movement).[120]

The five stars are of course the planets. Shortly before, Seneca had said about the fixed stars that the Greeks had started to name them "less than

[117]Hipparchus may have used a more sophisticated reasoning than that. The reduction in weight as the center of the earth is approached is naturally predicted by any theory that postulates that gravity is a mutual attraction between bodies. In Section 10.7 we will analyze certain testimonia that suggest that such a theory did arise in Hellenistic science.

[118]See notes 105 and 106 immediately above.

[119]Further evidence for a Hipparchan planetary dynamics comes from Roman sources; see Sections 10.7 and 10.8.

[120]"Harum quinque stellarum, quae se ingerunt nobis, quae alio atque alio occurrentes loco curiosos nos esse cogunt, qui matutini vespertinique ortus sint, quae stationes, quando in rectum ferantur, quare agantur retro, modo coepimus scire; utrum mergeretur Iupiter an occideret an retrogradus esset (nam hoc illi nomen imposuere cedenti), ante paucos annos didicimus." (Seneca, *Naturales quaestiones*, VII, xxv §5).

fifteen hundred years ago". It is clear, therefore, that the "few years" of the present passage should not be taken literally, but in the context of a very long time scale.

It is generally believed that Seneca's book drew from essentially one source dating from the first century B.C. Therefore in the first century B.C. there was still the living memory of a new theory, through which scientists had "started to understand" planetary motions. The lack of astronomers in the period between Hipparchus and Seneca makes it very unlikely that the theory that Seneca refers to was unknown to Hipparchus. Thus Seneca's passage strengthens the case that, Ptolemy notwithstanding, Hipparchus had at least started to fashion a new planetary theory.

Seneca tells us more about this "new" theory in the sequel:

> There are some who have told us: "You are mistaken in thinking that any star stops on its track or turns backward. Heavenly bodies cannot be detained or turned back; they forever move forth; as they once were sent on their way, so they continue; their path does not end but with their own end. This eternal work has irrevocable motions: if ever [these bodies] stop, they will fall upon one another, for it is constancy and evenness that preserve them now. Why is it then that some of them seem to turn back? The appearance of tardiness is caused by the intervention of the sun (*solis occursus*), and by the nature of their paths and circles, so arranged that for a certain time they deceive the observer: just as ships, though moving under full sail, appear stationary."[121]

Planets cannot reverse their motion: heavenly bodies are kept in their orbits by the regularity of their motion; they cannot stop because, if they did, they would fall on one another (*alia aliis incident*). It sounds like the same idea presented more pointedly by Plutarch in the *De facie* for the case of the moon, but with a significant difference: gravity here seems to be viewed as a mutual action between bodies.

The sling argument mentioned by Plutarch, which in modern language we might phrase as the cancellation between gravity and centrifugal force, can explain quite easily the motion of the moon around the earth, at least if, as suggested in the Plutarchan passage, we just want an approximate

[121]"Inventi sunt qui nobis dicerent: 'Erratis, quod ullam stellam aut supprimere cursum iudicatis aut vertere. Non licet stare caelestibus nec averti; prodeunt omnia: ut semel missa sunt, vadunt; idem erit illis cursus qui sui finis. Opus hoc aeternum irrevocabiles habet motus: qui si quando constiterint, alia aliis incident, quae nunc tenor et aequalitas servat. Quid est ergo cur aliqua redire videantur? Solis occursus speciem illis tarditatis imponit et natura viarum circolorumque sic positorum ut certo tempore intuentes fallant: sic naves, quamvis plenis velis eant, videntur tamen stare' ". (Seneca, *Naturales quaestiones*, VII, xxv §§6-7).

description involving circular orbits. But the extension of this argument to planets runs into a serious obstacle: How it is that at the moment of a planetary station, when presumably there should be no centrifugal force, the planet does not start to fall on the earth? This is the reason for the interest in planetary stations and the gist of Seneca's question: how is it that some planets seem sometimes to reverse their motion, if heavenly bodies cannot stop or turn back without starting to fall upon one another?

Seneca explains retrogressions by alluding to combinations of several circular motions (*natura viarum circolorumque sic positorum...*). The retrograde motion of planets, arising for a while from such combinations, is but a deception (*ut certo tempore intuentes fallant*); in their real motion, planets never turn backwards.

Possibly, Seneca's source explained that the apparent motion of planets (their motion relative to the earth) results from the combination of two circular orbits, both centered at the sun and traveled respectively by the earth and the planet, while the "real" motion of the planet happens on the second of these orbits. This explanation accounts well for the occurrence of retrogressions, and, having been proposed by Aristarchus of Samos, at the time of Seneca's source it would have been known to Hellenistic astronomers for about two centuries. Of course, Seneca's words about "paths and circles" by themselves might admit other interpretations, and might indeed suggest an epicycle-based geocentric theory. But I think that the heliocentrism of Seneca's source can be discerned clearly enough from the following considerations.

First, heliocentrism is able to solve the dynamical problem mentioned by Seneca: the sling argument can be applied to planetary motion exactly as to lunar motion, by making the sun, rather than the earth, be the center. It is hard to see what the solution would be in a geocentric framework.

Next, a Ptolemaic-type epicycle theory would explain the motion of the planets without any reference to the sun, whereas the words *solis occursus* in our passage show that the sun played a role in the explanation given by Seneca's source.

Thirdly, Seneca's statement that planetary stations are just an illusion and the ship analogy imply that what is regarded as the true motion (in which the planets do not retrogress) is not the motion with respect to the earth. Indeed, the ship topos is used to illustrate the relativity of motion not only here but in several other ancient passages: Lucretius develops it quite explicitly.[122]

Finally, we know that Seneca's source envisaged the possibility that the

[122]"Qua vehimur navi, fertur, cum stare videtur; / quae manet in statione, ea praeter creditur ire. / et fugere ad puppim colles campique videntur, / quos agimus praeter navem velisque vola-

earth moves, because in an earlier passage Seneca mentions the rotating-earth explanation for the daily motion of the skies.[123]

Thus it seems very likely that Seneca's source applied to the revolution of planets around the sun the same idea mentioned by Plutarch in the case of the moon: the cancellation between gravitational attraction and centrifugal force. Because heliocentrism can explain planetary motion in this way, overcoming the difficulty inherent in planetary stations, the result was a dynamical justification of heliocentrism. Since Seneca's source must be talking about ideas from Hellenistic astronomy of the second century B.C. (the time of Hipparchus), the conclusion reached is consistent with the thesis that the *De facie* passage discussed in the preceding section was based on Hipparchus' book on gravity.[124]

10.8 Rays of Darkness and Triangular Rays[125]

Although we have no true astronomical works from the period between Hipparchus and Ptolemy, the Roman literature of the first centuries B.C. and A.D. contains, besides the Seneca passages, at least two expositions of astronomical arguments with some pretense of systematicity; they occur in Book II of Pliny's *Natural history* and in Book IX of Vitruvius' *De architectura*. Since astronomical activity in Rhodes went on after it ceased in Alexandria in 145 B.C., and since Rome maintained important trade and cultural ties with Rhodes down to the mid first century B.C., it would not be too surprising to find in these works traces of ideas of Hipparchus unknown to Ptolemy — the more so because, when Pliny mentions the foreign sources for Book II of the *Natural history*, Hipparchus heads the list and garners lavish and enthusiastic praise. In fact Pliny regrets that no one was left to carry on the astronomer's scientific legacy.[126] He briefly discusses an "immensely ingenious" theory capable of explaining the motion of planets, and concludes by saying: "This is the theory of the outer planets, and it is harder than the rest; never before me has it been divulged."[127]

mus" (*De rerum natura*, IV, 387–390). This passage parallels Seneca's, as shown in [Russo: Lucrezio] based on the respective contexts.

[123]Seneca, *Naturales quaestiones*, VII, ii §3.

[124]It has been generally assumed that this work of Hipparchus on gravity bore no relation to his astronomical interests. But already Heath wrote: "It is possible... that even in this work Hipparchus may have applied his doctrine to the case of the heavenly bodies" ([Heath: HGM], vol. 2, p. 256).

[125]The material in this section is drawn largely from [Russo: Vitruvio] and [Russo: Hipparchus].

[126]Pliny, *Naturalis historia*, II §95. Vitruvius, too, names Hipparchus (*De architectura*, IX, vi §3).

[127]"Haec est superiorum stellarum ratio; difficilior reliquarum et a nullo ante nos reddita" (II §71). This discussion of the motion of outer planets starts with "aperienda est subtilitas immensa et omnes eas complexa causas" (II §67).

Together with his later remarks about Hipparchus, this makes one think that the planetary theory that Pliny is "divulging" is that of Hipparchus. It is very unlikely that Pliny would have dared to read astronomer's works directly, but his statements prove that a planetary theory attributed to Hipparchus was explained in books that were still available in Rome in the first century A.D.

As to the contents of the theory, it is admittedly not easy to reconstruct it based on the pages of the *Natural history*. But it is clear that Pliny's sources include at least one that is scientific, because some of the data mentioned, such as the periods of planetary revolutions, are reasonably accurate.

Pliny, like Seneca, says that planetary stations are merely appearances,[128] which suggests that his source too espoused heliocentrism. Vitruvius adds a bizarre explanation (which he says he does not agree with) of the retrogressions and stations of the outer planets Mars, Jupiter and Saturn, involving a "darkness" that certain people supposedly claimed was caused by the sun.[129] It is not very likely that someone actually claimed that the sun sends out rays of darkness. It seems more plausible that Vitruvius misunderstood the heliocentric argument transmitted by Seneca and alluded to by Pliny. The source may have contained some remark (similar to the one found in Seneca) to the effect that the motion of the planets is obscured by the sun — that is, cannot be perceived directly from earth — an explanation which Vitruvius could not grasp and took to mean a literal darkness (*obscuritas*). The heliocentrism of Vitruvius' source is corroborated by his statement that "the planets Mercury and Venus nearest the rays of the sun move round the sun as a centre".[130]

Vitruvius' most interesting passage is probably this:

> ... the sun's powerful force attracts to itself the planets by means of rays projected in the shape of triangles; as if braking their forward movement or holding them back, the sun does not allow them to go forth but [forces them] to return to it...[131]

Pliny has a parallel passage:

[128]"Hoc non protinus intellegi potest visu nostro, ideoque existimantur stare, unde et nomen accepit statio" (Pliny, *Naturalis historia*, II §70).

[129]"Id autem nonnullis sic fieri placet, quod aiunt solem, cum longius absit abstantia quadam, non lucidis itineribus errantia per ea sidera obscuritatis morationibus impedire. Nobis vero id non videtur." (*De architectura*, IX, i §11). Vitruvius then contradicts the explanation on the grounds that the sun sheds light, not darkness, and that the outer planets are visible even while retrogressing.

[130]Vitruvius, *De architectura*, IX, i §6, Gwilt translation.

[131]"solis impetus vehemens trigoni forma porrectis insequentes stellas ad se perducit et ante currentes veluti refrenando retinendoque non patitur progredi sed ad se regredi..." (Vitruvius, *De architectura*, IX, i §12).

> [The planets] are struck in the position we have said and are prevented by a triangular solar ray from following a straight path and are lifted on high by the burning force [of the sun]. [132]

The overlaps between the two texts are significant above all in view of the matching contexts: both authors are explaining the motion of the three outer planets. Both passages clearly display the idea of the sun's pull on the planets; Pliny includes the particularly interesting idea that this pull has the effect of preventing the planets from moving on a straight line.

The continuation of the Vitruvius passage is very obscure. It reads:

> and to be in the *signum* of the other triangle. It may be asked why the sun in its hotness pulls [the planet] back in the fifth *signum*, rather than in the second or third, which are closer. Therefore I will explain why this seems to happen. Its rays extend out into the universe using lines in the shape of a triangle with equal sides. This however does not happen either more or less [than] to the fifth *signum* from it. Therefore. . . [133]

To understand this passage it is essential to understand the sense in which Vitruvius, and above all his source, used the term *signum*, which we have not translated. It is generally assumed that *signum* means a zodiacal sign and this interpretation is supported by the fact that elsewhere, including in earlier passages in this chapter, Vitruvius uses the word in that sense. This might seem to settle the question, but notice that in the previous passage, about the strange "darkness", Vitruvius demonstrates difficulty in understanding his astronomical sources. Immediately after the passage just quoted he curtails the argument by appealing to Euripides' authority, another sign of his lack of familiarity with the subject. Moreover in this passage the words "lines" and "triangle"[134] clearly indicate

[132]"Percussae in qua diximus parte et triangulo solis radio inhibentur rectum agere cursum et ignea vi levantur in sublime" (Pliny, *Naturalis historia*, II §69).

[133]"in alterius trigoni signum esse. Fortasse desiderabitur, quid ita sol quinto a se signo potius quam secundo aut tertio, quae sunt propiora, facit in his fervoribus retentiones. Ergo, quemadmodum id fieri videatur, exponam. Eius radii in mundo uti trigoni paribus lateribus formae liniationibus extenduntur. Id autem nec plus nec minus est ad quintum ab eo signum. Igitur. . . " (Vitruvius, *De architectura*, IX, i §13). The penultimate word quoted is a generally accepted emendation of the manuscripts' "signo", but the text as received also has much the same translation ("to the fifth *signum* from that one", i.e., the one where the sun is).

[134]*Trigonum* is the Latin transliteration of the Greek word for triangle. In Vitruvius the word is generally assumed to mean the astrological *trine* or *trigon*, a configuration where the sun and a planet appear separated by 120° in the sky. Under this interpretation the passage is rendered essentially meaningless; there is no explanation for the trigonal rays of the sun, which according to Pliny prevent the planet from moving on a straight line, nor for Vitruvius' words "with equal sides". This astrological interpretation of *trigonum*, like that of *signum*, may even reflect Vitruvius' own thinking, but not that of his source.

a geometric construction. Thus we may suppose that Vitruvius is trying, with difficulty, to convey a geometrical argument from a Greek scientific source.[135] If this is the case, one hint to the meaning of Vitruvius' source may be gained by literally translating the more obscure terms into Greek. If we do this to the expression "second *signum*", the Latin term *signum* becomes *semeion* (σημεῖον), while the ordinal is indicated in Greek by the letter β; likewise "third" and "fifth" become the letters γ and ε. The Greek term *semeion*, as we know, meant not only a sign but also (from Euclid on) a point,[136] and the letters of the alphabet were used not only for ordinals, but to indicate the points of a geometric construction. In a context marked by the presence of triangles and lines, the literal translation into Greek of the Latin expressions "secundum signum", "tertium signum", "quintum signum" has a clear meaning: "point B", "point Γ", "point E" in a geometric construction. Could this have been the meaning, misunderstood by Vitruvius, of the expressions in the Greek original?

When he talks about diagrams that he does understand, Vitruvius uses expressions such as "where the letter A will be", or "let's draw a line from the letter S".[137] Thus he uses the letter not as a label for a point, but to indicate the actual place in the drawing where the letter is written. In one passage where *signum* is used in a sense that seems to correspond approximately to the meaning "point", he writes: "from this *signum* and letter C, let's draw a line to the center, where the letter A is".[138] "*Signum* and letter": it is clear that here too, Vitruvius does not use *signum* to mean a point, but simply a sign next to which is a letter. The absence of the abstract geometric notion of a point in Vitruvius' work should neither surprise us nor be held against him. Never until his time had a geometrical work been written in Latin (see beginning of Section 8.2), so it was not easy to express in that language the geometric notion of a point.

The difference between the abstract meaning that *semeion* had to the Greek mathematicians and the concrete notion of *signum* (a sign on the paper) that a Roman writer like Vitruvius may substitute for it can be important in the interpretation of passages of astronomical arguments. The same Latin term *signum* can in fact mean a sign of the zodiac (a meaning expressed in Greek by a different word, ζῴδιον). In all cases the Latin term

[135]One example of the scientific reliability of this source is the value Vitruvius gives for the period of revolution of Saturn: 29 years and 160 days. This is closer to the true value (29 years and 167) than the value Ptolemy would later adopt in the *Almagest* (29 years and 182 days).

[136]See page 181.

[137]"Ubi erit littera A" (Vitruvius, *De architectura*, IX, vii §2); "ab littera S ducatur linea" (Vitruvius, *De architectura*, IX, vii §6).

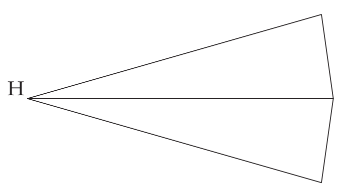
[138]"Ab eo signo et littera C per centrum, ubi est littera A, linea perducatur" (Vitruvius, *De architectura*, IX, vii §3).

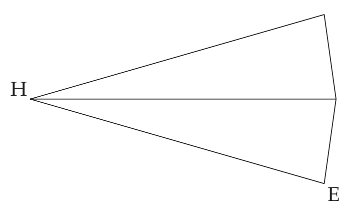
maintained a concrete meaning; of course the context would generally allow one to tell which *signum* was meant: for example, a sign on paper or a sign of the zodiac. But we see the problems that could arise in the case of a Greek scientific work making a statement about a σημεῖον in astronomical space. A Roman reader might think that, since in Latin a *signum* on the sky is a zodiacal sign, the same is true of a *semeion* on the sky; and even if he was aware that this is not so, he would lack the linguistic tools needed to correctly translate the text into Latin. As to the association between letter and *signum*, since to Vitruvius it is at most a matter of spatial proximity, it cannot be easily extended to the "signs" in the sky, since there are no letters there. Since in Greek the letters of the alphabet are used also as ordinal numerals, it would be natural in this case to assume this latter meaning for the letters and to intrepret expressions such as *semeion* B ("point B") as "the second sign".

Assuming that the source really did refer to the points in a geometric construction, can the diagram underlying Vitruvius' passage be reconstructed?

Note first that Vitruvius mentions triangles with equal sides (*paribus lateribus*). This might seem to refer to equilateral triangles, but it can equally mean an isosceles triangle, especially since the Greek adjective "isosceles" (ἰσοσκελής) means "equal-sided" and Vitruvius may have translated it componentwise instead of adopting the Greek word itself as became customary later. The surrounding sentence, "its rays extend out … using lines in the shape of a triangle with equal sides", suggests triangles whose equal sides are formed by rays originating in the sun, and so favors the meaning "isosceles". More than one such triangle, each sharing one ray-side with the next, would all have the same side length. Thus the geometric construction might involve adjacent isosceles triangles fanning out from the sun; note that the rays of the sun (*radii* in Latin) are also *radii* of a circle, since they are all of the same length.

The use of *alter* ("alterius trigoni") indicates that there were two triangles being considered. Thus we obtain the figure on the left, where H marks the position of the Sun.

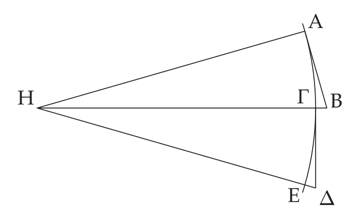




The last position considered for the planet is, in Vitruvius' words, at the fifth *signum*, which is a *signum* in the second triangle. In our interpretation,

the original meaning of this statement was that the planet, at the end of the motions described, would be at the point E, a vertex of the second of the two isosceles triangles (a *signum* of a triangle being simply one of its vertices). Thus we get the figure on the right on the previous page.

We must account for four other points in the construction, A, B, Γ and Δ. It is natural to think that two of these (together with E) were the triangle vertices other than H. Note that, since these are points on a circumference centered at the sun, their interpretation as successive positions of a planet is in accord with the heliocentric view that we seem to detect in Vitruvius' source. There remain to determine two other points, corresponding to two other positions considered for the planet. Vitruvius' statement that the sun forces the planet to return to it ("ad se regredi") in the fifth *signum* suggests that the last position considered before E (and presumably denoted with Δ) lay on the line HE, beyond the point E. Pliny, who seems to be using the same source, says that the sun prevents the planet from moving on a straight line; thus we may suppose that point Δ represented the position the planet would occupy if the sun were not pulling it. Thus Δ is the intersection of the half-line HE with the imagined extension of the previous bit of the orbit along a straight line (the tangent). The presence of two isosceles triangles suggests that the whole construction could be explained by repeating the same procedure twice (as usual in the exposition of iterative procedures by Greek mathematicians). Thus we get:



The meaning of the figure is clear. It shows how a planet's orbit (supposed circular) can be constructed as a succession of small strokes, each of which is obtained by composing two simultaneous displacements: one along the tangent to the circle (this would be the actual movement of the planet if, in the absence of the sun, it could continue on a straight line, as Pliny says) and another, directed toward the sun. The figure illustrates the notion transmitted in qualitative form by Plutarch: indeed, the motion of the planet arises as the result of a series of "thrusts toward the center" We have already seen that the technical tool of vector addition for displacements is present in Heron and in the pseudo-Aristotelian *Mechanics*, and indeed it is used in this latter work to explain how a uniform circular motion can be regarded as a continuous superposition of a displacement "according to nature", along the tangent, with one "contrary to nature", directed toward the center.[139] (We will see on page 373 that this description of motion under a central force was used again by Newton.)

The figure we have drawn explains the origin of the otherwise inexplicable statement by Pliny and Vitruvius that the planets are prevented from moving on a straight line by "sun rays shaped like triangles".

The way in which Vitruvius tries to explain what's special about the fifth *signum* ("id autem nec plus nec minus est ad quintum ab eo signo") seems to confirm that a *signum* meant a point in the source: the exact equality of distance implied by the words "neither more nor less" seems more appropriate to describing the position of a point than that of a zodiacal sign. Moreover Vitruvius' sentence matches the procedure throught which one obtains point E in the proposed construction: namely, as the point on the half-line HΔ that makes the side HE exactly equal ("nec plus nec minus") to the other side, HΓ, of the triangle.

Of course, Vitruvius' statement that the second and third *signa* are closer to the sun than the fifth is false when applied to points B, Γ and E of the preceding figure. Within our proposed reconstruction, this can be seen as a natural consequence of Vitruvius' error. If he interpreted the points of the geometric construction as signs of the zodiac and the letters used to denote them as ordinal numerals, he might have thought that the zodiacal signs were ordered starting from the one where the sun is.

10.9 The Idea of Gravity after Aristotle

Aristotle's geocentrism is closely connected to his notion of gravity. The center of the earth, which is also the stationary center of the cosmos, is the "natural place" for which all "heavy objects" yearn. "Light" objects, on the other hand, tend to go up (that is, away from the center), thanks to their different nature.[140] This view, like many others held by Aristotle, held sway in late Antiquity and the Middle Ages.

At the end of the second century B.C., judging from the conclusions reached in the previous sections, gravity stood on a very different footing. In this and following sections we will follow clues to the evolution of this idea in the third and second centuries B.C., interpolating logical steps to fill the gaps between the few extant testimonia.

[139] See page 289, including notes 101 and 102.

[140] Aristotle, *Physica*, IV, i; *De caelo*, I, iii; *De caelo*, IV. Plato had already put forth substantially equivalent ideas (above all in *Timaeus*, 62c–63e).

Aristarchus' heliocentrism was of course in explicit opposition to Aristotle's ideas. Did the critique of geocentrism also lead to a critique of the Aristotelian view of gravity?

Aristotle's theory had become unsustainable after the development of Archimedean hydrostatics. First of all, Archimedes had shown that there is no such thing as levity (lightness) as the opposite of gravity (weight). This eliminated one important rationale for regarding heavenly bodies as essentially different from earthly ones. Even more importantly, in the first book of *On floating bodies* Archimedes showed that simple postulates on gravity (essentially that gravity is a spherically symmetric pull toward the center of the earth, as Aristotle thought), together with simple postulates about fluids, necessarily imply the spherical shape of the oceans (in rest conditions). This was a momentous step: it showed that the spherical shape is not something that must be accepted as "natural" because of its perfection, being instead derivable from a few hypotheses on elementary forces. It was, for the first time, a deduction of a feature of the actual world from physical laws. There is no doubt that Archimedes' demonstration was also used to explain the form of the earth as a whole (though this extension is not discussed in the *On floating bodies*, whose subject matter does not demand it). Indeed, the idea that the earth was originally fluid is reported in several sources, and in particular by Diodorus Siculus, who explicitly relates the earth's shape to gravity.[141] Archimedes' theorem must have helped corroborate it, showing that the actual shape of the earth was just what would be expected from a fluid mass. This is a good example of how exact science can connect apparently distant subjects through logical ties: Archimedes' theorem not only cast light on the earth's geological past, but also had important astronomical and cosmological consequences.

Once gravity is used to explain the roundness of the earth, the next step is inevitable, namely explaining in the same way the obvious spherical shape of the sun and of the moon. That shape could not but be viewed, by anyone who had read Archimedes' treatise, as indirect evidence that these bodies, too, have a gravitational pull — not toward the center of the earth, of course, but toward their own center. We do not know who the first person to draw this conclusion was. Perhaps it was Archimedes himself, since he was very interested in astronomy and took Aristarchus' heliocentric hypothesis as a possibility at least, and so had no reason to re-

[141] Diodorus Siculus, *Bibliotheca historica*, I, vii §§1–2. The idea that the earth was originally fluid might have arisen from geological studies. On this subject our main source is Strabo, who reports ideas of Posidonius (*Geography*, II, iii §6). Seismic and volcanic phenomena, too, were as far back as Eratosthenes a nexus for the study of transformations of the earth's crust in geological time (Strabo, *Geography*, I, iii §4).

strict to the earth alone the causal relationship, which he so clearly pointed out, between sphericity and gravity. What cannot be doubted is that the conclusion was drawn by someone, since Plutarch says explicitly:

> Just as the sun attracts to itself the parts of which it consists, so does the earth. . . [142]

After the theoretical reason for the spherical shape of the earth, sun and moon was understood, the same shape was assumed to be shared by other celestial bodies, though it could not be observed directly.[143]

At this point the Aristotelian picture (which later became the Ptolemaic one) crumbled from inside. The universe no longer has a hierarchical structure, centered on the earth and based on a distinction between earthly and celestial bodies; it is made up of so many worlds, equivalent in important ways to one another. The multiplicity of worlds is closely linked with the rejection of the sky of fixed stars, with the notion of an infinite universe and with the relativity of movement.[144] These ideas, too, whose memory was to play such an important role in the early modern age, seem to have had (apparently prescientific) forerunners among the pre-Socratic philosophers.[145]

To return to gravity, the train of thought outlined above allows two possibilities: either one thinks that gravity has so many independent centers, one for each heavenly body, each capable of attracting things that belong to it and no others; or one thinks that there is also an attraction between different heavenly bodies. The first possibility was certainly mooted, since it is explicitly advanced in the *De facie* by Lamprias, to whom belongs the line of the dialog quoted just above.[146]

The idea of so many gravity centers, each associated with one world and not interacting with the others, while it may explain the shape of heavenly bodies, does not solve the problem of what motions may be hypothesized theoretically. One can ask, in fact: How would a mass move that is far away from all stars? This question, crucial to the establishment of a "dynamical" astronomy and to the formulation of the principle of inertia, is in

[142] ὡς γὰρ ὁ ἥλιος εἰς ἑαυτὸν ἐπιστρέφει τὰ μέρη ἐξ ὧν συνέστηκε, καὶ ἡ γῆ (Plutarch, *De facie quae in orbe lunae apparet*, 924E).

[143] See Cicero, *De natura deorum*, II, xlvi §117; Diogenes Laertius, *Vitae philosophorum*, VII §145. We will return to this question in the next section.

[144] See Section 3.7.

[145] See, in particular, these testimonia about Anaximander: Eusebius, *Praeparatio evangelica*, I, vii = [DG], 579:7–20 = [FV], vol. I, 83:27–40 (Anaximander A10); Stobaeus, *Eclogae*, I, x §12, 122:20 – 123:6 (ed. Wachsmuth) = [DG], 277b:3 – 278b:4 = [FV], vol. I, 85:1–8 (Anaximander A14).

[146] Plutarch, *De facie. . .* , 924D–F. Compare Plutarch, *De defectu oraculorum*, 424E–425C.

fact brought up by Plutarch in the same context of the polycentric theory maintained by Lamprias.[147]

We now turn to the question of whether the possibility was raised of a gravitational interaction between different astronomical bodies.

10.10 Tides[148]

Given that the Mediterranean is almost tideless, it is not an accident that the Greeks became interested in tides primarily when they started sailing the Atlantic and Indian Oceans, starting at the time of Pytheas and Alexander the Great.

The main source of information on the subject is Strabo. He tells us that Eratosthenes, based on his study of tides, criticized Archimedes' conclusion in the first book of *On floating bodies*, and said that the shape of the oceans was not exactly spherical.[149] This a very interesting point, because it establishes a link between the study of tides and that of gravity in the framework of Hellenistic exact science.

Since the Archimedean demonstration of the roundness of the oceans is irreproachable, Eratosthenes knew that sphericity is an inevitable consequence of the assumptions on gravity made by Archimedes. He could have reached a different conclusion only by changing the assumptions: more precisely, he must have eliminated the assumption that the earth's gravity is spherically symmetric. He knew the correlation of tides with the moon (again our source is Strabo, in the continuation of the passage just mentioned), so it seems that he explained the ocean's ebb and flow by abandoning Archimedes' assumption of spherical symmetry and postulating an action of the moon that affected gravity on earth. The most obvious phainomenon that a theory of tides must explain is that oceans rise and fall twice a day, at times that correlate with the position of the moon in the sky. On open shores, high tide comes soon after the moon reaches the middle of its trajectory from horizon to horizon, whether up in the sky or under the earth (these are called the upper and lower transits over the local meridian). Neglecting this lag, now called the *high water interval*, Eratosthenes might have postulated an influence of the moon as follows: the tide is highest at locations and times when the moon is "seen"

[147] Plutarch, *De defectu oraculorum*, 425C–D. Actually Plutarch's subject is the motion of "a rock that some people assume to exist outside the cosmos"; the *theoretical* nature of the problem of the motion of such a mass is clear enough from the context, even if Plutarch cannot grasp it.

[148] The material in this section is drawn in part from [Russo: FR].

[149] Strabo, *Geography*, I, iii §11.

directly overhead or underfoot, near the zenith or the nadir.[150] In any case, an action of the moon extending all the way to the earth must have been hypothesized in the third century B.C. This was accompanied by the idea that the earth also acts on the moon, which was certainly present in Hellenistic astronomy (as we saw starting on page 286, it is documented in Plutarch). Thus the action between the earth and the moon would have been regarded as reciprocal, a significant conceptual leap.

Reciprocity might be thought to hold only for earth and moon, but if extended to other heavenly bodies it would lead to a notion of gravity no longer as an attraction toward one or many centers, but as a reciprocal attraction between bodies. This extension, too, was accomplished, and is attested in the passage of Seneca examined in Section 10.7. It, too, was probably motivated by the theory of tides, as we will now see.

Besides the semidiurnal cycle, tides are subject to a monthly cycle, related to the phases of the moon. *Spring tides* — the tides of highest amplitude in a monthly cycle — occur near the full and new moons, while *neap tides* — of lowest amplitude — occur at the first and third quarters. We have no information on whether Eratosthenes knew the monthly cycle, but the relationship between tides and phases of the moon was probably known empirically to dwellers of ocean shores from very ancient times, and it was articulated before Eratosthenes by Pytheas[151] and perhaps by an earlier Massaliote, Euthymenes.[152]

Ever since Parmenides, in the first century B.C., explained the phases of the moon based on the relative positions of sun, moon and earth, the knowledge of a relationship between tides and phases of the moon could *potentially* mean a recognition of the sun's role in tides.[153] A theoretical explanation of the monthly cycle is in fact not very difficult, if one attributes tides to a major effect of the moon and a minor effect of the sun, and admits that each of these two effects is maximal when the body is at the zenith or the nadir. For then the solar and lunar effects add up when the two bodies are aligned with the earth (Figure 10.3) — that is, at full

[150]In fact this reasoning helps explain the high water interval as well; in the open ocean where there are no obstacles to the flow of water, high tide follows the moon's transit over the meridian very closely, whereas in bays or other shores separated from the open ocean by geographical features, time is needed for the water that has risen in the ocean to flow in. The lag can be of many hours.

[151]Aetius, in Stobaeus, *Eclogae* I, xxxviii, 252:18–19 (ed. Wachsmuth) = [Pytheas/Roseman], 102 = [DG], 383b:4–7. The correlation attributed to Pytheas is incorrect and may reflect a doxographer's misunderstanding of the mariner's accurate observations. In any case it shows that Pytheas related tides with phases of the moon.

[152][DG], 634 = Pseudo-Galen, *De historia philosopha*, lxxxviii.

[153]But this seems to be a non-obvious step. For example, chapter 12 of *Le monde de M. Descartes, ou Le traité de la lumière* contains a clear statement of the relationship of phases of the moon with tides, but explains it using the moon alone.

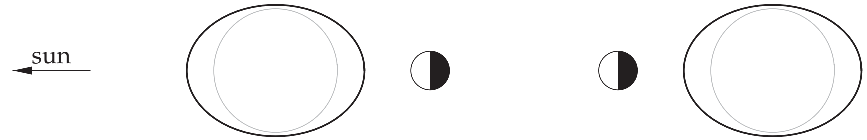


FIGURE 10.3. Spring tides: at new moon and full moon, when there is rough alignment, the effects of moon and sun cooperate: each body individually would cause high tide at the same locations on the surface of the earth, namely those that see the body directly overhead (zenith) or underfoot (nadir).

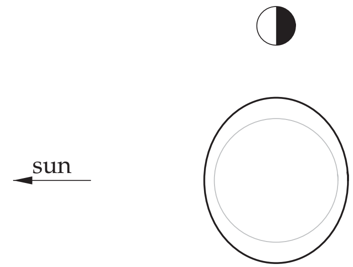


FIGURE 10.4. Neap tides: at quadrature, the sun's effect partly cancels out the moon's, because the sun would be causing high tide at points where the moon is causing low tide.

moon, when they're on opposite sides of our planet, and at new moon, when they're on the same side — while they work at cross-purposes when sun and moon are seen at right angles from the earth (Figure 10.4), the moon's effect predominating but being decreased by the sun's.

Did Greek scientists ever formulate the astronomical explanation for tides just stated, which we call the *lunisolar theory*? Posidonius, in the first century B.C., wrote a work on tides that has perished, but on which we have three important testimonia. One, by Strabo, is well-known:[154] it contains an accurate description of tidal phenomena that includes the daily, monthly and yearly cycles and correlates them with astronomical facts (for instance, it says that the highest tides occur at full moon and new moon); but it does not mention hypotheses or postulates from which one could deduce these phenomena.

Pliny also has a similar phenomenological description, but he adds two important pieces of information. First, just before talking about the moon's

[154]Strabo, *Geography*, III, v §8.

attraction, he declares that the moon and the sun are the causes of tides.[155] Second, after stating that spring tides come with the full and new moons, he says that they actually come a few days later, as also high tides come with a certain delay ("of about two hours") in the semidiurnal cycle.[156]

This material is probably from Posidonius, though Pliny does not say so explicitly.[157] In any case the ideas just discussed would be difficult to understand unless an astronomical theory, filtered out by Pliny, were present in his source. The two statements would then acquire a clear meaning and a precise role in the context of the lunisolar theory. The mention of delays in spring tides is particularly revealing. Pliny does not simply say that spring tides come a few days after the new moon and full moon; he first says they come *at* those times, and then "corrects" that with the statement that they come a few days later. Probably his source first expounded the purely astronomical model, which predicts that high water occurs when the moon crosses the meridian and that spring tides occur when the moon is full or new, and then mentioned the need to correct the first of these statements to take into account the time necessary for water to flow.

The third testimonium is from the early Byzantine period and appears in the work of Priscian of Lydia called *Solutiones ad Chosroem*. A good part of Question VI is devoted to an exposition of Posidonius' work on tides.[158] Most likely Priscian knew this work only indirectly, but his account is the best we have. It adds two elements of crucial importance; namely, that the moon's action is more important that the sun's,[159] and spring tides are greater than neap tides because the sun's action is added to the moon's.[160]

These two statements, plus with those that we can glean behind Pliny's writing, plus Strabo's accurate phenomenological description, characterize without doubt the lunisolar theory. (Although the partial cancelation of the actions of the two bodies during quadrature is not mentioned by

[155] "Et de aquarum natura complura dicta sunt, sed aestus mari accedere ac reciprocare maxime mirum, pluribus quidem modis, verum causa in sole lunaque" (Pliny, *Naturalis historia*, II §212).

[156] "Nec tamen in ipsis quos dixi temporum articulis, sed paucis post diebus ... nec statim ut lunam mundus ostendat occultetve aut media plaga declinet, verum fere duabus horis aequinoctialibus serius" (Pliny, *Naturalis historia*, II §216).

[157] He does mention Posidonius, together with Pytheas, Eratosthenes and Hipparchus, as sources for book II, where this passage is.

[158] [Priscian/Bywater], pp. 69–76.

[159] Horum igitur causas requirens Stoicus Posidonius, ut et per se ipsum explorator factus huiusmodi reciprocationis, discernit magis causam esse eius lunam et non solem ([Priscian/Bywater], 72:10–12).

[160] Unde in plenilunio et coitu extollitur maxime unda, quoniam et lunae tunc magna adest virtus: in plenilunio enim totum eius in terram conuersum a sole illustratur; in coitu autem illuminata desuper a sole aequalem in ea quae sunt in terra uirtutem plenitudini praestat ([Priscian/Bywater], 73:4–8).

Priscian, it must have been in Posidonius; for if the two actions never opposed each other, there would be no way to know which is the greater.)

We thus conclude that an astronomical theory of tides, capable of explaining the monthly cycle by appealing to both a solar and a lunar action, was known to Posidonius. It is harder to establish its origin.

10.11 The Shape of the Earth: Sling or Ellipsoid?

Some histories of geography say that one of the shapes given the earth in Antiquity was a sling. Behind this strange claim are obscure statements in late sources. Agathemerus, writing probably in the third century A.D., says that Posidonius described the *ge* (earth or land) as sling-shaped.[161] The twelfth-century Byzantine archbishop Eustathius writes in a commentary on Homer: "Posidonius the Stoic and Dionysius say that the *ge* is shaped like a sling."[162]

We don't know what Posidonius really said. As to Dionysius, a poet from the third century A.D. (nicknamed Periegetes from the title of his short geographical poem, *Oikoumenes periegesis* or *Sketch of the world*), the expression he uses is *like* a sling (σφενδόνη ἐιοικυῖα).

Commentators have tried to save the day by taking *ge* to mean the dry land, a natural meaning of the word but one that makes the analogy no less enigmatic. As for a sling-shaped earth, the idea is just too odd to have cropped up more than once. Therefore it seems likely that Dionysius' statement is taken from Posidonius, who might have had in mind (given among other things his astronomical interests) the same dynamical analogy between the earth and a sling that Plutarch brings up.[163] Once the earth's motions were forgotten, the analogy was no longer understood, and it is natural that people tried to reinterpret it, straining to see in the continents a sling shape.

Strabo, in the *Geography*, is clearly referring to the whole earth when he makes the ground assumption ("hypothesis") that the *ge* is spheroidal (σφαιροειδής).[164] This term is usually interpreted to mean "exactly like a sphere". Thus H. L. Jones, who adds for good measure: "The spheroid-

[161] Ποσειδώνιος [τὴν γῆν φησι] σφενδονοειδῆ and he continues και μεσόπλατον ἀπὸ νότου εἰς βορρᾶν, στενὴν πρὸς ἕω καὶ δύσιν· τὰ πρὸς εὖρον ὅμως πλατύτερα τὰ πρὸς τὴν ᾿Ινδικήν (Agathemerus, *Geographiae informatio*, I, i).

[162] Eustathius, *In Homeri Iliadem*, vi, 446.

[163] See quotation and discussion starting on page 286. An echo of the original meaning may perhaps still be seen in Dionysius' verses: he writes that the χθῶν (earth or land) is ὀξυτέρη βεβαυῖα πρὸς ἠελίοιο κελεύθους, σφενδόνῃ εἰοικυῖα (Dionysius Periegetes, *Oikoumenes periegesis*, lines 6–7 = [GGM], II, 104–105).

[164] Strabo, *Geography*, I, i §20, and again at II, v §5.

icity [non-sphericalness] of the earth was apparently not suspected until the seventeenth century."[165] Strabo's use of the term can certainly bear that interpretation. But Strabo says he borrows his ground assumptions from the scientific treatises on the subject; therefore one cannot exclude the possibility that the word here has its (Hellenistic!) scientific sense, now generally borne by the expression "ellipsoid of revolution".[166] The suspicion that Strabo's sources declared the earth to be ellipsoidal finds some support in a passage of Diodorus Siculus about the genesis of our world, to the effect that the early earth, still fluid, was shaped by gravity and its own continual rotation.[167]

Diogenes Laertius reports that "the Stoics" gave not only the earth but also heavenly bodies a spheroidal shape.[168] Elsewhere he talks of heavenly bodies being spheroidal in certain cases and ovoidal in others,[169] saying this was the Epicurean belief. Aetius attributes to Cleanthes the opinion that heavenly bodies have a conoidal shape.[170] The introduction in a technical sense of terms that were previously in common use must perforce have led to confusion after the notion of linguistic conventionalism was lost.[171]

The ellipsoidal shape of the earth was "suspected" in the seventeenth century, as Jones says. Remarkably, this suspicion predates any measurement of *or* theoretical explanation for the polar flattening (for one thing, there was longstanding debate over whether the ellipsoid was flattened or elongated[172]). If the idea that the earth is ellipsoidal arose neither from experimental data nor from theoretical arguments, what led to it? Couldn't it have been the assiduous reading of Strabo attested among seventeenth-century geographers?

[165] [Strabo/Jones], vol. 1, p. 40, note 2.

[166] See the Archimedes quotation on page 180 and note 32 immediately thereafter.

[167] Diodorus Siculus, *Bibliotheca historica*, I, vii §2. Note that when Archimedes proves in the first book of *On floating bodies* that the spherical shape of the oceans follows from gravity, he specifies that this is true "in rest conditions".

[168] Diogenes Laertius, *Vitae philosophorum*, VII §145.

[169] Diogenes Laertius, *Vitae philosophorum*, X §74.

[170] In Theodoret, *Graecarum affectionum curatio*, VI, xx = [DG], 344b:1.

[171] Thus in the passage just cited, as in others such as [DG], 312b:22–23 and 329a:1–2, the doxographer's statements that certain authors gave objects a conical or conoidal shape might have arisen from a misunderstanding of references to conics.

[172] The second possibility was still maintained by the famous Giacomo Cassini (1677–1756).

10.12 Seleucus and the Proof of Heliocentrism[173]

Seleucus of Babylon, already encountered on page 88 in connection with the infinity of the universe, was an astronomer from the second century B.C. about whom not much else is known.[174] But Plutarch offers a very interesting testimonium, whose import appears to have been neglected by historians of science:

> Was [Timaeus] giving the earth motion . . . , and should the earth . . . be understood to have been designed not as confined and fixed but as turning and revolving about, in the way expounded later by Aristarchus and Seleucus, the former assuming this as a hypothesis and the latter proving it?[176]

The passage refers to two types of terrestrial motion, rotation and revolution.[177] The verb ἀποφαίνομαι appearing at the end of the passage allows different possibilities for what Seleucus actually did, but the contrast with "as a hypothesis" clearly implies that he found new arguments in support of these motions.

To state, as Seleucus did, that the sun really is fixed and the earth is moving is equivalent to stating that planetary stations and retrogressions don't just disappear under the assumption that the sun is stationary, as Aristarchus said, but that they really don't exist. That retrogressions and stations are merely apparent is repeated by pre-Ptolemaic Latin sources,

[173] The material in this section is drawn in part from [Russo: Seleuco] and [Russo: FR].

[174] See [Russo: Seleuco] for an analysis of the testimonia on him.

[176] Plutarch, *Platonicae quaestiones*, 1006C.

[177] Thus [Schiaparelli], p. 36 and [Heath: Aristarchus], p. 305. Nevertheless, Dreyer and after him Neugebauer took the words στρεφομένην καὶ ἀνειλουμένην, which we have translated "turning and revolving about", to mean only the earth's daily rotation. They offered no arguments for this position, and indeed avoided translating the expression altogether, replacing it by a single word, perhaps to avoid raising doubts in the reader's mind ([Dreyer], p. 140; [Neugebauer: HAMA], p. 611). It is true that, taken in isolation, each of these two Greek verbs might refer either to rotation around an axis or revolution about an external point (as can "rotate" and "revolve" in English, outside the narrowest astronomical convention). But if Plutarch meant rotation only, why two verbs? The idea of rotation and revolution is further clarified by the contrast with the possibility of an earth that is *fixed* (μένουσαν) and *confined* (συνεχομένην). Finally, Plutarch specifies that the motions referred here are the ones already attributed by Aristarchus to the earth, which we know from an unequivocal passage (Plutarch, *De facie*. . . , 923A) to be both rotation and revolution.

There is every likelihood that Copernicus, in choosing a word to designate the earth's movement around the sun, consciously selected *revolvo* as a calque on Plutarch's ἀνειλέω. The prefixes match, and both roots (which incidentally are cognate) bear the primary notion of "rolling" — not exactly a perfect fit for the idea being described. For an independent choice, other Latin roots might be more appropriate, whereas for a calque, nothing else will do. In view of this and of Copernicus' explicit references to Plutarch as a source, it is ironic that in the twentieth century we should have come to deny that Plutarch's verb might mean "revolve" in the Copernican sense. (I thank Maria Grazia Bonanno for bringing to my attention the correspondence between ἀνειλέω and *revolvo*.)

including Pliny and Seneca,[178] suggesting that the notion of heliocentrism as a physical reality, far from being exceptional, was well-known. Thus we might hope to find traces of Seleucus' proof in the literature.

One argument in favor of heliocentrism is what we reconstructed in Section 10.7 based on a passage of Seneca. With the sun as the reference, the planets' motion admits a simple dynamical description, where centrifugal force balances attraction. In a geocentric model this is not so easy to do: if the planets are attracted by the earth, why wouldn't they fall when they stop in the sky? And if not attracted by the earth, why don't they go off forever? One is tempted to deduce that only the motion around the sun is real. Since classical literature contains no other arguments in favor of heliocentrism, it is reasonable to conjecture that the proof that Plutarch attributes to Seleucus is based on the argument just given, which is reported by Seneca.

But it may be objected that the preceding argument is not a true proof of heliocentrism. Even if only heliocentric astronomy allows a dynamic description of planetary motion, there is always the possibility of rejecting both heliocentrism and the dynamic description. An Aristotelian thinker might insist, without incurring in contradiction, that the moon and the planets are not subject to *either* centrifugal force *or* attraction, and that they simply move according to their nature. As to the earth, if one does not accept that it revolves around the sun or that it is attracted toward the sun, it can very well stay at rest, most reassuringly.

How can one gainsay the Aristotelian view, proving that the earth is in fact subject to two forces, an attraction toward the sun and a centrifugal force, rather than to neither? An astronomer from the second century B.C., in possession of the ideas that we have reconstructed through Plutarch's and Seneca's testimony, might revisit the analogy with the pebble spun in a sling, and extend it. If instead of the pebble on a sling one spins an easily deformable object (say a ragball) attached to a rope, the object is stretched by the opposing centrifugal and centripetal actions, whose presence is thereby made manifest. Does something similar happen with the earth? Solid land is not easily deformable, but the oceans are. Because solar tides can be detected and work precisely to raise the level of the oceans toward the sun and the antipodal point, they provide a physical proof of heliocentrism than can hardly be denied.

Several testimonia provide support for our reconstruction of Seleucus' proof. Aetius says that Seleucus connected tides with the earth's motion.[179] Other authors portray the Babylonian astronomer as a great authority on

[178]Pliny, *Naturalis Historia*, II §70. The Seneca passage was discussed in Section 10.7.
[179]See quote and discussion on page 315.

tides. A particularly interesting passage in Strabo reveals that Seleucus, studying tides in the "Erythrean Sea" (which probably means today's Arabian Sea[180]) went beyond an understanding of daily and monthly cycles: he found a correlation between the *diurnal inequality* (defined as the height difference between consecutive high tides) and the seasons. Specifically, he found that spring tides show maximum diurnal inequality around the solstice, and minimum during the equinoxes, thus giving rise to a yearly cycle.[181] A theoretical explanation of the effect described by Seleucus is easily provided within the framework of the lunisolar theory we encountered on page 307. Suppose, for example, that we have full moon at the northern summer solstice; ignoring the high water interval, we then have a highest spring tide at point A on the Tropic of Cancer where it is noon, and at the antipodal point B on the Tropic of Capricorn, where it is midnight. The next high tide on the same spot is rather less severe, since point A has moved to A' and no longer has the moon and the sun on the zenith-nadir axis, but aslant (Figure 10.5, left). Thus the diurnal inequality is large. On the other hand, an equinoctial spring tide (Figure 10.5, right) has diurnal inequality close to zero, as can be seen by comparing the same two points A and A' on the Tropic of Cancer (or, for that matter, any two

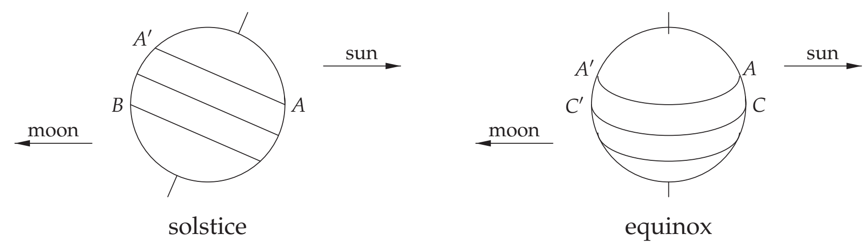


FIGURE 10.5. Configuration of spring tides.

points of same latitude where it is respectively noon and midnight). In this situation the highest spring tide is at points C and C' on the equator. On other days of the year we have an intermediate situation.

The interesting thing is that in the Arabian Sea the actual behavior of tides (which may in general depart far from what might be expected on the basis of simple models) agrees both with Seleucus' description and

[180]The expression was also used for today's Red Sea, but since Strabo sometimes says that Seleucus came from the Erythrean Sea (*Geography*, III, v §9) and sometimes from Babylonia (*Geography*, I, i §§8–9; XVI, i §6), it seems that in this case the Arabian Sea was meant.
[181]Strabo, *Geography*, III, v §9.

with the theoretical scheme just discussed, as was recognized in 1898 by G. H. Darwin, one of the founders of modern tidal theory.[182]

Strabo does not report an astronomical explanation for the phenomenon studied by Seleucus, but there are hints that the Babylonian astronomer knew the one we just gave. First, we have seen that the lunisolar theory, on which the explanation is based, was known to Posidonius, and, in view of the interruption in scientific activities already mentioned several times, it is unlikely that the theory was created later than the second century B.C. Also the testimonia clearly show that Seleucus was an astronomer and not a naturalist interested only in phenomenological description of tides: the tradition ranks him with "mathematicians",[183] and, what is more, Hipparchus regarded him as an authority on tides.[184] Then Pliny, who seems to draw on sources that knew the lunisolar theory, seems to refer implicity to Seleucus when he mentions the diurnal inequality and the fact that it vanishes only during the equinox.[185]

It is likely, then, that Seleucus not only observed the yearly cycle of diurnal inequality but actually explained it using the lunisolar theory. Thus he would have been in a position to formulate a dynamical justification of heliocentrism based on the tide's solar component, as conjectured earlier in this section. But the tide depends more on the moon than on the sun. Can lunar tides be explained in a similar way?

Besides the obvious analogy between sun and moon, other considerations might have led our ancient astronomer to an affirmative answer. As we read in Plutarch, the moon does not fall onto the earth because an attractive force toward the earth is balanced by a centrifugal force. Once tides led to the realization that the earth is also attracted by the moon, it would be natural to explain that the earth, in turn, does not fall on the moon because a centrifugal force acts on it — a force arising from an earthly motion. The analogy of the sling also has something to contribute here. Anyone who has tried to spin around a weight at the end of a string has noticed that it is impossible to do this while keeping perfectly still; likewise a hammer thrower never remains immobile, but swings his own body in a small circle as he spins the hammer in a larger one.

All this would have led to the hypothesis that what seems to be the revolution of the moon around the earth is in fact a circular motion of both bodies around a common center; that is, it would have suggested that the earth, besides its daily and yearly movements, also has a monthly

[182] [Darwin: Tides], pp. 84–85.

[183] See for example Strabo, *Geography*, XVI, i §6, where Seleucus is linked to the Chaldeans, famous among "mathematicians".

[184] Strabo, *Geography*, I, i §9.

[185] Pliny, *Naturalis historia*, II §213.

movement around the barycenter of the earth-moon system,[186] enough to cause a centrifugal force which, together with attraction toward the moon, causes lunar tides.

Now consider Aetius' testimony on Seleucus, which appears in a list of explanations for the phenomenon of tides:

> Seleucus the mathematician (also one of those who think the earth moves) says that the moon's revolution counteracts the whirlpool motion of the earth.[187]

These words have long been obscure: what is the "whirlpool motion" (δίνος or δίνη) of the earth? The usual translations of this passage say "whirling" or "rotation", but if just the earth's rotation were meant, why the use of a noun whose primary meaning is a whirlpool or eddy, rather than the nouns attested elsewhere for rotation?[187a] I believe that what is meant is the earth's revolution along a very small circle (that is, around the earth-moon barycenter), a wobbling very much like that of a largish object caught near the center of a whirlpool. So the moon's revolution *counteracts*, or is counterposed to, the earth's wobbling: as the moon describes a large orbit, the earth describes a small one, both remaining always opposed in relation to the center of the orbit, just as the hammer thrower moves in a small circle, keeping diametrically opposite the projectile in its circular trajectory.[188]

The fact that the analogy between the earth and a sling goes back to Posidonius, who is the author of the most famous exposition of the ancient theory of tides, may be a further piece of indirect evidence in support of our reconstruction.

10.13 Precession, Comets, etc.

The main result of Hipparchus mentioned by Ptolemy is the discovery of the precession of the equinoxes.[189] The precession is so slow that any available observational data would have given Hipparchus displacements

[186] These qualitative arguments do not allow one to pinpoint the center of rotation, but they indicate that it is closer to the more massive of the two bodies. The barycenter, a notion present from beginnings of Greek statics, would be the most natural candidate.

[187] Σέλευκος ὁ μαθηματικός ... κινῶν καὐτὸς τὴν γῆν, ἀντικόπτειν αὐτῆς τῷ δίνῳ φησὶ τὴν περιστροφὴν τῆς σελήνης. (Stobaeus, I, xxxviii §9, 253:16–18, ed. Wachsmuth = [DG], 383b:26–34; or, with minor variations, pseudo-Plutarch, *De placitis philosophorum*, III, xvii = [DG], 383a:17–25).

[187a] Compare note 177 on page 311.

[188] This interpretation of the Aetius passage was first published in [Russo: AS] and [Russo: FR].

[189] Ptolemy, *Almagest*, III, i, 192 (ed. Heiberg).

of only a few degrees.[190] Nevertheless, the astronomer, who is known to have been very rigorous in his use of experimental data,[191] dared to extrapolate from a tiny arc the existence of a circular uniform motion with a period of 26,000 years. If his astronomy was "dynamical", any old top might have given him the idea of sifting through observational data for the existence and periodicity of precession.[192]

Phenomenologically, comets have very little in common with planets; their appearance is quite different and even if the periodicity of a comet can be noticed (which is far from easy), the trajectory remains for the most part hidden and is different from that of any other comet. A purely descriptive astronomy precludes a theory of comets. On the other hand, a dynamical astronomy based on some sort of gravitational theory might naturally ask whether there are things besides the planets going around the sun, perhaps with very long orbits. It is not an accident that there is no theory of comets in the *Almagest* and that the modern theory was created by Halley only after the development of Newton's theory of gravitation. But Seneca writes:

> For he [Apollonius Myndius] says that the Chaldeans reckon comets among the planets and know their orbits.[193]

> Apollonius Myndius thinks otherwise. He says that a comet is not made up of many planets, but rather that many comets are planets. He says: "A comet is not a false appearance nor an extension of fire in the neighborhood of two stars, but a heavenly body in its own right, like the sun and the moon. Its shape is just like that, not enclosed in a circle but more elongated and stretched out. Moreover it has an orbit that is not openly visible; it crosses the upper regions of the universe and appears only when it reaches the lowest point of its orbit.[194]

[190] Ptolemy says that Hipparchus noticed a difference of less than three degrees between his own observations and those of Meton, of 431 B.C. (*Almagest*, VII, ii, 15–16, ed. Heiberg).

[191] Ptolemy (*Almagest*, IX, ii, 211, ed. Heiberg) concedes this much to Hipparchus in the midst of his efforts to deny him credit for a planetary theory (compare page 286).

[192] Tops display precession clearly. They were common toys; see, for example, Callimachus' epigram in *Anthologia graeca*, VII, 89.

[193] "Hic enim ait cometas in numero stellarum errantium poni a Chaldaeis tenerique cursus eorum" (Seneca, *Naturales quaestiones*, VII, iv §1). Recall that Seleucus was one of the Chaldeans (see note 183 above).

[194] "Apollonius Myndius in diversa opinione est. Ait enim cometen non unum ex multis erraticis effici, sed multos cometas erraticos esse. Non est, inquit, species falsa nec duarum stellarum confinio ignis extentus, sed proprium sidus cometae est, sicut solis ac lunae. Talis illi forma est, non in rotundum restricta sed procerior et in longum producta. Ceterum non est illi palam cursus; altiora mundi secat et tunc demum apparet cum in imum cursus sui venit." (Seneca, *Naturales quaestiones*, VII, xvii §§1–2).

The statement about the "shape" probably referred to the orbit in the source, though Seneca may not be aware of it. For even admitting that an astronomical source would linger and say that the tail of a comet is "more elongated" than a circle, we would lose the clear logical connection with the next sentence, which evidently implies in any case that the *orbit* is elongated.

The notion that comets are celestial bodies of the same type as planets, which Seneca attributes to Apollonius of Myndius, is mentioned also by Aetius, who attributes them to some Pythagoreans,[195] and to Pliny, who, without attributing the opinion to anyone in particular, says of the comets: "Some move like planets, others remain fixed."[196] It may be suspected that the immobility option (which is contrary to all observable facts) was added by Pliny, to whom the notion that all comets without exception shared the planets' vagrancy may have seemed too far-fetched. But the association between comets and planets must have been a widespread idea in pre-Ptolemaic times, because, other than in Pliny, Seneca and the Pythagoreans mentioned by Aetius, it can be found also in Manilius. This character, who wrote an astrological poem some time in Augustus' and Tiberius' reign, mentions three different theories about comets; according to one (the most interesting for our purposes), the sun periodically attracts the comets to itself and then lets them go, as it does with Mercury and Venus.[197]

10.14 Ptolemy and Theon of Smyrna

The preceding sections have argued that Hipparchus, and perhaps other astronomers in the second century B.C., reached a sort of "dynamical heliocentrism" based on the principle of inertia and the equilibrium between centrifugal force and the gravitational pull of the sun.

The greatest obstacle to accepting this picture is that these ideas are nowhere to be found in the *Almagest*. But the *Almagest* itself offers indirect evidence for the proposed reconstruction, in the form of remnants of scientific ideas not understood by Ptolemy and the absence of other ideas whose existence in Hellenistic science is well documented. Dennis Rawlins makes a strong case that several technical elements of Ptolemaic astronomy can only be explained as derivatives of an earlier heliocentric model.[198]

[195] In Stobaeus, *Eclogae* I, xxviii, 227:8–10 = [DG], 366b:6–10.

[196] "Moventur autem aliae errantium modo, aliae immobiles haerent" (*Naturalis historia*, II §91).

[197] Manilius, *Astronomica*, I:867–875.

[198] [Rawlins: AHPE].

Among the consequences of the cultural gap separating Ptolemy from Hellenistic astronomy was the loss of the idea of relative motion. Indeed, the idea that one can choose the reference system, present for example in Euclid, Herophilus and one of Seneca's sources,[199] was completely foreign to Ptolemy. Nevertheless he reported recognizably relativistic opinions,[200] which he himself understood simply as a convenience for describing observed motions, with no consequences to his concept of space and motion, which is squarely Aristotelian. In just the same way physicians like Rufus Ephesius, though diligently transmitting Herophilus' neologisms, could no longer grasp their nature.

Ptolemy not only did not use gravity (or any other dynamical idea) in astronomy; he also approached the earth's sphericity in a purely geometric and descriptive way, seemingly unaware of the explanation for it that was well known in Hellenistic times.[201]

The *Almagest* mentions (and contests, of course) the opinion that the "fixed" stars have a uniform linear motion.[202] To Ptolemy the motion in question was the apparent movement in the sky, but we may conjecture that his source was not making such a bizarre claim; it was instead referring to the "real" motion whose existence Hipparchus suspected on the basis of general principles and for whose detection he sought to enlist the help of posterity (page 88).

Ptolemy says, in the passage where he denies that Hipparchus had taken steps toward a planetary theory:

> All that he did was to make a compilation of the data arranged in a more useful way[.][203]

Fitting a theory to a great deal of experimental data of course requires a lot of data reorganization, and these manipulations were not intelligible to imperial-age scholars, who no longer created theories based on experimental data, but at best used them. In Ptolemy's time it was thought that a theory must be put forth in a purely deductive fashion; Ptolemy's exposition is of this type and we have seen that Galen criticized Herophilus' treatment of heartbeats for the same reason.[204]

The preceding discussion suggests that the guiding ideas that we have descried in Hipparchus' astronomy did not remain at a qualitative stage, but that, on their basis, a quantitative description of planetary motions

[199] See pages 177, 188 and 296.
[200] See page 84.
[201] Ptolemy, *Almagest*, I, iv, 14–16 (ed. Heiberg); for the explanation of sphericity see Section 10.8.
[202] Ptolemy, *Almagest*, I, iii, 11 (ed. Heiberg).
[203] Ptolemy, *Almagest*, IX, ii, 210 (ed. Heiberg), Toomer translation.
[204] See page 154.

was begun. This conjecture finds support in Seneca's statement that "we have recently started to understand" the motions of the planets, in a statement of Pliny that seems to imply that the study of Mars' motion had been particularly difficult,[205] and in Ptolemy's report of Hipparchus' dissatisfaction with the incomplete accord between theory and experimental data.[206]

Theon of Smyrna, a contemporary of Ptolemy, wrote:

> [The sun] is the place that animates the cosmos, as cosmos and living being — as if, blazing-hot, it were the heart of the universe, because of motion and magnitude and the common journey of all that is around it. . . .[207]
>
> The center of the magnitude is the earthly, cold and immobile one; but the center that animates the cosmos, as universe and as living being, is that of the sun, which is said to be the heart of the universe and the place whence derives the universe's soul, which reaches out all the way to its edge.[208]

These two notions — the particular role of the sun in animating the world and the existence of a "cosmic sympathy" between heavenly bodies — survived for a long time, and have been regarded as purely philosophical, mostly "Stoic", in origin. They are reported, for example, by Macrobius, though he attributes them to Pythagoras.[209] They may have been remnants of a gravitation-based dynamical astronomy, no longer understood.

If we look into why these ideas were attributed to the Stoics, we find that the primary reason is that their origin can be traced back to Posidonius — just as in the case of the analogy between earth and sling. Note that Posidonius (who, besides having built planetaria, seems to have been the last scholar to be seriously interested in tides[210]) headed a school at Rhodes not long after Hipparchus had been active there. As for the connection with Pythagoras, besides the general tendency of neo-Pythagoreans to credit him as the source of all sorts of knowledge, it may be explained also by the confusion between Hipparchus and Hippasus (page 241).

[205] "Multa promi amplius circa haec possunt secreta naturae legesque, quibus ipsa serviat, exempli gratia in Martis sidere, cuius est maxime inobservabilis cursus[.]" (Pliny, *Naturalis historia*, II §77). Note that Pliny's sources used the concept of "laws of nature" and expressed it in the same terminology that was transmitted down to the modern age (in part by writers like Pliny).
[206] Ptolemy, *Almagest*, IX, ii, 210 (ed. Heiberg).
[207] Theon of Smyrna, *Expositio rerum mathematicarum ad legendum Platonem utilium*, III, xxxiii, 187: 14–18 (ed. Hiller).
[208] Ibid., 188:2–7.
[209] In particular in his commentary on Cicero's *Somnium Scipionis*.
[210] See Strabo, *Geography*, III, v §9.

10.15 The First Few Definitions in the *Elements*[211]

As already discussed, there is much to suggest that Euclid subscribed to the nominalist and constructivist blueprint (see particularly pages 181 and 185), but this is apparently contradicted by the first few definitions in Book I of the *Elements*, of which we quote the first four:[212]

> A point is that which has no part.
>
> A line is breadthless length.
>
> The extremities of a line are points.[213]
>
> A straight line is a line that lies equally with respect to the points on itself.[214]

Such definitions, and the next ones for a surface and a plane, are clearly Platonist-essentialist, and fit better with the cultural imperial age than with that of the early Hellenistic climate of the period: they find no parallel in the works of Archimedes and Apollonius.

The *Elements* have reached us with interpolations from late Antiquity,[215] and it is not unreasonable to suspect that these definitions are instances thereof. One of the few definitions in the *Elements* methodologically analogous to those just considered is found right at the beginning of Book VII:

> An unit is that by virtue of which each of the things that exist is called one.[216]

This "definition", which is clearly of Platonist character, is attributed by Iamblichus (circa 300 A.D.) to "more recent" writers (οἱ νεώτεροι).[217] It is possible, then, that similar interpolations of Platonist definitions by later authors occurred for the fundamental geometric entities.

[211] The material in this section is drawn from [Russo: Elementi] and [Russo: Elements].

[212] The word "line" in English is ambiguous. In this section it will always mean a not-necessarily-straight line (γραμμή). We use "straight line" to translate ευθεῖα.

[213] σημεῖόν ἐστιν, οὗ μέρος οὐθέν. γραμμὴ δὲ μῆκος ἀπλατές. γραμμῆς δὲ πέρατα σημεῖα (Heath translation).

[214] εὐθεῖα γραμμή ἐστιν, ἥτις ἐξ ἴσου τοῖς ἐφ' ἑαυτῆς σημείοις κεῖται.

[215] For example, Theon of Alexandria mentions a theorem whose demonstration he inserted in his edition of the *Elements* (*Commentary on the Almagest*, on I, x = [Theon/Rome], II, 492:6–8). We also know that Heron wrote a popularizing commentary to the *Elements*, and some passages that appear in all our manuscripts of the *Elements* have been positively identified as interpolations from Heron's commentary. For instance, the Arabic commentator an-Nairīzī attributes proposition 12 of Book III to Heron (see [Euclid/Heath], vol. II, pp. 28–29), and Proclus does the same regarding an alternative proof of proposition 25 of Book I (Proclus, *In primum Euclidis Elementorum librum commentarii*, 346:12–15, ed. Friedlein).

[216] Μονάς ἐστιν, καθ' ἣν ἕκαστον τῶν ὄντων ἓν λέγεται (Heath translation).

[217] Iamblichus, *In Nicomachi Arithmeticam introductionem*, 11:5 (ed. Pistelli).

In seeking objective support for this suspicion, we can use documents of two types: papyrus finds and the testimonia of authors who had access to earlier versions of Euclid's works than the ones available today.

Among the few papyri containing fragments of the *Elements*, only two are relevant to the definitions from Book I. One, probably written by a school child, contains the first ten definitions almost exactly as they have come down to us, but it makes no reference to Euclid.[218] Thus it shows that at the time of its writing — the third century A.D. — the definitions of fundamental geometrical entities were taught in the form that we know from the *Elements*, but not that they appeared in the *Elements* or that they were associated with Euclid. The second papyrus, chronologically more relevant, comes from Herculaneum[219] and does not contain "definitions" of fundamental entities, but only one for a circle. This latter is quite correct, but does not coincide with the one present in the known versions of the *Elements*: in particular, the term *circumference* (περιφέρεια) is used in the papyrus without being explicitly defined, whereas in the extant *Elements* a definition of this term is included in the course of the definition of a circle.[220] This shows that in at least one case a definition of a term originally left undefined was interpolated in the *Elements*.

Sextus Empiricus wrote before Euclid's work took the form that has come down to us. He discusses definitions of geometric entitites several times. The importance of his testimony is increased by the fact that he seems to report the definition of a circle not in the form present in today's *Elements*, but in that found in the Herculaneum papyrus;[221] this leads us to think that he had an edition of the *Elements* that was if nothing else less corrupt than ours.[222]

When Sextus reports Platonist-essentialist definitions of fundamental geometrical objects similar to the first few in Book I of the *Elements*, they usually differ from the latter in telling ways. Let's examine, for example,

[218] P. Michigan III, 143.

[219] P. Herculaneum 1061.

[220] The text transmitted by all manuscripts of the *Elements* (I, definition 15) is: "A circle is a plane figure contained by one line, called the circumference, such that all straight lines emanating from one point inside the figure and falling upon it — upon the circumference of the circle — are equal to one another" (Κύκλος ἐστὶ σχῆμα ἐπίπεδον ὑπὸ μιᾶς γραμμῆς περιεχόμενον, ἣ καλεῖται περιφέρεια, πρὸς ἣν ἀφ' ἑνὸς σημείου τῶν ἐντὸς τοῦ σχήματος κειμένων πᾶσαι αἱ προπίπτουσαι εὐθεῖαι πρὸς τὴν τοῦ κύκλου περιφέρειαν ἴσαι ἀλλήλαις εἰσίν). Note the gauche amplification "upon the circumference", especially pointless in Greek since the pronoun ἣν cannot be misunderstood in view of its position and gender. (Heath omits from his translation the two references to "circumference" because they were declared spurious by Heiberg.)

[221] Sextus Empiricus, *Adversus geometras* (= *Adv. math.*, III), §107.

[222] Heiberg, for a variety of reasons, concluded that Sextus still had access to the original edition of Euclid: see the Prolegomena to the critical edition of the *Elements* in [Euclid: OO], vol. V.

his passage about the definition of a point. He writes that mathematicians, in describing (ὑπογράφοντες) geometrical entities, say that

> a point [*stigme*] is a “sign” [*semeion*] having no parts and no extension, or the extremity of a line[.][223]

Since the sentence “a point is a ‘sign’ having no parts” is very similar to Definition 1 in the *Elements*, and the characterization of a point as the extremity of a line coincides with Definition 3, it is generally thought that Sextus is citing Euclid in this passage. But a third property of points is mentioned, namely their extensionlessness, and further, none of the three properties is called a definition (ὅρος), but rather a description. By contrast, when Sextus reports the definition of a circle (which we can probably assume to go back to Euclid, because of the testimonium from the Herculaneum papyrus), he talks instead of mathematicians defining (ὁριζόμενοι) a circle.[224] Now, in the *Elements* many statements are called definitions, but nothing is called a description. This indicates that in the case of a point Sextus Empiricus is alluding not to Euclid but to someone else.

What Sextus says about points is much closer to a passage from a work titled *Definitions of terms in geometry*, with which Heron’s name has long been associated. This text, after a dedication where the author states his purpose of *describing* the technical vocabulary of geometry,[225] consists of a hundred or so paragraph-length sections, each illustrating and characterizing one geometric concept. Little of this material qualifies as definitions, belying the work’s traditional name.[226] The first section starts:

> A point is that which has no part, or an extremity without extension, or the extremity of a line.[227]

Thus we see here the two telltale differences present in the Sextus Empiricus passage but not in the *Elements*: the use of the verb “describe” (ὑπογράφων) and the characterization of the point as devoid of extension.

Getting back to the Sextus passage quoted at the top of this page, here is how it continues:

[223]στιγμὴν μὲν εἶναι σημεῖον ἀμερὲς καὶ ἀδιάστατον ἢ πέρας γραμμῆς (Sextus Empiricus, *Adversus geometras* (= *Adv.~math.*, III), §20. For the terms *stigme* and *semeion* see page 181.

[224]Sextus Empiricus, *Adversus geometras* (= *Adv. math.*, III), §107.

[225]This passage is quoted on page 325.

[226]*Heron’s Definitions of terms in geometry* (Ἥρωνος ὅροι τῶν γεωμετρίας ὀνομάτων) is the title it bore in the Byzantine compilation where it was preserved. Though the work is still generally attributed to Heron, Knorr gives good reasons to think that it belongs to Diophantus ([Knorr: AS]). It first appeared in print in 1570 as an adjunct to Dasypodius’ edition of Book I of the *Elements*; our references are to Heiberg’s edition in [Heron: OO], vol. IV.

[227]σημεῖόν ἐστιν, οὗ μέρος οὐθέν ἢ πέρας ἀδιάστατον ἢ πέρας γραμμῆς (*Heronis Definitiones*, 14:11–12, ed. Heiberg).

> A line is a length without width or the extremity of a surface; a surface is the extremity of a body, or a width without depth.

All of this appears both in the *Definitions* and in the *Elements*, except for “a surface is the extremity of a body”, which is not in the latter, showing again that Sextus was not relying on the *Elements*. In discussing the notion of a straight line, too, Sextus reports both the definition that appears in the *Elements*[228] and another definition (based on the invariance of the line with respect to rotations that fix two points) that is missing in Euclid’s work but appears in *Definitions*.[229] We can conclude that when Sextus reports “descriptions” of basic geometric entities his source usually seems to be the *Definitions* rather than Euclid.[230]

As we have seen, Sextus knew that to avoid infinite regression one must either eschew definitions altogether or accept that some entities must remain undefined.[231] Since he could hardly have broached this subject without taking into account Euclid’s *Elements* (a foundational work for all later mathematical developments), this testimony of Sextus, too, suggests that in the version(s) of the *Elements* known to him certain things were left undefined. This would explain why on the subject of fundamental geometric entities Sextus had to use the *Definitions*.

The definitions of fundamental geometric entities (point, line, straight line, surface and plane) given in the *Elements* all closely follow passages from the *Definitions*. We therefore narrow down our suspicions by conjecturing that they are interpolations drawn from the *Definitions of terms in geometry*.

What settles the issue for me is the definition of a straight line found in the *Elements*, and quoted at the beginning of this section. This very murky statement[232] seems to mean, if anything, that a straight line is “seen” in the same way from all its points; that is, that there are rigid motions that leave the line invariant and take any point to any other. But this property (which as we know also interested Apollonius[233]) does not fully charac-

[228]Sextus Empiricus, *Adversus geometras* (= *Adv. math.*, III), §94.

[229]Sextus Empiricus, ibid., §98; *Heronis Definitiones*, 16:21 – 18:6 (ed. Heiberg).

[230]This possibility could not have occurred to earlier scholars such as Heiberg and Heath, who thought that Sextus Empiricus predated Heron (the correct dating for Heron having been established later by Neugebauer). Note that if Knorr’s attribution of the *Definitions* to Diophantus becomes established (note 226), the fact that Sextus seems to cite his work would help solve the vexing question of Diophantus’ dating. (The argument generally used to assign Diophantus to the third century A.D. goes back to Tannery, but its inconsistency has been demonstrated in [Knorr: AS]. Knorr (op. cit., note 23) also considers the possibility that Diophantus was a source for Heron, in particular for arithmetic terminology.)

[231]See page 182.

[232]Heath concludes his discussion of it with the words “the language is thus seen to be hopelessly obscure” ([Euclid/Heath], vol. I, p. 167).

[233]See page 98.

terize straight lines: in the plane it is also shared by the circumference. It is strange that Euclid should not realize that a circumference, too, "lies equally with respect to the points on itself".

Now, in the *Definitions* the long paragraph on straight lines starts thus:

> A straight [εὐθεῖα] line is a line that equally with respect to [all] points on itself lies straight [ὀρθή] and maximally taut between its extremities.[234]

The origin of this characterization can be traced fairly easily. Already the Stoics had defined a straight line as a line taut between its endpoints.[235] Archimedes took it as a postulate that among lines sharing the same extremities the straight line was the shortest.[236] In the imperial period a "straight line", εὐθεῖα, (which in Euclid, unless otherwise specified, means a segment) came to mean one that extends endlessly in both directions.[237] A person wishing to use the Stoic definition or the Archimedean postulate to characterize the "new" infinite straight line of course could not base a statement on one pair of endpoints only, but must instead say that the straight line has this tautness property "equally with respect to all points on itself". The statement found in the *Definitions* is thus clear in this post-Euclidean context.

In view of the difficulty in memorizing long chunks of text and of the type of teaching prevailing in the imperial age, one can imagine the poor students being encouraged to work from crib sheets abridged from the *Definitions*, where each entry was truncated as soon as the syntax allowed it.[238] Such a crib sheet, from being copied together with the *Elements*, might easily have merged eventually with the Euclidean text. The fact that by this procedure we obtain exactly the definitions found in the *Elements* — even "A straight line is a line that lies equally with respect to the points on itself", which no mathematician has ever been able to make proper sense of — demonstrates the plausibility of our conjecture.

We should every so often pause to consider the legions of students who throughout the centuries were forced to memorize a half-sentence by teachers who did not know the second half that could have made it meaningful.

Proclus seems to have preserved, through channels not easily identifiable, a memory of the relationship between Archimedes' shortest-line

[234] εὐθεῖα μὲν οὖν γραμμή ἐστιν ἥτις ἐξ ἴσου τοῖς ἐπ' αὐθῆς σημείοις κεῖται ὀρθὴ οὖσα καὶ οἷον ἐπ' ἄκρον τεταμένη ἐπὶ τὰ πέρατα (*Heronis Definitiones*, 16:22–24, ed. Heiberg).

[235] Simplicius, *In Aristotelis Categorias commentarium* ([CAG], vol. VIII), 264:33–36 = [SVF], II, 456.

[236] Archimedes, *De sphaera et cylindro*, 10:23–25 (ed. Mugler).

[237] Straight lines form one class of infinite lines in a classification of Geminus reported by Proclus (*In primum Euclidis Elementorum librum commentarii*, 111:1–12, ed. Friedlein).

[238] Papyrus Michigan iii, 143, already mentioned, may have been a late examplar.

postulate and the definition of a line given in the *Elements*. He struggles to show, using strange arguments whose origin remains mysterious, that the *Elements* statement is just a reformulation of the shortest-line postulate.[239]

Points and lines are each defined twice in the *Elements* (Definitions 1 and 3 for points, 2 and 6 for lines). Even this duplication, which amounts to a clear logical incongruence, is easy to explain in the scenario discussed above. In the *Definitions*, the notion of a point was explained with a list of many characterizations, and likewise the line. It is then understandable that the abridger, faced with the task of choosing one sentence to be kept as a "definition", should sometimes have hesitated and chosen two just in case.

Significant support for our thesis can be found in the very preamble to the *Definitions*, which starts:

> In describing [ὑπογράφων] and summarizing for you, illustrious Dionysius, as concisely as possible, the technical terms presupposed in the foundations of geometry [τὰ πρὸς τῆς γεωμετρικῆς στοιχειώσεως τεχνολογούμενα], I will lay down the beginnings and general structure according to the teachings of Euclid, the author of the *Elements* of geometric theory.[240]

This wording makes sense if we assume that one of the author's goals was to illustrate the geometrical entities left undefined by Euclid, that is, the "technical terms presupposed in the foundations of geometry". The fact that the author considers his own work as preliminary to the *Elements* provides strong support to the conjecture that either he or later editors prepended extracts from the *Definitions* to the Euclidean text; the tradition could hardly have failed to merge the texts at some later point.

This conjectural reconstruction is consistent with other available testimonia, in that no author who cites Euclid attributes to him the definitions we are considering, all the way down to late Antiquity.[241] This silence is significant if we consider it together with the testimonia of several ancient authors who quote definitions of fundamental geometrical entities, none of which come from the *Elements*. We have already examined the passages in Sextus Empiricus; the case is similar with Plutarch, for whom the straight line is still characterized by being the shortest line between two points, and not by the definition that we now find in the *Elements*.[242] Again

[239] Proclus, *In primum Euclidis Elementorum librum commentarii*, 109–110 (ed. Friedlein).

[240] *Heronis Definitiones*, preamble, 14:1–6 (ed. Heiberg).

[241] I have checked the *Thesaurus Linguae Grecae* for all passages containing Euclid's name, and all authors not included in the TLG corpus who to my knowledge were interested in mathematical definitions.

[242] Plutarch, *Platonicae quaestiones*, 1003E; *De Pythiae oraculis*, 408F.

when defining a point Plutarch does not use the *Elements*: he says instead that "a point is a unit in a location": an ancient definition, Pythagorean in origin, that appears in the *Definitions* but not in Euclid's book.[243]

An even more significant passage, for chronological reasons if nothing else, is found in Philo of Alexandria (first century B.C.). He makes a distinction among geometric concepts, placing on the one hand circles and isosceles triangles and polygons, for example, and on the other concepts like points and lines, which can be defined only in philosophical terms. He wonders:

> How could [geometry], giving definitions, say that a point is that which has no part, a line is breadthless length, a surface only has length and breadth, and a solid is three-dimensional, because it has length, breadth and depth? This is the stuff of philosophy...[244]

Clearly, in first century B.C. Alexandria, the type of Platonist-essentialist definitions that now head the *Elements* could not be found in geometric works.

There remains to explain what sources were used in compiling the *Definitions*. Consider the first entry: after the beginning quoted on page 322, it goes on with illustrative properties of a point and other divagations, some of which are found in Aristotle. The second entry says among other things that a line is the extremity of a surface; this characterization, interpolated as Definition 6 into our *Elements*, goes back to Plato and had already been criticized by Aristotle.[245] It is clear, then, that far from reflecting the method of Euclid's *Elements*, the *Definitions* draws heavily from pre-Hellenistic sources, though of course it also contains properly geometric material.

The thesis espoused in this section openly challenges the traditional view that Euclid was a Platonist. I think the idea of a Platonist Euclid has three chief causes: the lasting influence of the only commentary to Euclid preserved in Greek, by the neo-Platonist philosopher Proclus; the presence in the *Elements* of the definitions that we have been discussing; and the ascendance of Platonizing interpretations of Euclid, arising from the vigor that Platonist views have enjoyed in schools of mathematical thought ever since the imperial age.

[243]Plutarch, *Platonicae quaestiones*, 1003F; *Heronis Definitiones*, 14:15, where the point is said to be "like a unit having a location".

[244]Philo of Alexandria, *De congressu eruditionis gratia* §146, 102:15–25 (ed. Wendl) = [SVF], II, text 99

[245]Some relevant Aristotelian passages: *Physica*, IV, xi, 220a:15 ff. (the point as the extremity of a line); *Metaphysica*, V, vi, 1016b:24–30 (indivisibility as a characteristic feature of points); *De caelo*, III, i, 300a; *Topica*, VI, vi, 143b:11 (line as extremity of a surface). See also footnote 243 above.

Karl Popper seems to have implicitly reached the conclusion we have articulated in this section. For if we apply his already quoted thoughts about the Platonist-Aristotelian method[246] to geometry, and discard (as we must) the possibility that Euclid's method, which remained for two thousand years the very model of scientific method, might rate as "empty verbiage and barren scholasticism", we must deduce that the definitions that we have been discussing — which are Platonist and Aristotelian not only in tenor and methodology but often in actual wording — cannot be Euclidean. Strangely enough, Popper did not draw this conclusion from his perceptive analysis, but retained the traditional idea of a Platonist Euclid.[247]

[246]See page 180.

[247]See [Popper: OSE], Addendum 1 (which appears in vol. 1 of the third and later editions). The scientific importance of the *Elements* obviously cannot escape Popper, so he must deduce from his assertion of Platonism (which he supports, in particular, with passages from Proclus) the consequence that Plato was the "founder of modern science". Since Popper also says that Plato founded the essentialist method used in Aristotelian definitions, this latter statement is hard to reconcile with the passage quoted on page 180.

11 The Age-Long Recovery

11.1 The Early Renaissances

The memory of Hellenistic science survived thanks to a series of revival periods when interest in ancient knowledge was rekindled for a time in a certain geographical area. The resumption of scientific studies in imperial times, already extensively discussed, can perhaps be viewed as the first of these renaissances. The second occurred around the early sixth century A.D. and had as its protagonists Simplicius, John Philoponus, Eutocius, Anthemius of Tralles and Isidore of Miletus. For our narrative a few observations about this cultural reawakening will suffice.

All these authors showed great interest in Hellenistic science: Eutocius wrote a commentary on some works of Archimedes and Apollonius and even believed he had found a lost proof by Archimedes.[1] John Philoponus, like Simplicius, is known chiefly for his commentaries on Aristotle, but he also studied mathematics and wrote a work on the astrolabe. Isidore of Miletus, best known for having designed (with Anthemius) the Basilica of Hagia Sophia in Constantinople, edited Archimedes' works and wrote, among other things, commentaries on Heron.[2] His work about marvelous

[1]Eutocius, *In Archimedis sphaeram et cylindrum*, II, iv = [Archimedes/Mugler], vol. IV, 88–89. But Archimedean authorship of the fragment found by Eutocius is very doubtful.

[2]According to a passage inserted by a copyist into Eutocius' commentary on Archimedes' *On the sphere and cylinder*, one of the works commented by Isidore was a treatise of Heron called Περὶ καμαρικῶν, meaning that it dealt with vaults and/or domes (Eutocius, *In Archimedis sphaeram et cylindrum*, II, i = [Archimedes/Mugler], vol. IV, 62:1–4). Specifically, the interpolator says that in this commentary Isidore described a parabola tracer of his invention; this suggests that one of the subjects dealt with in Heron's work was vaults or domes with a parabolic section. It might be

mechanisms, too,[3] is clearly based on Hellenistic sources.

All these scholars had been disciples of one master, Ammonius Hermiae in Alexandria. Anthemius, to whom Eutocius dedicated his commentary on Archimedes, had Isidore as a collaborator and successor. This school had Hellenistic works unknown to the Alexandrian scholars of the first few centuries A.D.: Diocles' work on burning mirrors was unknown to Pappus but is mentioned by Eutocius. Simplicius (page 291) is our only explicit witness about Hipparchus' lost work on motion under gravity. Priscian of Lydia, a neo-Platonist philosopher from the same time, is our fullest source for ancient knowledge about tides, including Posidonius' work on the subject.[4] Probably when the cultural center of gravity shifted from Alexandria to Byzantium — where Anthemius and Isidore, among others, worked — scholars became acquainted with works preserved in the East which had never been part of the Alexandrian tradition. Cultural exchanges with Eastern countries that had stayed outside the Roman empire are illustrated by a famous episode: When Justinian had the Athenian philosophical academy closed in 529, the Sassanid king Chosroes I invited the newly unemployed philosophers to his Persia; among those who accepted were Simplicius, Damascius (who was head of the academy at the time it closed) and Priscian of Lydia.

The level of scientific originality of the authors in this period is practically nil. Eutocius' comment on Archimedes is invaluable to us because of its references to otherwise unknown Hellenistic mathematical works, but it is never original. Anthemius' treatment of conics is shabbily pedestrian in its mathematics, when compared with Apollonius of Perga; Simplicius gives signs of misunderstanding Hipparchus' work.[5] However, because works of this time contain bits of knowledge not present in surviving earlier sources, certain scientific and technological results have often been dated to the sixth century.[6] Because Philoponus records the important fact

valuable to the study of architectural history to reconstruct the ups and downs of Heron's treatise and Isidore's commentary, both of which are lost. One might suspect that a Hellenistic architectural work with a commentary by the famous architect of Hagia Sophia (a building particularly prized for its dome) may have disappeared not when its contents stopped being of interest in Byzantium, but when they started being of interest elsewhere. See below, in particular note 27 on page 335.

[3]Only a few pages by Anthemius of Tralles survive ([MGM], pp. 78–87 or [Anthemius/Huxley]).

[4]See page 308.

[5]For example, Simplicius, following Alexander of Aphrodisias, argues from Aristotelian natural philosophy to criticize Hipparchus' statement that the weight of an object decreases as it gets nearer the center of the earth. Compare footnote 116 on page 292.

[6]For example, we read in [Vogel], p. 791, that Anthemius "out-distanced Apollonius on several points" in his work on burning mirrors. This is evidently based on a fragment that shows Anthemius using the focal property of parabolas, which is not stated in Apollonius' *Conics*. After the publication of [Diocles/Toomer] we have proof that this knowledge goes back to Hellenistic times, and probably was well-known to Apollonius. But it would have been enough to consider the

about gravity which is popularly associated with the leaning Tower of Pisa, certain authors have even credited him with being the first to contradict Aristotelian mechanics.[7]

Next came what has been called the Islamic Renaissance. Interest in science started in Islam during the eighth century, following the Abbasid revolution, under Caliphs al-Manṣūr and Hārūn ar-Rashīd, and reached its heyday in the ninth century, under Caliph al-Ma'mūn.[8] It was then that extant Arabic translations of Hellenistic scientific treatises started being made. A copy of Euclid's *Elements* was requested for this purpose from the Byzantine Emperor by the Caliph (either al-Manṣūr or al-Ma'mūn, depending on the source). Byzantium, too, experienced around that time a renewal in scientific interest, perhaps under the stimulus of the Islamic revival. Among the numerous works published in Byzantium during the ninth century were many editions of scientific texts: we have mentioned (page 52) one containing Archimedes' works, based on that of Isidore.

Islamic scientists — to whom we are indebted in a fundamental way for the survival of science — devoted themselves above all to the exegesis of scientific works from the imperial period; they regarded Ptolemy and Galen as the highest authorities in astronomy and medicine, respectively.

Optics was recovered primarily thanks to ibn Sahl, whose one extant work dates from around 893, and ibn al-Haytham, also known as Alhazen (ca. 965 to ca. 1039). Both wrote about mirrors of different shapes and also lenses. Alhazen's *Optics*, after treating other subjects in close emulation of Ptolemy's homonymous work, discusses the theory of spherical lenses.[9] It was Alhazen who, based on the observation that light is not emitted but only received by eye, banished the notion of a "visual ray" from Islamic optics. Whereas Alhazen, like Ptolemy, did not apply the theory of conics to optics, ibn Sahl before him had done so systematically, and considered not only parabolic and elliptic mirrors but even plane-convex and biconvex lenses bounded by hyperboloids.[10]

methodological chasm that separates the two authors to realize not only that Anthemius could not have out-distanced Apollonius but that he could not have been the discoverer of the focal property.

On the technological side, we read in [Maier], p. 66, that Anthemius of Tralles "even discovered the principle of the steam engine".

[7]See the articles in to Philoponus' knowledge of mechanics on [Sorabji]. We will return page 351.

[8]For extensive coverage of the history of Arabic science, see [Rashed: HAS].

[9]Alhazen, *Kitāb al-manāẓir* (*Book of optics*), VII = [Rashed: GD], pp. 83–110. For the relationship between Alhazen and Ptolemy in their exposition of earlier topics, see [Smith]. Recall (note 38 on page 64) that we lack parts of Ptolemy's *Optics*, including everything after the treatment of refraction through plane and cylindrical surfaces (the preceding topic being reflection in plane and spherical mirrors).

[10]Ibn Sahl, *Kitāb al-ḥarrāqāt* (*Book of burning instruments*). This work, written between 982 and 984, was recognized by R. Rashed in a manuscript in Teheran and published in [Rashed: GD]. See also [Rashed: PA].

Interest in Hellenistic science is simultaneous with the development of several industries: textiles, paper, metals, ships. Even "alchemy" is closely connected with productive processes: arrays of stills, for example, were used on an industrial scale for the production of rose-water.[11] Agriculture, much more advanced than it was in the Christian West at the same time, made use of irrigation devices powered by natural energy sources.[12]

Links between science and technology, often claimed to have been nonexistent before the modern age, are clear to Arab thinkers. Ibn Sina, or Avicenna (died 1037) lists the practical sciences that depend on geometry: geodesy, the science of automata, the kinematics of weights, the science of weights and balances, the science of measuring instruments, the science of lenses and mirrors, the science of water transport.[13] Among Hellenistic technological products that interested Arab scientists were automata (in Islam a guild of automata builders flourished for centuries[14]) and geared mechanisms.[15]

The earliest period in Western Europe to which the name "Renaissance" has been applied is the twelfth century.[16] It was then, for example, that Greek scientific works were first translated into Latin. At the beginning of the century Bernard of Chartres encapsulated his generation's relationship with the Ancients in a witticism destined to become very popular: "We ourselves are dwarfs, but by standing on the shoulders of giants [who came before us] we can see further than they." Tellingly, the opinions of giants studied and discussed during that time were not just those that later came to be regarded as canonical for the whole of "Antiquity". Thus, William of Conches' *Dragmaticon philosophiae*, written around 1140, reports statements such as that the fixed stars have an intrinsic motion too slow to be noticed within a human lifespan, and that the sun has an attractive force.[17]

In the twelfth and thirteenth centuries the Iberian peninsula and Sicily, taken back from Islam, and Southern Italy, which had stayed in contact with Constantinople all along, were important meeting points between

[11][Holmyard], p. 49.

[12]For the diffusion in Islam of wind mills and water wheels (Bassora even boasted a tide mill), see, for example, [Hill: E], pp. 780–784.

[13]Avicenna, *Resā 'il fī 'l-hikmet* (*Treatise on wisdom*), Constantinople, Y.H. 1298, p. 76. The passage is quoted, translated and discussed in [Philo/Carra de Vaux], p. 13.

[14]For the Arabic tradition of building "marvelous mechanisms", which goes back to the eighth century, see [Hill: MAS].

[15]Price has remarked on the close affinity between the Antikythera machine (Section 4.8) and a similar mechanism described by al-Bīrūnī around 1000 A.D. ([Price: Gears], pp. 42–43).

[16]The notion of a "twelfth century Renaissance" was discussed in [Munro], and more extensively in [Haskins].

[17]William of Conches, *Dragmaticon philosophiae*, III and IV.

European culture and the scientific tradition that went back to Hellenistic times. In the states arising from the Reconquest it was in fact possible to find Hellenistic works and Arab scholars able to understand their content, at least in part. Arabic translations of Euclid, Galen and Ptolemy started to spread throughout Europe. Another channel was opened violently by the Fourth Crusade, with the sack of Constantinople in 1204 and the consequent foundation of the Latin empire, one of whose consequences was a major dispersion of manuscripts hitherto held at Byzantium.[18]

Two European cultural centers of the thirteenth century that applied themselves to the task of reclaiming ancient scientific knowledge were Paris and Oxford, the latter being where the exceptionally important work of Robert Grosseteste (1168–1253) and Roger Bacon (ca. 1220–1292?) took place. Bacon's masterpiece, the *Opus maius*, organizes the knowable into seven parts. Not counting a *pars destruens*, focusing on the identification of sources of error, the second part, requisite for all other studies, is the command of Greek, Arabic and Hebrew. Only those who could read these tongues, says Bacon, could acquire essential knowledge that had remained hidden to his day from Latin speakers.[19] The same was said on various occasions by Robert Grosseteste (compare citation on page 347). Now, it is scarcely conceivable that all Greek, and especially Arabic and Hebrew, manuscripts extant in Europe in the thirteenth century survived until the invention of printing and were then published — if nothing else, readership would have been too limited to make such an enterprise viable. Therefore it is likely that on certain topics what we get from Grosseteste and Bacon comes from sources now utterly lost or as yet undiscovered. That Bacon was in the past held by many to be not only a pioneer in mathematical geography but also the inventor of lenses, gunpowder and such like shows how hard it is to track the scientific and technological sources of these ideas. Down to the present Arabic manuscripts have been "discovered" containing translations of Greek works that had remained outside the Western tradition since the imperial age, yet obviously were preserved in Muslim lands down to our days.[20] Thus we must always keep in mind the possibility that thirteenth-century scholars had access to ancient works that we have not even heard of.

What is probably the most famous document of medieval technology dates from around 1230: the sketchbook of Villard de Honnecourt.[21] In

[18] See, for example, [Vogel], p. 274.

[19] Bacon returns to this point (and to the sacrifices he had to make to procure costly manuscripts in the "scientific" languages) in a letter to Clement IV, published in [Bacon/Gasquet] and [Bacon/Bettoni].

[20] See, for example, note 28 on page 62, note 26 on page 103 and note 10 on page 331.

[21] [Villard de Honnecourt].

addition to many religious and architectural objects, it illustrates some remarkable devices that we have already encountered: for instance, on folio 22v we see a water-operated saw, a screw-jack to lift heavy weights and a simple automaton. Folio 9r shows and describes a gimbal suspension, which has been called the first of its kind but is essentially a copy of the figure that appears in Philo's *Pneumatics*.[22] The sketchbook's most fascinating device may be another one found on folio 22v, whose caption reads: "How to make an angel keep its finger pointing toward the sun". It is based on an escapement, according to one plausible interpretation.[23]

Among technological finds that spread around Europe in the thirteenth century are lenses (about which we will say more later) and mechanical clocks based on reduction gears[24] and escapements. Price, who was the greatest specialist on the subject, remarked on the close analogy between Hellenistic planetaria and a number of Chinese astronomical clocks from between the second and the ninth centuries A.D., as well as the recurrence of particulars found in the Antikythera machine (e.g., the shape of the gear teeth) in Islamic clocks and European clocks of the fourteenth century.[25]

The thirteenth century also saw the first astronomical tables made in Europe: the famous *Alfonsine Tables* (1252), compiled in Spain by Christian, Jewish and Muslim scholars by order of the Castilian king Alfonso X. Soon thereafter gunpowder was introduced to western Europe.[26]

Scientific knowledge gave Spain and Portugal a mighty edge that lasted

[22] Philo, *Pneumatica*, lvi = [Philo/Prager], p. 216; Prager remarks on the identity on pp. 26–27. Priority for Villard de Honnecourt is claimed for example in the exhibit of the Bibliothèque nationale de France on him, at http://classes.bnf.fr/villard/analyse/inv/index3.htm.

[23] China had mechanical clocks with escapement mechanisms at least as early as the eleventh century.

[24] Heron, describing reduction gears (cf. Figure 15 on page 98), systematically remarks that they slow down the movement by a factor equal to the mechanical advantage.

[25] [Price: SSB], chapter 2; [Price: Gears], pp. 42–43, and on p. 44: "It seems quite clear that the tradition of the geared calendrical work must have been continued from Greco-Roman times to Islam." However, he does not exclude the possibility of independent reinvention of complicated clockwork; after discussing the differential gear, which is not attested after Antikythera until the sixteenth century, he writes: "Perhaps there is a particular sort of inventive mind that has its particular brilliance in the perception of such things as the complex relationship of a gear system of or an involved mechanism. . . . I think that several times in history such genius has made geared astronomical clockwork so far ahead of his time that after him the development has rested for awhile to emerge with a tradition augmented more by stimulus diffusion than by direct continuation of the idea" ([Price: Gears], p. 61). But we will see in the next section (note 55) that the possibility that the differential gear was an independent invention of the modern age can be discarded.

[26] The invention of gunpowder is generally attributed to the Chinese, who had known it for centuries by the time it was introduced to western Europe (probably from Byzantium). But perhaps the difference was not great between gunpowder (saltpeter, sulfur and charcoal) and "Greek fire" (saltpeter, sulfur and naphtha, which seems to be the composition stated in a ninth-century treatise: see, for example, [Ensslin], pp. 49–50). Greek fire is usually considered to be a seventh-century invention, but Ensslin lists a number of sources that attest its use in the fifth century A.D.

for centuries. This is mostly clearly seen in navigation. In the fourteenth century the Spanish and Portuguese, thanks to mathematical geography learned from the Arabs, were still the only Europeans able to draw trustworthy geographical and nautical maps (as shown by the *Catalan Map* of 1375). The Portuguese prince Henry the Mariner (1394–1460) was one of the first to support the use of astronomical methods for high seas navigation.

11.2 The Renaissance

Starting in the mid fourteenth century, a flow of Greek documents coming from Constantinople made its way to Italy and from there to the rest of Europe, triggering what we now know simply as The Renaissance. The flow increased in the early fifteenth century. Two hundred thirty-eight, for example, were the manuscripts brought back by Giovanni Aurispa from his voyage of 1423.[27]

Renaissance intellectuals were not in a position to understand Hellenistic scientific theories, but, like bright children whose lively curiosity is set astir by a first visit to the library, they found in the manuscripts many captivating topics, especially those that came with illustrations: anatomical dissections, perspective, gears, pneumatic paraphernalia, large bronze casts, war machines, hydraulic devices, automata, "subjective" portraits, musical instruments.

The most famous intellectual attracted by all these "novelties" was Leonardo da Vinci, who not only took an interest in all the things just listed, but even ventured — unsuccessfully, alas — to master Archimedes' works. He fared much better when he tried to put in practice some of the ideas contained in ancient works, especially when he could use his extraordinary gifts of observation and depiction: for instance, in trying to recover anatomy through the dissection of corpses and in making observations in hydraulics.

For a while now Leonardo da Vinci has no longer appeared as a lone genius, but as the most important representative of a milieu where for many decades the same subjects had been pursued, the same books had been prized and similar drawings had been made.[28] Many of Leonardo's

[27] Aurispa, above and beyond being a humanist, was one of the many merchants who devoted themselves to the profitable traffic of manuscripts from Constantinople to Italy in the early fifteenth century. We may be sure that a part of the books that reached Italy at that time was lost after a few generations. Incidentally, the ideas that sprouted in those years among Italian artists included perspective and the possibility of building larger domes by making their section parabolic (or nearly so) rather than semicircular.

[28] This reappraisal started with [Gille: IR].

favorite technological subjects had earlier occupied, in the first half of fifteenth century, the Sienese Mariano Taccola, who was particularly keen on Philo of Byzantium's works on pneumatics and military engineering. In the same period was made what is probably the first translation of a work of science in a modern European language: the Italian version of Philo's *Pneumatics* that opens the anonymous manuscript *Hydraulic and war machines*.[29] The second half of the century saw the appearance of the *Treatise on architecture, engineering and the art of war* by the sculptor, architect and engineer Francesco di Giorgio Martini, also from Siena, which has drawings of water wheels fed by pressure pipes, vacuum and pressure pumps, endless worms, rack-and-gear mechanisms, many other elements from Hellenistic technology and even a vehicle with a steering wheel.[30]

Whereas Taccola and Francesco di Giorgio focused mainly on Philo and Vitruvius, Leonardo, like other engineers of his time, was also very interested in Heron. Often in the past the same critics who pooh-poohed Heron's "useless toys" fawned on the Leonardo's "futuristic" technical drawings, many of which turn out to have been either copied from or closely inspired by Heron: screws, reduction gears, screw threaders, automatic pounders, wind wheels, syphons, "Heronian" fountains, devices moved by rising hot air, water levels...[31] For other things, such as the flat-mesh conveyor belt and the repeating crossbow, Leonardo follows Philo of Byzantium. In numerous other notes, he clearly shows himself in debt to ancient sources: in his observations on optics, or on the origin of sea fossils found far inland; or yet in drawings of wheel boats, burning mirrors, crossbows, hydraulic saws, ball bearings.... The list goes on.

The oft-heard comment that Leonardo's genius managed to transcend the culture of his time[32] is amply justified. But his was not a science-fiction voyage into the future so much as a plunge into a distant past. Leonardo's drawings often show objects that could not have been built in his time because the relevant technology did not exist. This is not due to a special genius for divining the future, but to the mundane fact that behind those drawings (and Francesco di Giorgio's) there were older drawings from a time when technology was far more advanced.

[29] *Macchine idrauliche, di guerra, etc.* The manuscript continues with extracts from Vitruvius and compilations of various kinds, including one on incendiary substances, and concludes with the transcription of Taccola's *De ingeniis*, which is based on Philo's writings and deals with pneumatics and military technology. The contents of the manuscript (preserved in the British Library as Additional Manuscript 34113) is described in [Philo/Prager], pp. 112–113.

[30] On Taccola and Francesco di Giorgio see [Galluzzi], for example.

[31] It is enough to compare the respective drawings to reach this conclusion. Not many have done so, because of the limited availability of Heron's books.

[32] The *Encyclopaedia Britannica* says: "his notebooks reveal... a mechanical inventiveness that was centuries ahead of his time" (15th edition, Micropaedia, sub "Leonardo da Vinci").

Leonardo's written explanations are often not on par with his sketches. In the Leicester Codex[33] there is a drawing of a machine moved somehow by steam. Here is Bertrand Gille's comment about it:

> The drawing is quite striking. Except for the explanation given on the same page, one could swear that it shows a primitive steam engine. But it is nothing of the sort. ... There are many figures of this curious device — curious above all because of its similarities [with later devices].[34]

Gille does not seem troubled by the notion that a drawing should not have been understood for what it is by its own author, nor yet by many subsequent generations.

Knowledge about Hellenistic technological elements spread in Europe above all through Philo's and Heron's works. Some of this knowledge, such as the shape of various kinds of gear, could be easily reconstructed from the manuscripts' illustrations, but the situation was not always so favorable. Regarding the pumps drawn by Francesco di Giorgio, Gille writes:

> The drawings that Francesco left us of vacuum and pressure pumps and of a hydraulic saw show that he ran into the same difficulties as his predecessors, namely the impossibility of a working implementation. Only developments in metallurgy and metal turning and the use of proper lubricants would allow [the rod-and-crank system] to acquire its full intended range of use. [35]

The use of oil to lubricate pumps is mentioned by Vitruvius, in a passing reference that shows it was standard in the working implementations of his time.[36] In Francesco de Giorgio's day, much worse than the lack of proper lubricants was the backward state of metalworking: many Hellenistic designs were to be made in metal, and so could not be imitated in the Renaissance for the loss of good techniques of metal smelting, casting, turning and grinding. Ludovico Sforza called Leonardo to Milan and charged him with the creation of a great bronze statue, but the manufacture of this colossus remained a fantasy vainly pursued for many years. The Sforzas' interest in the development of casting techniques was not just esthetic: many advances, especially in artillery, were blocked by the

[33] Folio 10r.

[34] [Gille: IR], p. 179.

[35] [Gille: IR], p. 103.

[36] Vitruvius, *De architectura*, X, vii §3.

inability to recapture the ancient technology that had allowed the casting of molten metal into colossi.[37]

Although the methodology of science remained well beyond the grasp of Renaissance intellectuals, there was widespread interest in certain scientific theories, especially those, such as mathematical geography and astronomy, that were essential to navigation.

Already in 1406 Jacopo Angelo had translated Ptolemy's *Geography* into Latin, and later he brought a copy of the original from Constantinople to Europe. The *Geography* was finally printed in 1477. To appreciate the importance of this work it is enough to compare a map prior to that date (apart from those made in Islamic and Iberian lands[38]) with those that came after: for instance the *Hereford Mappa Mundi*, drawn around 1300 in England and showing an oceanless flat world centered in Jerusalem and crammed with unrecognizable continents separated by thin water lines, versus the 1492 map engraved by J. Schnitzer in Ulm.

The rediscovery of mathematical geography breathed new life into an old Hellenistic idea: reaching the Indies by sailing west.[39] Just seven years after the *Geography* was published Columbus presented his plan to the king of Portugal, and another eight years later he bravely embarked on his enterprise.

The next milestone in mathematical geography was the recovery in the sixteenth century of Erathostenes' estimate for the size of the earth. This was in all likelihood the basis for the length of the degree of the meridian adopted during that century by Portuguese navigators.[40]

At the same time that mathematical geography was being rediscovered, so was ancient astronomy. Girolamo Fracastoro and Giovambattista Amici proposed again, independently of one another, the theory of concentric spheres of Eudoxus of Cnidus. The study of the classics also led to the rediscovery of the earth's motions: the first modern work to propose the daily rotation was written around 1525 by the humanist Celio Calcagnini, professor of belles-lettres at Ferrara.[41]

Aristarchus' heliocentric theory was first revived by Copernicus in his *De revolutionibus orbium caelestium*, published in 1543. That Copernicus was following the ancient thinker was quite obvious to his contempo-

[37] We don't know why this technology was developed in the first place, but we can be sure that in the third century B.C. Rhodes was no less interested in naval technology than in large ornamental statues. See pages 114 and 116.

[38] See page 335.

[39] Attempts to circumnavigate the globe are recorded in Strabo; see note 72 on page 114.

[40] [Taylor], p. 547.

[41] *Quod caelum stet, terra moveatur, vel de perenni motu Terrae*, published posthumously in his *Opera aliquot*, Basel, 1544. His arguments for proposing the earth's rotation consist essentially of a series of classical citations, ranging from Virgil to Archimedes.

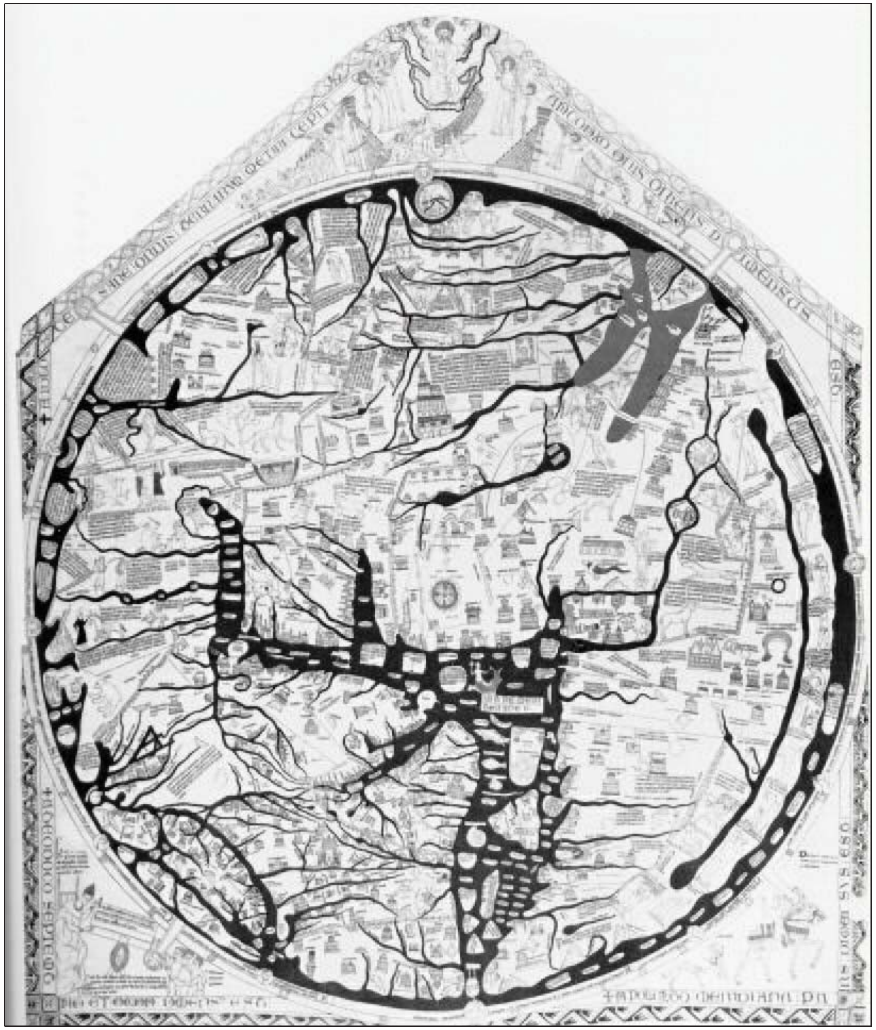
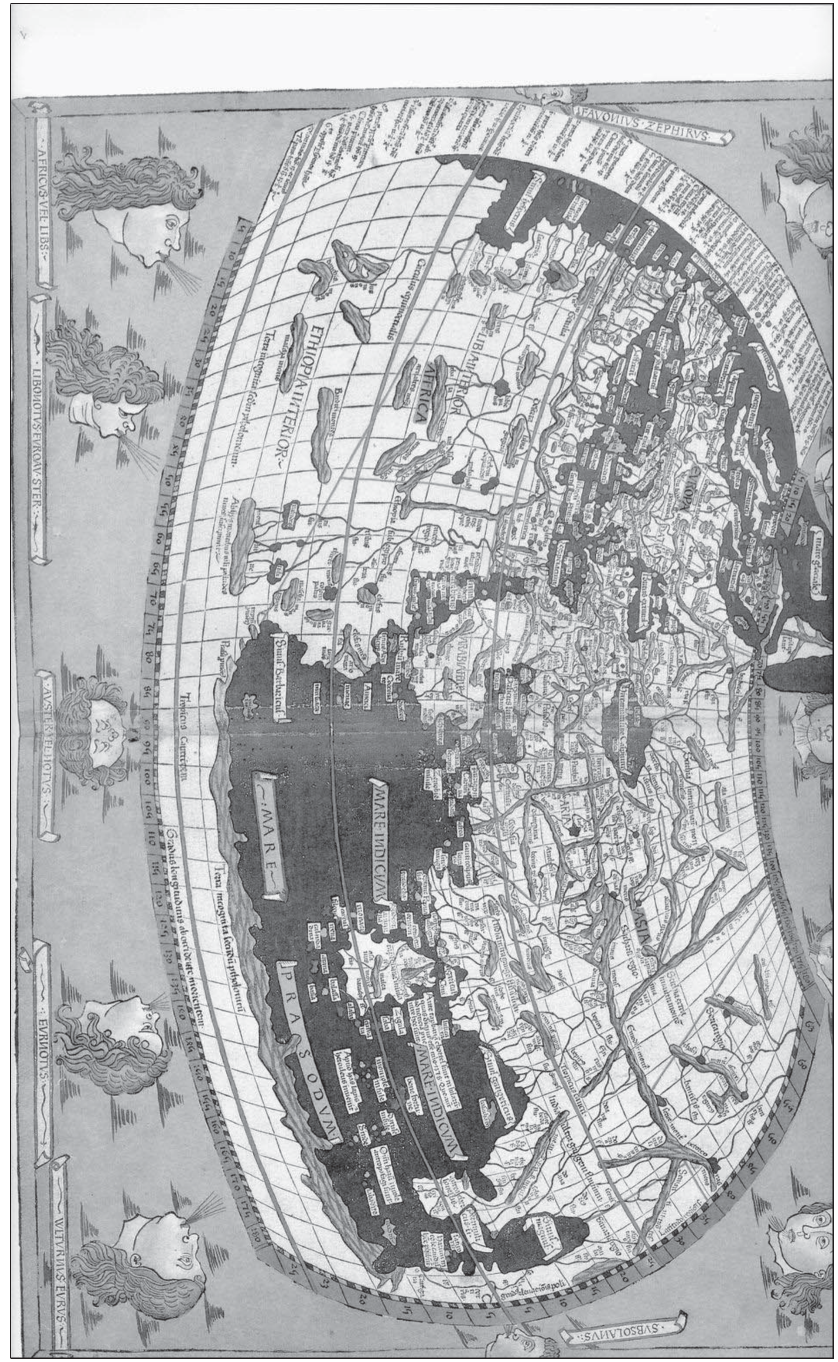


FIGURE 11.1. Above: Hereford Mappa Mundi. Overleaf: Schnitzer's 1492 map.



raries, who could do mathematics and astronomy only through the attentive study of the Hellenistic works that had resurfaced. Note that Copernicus, in putting forth again the heliocentrism of Hellenistic times, at the same overcame the Aristotelian concept of gravity, by co-opting the polycentric theory (capable of explaining the roundness of the sun, moon and earth) mentioned by Plutarch.[42]

Copernicus did much more than overcome Ptolemy's geocentrism and views on gravity by taking up Aristarchus' ideas. From the perspective of science the key point is that he was able to create an algorithm, based on a system of epicycles, to calculate the apparent motion of planets: thus he was the first person able to reconstruct Ptolemy's mathematical astronomy.[43] This reconstruction was far from trivial, because the *Almagest* contains no hint about how to derive the algorithm whose use it describes, and moreover the algorithm needed fine tuning in view of errors accumulated in the intervening 1400 years.

Because of the choice of accepting the "hypotheses" of Aristarchus, this fundamental work of recovery took on the odd guise of a battle against Ptolemy. Copernicus' opponent was not in fact Ptolemy, but the syncretic image derived from his system by European culture. The Ptolemaic system was not understood in its true function as an algorithm to predict the motion of planets, because nobody before Copernicus had been able to use it that way. Ptolemy's name had instead become associated with a complex cosmology, such as we see in Dante, blending certain aspects of Ptolemaic astronomy (geocentrism, the use of circles to describe motions) with features from Aristotelian natural philosophy and Christian religious tradition. Thus the cultural battle for heliocentrism acquired a huge ideological charge, since it was necessary to overcome that cosmology and, by recognizing the earth as a "heavenly" body, conversely affirm the "earthly" nature of astronomical phenomena.

Kuhn has called attention to the fact that only after the Copernican revolution took hold was it possible to observe the appearance of new stars and the motion of comets across the putative planetary spheres. Though visible to the naked eye, these phenomena had been ignored so long as the Ptolemaic paradigm (with which they are incompatible) held sway.[44] Since we know that novae were observed and recorded in Hellenistic times, as were the elongated orbits of comets (see Sections 3.7 and 10.9), we have

[42]Copernicus, *De revolutionibus orbium caelestium*, I, 9. For the polycentric theory and the Plutarchan passages where it is mentioned, see page 304. Recall that Copernicus cites Plutarch already in the preface to his work.

[43]This is underlined in [Neugebauer: ESA], pp. 241–242, but Neugebauer is totally indifferent to the ideological import of the "Copernican revolution".

[44][Kuhn: SSR], p. 115–116; [Kuhn: CR], pp. 206–209.

further proof that the Ptolemaic paradigm only became prevalent in the imperial period. Kuhn does not seem to have realized this consequence of his observation.[45]

The sixteenth century was a time of great interest in other aspects of Hellenistic culture as well. The study of anatomy was resumed: as in the case of astronomy, the main step was the rediscovery in Byzantine manuscripts of the Hellenistic science of the third century B.C., in contrast with the tradition originating in the imperial age, which had already been reacquired in part through the Arabs of Spain. Just as Ptolemy was superseded by the use of Aristarchus' ideas, so was Galen superseded through the partial recovery of Herophilus. Among the leaders in this process were Vesalius (1514–1564), Cesalpino (1519–1603) and Fallopius (c. 1523–1562). Vesalius revived the ancient practice of dissection, recognizing that Galen's belief that one could do without it represented a grave limitation of his science. Cesalpino is known today as the founder of modern scientific botany and of botanical taxonomy, but in his time he was best known as a commentator of Aristotle. Among his important contributions to anatomy was the description of cardiac valves (earlier described by Herophilus) and their function in blood circulation.[46] Fallopius, whose name became associated with anatomic features described by Herophilus, such as the Fallopian tubes and the Fallopian acqueduct (canalis facialis, the orifice in the temporal bone that admits the facial nerve), wrote that "Herophilus' authority on anatomical matters is gospel to me", and again:

> When Galen contradicts Herophilus, it is to me as if he were contradicting medical gospel.[47]

These would be strange statements if Fallopius' acquaintance with Herophilus were, like ours, limited to late references and fleeting citations, many of them in Galen. One wonders whether he and Cesalpino made some of their anatomical discoveries in Herophilus' writings. If a work of

[45]A certain parallel can be drawn with the fact that not only were fossils ignored during the Middle Ages, but ancient observations of and theoretical explanations for fossils have tended to be ignored in recent times (see pages 161–163). Thus we see a twofold operation of the fertile principle that phenomena incompatible with current paradigms tend to be systematically ignored. It seems that the principle is equally applicable to science proper and to scientific historiography.

[46]*Andreae Caesalpini Peripateticarum quaestionum libri quinque*, Florence, 1569; see also his *Quaestionum medicarum libri duo* (with other works), Venice, 1593. Those who would have William Harvey (1578–1657) as the discoverer of blood circulation maintain that Cesalpino's description of the circulatory system was incomplete.

[47]"Herophili authoritas apud me circa res anatomicas est Evangelium." "Quando Galenus refutat Herophilum, censeo ipsum refutare Evangelium medicum." Both passages come from the *Observationes anatomicae* and are quoted in [von Staden: H], p. xi.

the father of anatomy had survived into the sixteenth century[48] and made its way into the hands of some lucky physician, it would not be to the advantage of the possessor of such a treasure to divulge its contents by having it published. Of course, it is very hard to turn specific suspicions of this sort into proofs, but there is no reason to believe that all Hellenistic works that survived down to the Renaissance still exist today, or that those that exist have all been published or are even necessarily known.

An annotation by Leonardo in a manuscript contains the sentence “You will get through Borges the Archimedes from the bishop of Padua and through Vitellozzo the one from the village at San Sepolcro.”[49] Some of Leonardo’s sources on Archimedes must have contained information now lost: for instance, he describes and draws an otherwise unknown steam cannon (the *architronito*, or “mega-thunder”), crediting it to Archimedes,[50] and he knows biographical details concerning a stay of Archimedes in Spain[51] and concerning his tomb[52] of which we have no other record.

There is more in Leonardo’s manuscripts that seems to be based on now vanished sources. It would be nice to find out, for example, where he read about the magnification of faraway objects and the possibility of building an instrument that could be used to study the features of the moon’s surface.[53] Perhaps his source was the same that allowed Girolamo Fracastoro, a bit later (1538), to be more specific and say that this is done with two lenses, not least because Fracastoro too (a physician, poet and humanist) talks about using the instrument to observe the moon.[54] What is certain is that neither author was able to describe in detail, and much less build, the telescopes of which they write.

Mechanical technology, too, continued to move forward in the sixteenth century, with the building for amusement of various self-propelled mechanisms (which gave rise to such things as clocks with jackwork) and the

[48]Herophilus’ writings were still being cited, apparently directly, in Constantinople in the sixth century, and works of the Herophilean school, particularly those of Demosthenes Philalethes, were available in the West in the fourteenth century. See [von Staden: H], pp. 68–69, 573.

[49]Codex L of the Institut de France, 2a = [Leonardo/Richter], vol. II, p. 428.

[50]The passage is quoted by Gille, who adds: “Such experiments … led Leonardo to conceive the war engine he called the *architonitro*, whose paternity he attributes to Archimedes, for reasons not too clear. It seems this was simply the same as the famous seventeenth-century experiment of making a cannon shoot by filling it with water and heating” ([Gille: IR], p. 179).

[51]Codex Ashburnham 2037 (ex codex B), 12 b = [Leonardo/Richter], vol. II, p. 451. Trips of Archimedes after his return from Egypt, not known from any ancient source presently available, are also mentioned by Torelli in his biography of the scientist at the beginning of [Archimedes/Torelli].

[52]Codex Arundel (British Museum), 279b = [Leonardo/Richter], vol. II, p. 446.

[53]Codex E of the Istitut de France, 15b = [Leonardo/Richter], vol. II, pp. 140–141.

[54]“Per dua specilla ocularia si quis perspiciat, alteri altero superposito, maiora multo et propinquiora videbit omnia” (G. Fracastoro, *Homocentrica sive de stellis*, II, viii); the reference to the moon is in III, xxiii. This is the book where the author puts forth Eudoxus of Cnidus’ theory of concentric spheres.

reproduction of ancient devices, like the differential gear, which were to be of great importance to productive technology.[55]

Despite all these advances, the general picture remained prescientific. Specific scientific results were grasped from Greek or Arabic manuscripts, but not the methodology that had led to them. The study of Hellenistic books brought with it the rediscovery of elements of exact science and technology, of anatomy, of philology and even the possibility of ocean voyages, thus putting Europe on the route to modern civilization, but the conceptual framework was sought in the “Ancients” as a whole. Neo-Platonist and neo-Aristotelian schools were born; classical reading was put on a pedestal, justified by theoretical reflections on the superiority of the Ancients, all of whose writings shared the same lofty status; Pliny’s *Natural history* was printed and read side by side with Hellenistic scientific works, with no awareness of the abyss that separates the two. In other words, the Renaissance accepted the idea (born in imperial times) of a Greco-Roman civilization that in fact confused completely different cultures.

The retrieval of Byzantine manuscripts by the West did not end when Constantinople fell to the Turks in 1453; it continued on for centuries. We mentioned on page 221 the importance for philology of the eighteenth-century find of ancient scholia to the *Iliad* in a codex preserved in Venice. Besides this city, which had always had special ties with Byzantium, manuscripts could also come from regions of the Byzantine empire, such as Dalmatia, that had been conquered by the Venetian state.

11.3 The Rediscovery of Optics in Europe

Optics was the first Hellenistic scientific theory attempted to be recovered. The usefulness of the ancient science of *perspectiva* — this was the name given in Latin to optics in the narrow sense, that is, the *science of sight* — quickly became obvious, even though its first applications to painting had to wait yet a century or so. But perspective was not all there was to it.

[55]The first application of differential gears in production (to a threading machine) came centuries after the introduction of the same mechanism in astronomical clocks. Any doubt that it was an independent reinvention rather than the resumption of Hellenistic knowledge disappears when we observe that in sixteenth-century astronomical clocks differential gears were employed to transform synodic months into sidereal months, the exact same use to which they were put in the Antikythera machine; see [Price: Gears], pp. 60–61. Since today we know about the existence of differential gears in Antiquity only from twentieth-century underwater archeology, this gives yet another proof that the sixteenth century still knew certain things about classical technology that were lost later.

In Section 10.2 we mentioned the dispersion of light, an area where "ancient science" usually gets very little credit. Since it is a common opinion that Newton was the father of the theory of dispersion (the dependence of a medium's refractive index on the color of incident light), we will give him the floor. In his *Opticks*, at the beginning of his explanation for the rainbow, he writes:

> ... this Bow is made by Refraction of the Sun's Light in drops of falling Rain. This was understood by some of the *Antients*, and of late more fully discovered and explained by the Famous *Antonius de Dominis* Archbishop of *Spalato*, in his book *De Radiis Visûs & Lucis*, published by his Friend *Bartolus* at *Venice*, in the Year 1611, and written above twenty Years before. For he teaches there how the interior Bow is made in round Drops of Rain by two refractions of the Sun's Light, and one reflexion between them, and the exterior by two refractions, and two sorts of reflexions between them in each Drop of Water, and proves his Explications by Experiments. . . . The same Explication *Des-Cartes* hath pursued in his Meteors. . . . But whilst they understood not the true Origin of Colours. . . [56]

Six pages later comes a report of several of de Dominis' experiments with globes full of water, which today are usually attributed to Newton. We must conclude that, despite Newton's reservations, the modern theory of dispersion did not start with him or Descartes, but with the Dalmatian archbishop. Since his *De radiis* was written no later than 1590 — as we know from the preface, by the editor Giovanni Bartolo, to the book's first edition, of 1611 — his optical experiments must have been started around the same time, if not earlier, than Galileo's first experiments (1586). Thus de Dominis not only pioneered the modern theory of dispersion: it seems he must be regarded as one of the founders of the "experimental method" that, in the common opinion, is exclusive to modern science.

De Dominis' work, from the title onward, uses Hellenistic terminology: it talks of visual rays, which had been abolished by the Arabs back in the eleventh century.[57] Even more tellingly, his explanation for the rainbow had already been given at the turn of the thirteenth century by Dietrich (Theodoric) of Freiberg[58] and, apparently independently, by the Arabic

[56]Newton, *Opticks* (1704), Book I, Part II, Prop. IX, Prob. IV, pp. 126–127 (or p. 169 in the most common reprints of the 1730 edition).

[57]See page 331.

[58]Dietrich of Freiberg, *Tractatus de iride et de radialibus impressionibus* = [Dietrich of Freiberg], pp. 115–268.

writer Kamal al-Din al Farisi,[59] both of whom described the same experiments with water globes reported by de Dominis.[60]

In his book, de Dominis — who at the time of its writing, around 1590, taught optics and other subjects in Padua, where Galileo was to arrive a few years later — also treated the theory of lenses and (finally!) gave an explanation for how telescopes work, though it seems that the instrument did not exist then. Since the book came out in 1611, when telescopes were the order the day thanks to Galileo's discoveries of the previous year, doubts have been cast on the date of writing stated by Bartolo, but what is unarguable is that around 1590 the theory, if not yet the practice, of the telescope was spreading around Europe, because the second edition of Della Porta's *Magia naturalis* (1589) makes very precise references to it.[61]

It seems that the development of optics was plagued by amazing bad luck: the "Ancients" knew how to make good lenses but did not know what to make of them and kept them as baubles,[62] later intellectuals — not just Leonardo and Fracastoro,[63] but also Roger Bacon and Grosseteste centuries earlier — knew many uses for them, yet could not build them and had never even seen such things. Some medieval manuscripts even show astronomers looking at the sky through long tubes; the incongruity has been addressed by postulating that these were empty sighting-tubes![64] (See Figure 11.2.)

Consider that Bacon, in the fifth book of the *Opus maius*, waxed enthusiastic about the Ancients' ability to enlarge small objects and to bring faraway ones close, using appropriate configurations of lenses and mirrors — though he himself is unable to present a reasonable theory even in

[59][Rashed: MST].

[60]Crombie is convinced that Dietrich of Freiberg did perform the experiments that Newton credits de Dominis with ([Crombie: AG], pp. 122–125), and he calls them "an outstanding example of the use of the experimental method in the Middle Ages" (p. 122). The fact that Dietrich and al Farisi offer the same explanation Crombie calls "a curious coincidence", but explains through the use of common sources (ibid., p. 124). See also [Ziggelaar] for arguments for the non-originality of de Dominis' optical treatise.

[61]We met Della Porta when discussing the resurrection of the steam engine recorded by Heron, and we will meet him again in connection with Philo of Byzantium's thermoscope. This playwright and student of the classics alludes in his works to many other objects of "modern" technology, such as the "magic lantern" (which had also been a subject for Alhazen).

[62]See page 271.

[63]See page 343.

[64]The matter is summarily dealt with in [Zinner], pp. 214–215, where two such figures are mentioned: one from a manuscript of 982 (extant copy, in Sankt Gallen, from the 13th or 14th century) and one from a 1241 manuscript now in Munich. Price gallantly allows as how the idea that these are depictions of telescopes "cannot be summarily discarded merely because of the great improbability of the invention having been made so early" ([Price: Instruments], p. 593), and then tacitly discards it. In fact the illustrations may represent a residual iconographic tradition that was no longer understood.

FIGURE 11.2. Triptych illustration from medieval manuscript (1241). The person with the tube is named as Ptolemy. Bayerische Staatsbibliothek, CLM 17405, fol. 3.

the case of a single lens. Consider the lucid account, in Grosseteste's *On the rainbow* (ca. 1230), of the possibility of using refraction phenomena to build magnifiers and telescopes:

> The main parts [of optics] are three, according to the ways in which the rays can reach what is seen. Either the course of the ray is straight, through a uniform transparent medium lying between the viewer and the object seen; or its course is straight toward ... a mirror, in which it is reflected and so reaches the object seen; or the ray goes through several transparent media of different natures, at the boundaries of which it bends at an angle, thus reaching the object not through a direct line, but through several straight lines joined at angles.
>
> The first part is the province of the science called "of sight", and the second, of that called "of mirrors". The third part has remained untouched and unknown among us [Latin speakers] until now. But we know that Aristotle studied it, that because of its subtlety it is much harder than the others, and that by the depth of the phenomena [it explains] it was the most admirable. Indeed this part of optics, if perfectly known, shows the way to make things that are very far away

> appear as if placed very close; large things placed nearby appear very small; and small things placed at a distance appear as large as we please...[65]

The source that Grosseteste mentions (and attributes to Aristotle), if real, naturally used a Greek term to indicate the study of refraction. This term, if known in thirteenth-century Oxford, would have a good chance of being transmitted to later generations. The term *dioptrics*, used by modern scholars, was spelled out in Greek (διοπτρική) in Kepler's work of 1611,[66] and this, together with its etymology, which matches well the phenomenon of light passing through a transparent medium, indicates that it may have been precisely the term transmitted from ancient works. The same term is attested in ancient works still extant, but with an apparently different meaning, namely the technology relative to the construction and use of dioptras.[67] The two meanings would naturally coincide if there were ancient dioptras that employed refraction (lenses).

The modern study of refraction phenomena was not limited to the study of Ptolemy's *Optics*. Whereas Ptolemy believed that the angle of refraction varies quadratically with the angle of incidence (see figure on page 65 and surrounding text), modern scholars found the famous sine law, now known in English as Snell's law. This was attributed to Descartes — and still bears his name in French — because he was the first to publish it, in his *Dioptrics* (1637). Later it was realized that Willebrord Snell already knew the law in 1621 (as first documented by Huygens in his *Dioptrics*, of 1703), but Snell himself was apparently preceded by Thomas Harriot, in 1601 (neither man bothered to publish the result). Today we know that priority belongs to neither Snell nor Harriot: both were preceded at least by the Arabs, centuries earlier. It is true that "Snell's law" was not known to Alhazen, but it had been known a few decades earlier to ibn Sahl.[68] That Arabic knowledge about refraction seems to increase as time moves backward rather than forward raises the suspicion that it was based not on original experiments but on the reading of ancient sources.

In ibn Sahl the refraction law is expressed in terms of ratios of segments, not in terms of angles and their sines (though the sine function was introduced by the Arabs), and is applied as something well-known, without justification or any claim to originality. And one gets the impression that he knew that the law is an application of the principle of the shortest path.

[65] Robert Grosseteste, *De iride*, 73–74 (ed. Baur).

[66] Johannes Kepler, *Dioptrice, seu, demonstratio eorum qua visui ... accidunt*, Augsburg, 1611; this apparently the first modern use of the term.

[67] For example, Heron, *Dioptra*, xxxiv, 292:16 (διοπτρικὴ πραγματεῖα); Proclus, *In primum Euclidis Elementorum librum commentarii*, 42:4 (ed. Friedlein).

[68] It is clearly stated in the *Book of burning instruments* mentioned in note 10 above (page 331).

What is curious is that Grosseteste too seems aware that refraction, like reflection, can be described by a minimum principle,[69] even though his refraction law is patently wrong.[70]

11.4 A Late Disciple of Archimedes

The pace of study of resurfaced works of Hellenistic exact science picked up at the end of the sixteenth century. Pride of place goes to Galileo, who, after writing several commentaries on Archimedes, was the first person who attempted to build new scientific theories, in his *Dialogues and mathematical demonstrations concerning two new sciences* (1638).

Because Galileo is not widely read today and is often portrayed as the founder of a new and almost unprecedented method, we must stress that the aim of recovering Hellenistic science was openly and avowedly part of his work.[71] Love and enthusiasm for this far past were quite likely instilled in him by his father Vincenzo, who had boldly undertaken a similar, and extremely difficult, work of reconstruction in the field of music.[72]

At the end of the third day of the *Dialogues* — the day that contains Galileo's greatest contribution to the science of dynamics — there is a passage of great significance, a profession of what the author, now old, thinks was his greatest scientific achievement. Galileo congratulates himself by putting in Sagredo's mouth the following words:

> I do think we can grant our Academician [Galileo] that it is no idle boast when he says, at the beginning of this treatise of his, that he has founded a new science dealing with a very old subject. And seeing with what ease and clarity he deduces from one very simple principle the demonstrations of so many propositions, I marvel at how such a matter could have been left untouched by Archimedes, Apollonius, Euclid and so many other famous mathematicians and philosophers; the more so because plenty of thick books have been written about motion.[73]

[69] Robert Grosseteste, *De iride*, 75 (ed. Baur).

[70] Philosophers of science have, with good reason, criticized the notion of absolute scientific truth, but Grosseteste's law of refraction qualifies as an "absolute scientific falsehood" if anything does. He purports to obtain the refraction angle from the incidence angle independently of the refracting substance (*De iride*, 74, ed. Baur). In my opinion, this absurdity can only come from a misunderstanding of a source, since any experimenter is aware that the effects of refraction tend to disappear when the two media are very similar. Yet Grosseteste has often been regarded as one of the fathers of the experimental method, especially after the publication of [Crombie: RG].

[71] See [Drake: Galileo]. While earlier historians had sought the origins of Galilean science in its medieval precursors, Drake has clarified how the development of ideas was not continuous, but instead the same Greek sources were essential both for the Arabs and for Galileo. On this see [Drake: HGG], in particular.

[72] Among the works of Vincenzo Galilei, composer and music theorist, is a *Dialogue of ancient and modern music* (1581). He also published the hymns of Mesomedes, from the second century A.D. The model he offered for the interpretation of ancient music thus rediscovered was essential to the formation of Florentine opera.

Thus Galileo's highly ambitious scientific aim was the recovery, after so many centuries of oblivion, of the Hellenistic scientific method, consisting in the creation of hypothetico-deductive systems where natural phenomena can fit. With his well-honed critical mind, Galileo unequivocally names his own models — the great scientists of the golden period — without letting his obvious admiration blur into reverence for an undifferentiated "Antiquity". Indeed, he does not hesitate to take issue with both Aristotle and Ptolemy.

Galileo did succeed in breathing new life into two legacies from his distant masters, the experimental method and the deductive method. Yet he still lacked a command of the more refined Hellenistic mathematical tools. While he could use Euclidean proof techniques and geometric algebra, he never did grasp the so-called "method of exhaustion" and the theory of proportions (and indeed nobody would for another two centuries and more).

The crux of Euclid's definition of proportions is that it is equivalent to a construction of the notion of the ratio between magnitudes;[74] thus it is altogether foreign to a Platonist understanding of mathematics and definitions. If ratios between magnitudes are conceived of as preexisting, equality between them cannot but seem a self-evident notion, and Euclid is guilty of introducing an abstruse and superfluous complication — one which Galileo felt able to dispense with easily:

> I will add another way in which one should understand that four magnitudes are in proportion. It is the following. When the first is neither more nor less than is necessary in order for it to have to the second the same proportion that the third has to the fourth, we say that the first magnitude has to the second the same proportion that the third has to the fourth.[75]

[73] Galileo Galilei, *Discorsi e dimostrazioni. . .*, end of third day = [Galileo: Opere], vol. VIII, pp. 266–267.

[74] See page 46, where the definition is given, and page 181.

[75] Galileo Galilei, *Sopra le definizioni delle proporzioni d'Euclide*, at Salviati's seventh turn = [Galileo: Opere], vol. VIII, p. 353. The passage caps Galileo's critique of Euclid's definition (on the grounds that it is impossible to apply and is "more of a theorem to be proved than a definition to be given").

This dialogue was first published by Vincenzio Viviani in his edition of Book V of the *Elements* (*Quinto libro degli Elementi d'Euclide, ovvero Scienza universale delle proporzioni spiegata colla dottrina del Galileo. . .*, Venice, 1674), with the subtitle: "To be added to the four *Dialogues and mathematical demonstrations concerning two new sciences*."

The circularity of this "definition" makes it clear that in Galileo's time we were still far from regaining the ability to build true scientific theories.

As to the seminal reestablishment of the experimental method, the common attitude of denying any indebtedness to ancient science also denies Galileo the merit of his hard, deliberate work in this direction. But a brief recapitulation of the main phases of his experimental work can help set the record straight.

Galileo's first known experiments date from 1586, and tried to rebuild the experimental basis for Archimedes' *On floating bodies*. They culminated with the construction of a hydrostatic balance, described in *La bilancetta*. Galileo had already realized how important it was not just to read ancient scientific texts, but to grapple with their concrete component.

His second experimental scientific work, to our knowledge, is the first on the motion of bodies, described in the *De motu*. Since the theory derived therefrom by Galileo does not differ from the one that Simplicius attributes to Hipparchus,[76] there is no reason to believe that the latter's theory was founded on a less solid experimental basis.

As to the momentous observation that the time a body takes to fall does not depend on its weight (if one neglects air resistance), it was thought for centuries that Galileo reached it by dropping weights from the tower of Pisa. Surely a simpler way would have been to read it in Philoponus' commentary to Aristotle's *Physics*.[77] And it is not easy to maintain that the experimental method, having eluded Hellenistic scientists, was invented in the sixth century A.D. by a theologian and commentator of Aristotle. Since some of Philoponus' statements on motion under gravity are akin to those that Simplicius attributes to Hipparchus, and since both commentators probably had access to the same sources,[78] one might conjecture that the invariance of fall time was already mentioned in Hipparchus' work on gravity[79] — if only because it is hard to see how else it would be known to Lucretius,[80] Hipparchus having been the last Hellenistic scientist who studied motion under gravity, at least to our knowledge.

[76] As observed in [Koyré: EG], pp. 70 and 100. In the *De motu*, Galileo cites the relevant Simplicius passage on Hipparchus (which we discussed on page 292). He says he read it only after having formulated the same theory independently ([Galileo: Opere], vol. I, pp. 319–320).

[77] John Philoponus, *In Aristotelis Physicorum libros commentaria*, 683 in [CAG], vol. XVII.

[78] For the connection between Philoponus and Simplicius, see p. 329.

[79] The real difficulty that must be overcome in reaching this invariance result is the need to set aside the effects of air resistance. Thus the result is within reach of a theory based on the principle of inertia and the notion of friction, and our earlier considerations (Section 10.6) make it plausible that Hipparchus had gotten there. Philoponus (ibid., 642) says that a projectile receives at the moment of launching a δύναμις κινητική, which he calls "incorporeal" — an adjective that, as we saw in Sextus Empiricus, had been used since imperial times to describe the entities of Hellenistic scientific theories. Philoponus also uses for the same notion another name that was to have a bright future: κινητικὴ ενέργεια, kinetic energy.

Suitable adaptation or alteration of experimental conditions to facilitate measurements is customarily regarded as one of the fundamental features of the Galilean method. For the study of motion under gravity, the object of Galileo's most important experiments, the key alteration was the use of an inclined plane. His first consideration of inclined planes appears already in the youthful *De motu*. Most interesting is a statement about the particular case of horizontal planes:

> And in this situation [i.e., in the absence of friction], any movable body lying on a plane equidistant from the horizon will be moved by a minimal force, that is, by a force smaller than any arbitrary force.[81]

Though Galileo did not foresee all its consequences at the time, this sentence marks the decisive step toward the supersession of Aristotelian physics and the formulation of the principle of inertia. We have already seen how Heron had introduced the subject:

> We demonstrate that a weight in this situation [i.e., on a level, frictionless plane] can be moved by a force less than any given force.[82]

Heron's demonstration, treating a horizontal plane as the limiting case of an inclined plane whose slope approaches zero, was to reappear in the *Dialogues*. Now, the possible influence of Heron's *Mechanics* on Galileo is generally dismissed because that work, apart from extracts contained in Pappus, is held to have been unknown to Europe until the end of the nineteenth century, when it was found in Arabic translation. But correspondences between several passages of Galileo — on subjects such as friction or motion on inclined planes — and closely analogous passages in Heron not relayed by Pappus should make one suspect that the two texts were not independent.[83] And obviously, given that even today not all manuscripts in Italian public libraries have been catalogued, it is absurd to claim exhaustive knowledge of those found in private libraries four hundred years ago.[84]

[80] Lucretius, *De rerum natura*, II:225–239. Clagett writes that Philoponus' passage "appears to indicate that he had dropped bodies of different weight" ([Clagett: SM], p. 546). He continues: "It is obvious, then, that neither Stevin nor Galileo was the first to perform such as experiment; nor in all likelihood was Philoponus. But Philoponus does give us the first record of such an experiment used to refute or confirm a dynamic law." If we wish to attribute the experiment to the oldest available source that mentions its result, credit should go to Lucretius.

[81] "Quae omnia si ita disposita fuerint, quodcumque mobile super planum horizonti aequidistans a minima vi movebitur, imo et a vi minori quam quaevis alia vis" ([Galileo: Opere], vol. I, p. 299).

[82] Heron, *Mechanica*, I, §§20. We discussed this passage on page 289.

[83] Some such correspondences are analyzed in [Voicu]. The subject will be discussed at length in a forthcoming work of mine.

[84] In the introduction to [Heron/Carra de Vaux], the editor and discoverer of the Arabic version of the *Mechanics* bemoans the obstacles met in mapping the fortunes of manuscripts, "especially

Galileo's interest in hydraulic experiments first arose during his Paduan period; in particular, the work that led Galileo to patent a water-lifting machine dates from 1593–94.[85]

Among the experiments that most interested him in subsequent years were those relative to the syphon principle and the operation of vacuum pumps. At the time it was not yet possible to build Ctesibian pumps to sufficient precision, and in particular they could not be made with metal cylinders.[86] In these conditions it is easy to imagine that the experiments that gave Galileo food for thought were primarily those of Hellenistic scientists, described in ancient works. Thus it is not surprising that the theoretical conclusions often had the same origin. For example, the explanation for water pumping given by Galileo on the first day of the *Dialogues*, based on the cohesion of water particles,[87] restates that of the *Pneumatics* of Philo of Byzantium for the drinking straw and the syphon.[88]

Regarding catoptrics, the experimental facts that most interest Galileo and his students are those regarding burning mirrors: on the first day of the *Dialogues*, Sagredo says he has seen a spherical mirror melt lead, and he imagines from this the enormous power of the Archimedean mirrors, which he knows were parabolic.[89] The *Dialogues* came out only six years after Bonaventura Cavalieri's important treatise on *The burning mirror*,[90]which treated many applications of the classical theory of conics, in-

when it comes to Roman libraries", and says he found references to Greek copies of the *Mechanics* (none of which he could locate) in several Roman libraries, in Venice's Biblioteca Marciana and at the Escorial. The Venice lead turned out to be false (a copy of the *Pneumatics* wrongly classified as the *Mechanics*), but the other manuscripts seem to have been lost track of.

A holograph of Christopher Clavius, dating from 1579 or 1580 and apparently referring to mathematics courses he would presumably be teaching at the Collegio Romano, details the mechanics syllabus as consisting of the "mechanical questions of Heron, Pappus and Aristotle" ([Baldini], p. 175). Since the document is essentially a list of texts, the explicit reference to Pappus shows that "Heron" did not mean just the passages of Heron included in Pappus.

85 The patent letter is printed in [Galileo: Opere], vol. XIX, pp. 126–129.

86 See for example [Usher], p. 332, where it is said that the use of metal (cast iron) cylinders in vacuum pumps is attested from the second half of the seventeenth century. Before that, though metal was used for other parts of the pump, the cylinder was wooden. Clearly techniques for metal turning and grinding, needed to make surfaces regular enough to ensure a tight fit between cylinder and piston, did not regain the level they enjoyed in Hellenistic times until after 1650.

87 Galileo Galilei, *Discorsi e dimostrazioni...*, day 1, at Sagredo's 12th turn = [Galileo: Opere], vol. VIII, pp. 64–65.

88 Philo of Byzantium, *Pneumatica*, iii = [Philo/Prager], 81 + 129–130. But the cohesion argument is not in Heron's *Pneumatica*, and moreover Empedocles already knew that air pressure can overcome the weight of water, as happens with the clepsydra (note 77 on page 76), the syringe and the syphon. So perhaps Galileo was led into error by Philo's text, which survived to modern times only in very corrupt Latin and Arabic translations. (The explanation chosen by Galileo is clearer in the Latin text, which seems to be the farther from the original; see [Philo/Prager], p. 81.)

89 Galileo Galilei, *Discorsi e dimostrazioni...*, day 1, at Sagredo's 25th turn = [Galileo: Opere], vol. VIII, p. 86.

90 See note 88 on page 118.

cluding parabolic mirrors (using newly relearned knowledge about the parabola's focal property), and even demonstrated that a body under the action of gravity follows a parabolic trajectory.

Skipping over Galileo's prominent work in observational astronomy (because, as we know, many don't grant such things the status of "experimental science"), we turn to his interest, in the next few years, in other Hellenistic experimental subjects such as statics, aerostatics and thermology. We single out his studies on the expansion of heated gases (a subject that around that time also interested Della Porta and van Helmont[91]), which led to the reconstruction of experiments and devices described by Philo and Heron, opening the way to the construction of modern thermometers.[92]

Galileo's most famous experiment involved the motion of a body along an inclined plane. Let's see now what he says about the crucial matter of measuring the time of descent:

> For the measurement of time, we employed a large vessel of water placed in an elevated position; to the bottom of this vessel was soldered a pipe of small diameter giving a thin jet of water, which we collected in a small glass during the time of each descent, whether for the whole length of the channel or for a part of its length; the water thus collected was weighed, after each descent, on a very accurate balance; the differences and ratios of these weights gave us the differences and ratios of the times, and this with such accuracy that although the operation was repeated many, many times, there was no appreciable discrepancy in the results.[93]

Galileo understood well how important it was, in order to rebuild the experimental basis of ancient science, to find a replacement for the precise water clocks whose technology had long ago been lost. But the fact that his measurements, though repeated so often, never showed discrepancies cannot but make a modern physicist wonder. Galileo's description of his timepiece, so more rudimentary to Ctesibius' clocks, strengthens the suspicion that measurements of time via buckets and glasses were complemented in other ways. A sentence a few lines before the quoted passage, in the description of the experimental setup, provides a clue to this "complementation":

91 See note 60 on page 279.

92 Galileo's *thermoscope*, essentially a gas-expansion thermometer, is the device described by Philo of Byzantium (see page 280).

93 Galileo Galilei, *Discorsi e dimostrazioni...*, third day, at Salviati's first turn after Corollary I = [Galileo: Opere], vol. VIII, p. 213 (Crew and de Salvio translation).

> We repeated [the ball's run] several times in order to be sure of the elapsed time, and it never presented a deviation exceeding one tenth of a heartbeat.[94]

Evidently, the pulse was the most commonly used clock. It is worth pausing to consider how far we still were, in spite of Galileo's efforts, from the refined measuring instruments (the basis of any true experimental method) that had once allowed Herophilus to study scientifically the heartbeat and to use the theory thus built in making diagnostics.[95]

In spite of its mathematical and technological limitations, the recovery work carried out by Galileo was extraordinary, especially in terms of methodology. He borrowed from ancient science the humility of the scientific method, which is content with tackling well-circumscribed problems (such as motion under gravity or hydrostatics) through the tools of mathematics and experimentation, and resists the temptation to pursue overarching natural philosophy explanations for how nature behaves. For almost two thousand years, no scientist in this mold had been seen.

Another point to be stressed is that the rift between mathematics and physics, whose absence is often read as a sign of the primitive state of Hellenistic science, is not yet present in Galileo's work. He follows exactly the example of his distant masters, blending together mathematics and experiments.[96] *Physics* as opposed to *mathematics* had not arisen yet.

11.5 Two Modern Scientists: Kepler and Descartes

Galileo's lucid rationalism represents a methodological attitude exceptional among early modern scientists. Kepler, who played a giant role in the formation of modern science, had quite a different approach. His eclectic spirit drew on an ample gamut of sources — Hellenistic science,

[94] Ibid.

[95] See pages 145–148 and 154. This observation may seem contrary to the statement, contained in a letter of Vincenzo Viviani to Leopoldo de' Medici (published in [Galileo: Opere], vol. XIX, pp. 647–659), that Galileo thought of using a pendulum to measure the heartbeat. But of course a pendulum only lets one measure intervals of time equal to several periods or more, which is why Galileo used not a pendulum but a water clock (besides his pulse and possibly other methods) for the experiments on gravity. The frequency of the pulse can be obtained from the overall time of many beats (which does lend itself to measurements with a pendulum), but this will not yield the ratio between systolic and diastolic intervals, which had been studied by Herophilus. To measure the overall time of many systoles or diastoles one must have a stopwatch that can be stopped and started at will, as is the case with a water clock.

[96] In this connection it is illuminating to read the pages in the second day of the *Dialogues and demonstrations* (in particular pages 169–170) showing how the shape of machines and animals depends on their dimensions. I don't know of a similarly limpid and example-rich account in ancient literature, though the general idea is present in Vitruvius (*De architectura*, X, xvi §5).

Aristotelian and neo-Platonic philosophy, neo-Pythagorean numerology, astrology, alchemy — and amalgamated it all with the glue of theology.

His scientific method may be illustrated with excerpts from his writings. About tides, for example, he writes:

> Experience shows that everything that is made of moisture swells up when the moon waxes and shrinks back when it wanes.[97]

Kepler derives the distances of the planets from their correspondence with Platonic solids and metals: he believes in the celestial musical harmonies and the crystalline sphere of fixed stars. Here is how he describes the structure of the universe:

> The philosophy of Copernicus matches the main parts of the world [universe] to different regions of the world's shape. For just as the sphere, image of God the Creator and archetype of the world (as shown in Book I), has three regions, symbolizing the three persons of the Holy Trinity — the center corresponding to the Father, the surface to the Son and the in-between to the Holy Spirit — so three main parts of the world were created, each in its part of the spherical shape: the sun in the center, the sphere of fixed stars on the surface and finally the planetary system in the region in between.[98]

Later he relates the masses of the three zones:

> Since these three bodies are analogous to the center of the sphere, the surface and the in-between, the symbols of the three persons of the Holy Trinity, it is plausible that each one of them has as much matter as each of the other two[.] [99]

Continuing in this vein, he manages to determine the thickness of the sphere of fixed stars as one twelve-thousandth of the diameter of the sun: in his reckoning, a bit over 2000 German miles.

The eclecticism of Kepler and his colleagues had the important function of allowing the recovery of some ancient scientific knowledge that had been disguised and filtered through nonscientific traditions. Some other Keplerian passages will help make this point clear:

> The perfection of the world consists in light, heat, motion and the harmony of motions … Regarding light, the majestic sun itself is

[97] J. Kepler, *De fundamentis astrologiae certioribus*, thesis XV (Prague, 1601) = [Kepler: OO], vol. I, p. 422.

[98] J. Kepler, *Epitome astronomiae copernicanae* (Linz, 1618), book IV, part I, section I = [Kepler: OO], vol. VI, p. 310.

[99] J. Kepler, *Epitome …*, book IV, first part, near end of section IV = [Kepler: OO], vol. VI, p. 334.

light, and as it were the eye of the world; and it lights up, colors and beautifies everything else in the world, like a fountain of light or a very bright torch[.]

Regarding fire, the sun is the hearth of the world . . . The sun is fire, as the Pythagoreans said . . . [It is] as if a vegetative faculty were present not only in earthly creatures but also in all of ether throughout the whole expanse of the universe, a supposition thrust upon us by the sun's obvious heating energy. . . ; it is plausible that this faculty inheres in the sun as the heart of the world, and that it diffuses from there, as vehicle of light, together with heat, throughout the enormous amplitude of the universe, just as, in an animal, the seat of heat and of vital faculties lies in the heart[.]

Regarding motion, the sun is the prime cause of movement of the planets and the prime mover of the universe[.][100]

So it is the sun that makes the planets move in circles. . . But how can it do this, if it lacks hands with which to grip them. . . ? Instead of hands it uses the power of its own body, sent out in straight lines through the vastness of the word[.][101]

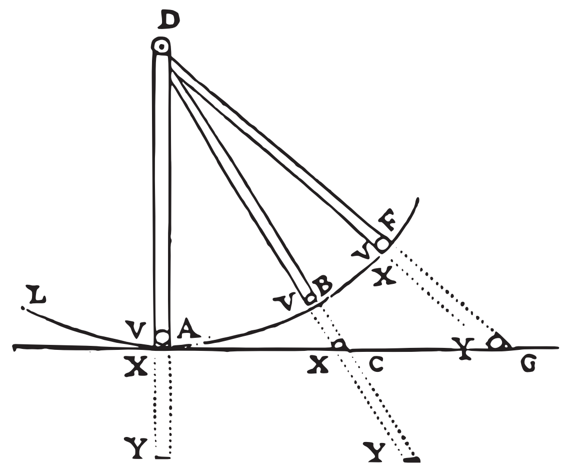
We recognize some of Kepler's sources: the last quote echoes passages of Vitruvius and Pliny that we have discussed; the rest we have seen, in almost the same words, in Theon of Smyrna.[102] The idea of a gravitational interaction between the sun and the planets was transmitted from ancient to modern science thanks to the interest that scientists like Kepler had in authors such as Pliny and Theon of Smyrna. Galileo's rationalism had led him to reject as foolish the notion of gravitation, which in his time appeared in works belonging to definitely unscientific traditions (Hermetic and astrological texts, for instance), where it exemplified astral influences and was yoked to religion and magic.

Of course some of Kepler's sources were much more "scientific" than the ones we recognize in the passages just quoted. He used Apollonius of Perga and Pappus, and one of the classical works that drew him strongly was Plutarch's dialogue *De facie quae in orbe lunae apparet*, of which he even published an annotated Latin translation.[103] One may ask whether ancient sources helped Kepler in the long and arduous road that led him to discover that planetary orbits are elliptic, as they helped him recognize the motor role of the sun. The approach attested by the passages quoted

[100]J. Kepler, *Epitome . . .*, book IV, first part, section I = [Kepler: OO], vol. VI, pp. 310–311.
[101]J. Kepler, *Epitome . . .*, book IV, second part, section III = [Kepler: OO], vol. VI, p. 344.
[102]See pages 297–298 for Vitruvius and Pliny, and page 319 for Theon.
[103]This appears in [Kepler: OO], vol. VIII.

earlier, involving in particular a belief in the perfection of the spherical shape, does not seem too likely to have led to the ellipticity of orbits on its own, particularly since the observed data could be described equally well through a system of epicycles.

To take another example of the importance of sources in determining method, we can contrast Descartes' *Geometry* with his *The world*, subtitled *Treatise on light*. Let's examine, in this second work, the passage about a pebble revolving around in a sling: a subject of great interest, and one about which, as far as we know, the only sources available were literary and from the imperial age.



For example, suppose a stone is moving in a sling along the circle marked *AB* [see Descartes' drawing above], and consider it exactly as it is at the instant it arrives at the point *A*. You will readily find that it is in the process of moving. . . toward *C*, for it is in that direction that its action is directed in that instant. But nothing can be found here that makes its motion circular. Thus, supposing that the stone then begins to leave the sling and that God continues to preserve it as it is at that moment, it is certain that He will not preserve it with the inclination to travel in a circle along the line *AB*, but with the inclination to travel straight ahead toward point *C*.

According to this rule, then, we must say that God alone is the author of all the motions in the world in so far as they exist and in so far as they are straight, but that it is the various dispositions of matter that render the motions irregular and curved. Likewise, the theologians teach us that God is also the author of all our actions, in so far as

> they exist and in so far as they have some goodness, but that it is the various dispositions of our wills that can render them evil.[104]

Descartes, like Kepler, must have read the *De facie* and studied carefully the passage we quoted on page 286. But where Plutarch talks about bodies "turned aside by something else", Descartes finds it natural to link the *something else*, the cause of deviations from straightness, with the powers of evil, the cause of misdeeds. His method strikes today's reader as much more "scientific" when, writing about geometry, his source is not Plutarch, but the tradition of Euclid and Pappus.[105]

Historians of mathematics are quick to point out how much Cartesian mathematics differs from its Hellenistic counterpart. C. Boyer and others place the novelty of Descartes's method in that he abandoned the homogeneity principle,[106] which prevented "Greek mathematics" from dealing with expressions such as $x^2 + x$ or $x^2 + x^3$:

> In one essential respect, he [Descartes] broke from Greek tradition, for instead of considering x^2 and x^3, for example, as an area and a volume, he interpreted them also as lines. This permitted him to abandon the principle of homogeneity, at least explicitly, and yet retain geometrical meaning.[107]

But abandoning the homogeneity principle was not an unprecedented step. Neugebauer writes:

> [T]hat Heron adds areas and line segments can no longer be viewed as a novel sign of the rapid degeneration of the so-called Greek spirit, but simply reflects the algebraic or arithmetic tradition of Mesopotamia.[108]

In truth Heron's method differs somewhat from Descartes; Heron, to add x^2 and x, represents the summands as a square and a segment, using these as graphical tokens for algebraic quantities, the fundamental entities in the Mesopotamian tradition. By contrast, Descartes, to whom the fundamental entities remain geometric, can only add after making both summands into segments. It seems to me that our current procedure is in substance closer to Heron's than to Descartes'. In any case it is clear that, clichés aside, judgements about what mathematical procedures are "modern" are both subjective and liable to fluctuate.

[104] *Le monde de M. Descartes, ou Le traité de la lumière*, chapter 7, at 80% (Gaukroger translation).
[105] The main source of Descartes' *Geometry* is Pappus' *Collectio*.
[106] See note 47 on page 45.
[107] [Boyer], p. 371 (1st ed.), pp. 337–338 (2nd ed.).
[108] [Neugebauer: ESA], p. 146.

11.6 Terrestrial Motion, Tides and Gravitation[109]

In Section 10.10 we reconstructed an ancient theory that explained the main features of tides by combining the moon's action with the sun's. The testimony of Priscian of Lydia shows that the theory was expounded in works accessible in the Byzantine period. Therefore it is not too surprising that it should have outlasted the Middle Ages, winding its way through paths not easy to identify, to the confines of the Venetian Republic, where it resurfaces in a treatise of Jacopo Dondi, written about 1355.[110] After that the theory was kept alive mainly, but not exclusively, in Paduan lands, and there it was the subject of many writings and university courses. In the sixteenth century it appeared in print several times: first in a booklet by Federico Crisogono or Grisogono, of Zara (Zadar) in Dalmatia,[111] and then in several other works, of which we mention those by Delfino in 1559, Raimondo in 1589 and Duré in 1600.[112]

Kepler, in a scientific note to his fantastic novel *Somnium* (*The dream*), shows he knows that tides are caused by the moon's and sun's attraction, and that spring tides at full moon and new moon are caused by the sum of the two actions.[113] These ideas, playing no role in Kepler's astronomy,[114] being absent from all his astronomic works, and agreeing with those found in so many sixteenth-century books, were not reached by the astronomer on his own, but as a result of reading. This has not usually been recognized, probably because historians of science tend to be unaware of the sixteenth-century publications that contained the lunisolar theory;[115] it is easier to think that Kepler was filled with a magical intuition good for his novel alone and forgotten soon thereafter.

[109] The material in this section is drawn largely from [Bonelli, Russo] and [Russo: FR], which the reader should consult for details.
[110] *De fluxu et refluxu maris*, published in [Dondi/Revelli] and available at http://mat.uniroma2.it/simca/Testi/Dondi.pdf. Dondi also wrote a pharmacopeia and a philological work.
[111] *Tractatus de occulta causa fluxus et refluxus maris*, a short work included in a collection of Crisogono's writings published in Venice in 1528 (*Federici Chrisogoni... de modo collegiandi et pronosticandi et curandi febris...*) and reprinted in G. P. Gallucci's *Theatrum mundi et temporis* (Venice, 1588), which is today a less rare book.
[112] Federico Delfino, *De fluxu et refluxu aquae maris*, Venice, Academia Veneta, 1559; Annibale Raimondo, *Trattato utilissimo e particolarissimo del flusso e del riflusso del mare*, Venice, Domenico Nicolini, 1589; Claude Duré, *Discours de la vérité des causes et effects des divers cours, mouvements, flux, reflux et saleure de la mer Océane, mer Méditerrannée et autres mers de la Terre*, Paris, Jacques Rezé, 1600.
[113] J. Kepler, *Somnium seu opus posthumum de astronomia Lunari*, in [Kepler: OO], vol. VIII, p. 61, note 202.
[114] Kepler believed the sun acted on planets, but to make them revolve (like Descartes), rather than through an attraction.
[115] A recent and authoritative scientific history of tides [Cartwright] makes no mention of Dondi's work, while Crisogono's is mentioned in a note (no. 4 on p. 23) as if it had never been printed and had been unearthed in [Bonelli, Russo].

The tradition of writings on the lunisolar theory of tides concluded in 1624, with the work *Euripus, or the ebb and flow of the sea*,[116] by the same archbishop de Dominis that we have already encountered as an expert in experimental optics.[117] In the few pages of this work, as in the others, tides are ascribed to the action of the moon and the sun ("Therefore we hold that ... the sun and the moon have a strong force, magnetic as it were...");[118] de Dominis also states that high tide occurs simultaneously at antipodal points, and shows how the monthly cycle of spring and neap tides can be explained by the joint action of the two bodies.

One particular makes the account in the *Euripus* unique. De Dominis, after discussing the lunisolar theory in its traditional form, deduces from it the observation that, given that when the sun or the moon are directly above a point on the tropic of Cancer, the antipodal point is on the tropic of Capricorn, the two daily tides should be unequal and the diurnal inequality should be greatest at the solstice and least at the equinox.[119] What makes this all the more fascinating is that de Dominis, far from writing in support of the theory that he expounds, in fact disproves it on the grounds that its consequences are false! He was convinced that the two daily tides are invariably equal. Knowledge of the regime of diurnal inequality in such distant places as the "Erythrean Sea", of which Strabo wrote in connection with Seleucus (see page 313) had vanished in Europe. We know from no less an authority than G. H. Darwin that the import of Seleucus' achievement would have been impossible to appreciate even in the early nineteenth century.[120] The discussion in the *Euripus* is thus in a sense complementary to the Strabo passage, in that it includes a theoretical explanation, not found in Strabo, for the observable phenomena described by the ancient author but rejected by de Dominis.[121]

If it is already less than likely that our religious reformer and jurist could have created an original mathematical theory that correctly explains the annual variation of diurnal inequality, it beggars belief that he should have

[116]Marco Antonio de Dominis, *Euripus, seu de fluxu et refluxu maris sententia...*, Rome, 1624.
[117]See pages 345–346.
[118]"Itaque dicimus luminaria illa duo Solem & Lunam habere vim magnam, quasi magneticam..." (M. de Dominis, *Euripus*, 5).
[119]M. de Dominis, *Euripus*, 6–7. See Figure 10.5 (page 313) and the surrounding text.
[120][Darwin: Tides], p. 84: "The meaning of [the passage on Seleucus reported by Strabo] was obviously unknown to the Dutch commentator Bake — and indeed must necessarily have been unintelligible to him at the time when he wrote, on account of the then prevailing ignorance of tidal phenomena in remoter parts of he world".
[121]Naturally de Dominis (like any of his contemporaries who might be interested in physical geography) had to be familiar with Strabo's work. What happens is that after reporting Seleucus' observations, Strabo adds that Posidonius unsuccessfully tried to verify in Cádiz the phenomenon Seleucus noticed in the "Erythrean Sea". So probably de Dominis deduced not that the two seas have different regimes, but that Seleucus should be disregarded.

done so while denying that the diurnal inequality exists. The explanation is that he merely transmitted (not terribly well, it must be said) the elements of an ancient theory born from observations made in seas he knew nothing about. And it is not surprising that a high ecclesiastical figure, who was successively the bishop of Segna (Sinj) and the archbishop of Spalato (Split), and the scion, to boot, of an illustrious Dalmatian family going back to at least the thirteenth century,[122] should have outdone Galileo in the procurement of relevant manuscripts, if not in scientific originality.

From being a Catholic archbishop, de Dominis converted to Anglicanism and moved to England. Later he reverted to Catholicism and returned to Rome, but ended his days enclosed in the Castel Sant'Angelo, awaiting the outcome of an ongoing process against his person. Following his posthumous condemnation to the stake and to *damnatio memoriae*, his body and books were burned together in the Campo de' Fiori, and it seems that historians of science, including those outside Catholicism, have generally abided by the Inquisition's sentence.[123] But this should not obscure the fact that his scientific works were read with attention by other scientists of the early modern age, in England more than anywhere else.

While the basic lunisolar theory never quite died, Seleucus' elaboration of it resurrected in the early modern age. As we saw in Section 10.12, Seleucus was known to have provided an explanation for tides involving an earthly "whirlpool motion":

> Seleucus the mathematician (also one of those who think the earth moves) says that the moon's revolution counteracts the whirlpool motion of the earth.

Naturally, this passage must have been thoroughly scrutinized in the sixteenth century, appearing as it does in the *De placitis philosophorum*, a work then regarded as authored by the influential Plutarch[124] — the connection between Seleucus and tides being reinforced by the Strabo passage discussed on page 313. But what to make of the earth's "whirlpool motion"?

The important botanist, physician and anatomist Andrea Cesalpino, like the vast majority of his contemporaries, believed in the Ptolemaic system. Yet he decided that tides are caused by a motion of the earth, not rotation or revolution but a "small" motion introduced ad hoc, and supposedly

[122]On the archbishop's eventful life and political and religious writings, which are among the main sources for the concept of jurisdictionalism, there are Ljubic's nineteenth century works (in Serbo-Croatian) and the books [Malcolm], [Russo: de Dominis].
[123]There is no literature on de Dominis' scientific writings, other than the already cited [Ziggelaar] and some allusions to his theory of the rainbow, made inevitable by Newton's references to it (see page 345).
[124]See note 187 on page 315.

imparted onto earth from the heavens.[125] Before him, the passage was used by Celio Calcagnini, a highly regarded translator of Plutarch, in his already-mentioned essay reviving the daily rotation of the earth;[126] being aware of only one earthly motion, Calcagnini of course deduced that tides were caused by the daily rotation. Not surprisingly, neither he nor Cesalpino were able to fashion a theory able to account for observations.

Supporters of heliocentrism also made use of the *De placitis* testimonium, putting it together with the Plutarchan passage we discussed on page 311:

> Was [Timaeus] giving the earth motion ..., and should the earth ... be understood to have been designed not as confined and fixed but as turning and revolving about, in the way expounded later by Aristarchus and Seleucus, the former assuming this as a hypothesis and the latter proving it? [127]

The conclusion that Seleucus' proof involved a recognition of tides as an effect of the motion of the earth would have leapt out. They must have felt that the way to solve the problem most dear to them was to reconstruct Seleucus' proof, thus completing what Copernicus had started when he revived Aristarchus' idea. The first man down this road was Paolo Sarpi, who thought that tides must be caused by the combination of the two earthly motions he know about: rotation and revolution. He introduced the analogy (later taken up by Galileo) between the motion of the oceans as a cause of tides and the nonuniform motion of a basin full of water.[128]

Galileo, whose main scientific aim was precisely the demonstration of the earth's motions, fully welcomed not only the hints contained in the ancient passages but also the insights of his friend Sarpi. He devoted to the problem the fourth and last day of his *Dialogue concerning the two chief world systems*.[129] Unfortunately, he did not understand the *De placitis* passage any more than his predecessors, and, believing that it opposed the earth's rotation to the moon's revolution, concluded that Seleucus, though

[125] A. Cesalpino, *Peripateticarum quaestionum libri quinque*, Venice, 1571, book III, question V.

[126] See note 41 on page 338. His translation of Plutarch's *De Iside et Osiride* heightened the Renaissance intelligentsia's interest in Egypt.

[127] Plutarch, *Platonicae quaestiones*, 1006C.

[128] The idea was that the earth's rotational velocity would alternately work for and against the translational motion, thus making the velocity of a point on the earth oscillate periodically. This appears in Sarpi's *Pensieri naturali, metafisici e matematici*, contained in a manuscript of 1595; see especially thoughts 569, 570, 571, reported in the introductory essay to [Galileo/Sosio], p. lxxvii.

[129] Tides are the main theme of the book, which Galileo originally called *Dialogue on the ebb and flow of the tide*. The title was changed in deference to the Inquisition's stipulation (in allowing publication) that heliocentrism be discussed therein solely as a hypothesis, and that no stress be laid on what the author regarded as the physical proof of the earth's motion.

having the right intuition, misapplied it crassly.[130] Therefore Galileo tried to rediscover independently the link between earthly motions and tides, based only on Sarpi's hints. Alas, he knew of only two motions, with daily and yearly periodicities, so his efforts to deduce from them a phenomenon that has a monthly component could not but end in failure.

Those scholars who, based on the *De placitis* passage about Seleucus, maintained that tides were caused by earthly motions, felt that this explanation contradicted the lunisolar hypothesis, which reflects another part of the ancient theory. Thus it is that Galileo inveighs against "a certain prelate" for having "published a little tract saying that the moon, wandering through the skies, attracts and lifts to itself a mass of water, which follows it around."[131]

Nonetheless, the mutual consistency that united the two lines of thought (thanks to their common Hellenistic origin and regardless of the thinking of their rediscoverers) eventually led to their merging. The Genoese Giovanni Battista Baliani, a student and friend of Galileo's, trying to salvage the essence of the theory expounded in the *Dialogue concerning the two chief world systems*, proposed an interesting modification thereof. He reasoned that if tides, with their monthly cycle, depend on the earth's motions, there must be an earthly motion of monthly periodicity. The conclusion is sound, but Baliani felt that the only way to make the earth move in synchrony with the apparent revolution of the moon was to declare our planet a satellite of the moon!

Baliani's odd theory, born logically from the conjuntion of observational data with our passage in the *De placitis*, had an important and not generally recognized role as one of the links between Galileo's and Newton's theories of tides. The next step was taken by the one of the greatest scientists of his time, John Wallis, who, in an article published in 1666,[132] revisited both Galileo's idea of regarding tides as an effect of the nonuniformity of the earth's motion, and Baliani's elaboration, which Wallis

[130] "More is the wonder that, while some have thought to place the cause of ebb and flow in the earth's motion, and so displayed uncommon perspicacity, they should have then missed the mark ... The idea (reportedly held by an ancient mathematician) that the earth's motion, coming up against the moon's, causes the ebb and flow because of this contrast, is completely foolish, not only because no explanation is given (nor can one be found) of how tides should follow, but also because it is manifestly false: seeing as the earth turns not in the opposite direction to the moon's motion, but in the same." (*Dialogo sopra i due massimi sistemi del mondo, tolemaico e copernicano*, in [Galileo: Opere], vol. XIX, p. 486).

[131] Galileo, *Dialogo sopra i due massimi sistemi*, [Galileo: Opere], vol. XIX, p. 415. There was good reason to refrain from naming de Dominis, whose very memory was damned.

[132] "An essay ... exhibiting his hypothesis about the flux and reflux of the sea", *Philosophical Transactions* **16** (August 1666), 263–289.

thought overzealous rather than absurd.[133] Earth and moon are linked by a mutual influence, and so, according to Wallis, must be considered together as a single body. What revolves around the sun is neither the earth's center (as Galileo and others maintained) nor the moon's center (as Baliani alone had dare believe), but the barycenter of both bodies. It follows that, in order to compute the non-uniformity of the earth's motion, to which Wallis attributed tides, one must consider not only rotation and revolution, as Galileo did, but also the monthly motion around the barycenter of the earth-moon system. Thus Wallis achieved an important feat: introducing into the theory, in a natural way, all three periodicities observable in tides.

Thus the fragmentary information about Seleucus' studies that reached the modern age, in spite of its incompleteness, led, through the hard work of several generations of scientists, to the rebirth of another key element of the ancient theory.

11.7 Newton's Natural Philosophy

In analyzing the links between Hellenistic and modern science we must perforce dwell on Newton, often considered the primary founder of the latter. We start by recalling how Newton talks about space in the *Principia mathematica*:

> All things are placed ... in space as to order of situation. It is from their essence or nature that they are places; and that the primary places of things should be movable, is absurd. These are therefore the absolute places; and translations out those places, are the only absolute motions.
>
> But because the parts of space cannot be seen, or distinguished from one another by our senses, therefore in their stead we use sensible measures of them. ... And so, instead of absolute places and motions, we use relative ones; and that without any inconvenience in common affairs; but in philosophical disquisitions, we ought to abstract from our senses, and consider things themselves, distinct from what are only sensible measures of them. For it may be that there is no body really at rest, to which the places and motions of others may be referred.[134]

[133] Baliani had not published his theory of tides; Wallis got wind of it through Riccioli's *Almagestum novum*.

[134] Newton, *Philosophiae naturalis principia mathematica*, Definitions, scholium, at 35%, Motte translation (as revised by Cajori).

Newton's ideas, though seemingly close to and inspired by Aristotle's, differ from them in an essential respect. Aristotle's absolute space was simply that of everyday experience, fixed with the earth, and thus directly linked to empirical data. We have already seen how it was incompatible in substance with the assumption that the earth moves.[135] It is not an accident that Ptolemy, who reverted to an Aristotelian notion of space, rejected the earth's motions, or that Galileo, embracing heliocentrism, repudiated absolute space and arrived at the principle of relativity. In Newton, by contrast, absolute space coexists with Aristarchan heliocentrism (which had of course triumphed irreversibly after Kepler's work), so that it no longer corresponds to any empirical datum. It follows that the motions that are the object of Newton's *axioms or laws of motion* are beyond our perception. The Nature that concerns his natural philosophy transcends experience, unlike Aristotle's, and the first step toward it is to "abstract from our senses". This is not, mind you, an abstraction arising from the substitution of a theoretical model for real objects, but one imposed by the need to completely give up "sensible measures". In this scheme there can be no correspondence rule between relative movements and absolute ones, because the latter refer to a fixed space beyond perception, and bear no relation to phenomena. Thus Newton is forced, as he himself admits, not to deal with "common affairs". If this is the "experimental method" characteristic of modern science, it is likely that it was indeed unknown to scientists like Archimedes, Ctesibius and Herophilus.

(It is true that the *Principia* later does characterize absolute space, in the following way. Newton establishes that the barycenter of the solar system is at rest or in uniform motion, using an implicit, and arguably reasonable, assumption of isolation. At the same time — and here is the crux — he opts for rest over uniform motion, a gratuitous choice for which there is no logical justification in his scientific mechanics, but which is very important from the viewpoint of his metaphysical notion of space.[136])

Building, with full methodological coherence, on his notion of space, Newton next talks of motion and force:

> [E]ntire and absolute motions can be no otherwise determined than by immovable places.... Now no other places are immovable but

[135] See page 83.

[136] See *Principia*, Book III, proposition XI / theorem XI and its proof. The selection is effected through the immediately preceding hypothesis I, "That the centre of the system of the world is immovable", which is justified by a single argument: "This is acknowledged by all." In this metaphysical statement, based on an appeal to common sense, Newton prefers to talk of "the system of the world" and abandons previously defined terms. The proof of proposition XI / theorem XI shows that he conceives "the centre of the system of the world" as coinciding with the barycenter of the solar system.

> those that, from infinity to infinity, do all retain the same given position one to another; and upon this account must ever remain unmoved; and do thereby constitute immovable space.
>
> The causes by which true and relative motions are distinguished, one from the other, are the forces impressed upon bodies to generate motion. True motion is neither generated nor altered, but by some force impressed upon the body moved; but relative motion may be generated or altered without any force impressed upon the body.[137]

Thus Newtonian force (often claimed to be a cardinal novelty of modern science) is the efficient cause of absolute motion, that is, of displacement relative to "immovable space" — itself something whose only established property is that of not being connected to "sensible measures". And here moreover Newton seems to be saying that "true motion" is not possible in the absence of a causing force, in contradiction with the principle of inertia.[138]

In spite of the approach just exemplified, Newton eclectically borrows from Euclid a hypothetico-deductive expositive framework. His *Principia*, like the *Elements*, contain definitions and axioms. But consider the first few definitions:

> *Definition I*. The quantity of matter is the measure of the same, arising from its density and bulk conjointly.
>
> *Definition II*. The quantity of motion is the measure of the same, arising from the velocity and quantity of matter conjointly.
>
> *Definition III*. The innate force [*vis insita*] of matter is a power of resisting, by which every body, as much as in it lies, continues in its present state, whether it be of rest, or of moving uniformly forwards in a right line.
>
> *Definition IV*. An impressed force is an action exerted upon a body, in order to change its state, either of rest, or of uniform motion in a right line.
>
> *Definition V*. A centripetal force is that by which bodies are drawn or impelled, or any way tend, towards a point as to a centre.[139]

[137]Newton, *Principia mathematica*, Definitions, scholium, at 55%, Motte/Cajori translation.

[138]It may be objected that force is said to be necessary to cause motion and not to maintain it. But, supposing that Newton does admit true motion in the absence of force, how can force as the cause of motion provide the desired distinction between true and relative motion? For this reason I think that in this passage the Aristotelian view (upon which is based the characterization of true motion as the effect of a force) coexists with its opposite (according to which a force is needed only to alter motion). The origins of this second view will be discussed shortly.

[139]Newton, *Principia mathematica*, Definitions, Motte/Cajori translation.

In later developments force came to be a physical magnitude, but here it is defined as a disposition or action directed to a particular end. It seems that Newton does not mean to build scientific theories in the sense we have given the expression — in the sense of Euclid and Archimedes. His intention, to judge from the passage just quoted, would seem rather to develop a natural philosophy based on Aristotelian concepts such as efficient cause and final cause.

Here one must face what appears at first to be a very difficult problem. How did Newtonian mechanics, leaning on such foundations, develop into a true scientific theory?

First note that the deductive method, which was part of European culture mainly thanks to Euclid's *Elements*, is an effective antidote against the use of Peripatetic and theological approaches in science, and this in spite of the practitioner's own inclinations. Treated with massive doses of the method, even an amorphous "theory" is eventually forced to take on a deductive and logically coherent form. But this does not explain it all. Newtonian dynamics did not just evolve toward an internally coherent theory: it was applicable from the start as a model of real motion — and planetary motion, no less. There must have been other factors at work.

Let's read again the definitions that inaugurate the *Principia*. The first is meaningless, since density would not be definable except by indulging in a silly tautology. The second is unusable because it depends on the first. But the third and fourth are very interesting, because (despite the use of nonscientific language[140]) they graft into the Aristotelian framework an idea that is foreign to it: that of considering "according to nature" not just rest but also uniform straight motion (this being the effect of giving the name of "impressed force" to the efficient cause of a departure from such motion). This idea, as we know, heralded great developments. But we must note that it is being superimposed on, not superseding, the more purely Aristotelian idea attested in our earlier excerpt, where force was the efficient cause of "true motion". The third and fourth definitions are thus anomalous in comparison with the rest of the discussion.

The next definition is strange. Why should centripetal force be introduced immediately after the extremely general notion of impressed force? If this is just a descriptive expression, to be used later about forces directed toward a center, it makes no sense to place it among the very first definitions. If, on the other hand, Newton is introducing here a "law of

[140]These two definitions clearly belong to the essentialist (Platonist–Aristotelian) type discussed on page 179, and cannot identify any measurable physical quantity. Newton is perhaps conscious of this, since, in contrast with the first two definitions, he does not say that innate force and impressed force are measures of anything.

nature", it is not clear what law that is. One might think gravitation, but then why aren't bodies mutually attracted, rather than impelled toward a point? What are these points that attract things? And what are they centers of? It's all very mysterious.

Plutarch, in the *De facie quae in orbe lunae apparet*, had written:

> [T]o help the moon, that it may not fall [on the earth], there is its motion itself and the whizzing nature of its rotation, just as objects placed in a sling are prevented from falling by the circular motion. *For each body is guided by motion according to nature, if it is not turned aside by something else.*[141]

If the interpretation we gave to this passage in Section 10.6 is correct, Newton, in the third and fourth definitions of the *Principia*, succeeded in restoring the original meaning of the statement reported by Plutarch in the italicized sentence. In choosing a name for the *something else* Newton was more felicitous than Descartes: drawing from Aristotelian language, he speaks of *force*. A little further in Plutarch's dialog we read:

> . . . the center. For this is [the point] toward which all weights, from everywhere, are pressed, tending toward it and being moved toward it and striving toward it. [142]

The fifth definition in the *Principia* is virtually a translation of this. The question arises: Were the definitions that Newton poses at the outset of the foundational work of "modern science" influenced by Plutarch? The evidence, though indirect, is quite strong.

Acquaintance with the *De facie* probably dated from Newton's youthful years, around 1664, when he started his scientific career precisely by studying the appearance of the moon's disk. We know that the *De facie* played a role in guiding the formulation of the *Principia*, because Newton included long excerpts thereof (including the ones we've quoted) in the first draft of his book. They appear in the so-called *Classical scholia*, notes about classical matters that Newton wrote for the *Principia* but did not include in the published version.[143] And Newton opens his illustrative scholium to the fifth definition (of centripetal force) with what may well be an echo from the *De facie*:

> Of this sort is gravity, by which bodies tend to the centre of the earth; magnetism, by which iron tends to the loadstone; and that force,

[141]Plutarch, *De facie quae in orbe lunae apparet*, 923C–D.
[142]Plutarch, *De facie*, 923E–F.
[143]The *Classical scholia* were partly published for the first time in [Casini]. Newton's personal library naturally included Plutarch's *Opera Omnia* (number 133 in [Harrison]).

> whatever it is, by which the planets are continually drawn aside from the rectilinear motions, which otherwise they would pursue, and made to revolve in curvilinear orbits. A stone, whirled about in a sling, endeavours to recede from the hand that turns it[.][144]

With the loss of the original scientific treatises, Plutarch had become precious. Copernicus, too, had resorted to him in reconstructing Aristarchus' ideas (just as the inventors of the modern age had to start from Heron's "toys" in reconstructing Alexandrian technology). But when the source is a nontechnical writer of the imperial age, rather than Euclid or Archimedes, the effect is unmistakable. The metaphysical ingredient, largely absent from Galileo, looms large. For already the ancient writer, unable to make clear demarcations between scientific theory and the reality it models, tends to frame the discussion in pre-Hellenistic, usually Aristotelian, concepts: a mix that early modern scientists are wont to contaminate further with biblical influences (Newton was a passionate commentator of the Bible).

Plutarch is not the only literary classical source used by Newton. In the following passage we clearly recognize Seneca as the source, though it is not explicitly stated:

> The *Chaldeans*, the most learned astronomers of their time, looked upon the comets (which of ancient times before had been numbered among the celestial bodies) as a particular sort of planets, which, describing eccentric orbits, presented themselves to view only by turns, once in a revolution, when they descended into the lower parts of their orbits.[145]

The same applies to this very significant passage:

> Therefore the earth, the sun and all the planets which [are] in our system, according to the ancients have weight with respect to one another, and would fall toward each other because of mutual gravity and coalesce into one mass, if their fall were not prevented by their circular motions.[146]

Where does this lead us?

[144]Newton, *Principia mathematica*, immediately after Definition V, Motte/Cajori translation.
[145]Newton, *De mundi systemate liber*, 1, anonymous translation of 1728 (probably by Motte), as revised by Cajori. Compare the passages of Seneca quoted on page 316.
[146]"Igitur Terra Sol et Planetae omnes qui in nostro systemate ex mente veterum graves sunt in se mutuo et vi gravitatis mutuae caderent in se invicem & in unam massam coirent nisi descensus ille a motibus circularibus impediretur" (*Classical scholia*, folio 271r = [Casini], p. 46 or p. 37). Thus Newton, who of course was immune to the modern scholarly fear of committing anachronisms by attributing "Newtonian" notions to ancient authors, provides authoritative corroboration for the interpretations of Seneca's passages proposed in Sections 10.7 and 10.13.

As we learned in school, the conceptual leap from a purely descriptive astronomy to a gravitation-based theory lay in taking the sun, the moon and the planets and realizing — though these bodies are only a handful and there was no hope of observing others — that their regular motions did not depend on their "heavenly nature", but on their having "weight", and that these motions could be generalized through a theory that renders just as regular the motion of anything whatsoever: a stone, an apple or a mass of liquid swinging about the center of the earth. The ancient testimonia discussed in Chapter 10 and the use made of them by Kepler, Newton and others show that this leap was achieved only once in history: in Hellenistic science.

Two sets of factors were essential in creating the conditions for a modern gravitation-based dynamics to take shape as a scientific theory and evolve into modern physics.

The first set consisted of certain Hellenistic technical and methodological tools, found above all in the works of Euclid and Archimedes. Some of these tools were:

– the hypothetico-deductive method created in the *Elements*, providing the general conceptual framework to which there must conform any scientific theory wishing to use the results of classical mathematics — a use that is of course inescapable;
– the so-called "method of exhaustion", whose relation to the seed of infinitesimal analysis developed by Newton will be discussed in the next section;
– Archimedean mechanics, as laid out specifically in the treatise *On the equilibrium of plane figures*, which showed how to use the preceding methods to found a scientific theory of mechanics.

The second set of prerequisites were certain pieces of information on dynamics and gravitation which, with the loss of the original treatises, were found scattered throughout works generally written by scientifically incompetent authors and representing traditions that were far from scientific. The fragmentary and heterogeneous nature of these testimonia makes their complete identification difficult. Based on an examination of a small part of the still extant literature, one can list at least the following examples.

– The hints about inertia, centrifugal force and gravity (toward the earth) transmitted by Plutarch in the *De facie*. These hints included a few "exercises in dynamics" together with their qualitative answers.[147]

[147]Sambursky (who was first and foremost an experimental physicist) revealingly wrote that "some of [the conclusions in the *De facie*] call to mind classic exercises from Newton's Theory

– Other hints complementary to the first, including those found in the commentaries to Aristotle by Simplicius and Philoponus and in the Heronian and pseudo-Aristotelian *Mechanics*.
– Mentions of an attraction between planets and sun, and of the use of this idea in a heliocentric framework to create a "celestial mechanics" that could account for the motion of planets and the more elongated motion of comets. These were found in classical authors (Seneca, Pliny, Vitruvius) but also, as we shall see, in writings stemming from the neo-Pythagorean and Hermetic traditions of late Antiquity.
– Some testimonia on an ancient theory of tides based on gravitational interaction, whose memory had not been totally obliterated.

But all these prerequisites were not enough, since the technical tools provided by the first set were insufficient for mathematizing the information in the second. Two more elements were essential: a quantitative law of gravity and a mathematical theory that could derive from it the motion of the planets. The first of these elements will occupy us later in this section; the second was the theory of conic sections of Apollonius of Perga. Since orbits under a central gravitational field are conic sections, one can in large measure view the theory of gravitation mathematically speaking as a set of "exercises in the theory of conics".[148]

The recovery of Apollonius' theory had been one of the main goals of seventeenth century mathematicians. We have already mentioned Bonaventura Cavalieri's *The burning mirror* (1632), which applied the theory's rudiments to burning mirrors, lighthouses, acoustics and motion under gravity. In 1655 there appeared John Wallis' *Tractatus de sectionibus conicis*; but apparently the author had been able to study only the first four books of Apollonius' treatise, those that survived in Greek.[149] The next three books of Apollonius' *Conics* were first printed in Florence in 1661, in a Latin translation (from an Arabic recension) prepared by Abraham Echellensis and Giovanni Alfonso Borelli; the latter was reckoned by Newton among his own forerunners concerning the universal law of gravitation. This work of recovery continued after the publication of the *Principia* in 1687. A critical edition of the first seven books, containing the Greek text

of Gravitation": see [Sambursky: PWG], p. 209. Yet he did not consider the question of the dialog's sources, taking it to be "perhaps the first work in astrophysics ever written" (p. 205) — which would make Plutarch the founder of that science!

[148]For instance, Newton solves in the *Principia* the problem of finding a conic going through five given points. According to Heath this problem had been solved by Apollonius, but its proof was not included in his treatise, perhaps so as not to lengthen it too much. See [Apollonius/Heath], Introduction, chapter VI, p. cli.

[149]The first four books were a sort of introductory textbook; Apollonius' original results appear in the next four.

of the first four and a Latin translation of the next three based on multiple Arabic manuscripts, was finally prepared by Edmund Halley,[150] the friend of Newton's to whom we owe the discovery (or rediscovery) of periodic comets and their elliptic orbits, not to mention the completion of the secular observational experiment designed and set in motion by Hipparchus.[151] The eighth book was never found; we have an inkling of its contents thanks to a remark contained in the seventh.[152]

That the bits of gravitational theory recorded by literary men such as Plutarch and Seneca are only qualitative does not mean that Hipparchus and other mathematicians of the second century B.C. necessarily neglected the theory's quantitative aspects. If they developed them, it would likely have been in a direction not too far from that which Newton, based on indirect and partial knowledge of Apollonius, later took. For we should not forget that the gravitational theory whose partial outlines we have tried to reconstruct came into being a few decades after Apollonius' time, within the same scientific tradition, and that some Hellenistic astronomers did discover that comets are no more and no less than planets.[153] It is true that Ptolemy never uses conics in astronomy, but he also avoids any discussion of comets in the *Almagest* and he overlooks, in the *Optics*, the applicability of the theory of conics to mirrors, though such applications had been known and used systematically from the time of Dositheus (third century B.C.) down to the Arabs.[154]

But it should be noted that the mathematical formalism used by Newton is generally more elementary than Appolonius': for instance, he sometimes deals with central forces as if they cause a sequence of successive "thrusts toward the center" (Figure 11.3) — an ancient idea that we encountered in Chapter 10.

Newton's work is explicitly founded on all the prerequisites we have listed. If the universal theory of gravitational soon managed to reach the status of a scientific theory, despite the obvious frailty of its foundations, the reason, one suspects, is that the mutual coherence of its contributing elements was ensured by their common origin. The starkly Aristotelian statements made from the beginning of the *Principia* and onwards could not, for instance, spoil the proofs about conics carried out according to the Apollonian model. But it is not hard to imagine what kind of a natural philosophy would come out of Newton's ideas of space, time and force in

[150]*Apollonii Pergaei conicorum libri octo et Sereni Antissensis de sectione cylindri et coni*, Oxford, 1710.
[151]See page 89.
[152]See page 201, note 99.
[153]See the passages cited in Section 10.13.
[154]Many other aspects of Hellenistic knowledge seem to have been unknown to Ptolemy, as we have seen in Sections 10.5 and 10.14.

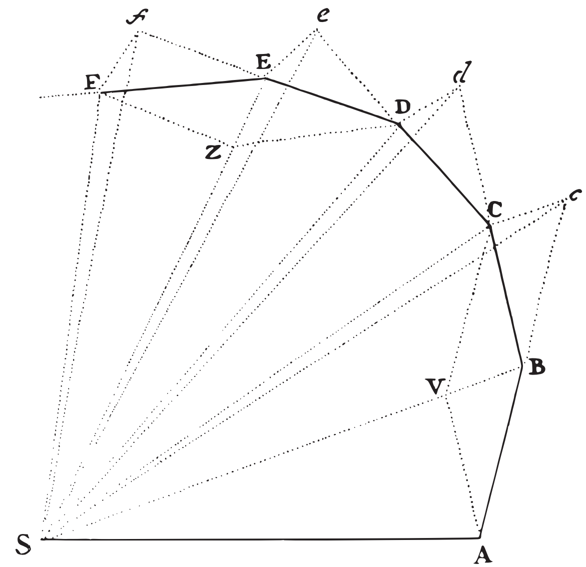


FIGURE 11.3. Figure illustrating Proposition I / Theorem I of Newton's *Principia* (Book I, Section II). Newton is proving that for central forces equal areas are swept in equal times. The proof starts: "For suppose time is divided into equal parts, and in the first part let the body by inertia go from *A* to *B*. In the second part of that time, the body would, if not hindered, move to *c* ... But when the body is arrived at *B*, suppose that a centripetal force acts at once with a great impulse, and, turning aside the body from the line *Bc*, compels it instead to move along the line *BC* [with *Cc* parallel to *SB*, so area $\triangle SBC$ = area $\triangle SBc$ = area $\triangle SAB$.] ... Now let the number of these triangles increase, and their breadth decrease to an infinite degree..."

the absence of all the elements listed earlier: we have over a million words in his pen to show us the directions he might have gone into. Newton's world view and methodology are easier to recognize today, thanks to an increasing awareness of his total written output. The cardboard image of a purely rational genius held sway for centuries because of general ignorance of a large fraction of his writings. When these writings started resurfacing, there were those who postulated a severe case of split personality; more recently we have started seeing more irreverent accounts as well.[155]

[155]For example, [White]. Newton's first exegetic work on the Apocalypse of St. John, written in the 1670s, was only published in 1994, as [Newton/Mamiani]. Its very existence was little known and indeed had been concealed by Samuel Horsley, the first editor of Newton's "complete" works (1779). The *Observations on the prophecies of Daniel and the Apocalypse of St. John*, written much later in Newton's life, were published in 1733 and enjoyed several reprintings, including the recent [New-

Newton was sharply aware of the importance of knowledge inherited from Antiquity, but his use of nonscientific sources and his ideological bent did not allow him to perceive the true origin of the relevant knowledge, namely Hellenistic science. In his view, the profundity of thought of the "Ancients" (which struck him inordinately) arose from an early Truth preserved in the esoteric tradition of priesthoods and religious sects. Here is how the origins of heliocentrism are presented in the *System of the world*:

> It was the opinion of not a few, in the earliest age of philosophy, ... that under the fixed stars the planets were carried about the sun; that the earth, as one of the planets, described an annual course about the sun, while by a diurnal motion it was in the meantime revolved about its own axis.... This was the philosophy taught of old by *Philolaus*, *Aristarchus* of *Samos*, *Plato* ... and of that wise king of the *Romans*, *Numa Pompilius*, who, as a symbol of the figure of the world with the sun in the centre, erected a round temple in honor of *Vesta*, and ordained perpetual fire to be kept in the middle of it.
>
> The *Egyptians* were early observers of the heavens; and from them, probably, this philosophy was spread abroad among other nations; for from them it was, and the nations about them, that the *Greeks*, a people more addicted to the study of philology than of Nature, derived their first, as well as soundest, notions of philosophy; and in the Vestal ceremonies we may yet trace the ancient spirit of the *Egyptians*; for it was their way to deliver their mysteries, that is, their philosophy of things above the common way of thinking, under the veil of religious rites and hieroglyphic symbols.[156]

The superiority of the Egyptians over the Greeks, and of sacred *rites* and *mysteries* over rational research, is a topos of Hermetic literature, a genre that dates from late Hellenistic times (when native Egyptians regained the upper hand in their country) and became very popular in Renaissance Europe.[157]

ton/Barnett]; they probably contributed to the false belief that Newton's writings on religion were the product of his declining years. There is in fact methodological continuity between Newton's religious works and the *Principia*; in this regard the introduction to [Newton/Mamiani] is very worth reading.

[156]Newton, *De mundi systemate liber*, 1, anonymous/Cajori translation.

[157]The role of such texts in the formation of modern scientific thought has probably not been sufficiently placed in the right perspective. There have been studies of the influence of Hermetic writings on Renaissance culture (especially after the famous [Yates] appeared) and on Copernicus, Newton and others. But to my knowledge no one has analyzed the possible contribution of remnants of Hellenistic science to Hermetic literature. Such an influence, if real, would help understand why so many of the founders of modern scientific thought cultivated Hermetic knowledge. Here we just remark that in document XVI of the Hermetic corpus — bearing the title "From Asclepius

Let's turn to the question that to many may seem the only crucial one: was the "Newtonian" law of gravitation, involving the inverse square of the distance, known in Antiquity? We don't have Hipparchus' work, but we can start from what Newton himself wrote:

> According to Macrobius, Pythagoras ... applied to the heavens the proportions found through these experiments [on the pitch of sounds made by weighted strings], and learned from that the harmonies of the spheres. And so, by comparing those weights with the weights of the planets, and the intervals in sound with the intervals of the spheres, and the lengths of string with the distances of the planets [measured] from the center, he understood through the heavenly harmonies that the weights of the planets toward the sun ... are inversely proportional to the squares of their distances.[158]

It is significant that Newton thought that the inverse square law was known to Pythagoras, even if the testimony of Macrobius does not quite justify this belief. Newton had probably read other, neo-Pythagorean, texts on this subject.

The idea of applying the inverse square law to deduce Kepler's laws predates Newton. It had occurred at least to Wren and Halley, as implied by a 1684 letter from the latter to Newton, and to Hooke, who put forth the law in a letter to Newton of January 6, 1680.[159] In fact Hooke's discovery of the universal law of gravitation was one of the two main causes of Newton's fierce animosity toward him. (The second had to do with optics, another of Newton's main scientific interests. Hooke, being aware of several phenomena caused by diffraction and interference, rejected Newton's corpuscular optics, and instead founded the wave theory of light. He discovered, for instance, the interference phenomenon now known as "Newton's rings", which is quite incompatible with Newtonian optics. He achieved a lot more: apart from his work as an architect, we

to King Ammon" and cited, for example, by Copernicus — we read that the sun draws all things to itself and, like a good charioteer, has bound the cosmos to itself, not letting it get away (*Corpus Hermeticum*, XVI §5, §7). A systematic search of the Hermetic, kabbalistic, astrological and alchemical literature that so fascinated Newton, and of which the neo-Pythagoreans were a chief source, would likely reveal many other interesting passages.

[158]"ut refert Macrobius ... [p]roportionem vero his experimentis inventam Pythagoras applicuit ad caelos et inde didicit harmoniam sphaerarum. Ideoque conferendo pondera illa cum ponderibus planetarum et intervalla tonorum cum intervallis sphaerarum, atque longitudines chordarum cum distantiis Planetarum ab orbium centro, intellexit per harmoniam caelorum quod pondera Planetarum in Solem ... essent reciproce ut quadrata distantiarum earum" (*Classical scholia*, f. 11v = [Casini], pp. 41–42 or 32–33). The reference is to Macrobius, *In Ciceronis somnium Scipionis*, II, i.

[159]Anyone can aspire to write interesting letters, and many succeed in doing so; but to receive such letters all the time is a much more ambitious goal, which not many contemplate and only an exceptional few achieve.

must mention his major contributions to the design of many scientific instruments, to the study of gases, to entomology, astronomy and crystallography. He was the discoverer of cells and gave them the name we use now. It is a legacy of Newton's long-armed hatred that even today our textbooks associate Hooke only with the study of elastic forces — a field where his investigations allowed the substitution, at least in certain cases, of a dynamometer-based definition of force for the one given in the *Principia*.)[160]

But the law itself goes back further. It was stated in 1645 by Boulliau, based on the argument that the sun's force, like the light it emits, must decay with distance in inverse proportion to the area reached.[161] Such considerations were not new even then, having been made by Kepler, though the latter rejected the analogy between the sun's motor force and light, imagining that force, spreading only over the plane of the ecliptic, was inversely proportional to distance.[162]

Continuing our brief backward history: the analogy between the sun's "virtue" and the light it sends out was drawn in the thirteenth century by Roger Bacon, who outlined a quantitative theory of any propagation along straight lines, and so arrived at the inverse square law, at least implicitly, since he attributed the weakening of the action with distance to the decrease in the cone (solid angle) under which the acted-on body is seen by the agent.[163]

We must conclude that knowledge of the dependence of gravitational force on the distance predated, in medieval and modern times, not only any connexion with Kepler's laws but even the statement of the second principle of dynamics. In other words, this property of gravitational force was known not only before it was used to explain any phenomenon, but even before anyone had properly established what should be understood by "force". This odd order of ideas becomes intelligible if we assume that Newton was right in thinking that the law was very old (although he undoubtedly overstates the case in throwing it as far back as Pythagoras).

[160]In recent years Hooke has benefited from a deep and rapid reevalution of his importance. See the recent biographies [Bennett et al.] and [Inwood] and references therein.

[161]*Ismaeli Bullialdi Astronomia Philolaica*, Paris, Piget, 1645, p. 23. The reference to Philolaus in the work's title shows that Boulliau, too, meant to reconstruct *Pythagorean* astronomy.

[162]J. Kepler, *Astronomia nova*, xxxvi.

[163]R. Bacon, *Specula mathematica*, III, ii. Johann Combach first edited and printed this work (Frankfurt, 1614), but it is now more readily available as the fourth part of the *Opus maius*. Bacon calls *multiplicatio secundum figuras* the law of dependence on distance of an action that radiates in all directions along straight lines, and he adds that the lines along which it radiates terminate in the concave surface of a sphere (op. cit., II, iii).

Kepler believed that the sun's "virtue" radiated out in straight lines and gripped the planets, dragging them in an orbital motion.[164] This is a far cry from our modern idea of force, but it is close indeed to the passages of Vitruvius and Pliny. It is hard to see why a hand-like "virtue" with which the sun grips the planets should decrease with distance, if it were not the case that this decay, too, had been suggested by classical sources.

Let's turn to the passage of Vitruvius that immediately precedes the ones studied in Section 10.8 (page 297). It reads:

> . . . ergo potius ea ratio nobis constabit, quod, fervor quemadmodum omnes res evocat et ad se ducit, ut etiam fructus e terra surgentes in altitudinem per calorem videmus, . . . eadem ratione solis impetus vehemens trigoni forma porrectis insequentes stellas ad se perducit. . .[165]

The Latin word *ratio* has multiple meanings: our "ratio" is one, and another is "reason, argument". The same is true of the Greek word *logos*, of which the Latin *ratio* is often a direct translation. Now, recall that we have established that Vitruvius used a Hellenistic scientific source for this discussion, but disfigured it seriously. Thus we must inquire what lies behind the "natural" meaning of the Latin, which is something like:

> Therefore we find the following reasoning [*ea ratio*] stronger: in the same way that heat calls and attracts everything to itself (as we see the grain shoot up in height during the hot months. . .), so for the same reason [*eadem ratione*] the sun's powerful force attracts to itself the planets by means of rays projected in the shape of triangles[.]

To try to separate Vitruvius' contribution from what was in the source, we can start by imagining that both occurrences of *ratio* correspond to *logos*, and that the original sense was that of our "ratio". Then the juxtaposition of "this ratio" (*ea ratio*) and "the same ratio" (*eadem ratione*) suggests the idea of proportionality, which Vitruvius seems to have missed, as he does not place these two expressions in parallel clauses.[166] The analogy with grain may be Vitruvius'; a scientific work would hardly have sandwiched it between ratios and triangles, in the midst of a mathematical argument.

We can therefore conjecture that in his scientific source there was a quantitative statement, expressed through an equality of ratios, linking the

[164]See the last passage quoted on page 357 (footnote 101).

[165]Vitruvius, *De architectura*, IX, i §12.

[166]However, if we read *qua* for *quod*, the two uses of *ratio* become parallel and the Latin lends itself naturally to the proportionality interpretation. Vitruvius' meaning may have been closer to the original than what is attested by our manuscripts.

force exerted by the sun with the spread of heat. It is possible that the source stated that the force decreases with distance in the same ratio as the intensity of light and heat, and thus in inverse proportion to the area reached: the same idea later stated by Roger Bacon, Kepler and Boulliau. The two-dimensional context of Vitruvius' work matches particular well Kepler's treatment (which, as we have already seen, appears to almost quote verbatim from Vitruvius in this matter of the force exerted by the sun).

Of course, the interpretation we have suggested for Vitruvius' passage is only one possibility. Boulliau might have taken his ideas from some other classical author or from medieval authors, such as Roger Bacon. However, in the absence of ancient sources, direct or indirect, it is unlikely that either he or Bacon would have examined the variation with distance in a "force" whose meaning they did not know yet.

Misunderstanding for reflections on nature what were in fact attempts to interpret ancient texts may have led astray many historians who have sought to trace the development of ideas in the dawn of "modern science".

11.8 The Rift Between Mathematics and Physics[167]

The terminology of Renaissance scientists still followed the Greek model. "Mathematics" meant the exact sciences as a whole.[168] When Copernicus proudly wrote, in the dedicatory letter of the *De revolutionibus*, that "mathematics is for mathematicians" (*mathemata mathematicis scribuntur*), he had no doubt that his theory of the solar system was part of math.

As an example of what was still understood by the word in the early seventeenth century, consider the second postulate in Simon Stevin's *On the theory of ebb and flow* (of tides):

> [We postulate] that the earth is entirely covered with water, without the wind or anything else hindering the ebb and flow.[169]

The assertion is blatantly and intentionally "false". Stevin obviously plans to build a model based on a simplification of reality, and he does not think

[167]This section contains material drawn from [Russo: Appunti].

[168]In fact, the Renaissance's extraordinary cultural unity caused the term to acquire an even wider meaning for some authors. Fra Luca Pacioli, in the front page of his famous *De divina proportione* (1509), addresses it to "every student of philosophy, perspective, painting, sculpture, architecture, music and other mathematics".

[169]Simon Stevin, *Van de spiegheling der ebbenvloet* (Leiden, 1608), p. 179 = [Stevin: PW], p. 333. The work is methodologically very interesting, but no more than middling in its technical content: it lays out a simple static model for tides, based on lunar effects alone, and does not even discuss the correlation between tides and phases of the moon.

that postulates must be "true". This short work (containing no formulas or quantitative arguments of any sort) was published in 1608 as part of his *Wisconstighe ghedachtenissen*, or *Mathematical works*, precisely because of its logical structure and the role played in it by postulates such as the one quoted.

The term "physics", too, had until the seventeenth century a sense similar to the Greek one: it was used for works in natural philosophy, or in the medical and biological sciences, while the practitioners of exact science called themselves mathematicians, when not philosophers.

We have seen that Galileo did not hesitate in ranking his "new science" of motion under gravity together with the scientific tradition of Euclid, Archimedes and Apollonius.[170]

But the ancient method was understood by very few of Galileo's contemporaries. It had not been utterly forgotten, but few scientists felt free (as Stevin did) to choose hypotheses for building models; much more frequently, the arbitrariness involved in setting ground assumptions was taken (as it had by Simplicius and Thomas Aquinas) as a quirk of mathematicians and a sign of the weakness of the mathematical method as compared to philosophy and theology, which could tell right from wrong. Here, for example, is a letter of April 12, 1615, from Cardinal Bellarmino to Brother P. A. Foscarini, who had tried to reconcile heliocentrism with Scripture:

> It seems to me that you, Father, and Mr. Galileo act prudently in staying with arguments *ex suppositione* rather than speaking absolutely.... For saying that, supposing that the earth moves and the sun is fixed, all appearances are saved better than with eccentrics and epicycles, is very well and involves no danger, and is enough for mathematicians: but wanting to claim that the sun really is at the center of the world and just turns around itself without speeding from east to west, and that the earth lies in the third heaven and turns with enormous speed around the sun, is a very dangerous thing, capable not only of annoying all philosophers and scholastic theologians, but also of injuring the Holy Faith by belying the Sacred Scriptures.[171]

As is well known, Bellarmino's recommendation was in part adopted by Galileo himself, though only as a ruse to try to avoid censure and condemnation.

Newton is often considered the founder of physics in the modern sense of the term. Indeed, although his *Philosophiae naturalis Principia mathema-*

[170]See the quote on page 350.

[171]This letter appears in [Galileo: Opere], vol. XII, pp. 171–172.

tica starts from definitions and axioms, following the practice of ancient science, it is clearly Newton's intention to move away from the model of ancient mathematics. On this point one can do no better than quote Roger Cotes, the editor of the second edition of the *Principia*. He writes in the book's preface (italics mine):

> [Those who possess experimental philosophy] derive the causes of all things from the most simple principles possible; but then *they assume nothing as a principle, that is not proved by phenomena. They frame no hypotheses, nor receive them into philosophy otherwise than as questions whose truth may be disputed.* They proceed therefore in a twofold method, synthetical and analytical. From some select phenomena they deduce by analysis the forces of Nature and the more simple laws of forces; and from thence by synthesis show the constitution of the rest. This is that incomparably best way of philosophizing, which our renowned author most justly embraced in preference to the rest[.][171a]

This passage is perfectly emblematic of the birth of modern *physics* as a science distinct from ancient *mathematics*. The ancient scientific method, which for so many centuries not even those who could not understand it had dared contradict explicitly, is here haughtily repudiated. Many of Newton's own assertions bear witness to the same attitude: the *regulae philosophandi* that open Book III of the *Principia* (particularly the fourth); the General Scholium that concludes the second edition of the same work (with the famous sentence "Hypotheses non fingo": I do not make hypotheses); and the considerations at the end of the *Opticks*.[172]

Newton's science, unlike classical natural philosophy, makes systematic use of instruments that are mathematical in the modern sense of the term. And yet what he does is physics and no longer mathematics, in the sense that he rejects hypotheses whose truth cannot be established; he is not content with a theory able to save the appearances, but instead seeks that substantial reality, beyond appearances, whose knowledge Simplicius and Thomas Aquinas, following Aristotle, had placed in the realm of physics. The word *hypothesis*, for Newton, had taken on the meaning that is now most current (and different from the classical one) — something still under debate, whose truth or falsehood will sooner or later will be definitively established. The Greek term *phenomenon*, too, had taken on its modern meaning: no longer a *phainomenon* — what is perceived, via the interaction of subject and object — but an objective fact, thought to be describable

[171a]Preface to second edition (Cambridge, 1713), at 8%, Motte/Cajori translation.

[172]Such as the statement: "For Hypotheses are not to be regarded in experimental Philosophy", in *Opticks*, second edition (1730), p. 404 in the most common reprints.

without any reference to the method by which it is observed. The awareness that different theories can save the same phainomena is abandoned for the conviction that phenomena unambiguously and definitively lead to "true principles". Although the technical structure of modern physics is built on results of ancient mathematics, its epistemology is profoundly affected by Aristotelian thinking and the theological tradition.

We have seen how the relativity of motion was introduced in Hellenistic science as an application of the much more general idea that different but equivalent explanations, based on different premises, can be offered for the same phainomena. Thus is it not surprising that with Newton we lose again the awareness that all motion is relative (which had been recovered by Galileo, at least in part, after seventeen centuries), and return to a an essentially Aristotelian conception of space.

The theoretical views put forth by Newton and Cotes spread together with Newtonian mechanics, leading to a split of exact science into two streams, mathematics and physics (in the modern sense). Both inherited from ancient mathematics the quantitative method and many technical results, and from ancient physics (which is to say, natural philosophy) the goal of producing absolutely true statements. The two streams diverged in the nature of their subject matter and the criterion of truth applied to their statements. Mathematical entities, though applicable to the description of concrete objects, were regarded as abstract, while physical entities were felt to be as concrete as the objects to which they applied. Whereas the assumptions of mathematics (called *postulates*) were seen as immediately obvious truths, those of physics (called *principles* or *laws*) were seen as true if and only if they were "proved by phenomena", to use Cotes' words. Other statements could be deduced from the initial ones; but whereas in mathematics the deductive method was essential and constituted the only way through which truths not immediately evident could be established, physics statements, though deducible from principles often enough, were also considered to be directly verifiable, and this lessened interest for the deductive method in physics, where it became optional.

The scope of what was considered mathematics or physics may seem to some extent arbitrary. For example, statics and optics ended up in physics, whereas geometry remained an essential part of mathematics. Work methods, which in Antiquity had been the same in all three, changed according to the new classification. In geometry, the ties to drawing wore off, and now Euclid's "problems" (constructions) are left out of the curriculum altogether.[173] In the various areas of physics, conversely, it was the de-

[173]Considered unworthy of appearing in mathematics textbooks for being too "concrete", such constructions were in part shunted to courses in specialized drawing, but not before they were

ductive method that became etiolated, and now even statements that are provable from simple principles are sometimes regarded as "experimental laws"; thus, in modern treatments of hydrostatics, the so-called principle of Archimedes is stated as an experimental law, whereas in Archimedes' treatise *On floating bodies* it was deduced as a theorem.[174] A similar slide can be documented in several other cases (for example, in statics) and may be suspected in others: for example, "Snell's law" is generally presented nowadays as an experimental truth, instead of being deduced from a minimum principle.[175]

The name "mathematics" continued to apply mainly to those areas in which Greek treatises were still essential. One good reason for this may be that when mathematics and physics went their separate ways, in the late seventeenth century, scientists were fluent in the deductive method — to them an essential feature of mathematics — only in fields where they could follow the classical model closely. The term "mathematics" was later extended to new subjects that arose organically from the old ones, but in the fast-paced development of mathematics in the eighteenth and early nineteenth centuries, the expansion of content away from the classics was accompanied by a drift away from the rigor of demonstrations as well.

A second, language-based, factor may have contributed to the fact that precisely those fields most directly linked to the Greek legacy were seen as dealing with "abstract" entities, and so labeled as mathematics: in these fields, the use of Greek-derived technical terms to denote theoretical entities made it easier to distinguish them from concrete objects. On subjects where complete Greek texts were not available, the use of terms from everyday language, such as "force" or "mass", favored instead a confusion between theoretical entities and concrete objects to which the theory was applied. The importance of this effect may not be readily appreciated by someone who is familiar (as we are) with conventional terminology, but we must keep in mind that in the late seventeenth century linguistic conventionalism was not even close to being recovered.[176]

stripped of their proofs. Thus logical rigor and practical applications, the two main features of science, are made to stand in exclusion of each other.

[174]See page 73.

[175]For the conjecture that the law was first obtained as a consequence of a minimum principle, see page 349.

[176]The recovery was a long process, with murky transitional stages that resist neat categorization and precise dating. A few examples will give an idea of the progression. Linnaeus' zoological nomenclature, in the eighteenth century, was an important first stage. In economics, the necessity to define terms was stressed by Malthus around 1820. Still after that, in Bolzano's *Paradoxien des Unendlichen*, published posthumously in 1851, infinite series are summed (leading to some of the "paradoxes of infinity" of the title) without any suspicion that the concept of summation of a series must first be defined; thus conventional terminology was still foreign to some. Conventionalism

The reassignment of meanings to "physics" and "mathematics", allied to methodological differences between modern and ancient science, was and is a source of pitfalls in the analysis of classical works. Believers in the absolute validity of the boundaries and categories of modern science have often been led astray by facile equations. Consider Euclid's *Optics*, a work automatically pigeonholed with physics on the basis of its title's modern meaning. Since physical theories, unlike mathematical ones, supposedly deal in concrete entities, modern scholars have again and again confused Euclid's visual rays with modern light rays. If Euclid's rays have any property not coinciding with those of our light rays, only one reason would spring to mind: Euclid was wrong. Here is what Giuseppe Ovio, a researcher in physiological optics, wrote in the introduction to his translation of Euclid's book, dating from 1918:

> These two books of optics . . . presuppose . . . a theory of vision where visual rays start from the eye and go toward the object. Nowadays, as everyone knows, this theory can no longer be sustained; we know instead that rays follow the reverse path, from the object to the eye.[177]
>
> According to Euclid, visual rays are set apart a certain distance from one another. Today this opinion brings a smile to our lips[.][178]

Ovio's was not an isolated opinion; a few years later Thomas Heath wrote:

> Euclid assumed that the visual rays are not 'continuous', i.e. not absolutely close together, but are separated by a certain distance, and hence concluded, in Proposition 1, that we can never really see the whole of any object, though we seem to do so. Apart, however, from such inferences as these from false hypotheses, there is much in the treatise that is sound. [179]

Even the meaning of "optics" changed: its object of study was no longer visual perception, but a natural object (light), felt to be describable in the absence of any reference to the way in which it is observed. The meaning of "phenomenon" changed in the same way. A rereading of the Dreyer quote on page 85 will give a further example of how hard it became to understand the ancient scientific method.

only became the norm in mathematics in the late nineteenth century. Physics still has a strong essentialist streak (see Section 11.11).

[177][Euclid/Ovio], p. 1.

[178][Euclid/Ovio], p. 15.

[179][Heath: HGM], vol. 1, pp. 441–442. Similar opinions have been expressed by many modern writers on the question; see, for example, [Enriques, de Santillana], [Heiberg: GMNA], [Ronchi] and the introduction to [Euclid/Ver Eecke]. All these authors consider false Euclid's statements about visual rays that differ from the statements about light rays accepted by modern optics. For the meaning of Euclid's hypotheses, see pages 58, 149 (especially note 17) and 183.

11.9 Ancient Science and Modern Science

At the base of modern science were Hellenistic science, on the one hand, and on the other the study of and experimentation with technological products having, in large measure, the same source. Yet modern science attained fairly soon a state that appeared much more powerful than that of its ancient counterpart. Why?

The exponential growth in scientific and technological knowledge and industrial production that started in seventeenth-century Europe — and at no other time or place in human history — hinged on many prerequisites, as we saw in Section 9.7, not all of them easy to identify. *One* of them was of course a sufficient mass of technological and scientific knowledge. This initial mass, or cultural capital, was inherited from a distant civilization, which (though lacking stock brokers and even a word for them) managed to create many cultural instruments of lasting usefulness. Since the introduction of writing, information can be preserved down the centuries and millennia, and this means that superficially unexpected similarities can coexist with profound differences between cultures.

What were the new factors that unchained the development seen in modern times?

Taking mathematics first, consider the common claim that the positional number system gave a decisive advantage to modern over ancient science. But this system was borrowed from the Arabs, who inherited it from the Indians, who in turn learned it from Hellenistic mathematicians. While its furthest origins go back to Old Babylon, it was reformulated and rationalized in the time of, and largely thanks to, Archimedes and Apollonius; in Hellenistic times it was systematically used (especially in base sixty) for trigonometrical and astronomical calculations — namely, those problems that could not be solved with ruler and compass.

Thus the question becomes: How did solutions with ruler and compass, which in Antiquity were considered simpler, get replaced by numerical calculations in the modern age?

In reality, numerical calculations got a significant lead over geometric methods only after the advent of printed tables of logarithms, in 1614. The speedup represented by these computational aids meant that instead of solving algebraic problems with ruler and compass, it became more convenient to turn even geometric problems into algebraic ones, inverting the relative standing that algebra and geometry had held since Hellenistic times. The bridge between geometric and algebraic problems was the assignment of coordinates to points. This, too, was not a radically new idea: Apollonius had already used what came to be called Cartesian coordinates. The novelty was that the systematic use of the algebraic form

reduced the drawing, now a mere “sketch” of the curve under study, to a subsidiary role, and primacy went to the curve’s equation, from which the desired results could be obtained through numerical computation. This allowed the study of a much broader mathematical phenomenology.

Shall we then conclude that the superiority of modern mathematics is based on a new idea, logarithms? Certainly not. That writing numbers as powers of the same base allows the reduction of time-consuming operations to easier operations on exponents is lucidly explained (en passant, as it were) in Archimedes’ *Arenarius*.[180] Nor was the practice of compiling numerical tables new, since Hellenistic astronomers made use of trigonometric tables. But in the seventeenth century we start seeing the compilation of numerical tables to a hitherto unmatched degree of precision and extension. Only the new, detailed tables of logarithms could make the ancient geometric calculation methods obsolete; but their preparation requires tremendous labor, hardly to be undertaken unless the expected use of the product exceeds a certain threshold. Moreover if it were not for printing it would not be possible to keep the tables reliable, and printing in turn only makes sense where the reading public is sufficiently wide. Thus the essential novelty is not to be found in new ideas, but in the achievement of a critical mass of interested individuals.

I believe that this discussion of methods of calculation exemplifies a much more general pattern: the factors that made modern science take off do not rest on radically new ideas, but rather on there being again, in early modern Europe, an opportunity for remnants of ancient culture to interact and develop, with the advantage of extension to a much wider social base. When there started to be mutual interaction between scientific and industrial development, the existence of wide markets became even more important.

But this left modern science with a serious weak spot. Since its results originated in the acquisition of external elements, created by a different civilization and not completely understood, it is not surprising that the science of Descartes, Kepler and Newton, despite its potential superiority (due to its applicability to a wider range of phenomena) was poorer than ancient science in its methodology. In the works of early modern science,

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[180] Archimedes, *Arenarius*, 147:27 – 148:26 (ed. Mugler, vol. II). Of course, to get an efficient numerical method from this it is necessary to take a geometric progression not of natural numbers like the one considered in the *Arenarius*, but of noninteger magnitudes whose ratio is close to unity (thus Napier’s table involved a geometric progression of ratio 0.9999999, while a table of decimal logs to, say, three decimal places involves a progression whose ratio is the thousandth root of 10). It is to be supposed that this step was within the reach of Hellenistic mathematicians, given the parallel development of the theory of proportions for integers and for magnitudes in the *Elements* and elsewhere. If it was not taken the reason is presumably to be sought in a relative lack of interest in numerical methods.

individual pieces of content either recovered from ancient science or derived from such were plunged in a foreign overarching framework based on theology and natural philosophy. The crystal sphere of fixed stars — which, as we recall, was first introduced to explain the rigid nightly motion of the heavens, then abandoned in the time of Heraclides of Pontus when the hypothesis of the earth's rotation was made, and then taken up again with the end of ancient science — did not disappear with the rise of heliocentrism: it still surrounded Kepler's universe. Likewise Newton tried to frame his "new" science in Aristotelian categories, in particular preserving a concept of absolute space that is virtually incompatible with the principle of inertia.

As we remarked on page 368, the eventual evolution of modern science into true scientific theories was ensured by the fact that its structure was circumscribed by technical elements that followed closely the surviving Hellenistic treatises, from which authors continued to draw. Nonetheless, the level of mathematical rigor remained for a long period far below what it was in Hellenistic times. Here is how Newton discusses the limit of the ratio between two infinitesimals (what he calls the "ultimate proportion" of "evanescent quantities"):

> Therefore if hereafter I should happen to consider quantities as made up of particles ... I would not be understood to mean indivisibles, but evanescent divisible quantities[.]
>
> Perhaps it may be objected, that there is no ultimate proportion of evanescent quantities; because the proportion, before the quantities have vanished, is not the ultimate, and when they are vanished, is none.... But the answer is easy; ... by the ultimate ratio of evanescent quantities is to be understood the ratio of the quantities not before they vanish, nor afterwards, but with which they vanish.[181]

Newton conceives "evanescent" quantities as real objects, which vary in real time: their ultimate ratio (in our language, the limit of their ratio) is thus the value the ratio takes at the moment in which the two values vanish. It is clear that Newton has no awareness of using a mathematical model. It is not an accident that his "method of ultimate ratios" is part of his *philosophy of nature*. Newton and his contemporaries were still far from mastering the technical methodology that, two thousand years earlier, had allowed Archimedes to compare infinitesimals of different orders within the rigorous structure of a hypothetico-deductive model.[182]

[181] Newton, *Philosophiae naturalis principia mathematica*, Book I, Section I, after scholium to lemma XI, Motte/Cajori translation.

[182] See, for example, Archimedes' *On spirals*, proposition 5, 17–18 (ed. Mugler), where the comparison between infinitesimals of different orders is an important step in determining the direction of the tangent at an arbitrary point of the spiral. This is the first known discussion of a topic in what we now call differential geometry, making Archimedes the founder of the subject.

This is how Boyer, in his *History of mathematics*, contrasts ancient and early modern infinitesimal methods:

> Stevin, Kepler, and Galileo all had need for Archimedean methods, being practical men, but they wished to avoid the logical niceties of the method of exhaustion. It was largely the resulting modifications of the ancient infinitesimal methods that ultimately led to the calculus[.][183]

How did modern infinitesimal calculus ever manage to work despite its lack of "logical niceties" (or, to put it more bluntly, in spite of logical contradictions)? Probably because it was created precisely by pruning the "logical niceties" from an actual scientific theory. The "practical men" of calculus considered raw "infinite" or "infinitesimal" quantities because they were not in a position to obtain rigorous demonstrations using only finite quantities, as Euclid and Archimedes did, and as contemporary mathematical analysis would again do. It was at that point, thanks to the "practical" mathematicians, that the idea was born that infinity was unfathomable to the "Ancients".[184] This misjudgement survived in the eyes of many historians of mathematics even after rigorous infinitesimal methods were reintroduced in the late nineteenth century.[185]

11.10 The Erasure of Ancient Science

Each step in the recovery of ancient knowledge was accompanied by a loss of historical memory. The assimilation of ancient ideas, indeed, consists in translating them into the idiom of one's own culture, recasting them into writings that tend to edge out the old ones, which often end up forgotten. Each of the renaissances discussed in Section 11.1 led to a partial replacement of earlier works on technical subjects by new works — in many cases of lower level; this sometimes limited the diffusion of the older works, and sometimes led to their complete disappearance. In imperial times, Heron's work on automata displaced Philo's and caused its disappearance. The recovery of ancient knowledge about refraction seems to have spelled, in the

[183] [Boyer], p. 354 (1st ed.), p. 322 (2nd ed.).

[184] Of course, when the modern recovery of the scientific method started, there reappeared mathematicians who "didn't understand infinity". Boyer states, with apparent amazement, that both "Gauss and Cauchy seem to have had a kind of *horror infiniti*"; see [Boyer], p. 565 (1st ed.), p. 516 (2nd ed).

[185] See the quotation by Kline on page 44.

early seventeenth century, the end of Ptolemy's *Optics*.[186] Gilbert's treatise made obsolete the earlier work of Pierre de Maricourt (Petrus Peregrinus) on magnetism and the compass;[187] one may suspect that the latter, in turn, had contributed to the loss of older works on the subject.[188]

But in none of these situations had it been denied or questioned that the study of ancient Greek sources was, in their sum, essential. Admiration for ancient science was imparted, unabated, by the writers of the imperial period to Byzantium and the Arabs; it revived in late medieval Europe in the passionate pen of Roger Bacon,[189] and continued to be shared by such men as Della Porta, Francis Bacon, Galileo and Newton.

In the eighteenth century something radically different took place. For the first time it was again possible to build coherent theories, which could be expected to evolve solely through the light of reason, without essential and constant recourse to poorly understood ancient sources. European science, confident of finally being able to walk on its own legs, underwent during the Enlightenment a phase of violent rejection of the old culture that had nurtured it, obliterating its memory. It was then that it came to be believed that pneumatics started with Torricelli, and the works of Heron and Philo of Byzantium fell into the oblivion in which they pretty much remain.[190] It was then that heliocentrism became "Copernican", for earlier it had always been linked with its creator, Aristarchus — as when Gilles de Roberval apocryphally published his book in defense of heliocentrism as if authored by the Greek astronomer (*Aristarchi Samii de mundi systemate partibus…*, Paris, 1644), or when Libert Fromond published a refuting tract called *Anti-Aristarchus* (Antwerp, 1631).[191] It was also then that Hellenistic technological inventions were forgotten or assigned the role of accidental "precursors" of their modern imitations.[192]

[186]The last document that directly cites this work is Ambrosius Rhodius' *Optica*, of 1611; after that it was considered lost.

[187]Gilbert's famous *De magnete…* (London, 1600), as the author acknowledges, owes much to Pierre de Maricourt's 1269 epistolary tract, which had until then been popular in manuscript ("Tractatus de magnete Peregrini de Maricourt ad Sygerum militem") and in print (*De magnete, seu Rota perpetui motus*, Augsburg, 1558; a truncated edition had appeared in the 1510s under the title *De virtute magnetis*, misattributed to Raymond Lull).

[188]In the thirteenth century, theoretical knowledge about magnetism and the compass was shared by other authors such as Roger Bacon and Albertus Magnus. The compass — but not its use in navigation — is attested in China in the first century A.D.

[189]See Section 4.10.

[190]See, for example, [Philo/Prager], p. 31.

[191]These books are mentioned in the preface to Heath's *Aristarchus of Samos: the ancient Copernicus*, a lovely work whose title nonetheless manages to get history backwards — as do all those who marvel at the "foresight" of Aristarchus the "precursor" of heliocentrism.

[192]Some scholars tried to resist this trend. See, for example, Louis Dutens, *Origine des découvertes attribuées aux modernes*, Paris, 1766.

The age-long history of thinking on gravitation, too, was erased from collective consciousness, and that force somehow became the serendipitous child of Newton's genius.[193] The new attitude is well illustrated by the anecdote of the apple, a legend spread by Voltaire, one of the most active and vehement erasers of the past.[194] (The immense popularity of this legend is worth dwelling on. It is hard to talk of universal gravitation without wondering what might have led to an idea so far removed from common experience as the mutual attraction of all bodies. But one can't reconstruct the genesis of this idea within the confines of modern science, which reclaimed it from an ancient tradition. Thus the erasure of gravitation's long history left a void that had to be filled by some other story: for the gullible, it could even be the notion that all it took was a genius seeing an apple fall.)

Newton's name became a pigeonhole for an incredible number of ideas, in a process already discussed.[195] Even one of the seven wonders of the ancient world suffered the magnetic attraction of his fame: lighthouses were sometimes called *Newtonian towers* in the eighteenth and nineteenth centuries.[195a]

Of course, in order for the new picture of the development of science to gain credit, it was necessary to forget the essential role played by the "missing links" between ancient and modern culture — intellectuals like Crisogono, Boulliau and de Dominis, whose works include early forms of important results of later science, but in a context that obviously cannot account for their origin. Not only that: it was necessary to conceal many of Newton's own writings, including those where he credits Pythagoras

[193]See Section 10.9 for a reconstruction of the subject's ancient lineage. Much information about the medieval and modern history of gravitation (which we discussed briefly on pages 376–379) can be found in [Duhem: TP], Chapter VII, Section 2.

[194]Voltaire tells the anecdote in the fifteenth of his *Lettres philosophiques*. To get an idea of the general tone of his polemic, it suffices to read the entry *système* in his *Dictionnaire philosophique* (see http://www.voltaire-integral.com/20/systeme.htm). There Voltaire inveighs against Aristarchus of Samos, whom he thinks not only scientifically mediocre but bigoted, wicked and hypocritical (this based on the passage of Plutarch we discussed on note 106 of page 82). After admonishing the wayward reader for thinking that perhaps heliocentrism predated Copernicus, after extolling ever more the depth, exactness, creativity and other qualities of Newton's genius, Voltaire tears at Dutens (see note 192) and all others who dare betray their own contemporaries by stressing what the ancients knew.

[195]See Section 11.7 and note 2 on page 270. Another example is the discovery that the earth is an oblate spheroid and the explanation for this shape. We read in [Whewell], vol. II, p. 111: "Newton's attempt to solve the problem of the figure of the earth, supposing it fluid, is the first example of such an investigation, and this rested upon principles which we have already explained, applied with the skill and sagacity which distinguished all that Newton did." Compare Section 10.11.

[195a]As attested, for example, in a letter from Farkas to János Bolyai of 1820; see [Dávid], p. 138.

and the ancient Egyptians with ideas now taken to be sudden creations of his own genius.[196]

The need to build the myth of an *ex nihilo* creation of modern science gave rise to much impassioned rhetoric. Still in the nineteenth century, the scientific and technological superiority of moderns over ancients was argued with a vehemence that may seem surprising to us today.

After the turnaround that occurred under the Enlightenment, ancient culture continued to be an essential influence on European science, but an unconscious one, like so many repressed memories. A number of important scientists were also archeologists and Greek scholars (Joseph Fourier, Thomas Young); a good part of the others, like the two Darwins and Freud, were cultured in the classics and read the ancients. But the influence of the study of Greek sources on scientific development often went unnoticed by the scientists themselves, as we have seen on several occasions and will see again in the next section. And the relinquishment of Latin as a language of science started to create a rift between scientific culture and humanistic studies that in due time would make most medieval and early modern writings largely inaccessible to all but classical scholars.

11.11 Recovery and Crisis of Scientific Methodology

The closer we get to the deepest aspects of Hellenistic science, which are the methodological ones, the longer they took to reappear.

One important methodological step in the evolution of modern mechanics was the introduction of *variational principles*, which correspond to ways to formulate a dynamical problem not as a search for solutions of ordinary differential equations with chosen initial conditions (Cauchy problems) but as a search for minimum points of appropriate functionals. Instead of deducing the future from the past (a process regarded as causal, if only unconsciously), the variational formulation in principle allows the whole motion to be obtained simultaneously.

This "radically new" way of setting problems was derived from its first attested example, transmitted by Heron of Alexandria and having to do with optics.[197] It was natural to draw ideas from Hellenistic science in trying to formulate the advances of modern dynamics within the lucid geometric framework that Archimedes used for the creation of mechanics.

Another important methodological loss that took a long time to be made good was that of the role of postulates and the criteria for their choice. Down to the late nineteenth century there was not a single counterpart to

[196]See pages 375–376.
[197]See pages 63–64 and the figure thereon.

the many Hellenistic hypothetico-deductive theories; we have seen how Descartes, Kepler and Newton operated in a very different mode. Indeed, even Euclid's *Elements*, having sojourned for so long in a milieu that had no notion of a scientific theory (and hence of the relations between theory and concrete objects), had been shoehorned into the same prescientific conceptual mold. Its five postulates were no longer conceived as the basis of a mathematical model for the use of ruler and compass, but as the Truth. Over the centuries there were countless attempts to derive the fifth postulate, an ugly duckling for not being obviously "true" like the rest, from the first four. This vain quest, launched in the imperial era,[198] only ended in the nineteenth century, with Lobachevskii and the so-called "non-Euclidean geometries". As is well-known, Lobachevskii discovered that if the fifth postulate is negated one can nonetheless obtain consistent theories. (Bolyai's work, independent of Lobachevskii's and covering much of the same ground, followed a few years later. Before both there had been observations in the same direction by Gauss, contained in private letters going back to 1799.)

But before this a non-Euclidean geometry had already been developed, albeit unconsciouly, by Johann Heinrich Lambert, in a *Theory of parallel lines*. This study, dating from 1766 (according to Johann Bernoulli grandson, who, two decades later, edited Lambert's unpublished works posthumously), considered whether a quadrilateral with three right angles might have an acute or obtuse fourth angle; studying these possibilities in order to exclude them, he actually deduced from them two non-Euclidean geometries, without realizing he had done so. Earlier attempts (notably by Saccheri) to demonstrate the fifth postulate by contradiction had already led to many "false" statements characteristic of non-Euclidean geometry, but Lambert's work is especially interesting, not least because of its influence on later developments: it palpably opened the way for the creation of consciously non-Euclidean geometries a few decades later. Lambert wrote:

> It seems remarkable to me that the second hypothesis [namely, that the fourth angle is obtuse] holds if instead of plane triangles we take *spherical* ones, for in this case not only the angles of a triangle add up to more than 180 degrees, but also the excess is proportional to the area of the triangle.[199]

[198]Proclus says that Ptolemy had a "demonstration" of the fifth postulate (*In primum Euclidis Elementorum librum commentarii*, 362:12 – 363:18 + 365:5 – 367:27, ed. Friedlein). Proclus himself gave another pseudo-demonstration (op. cit., 371:23 – 373:2). We know from an-Nairīzī's commentary on the *Elements* that Geminus had made a similar attempt earlier; see [Heath: HGM], vol. II, pp. 228–230.
[199]J. H. Lambert, *Die Theorie der Parallellinien*, in [Stäckel, Engel], p. 202.

This is a truly remarkable observation: it is (though he was unaware of that) equivalent to the statement that his results consequent upon the assumption of an obtuse fourth angle coincide with the spherical geometry of ancient times. Lambert had in fact redemonstrated some classical theorems.

The *Sphaerica* of Menelaus, from the first century A.D., is the oldest work in non-Euclidean geometry that has come down to us. It studies the surface of the sphere not as something immersed in three-dimensional space, but through its intrinsic properties (to use the technical term).[200] Each theorem, including those on spherical triangles, is proved following the scheme used in the *Elements* for plane geometry, but interpreting Euclid's straight lines (line segments) as arcs of great circles. Naturally, those theorems of plane geometry that depend on the existence of a parallel to a line through a given point are not present, being replaced by different theorems valid in the spherical case. In particular, the theorem cited by Lambert on the excess of the angle sum of a spherical triangle appears as proposition 11 in Book I of the *Sphaerica*.

To a Hellenistic mathematician, there would be no point even in posing the question whether one can construct a consistent geometry containing a theory of parallels different from the one in the *Elements*. This is because spherical geometry makes it obvious that the answer is yes.[201] It is reasonable to think, then, that Menelaus' *Sphaerica*, containing as it does an explicit alternative to the geometry of the *Elements*, may have pointed the way toward the recognition that such alternatives exist. And sure enough, the first modern edition of Menelaus saw the light of day in 1758, eight years before Lambert wrote his work.[202]

Hyperbolic geometry, too, is closely related to the spherical geometry of Antiquity, and it is not surprising that Lobachevskii devotes to the latter a good part of his *New principles of geometry*. Through his critical analysis of the Euclidean set of postulates, Lobachevskii made a landmark contribution to science as is generally agreed — but this precisely because he lived

[200] However, some theorems of intrinsic sphere geometry were already present in the work of Theodosius; see note 70 on page 55.

[201] For precision's sake we ought to point out that in spherical geometry (which has no "parallel" lines) what must be abandoned is not the fifth postulate in its original form, but an assumption that Euclid makes implicitly in the proof of proposition 16 of Book I (see note 43 on page 184). It may be objected that one must also modify the first postulate, since by interpreting lines as great circles in spherical geometry, uniqueness fails for lines passing through antipodal points. This fault is remedied by treating each pair of antipodal points as a single point (spherical geometry on the projective plane).

[202] The Greek text of Menelaus' work has perished. The 1758 edition is a Latin translation from Arabic and Hebrew manuscripts, edited by Edmund Halley. See [Menelaus/Krause] for a modern critical edition.

in a culture that had never before created an axiomatic system comparable to the Hellenistic ones.

How slowly the scientific method was recovered is hidden from most. For example, a student that takes a course in mathematical analysis and encounters several theorems bearing Cauchy's name never hears that the now-standard statements of these theorems do not correspond to actual theorems in that mathematician's works. For Cauchy studied numerical quantities, not the geometric magnitudes of Euclid, and for numbers there was no rigorous theory analogous to the Euclidean one. Thus the "Cauchy criterion" for the convergence of a sequence cannot be proved in the absence of a theory of real numbers (which Cauchy lacked). As we remarked on page 46, mathematical analysis became a scientific theory only after the Euclidean notion of proportionality was reinstated by Weierstrass and Dedekind, in 1872.

Up to that point, however, although mathematics had expanded widely, especially in the direction of analysis, the maximum rigor that it had managed to obtain in its base was that of Euclid, who remained unsurpassed after twenty-two hundred years. To settle accounts with this cumbersome character, it was necessary to finally face him on his own turf. This was first attempted by David Hilbert with his *Grundlagen der Geometrie*, which appeared in 1899, concluding a intense effort started by, among others, Pasch and Peano.[203]

Around the same time Peano formulated his axiomatization of arithmetic, systems of axioms were created for various other branches of mathematics, and several Hellenistic theories were revived, including propositional logic and semantics. In areas more distant from mathematics, too: besides dream theory, already discussed in Section 7.3, we may mention the new psychology of perception, founded on the essential need for active subject participation, or assent (συγκατάθεσις) in the terminology of Chrysippus.[204]

German-language authors were at the forefront of all these advances: they came from the same culture that almost single-handedly made classical philology into the rich structure it had become by the close of the nineteenth century. Among the new ideas that arose at the turn of the twentieth century from the fecund interaction of philology, history of science and epistemology was the rediscovery that scientific theories are underdeter-

[203] Hilbert tried to improve on Euclid's choice of postulates. Whether he succeeded is open to discussion; the greater rigor obtained came at the cost of denying the postulates any meaning that might relate them to experience. This created a problem of self-foundations in mathematics that has not proved solvable.

[204] See pages 175 and 213. The new psychology was born thanks to Franz Brentano (1838–1917), whose two main interests were psychology and the history of ancient philosophy.

mined: different theories can be used to explain the same phainomena. Henri Poincaré and Pierre Duhem (scientist, epistemologist and historian of ancient science) played a key role in this realization. Duhem denied, for example, the validity of so-called crucial experiments for "confirming" a theory, thus turning on its head their supposed absence from ancient science.[205] The work of recovery seemed finally to have come to a conclusion in the methodological dimension too.

In the following decades historians of science followed paths that diverged ever further from those of scientists and epistemologists. The study of ancient thought became the province of a few specialists, who talked but little with philosophers and scientists. It was then that the minimalist views that we have discussed took hold.[206]

For a while, in the first half of the twentieth century, a link — sometimes unconscious but nonetheless robust — between ancient and modern science was ensured through the high-school education of future scientists. Pride of place went to the study of Euclidean geometry, that unavoidable threshold into the scientific method that all schoolchildren had to cross, thus learning from an early age to recognize a theoretical model from concrete reality. Another component was the study of philosophy and the history of ideas, then part of the secondary school curriculum: for all their shortcomings, such courses at least made future scientists aware of an ancient, intricate and profound relationship between phenomena and theory. Their gradual and now virtually complete disappearance from secondary schools in the Western world has had, I believe, far-reaching consequences for scientific methodology. First mathematicians, starting with the French school, persuaded themselves that they could pursue an ideal of absolute rigor divorced from sensible reality; then the hypothetico-deductive method was abandoned in secondary school and in every area of knowledge other than pure mathematics practiced in an academic environment. Thus the ancient balance between rigor and applicability did not last after the Hellenistic tradition was abandoned. Physicists, on the other hand, abandoned the humility of ancient science: rather than creating theories capable of saving the phenomena in a circumscribed realm, they revived the age-old ambition of formulating all-encompassing "theories of everything" using a new version of the experimental method to research phenomena that save their own theories.

[205]See especially [Duhem: TP] and [Duhem: SPh]. Duhem regarded many ancient scientists (not excluding the imperial period) as conventionalists, and his position has been sternly criticized (on valid grounds in many specific cases; see in particular [Lloyd], chapter 11). But we should not forget that conventionalism arose in modern science thanks to intellectuals who, like Duhem, found it in ancient sources.

[206]See Section 9.1.

The departure of physics from the ancient scientific method started at the turn of the twentieth century, when an enormous increase in the range of observed phenomena demanded the creation of new scientific theories, toward which no light could come from reading Archimedes nor yet from browsing through all of Plutarch. Small-scale physics proved impossible to describe via classical mechanics: its phenomena do not conform either to the theory of particles or to that of waves. It was obviously necessary to build another theory, but the way in which that theory arose and developed shows how serious was the loss of the sure guide that had sustained us until then. Instead of proposing a third scientific theory, scientists such as de Broglie and Bohr postulated "particle-wave duality" and the "complementarity principle". Faced with the inapplicability of two mutually incompatible theories, a culture that still confused theoretical entities with real objects found it normal to attribute to nature the inconsistency of science itself.

Not surprisingly, an intellectual edifice built on the laying aside of the non-contradiction principle must claim citizenship in a country quite distant from mathematics. Thus de Broglie writes:

> In explaining scientific theories the "axiomatic" method is the most pleasing to reason, but in practice the least fruitful, except perhaps in the field of pure mathematics.[207]

We must conclude that for the creator of particle-wave duality, pleasing reason is a dangerous obstacle to the advance of science. De Broglie's ideas are deeply rooted in old traditions, also prized by Niels Bohr. This author, in his book *The unity of human knowledge*, illustrates his complementarity principle with the following comments, among others:

> Indeed, in renouncing logical analysis to an increasing degree and in turn allowing the play on all string of emotion, poetry, painting and music contain possibilities of bridging between extreme modes as those characterized as pragmatic and mystic. Conversely, already ancient Indian thinkers understood the logical difficulties in giving exhaustive expression for such wholeness. . . .
>
> [I]t is equally clear that compassion can bring everyone in conflict with any concisely formulated idea of justice. We are here confronted with complementary relationships inherent in the human position, and unforgettably expressed in old Chinese philosophy, reminding

[207][de Broglie], p. 170.

> us that in the great drama of existence we are ourselves both actors and spectators.[208]

And perhaps, too, when the Pythagorean school ran into an impasse in classifying the diagonal of the square, someone resorted to Eastern teachings and the inherently contradictory nature of the world, and proposed a solution: to declare the diagonal a profoundly ambiguous entity featuring a *duality* or *complementarity* between even and odd.

[208][Bohr], at 72% and 81%. His reference to old Eastern philosophies is an early example of a trend that became increasingly popular in the late twentieth century and led many authors to endeavors such as (to quote the subtitle of Fritjof Capra's *The Tao of physics*) "an exploration of the parallels between modern physics and Eastern mysticism"—which is to say, the age-old tendencies that ironically became known as *New Age*.

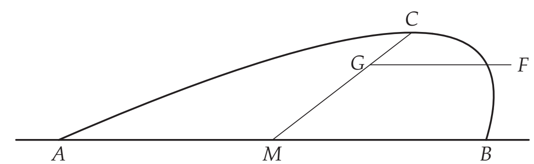
Appendix

We prove here the Archimedean results quoted in Section 2.7 as Lemmas 1 and 2. In his demonstrations, Archimedes used freely theorems that were well known in his day (just as we do). Since none of the works on conics written before Apollonius has survived, we do not know exactly how Archimedes would have written the proofs of these auxiliary results. Therefore we freely reconstruct demonstrations of the propositions taken for granted by Archimedes, and then, when dealing with the lemmas he actually demonstrates, we follow his proofs closely.

Preliminary Results Assumed by Archimedes

We denote by C the vertex of the segment of parabola of base AB, and by M the midpoint of AB. Our starting point is the following proposition:

Proposition. Given a point F in the plane, draw the parallel to AB and let G denote its intersection with the half-line CM originating at C, if this intersection exists. Then F lies on the parabola if and only if the intersection G exists and is such that $CM \cdot GF^2 = CG \cdot MB^2$.



Archimedes makes direct use of only one half of this equivalence (the "only if" part), but we give the Proposition in the form above because this allows the easy deduction, as corollaries, of all the results that Archimedes takes as known. We will not prove the Proposition here,[209] but only how the necessary corollaries follow from it.

Corollary 1. The line going through C and parallel to AB does not intersect the parabola except at C.

If this line intersects the parabola at a point F, we have $C = G$, and the Proposition implies $G = F$, that is, $F = C$.

Note that this corollary justifies our talking about *the* vertex.

Corollary 2. The line CM does not intersect the parabola except at C.

If CM intersects the parabola at a point F, we have $F = G$, and the Proposition implies $G = C$, that is, $F = C$.

Corollary 3. Every parallel to CM intersects the parabola in exactly one point.

If a parallel to CM intersects the parabola at F and F', consider the corresponding points G and G' along CM. Because of parallelism, $GFF'G'$ is a parallelogram, so $GF = G'F'$, so by the Proposition $CG = CG'$. It follows that $G' = G$ and $F' = F$.

Corollary 4. The line CM is parallel to the diameter (symmetry axis) of the parabola.

If we apply the Proposition to the particular case where A and B are symmetrical with respect to the diameter, symmetry implies that CM coincides with the diameter. Corollary 3 then implies that any line parallel to the diameter intersects the parabola in a single point. Thus it is enough to show that the direction of the diameter is the only one that has this property. To do this we again use again the Proposition in the case where CM coincides with the diameter. Given a point L of AB distinct from M, the half-line CL will intersect the parabola not only in C but also in the point F whose orthogonal projection G onto the diameter satisfies the relation $CG/CM = MB^2/ML^2$.

Corollary 5. A line CN joining the vertex C of a segment of parabola of base AB with a point N of the base, is parallel to the diameter of the parabola if and only if N is the midpoint of AB.

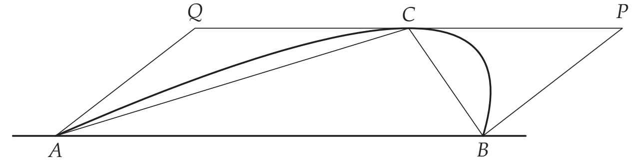
This is an immediate consequence of Corollary 4.

Lemmas Proved by Archimedes

Lemma 3. If C is the vertex of the segment of parabola of base AB, the area of the triangle ABC is more than half the area of the segment of parabola.

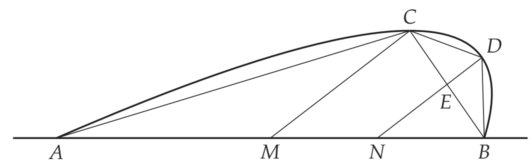
[209] The reader may enjoy reconstructing a "purely geometric" proof that reflects the following outline suggested by analytic geometry: Assume without loss of generality that the base AB is the x-axis. First deal with parabolas whose axis of symmetry is vertical; then reduce the general case to the particular case by applying a shearing transformation $y \mapsto y,\ \ x \mapsto x + ky$.

Archimedes demonstrates this by considering the parallelogram $ABPQ$ comprised between the base AB, the line passing through C and parallel to the base, and the parallels to the diameter passing through A and B, respectively:



This parallelogram encloses the segment of parabola, by the definition of the vertex and because, by Corollary 3, sides AQ and BP intersect the segment of parabola only once. The lemma follows easily, by observing that the area of the segment of parabola is less than that of the parallelogram $ABPQ$, which is twice that of the triangle ABC.

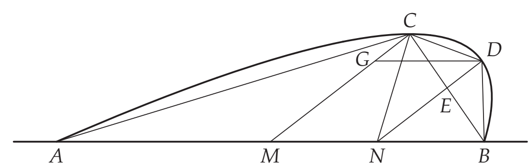
Lemma 4. If C is the vertex of the segment of parabola of base AB and D is the vertex of the segment of parabola of base CB, the area of triangle CBD is one-eighth that of triangle ABC.



Lemma 2 is proved by Archimedes using the Proposition, Corollary 5 and typical arguments of "elementary geometry", that is, the methods laid out in Euclid's *Elements*.

Consider parallels to the diameter of the parabola passing though C and D, respectively. By Corollary 5, the first of these lines intersects AB at its midpoint M, and the second intersects CB at its midpoint E. Let N be the intersection of AB with the line ED. Because MCB and NEB are similar triangles, N is the midpoint of MB.

Let G be the intersection of CM with the parallel to AB going through D. By the Proposition we have $CM/CG = MB^2/GD^2$. But GD is equal to MN (they are opposite sides of a parallelogram). We thus have $CM/CG = MB^2/MN^2 = 4$, and $DN = GM = \frac{3}{4}CM$. At the same time, since MCB and NEB are similar triangles, we have $NE = \frac{1}{2}CM$; thus $NE = 2DE$.



If one of two triangles of the same altitude has double the other's base, its area is also double. Applying this trivial observation to the equality $NE = 2DE$, we get

$$\text{area}\,CNE = 2\,\text{area}\,CED;$$
$$\text{area}\,NEB = 2\,\text{area}\,BED;$$

and adding the two relations,

$$\text{area}\,CNB = 2\,\text{area}\,CBD;$$
$$\text{area}\,CDB = \tfrac{1}{2}\,\text{area}\,CNB = \tfrac{1}{8}\,\text{area}\,ABC:$$

This concludes the proof of the lemma.

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