constant and variable capital

1 Definition

In *Das Kapital* Marx defined Constant Capital as that part of capital advanced in the means of production; he defined Variable Capital as the part of capital advanced in wages (Marx, 1867, Vol. I, ch. 6). These definitions come from his concept of Value: he defined the value of commodities as the amount of labour directly and indirectly necessary to produce commodities (Vol. I, ch. 1). In other words, the value of commodities is the sum of C and N, where C is the value of the means of production necessary to produce them and N is the amount of labour used that is directly necessary to produce them. The value of the capital advanced in the means of production is equal to C.

However, the value of the capital advanced in wages is obviously not equal to N, because it is the value of the commodities which labourers can buy with their wages, and has no direct relationship with the amount of labour which they actually expend. Therefore, while the value of the part of capital that is advanced in the means of production is transferred to the value of the products without quantitative change, the value of the capital advanced in wages undergoes quantitative change in the process of transfer to the value of the products. This is the reason why Marx proposed the definitions of constant capital C and variable capital V.

The definition of constant capital and variable capital must not be confused with the definition of fixed capital and liquid capital. Fixed capital is a part of constant capital which is totally used in production process but transfers its value to products only partially. Liquid capital is a part of constant capital which is totally used up and transfers its whole value within one production process. So constant capital is composed of both fixed capital and liquid capital, and on the other hand liquid capital belongs partly to constant capital and partly to variable capital.

Marx introduced the concept ‘value-composition of capital’, μ, which is defined as the ratio of constant capital C to variable capital V:

$$\mu \equiv \frac{C}{V}. \tag{1.1}$$

Marx knew well that the value composition of capital reflects not only material characteristics of the process of production but also the social relationship between capitalists and labourers. In fact definition (1.1) can be rewritten as

$$\mu = \frac{C}{N} \cdot \frac{N}{V} \tag{1.2}$$

C/N reflects the character of the process of production and *N/V* reflects the class relationship between capitalists and labourers. *C/N* is the ratio of the amount of labour necessary to produce the means of production to the amount of labour directly bestowed, which is completely determined by the material condition in the process of production, while *N/V* is the ratio of the amount of labour which labourers actually expend to the amount of labour that is necessary in order to produce commodities which labourers can purchase with their wages. If labourers are forced to work longer with less wages, this ratio must rise.

Marx proposed to call the value-composition of capital, insofar as it is determined by the material condition of the process of production, 'the organic composition of capital'. More explicitly, 'The value-composition of capital, inasmuch as it is determined by, and reflects, its technical composition, is called the *organic* composition of capital' (*Capital*, Vol. III, ch. 8). However, as shown above, the value composition of capital is not determined by the material condition of the process of production alone. So it is better to introduce the ratio *C/N* in the place of the organic composition of capital, which is determined only by the material condition in the process of production. In order to avoid confusion, I call this ratio the 'organic composition of production'. This is the ratio of dead labour to living labour, which Marx himself frequently used in *Das Kapital*.

2 Variable Capital and Source of Profit

In contrast to Smith, Ricardo and others, Marx attached great importance to analysis to find the source of profit. He found that source in surplus labour, which is the excess of labour expended by labourers over the value of commodities which labourers can obtain with their wages (*Capital*, vol. I, ch. 5). Using the notation introduced above, $N > V$ is the necessary condition for profit to exist. In order to illuminate this fact, he called capital advanced in wages *Variable Capital*. So the validity of this name depends on his analysis of the source of profit. How is it justified?

For simplicity we set up the simplest model which can reflect the fundamental characteristics of a capitalistic economy; these characteristics are the prevalence of commodity production, and the existence of class relationships between labourers and capitalists. There are only two kinds of commodities: the means of production (commodity 1) and consumption goods (commodity 2). In order to produce one unit of the ith commodity an amount of a_i unit of means of production and an amount of labour τ_i are necessary as input. Labourers are forced to work for T hours per day and earn the money wage rate w.

In order for profit to exist in both industries the following inequalities are necessary

$$p_1 > a_1 p_1 + \tau_1 w \tag{2.1}$$

$$p_2 > a_2 p_1 + \tau_2 w \tag{2.2}$$

where p_1 and p_2 denote the price of the means of production and consumption goods respectively. As labourers work for T hours a day at money wage w per hour, they can purchase an amount B of consumption goods.

$$B = \frac{wT}{p_2}, \qquad B/T = R \tag{2.3}$$

where R is the real wage rate.

In the first volume of *Das Kapital*, Marx assumed that all commodities are exchanged at prices exactly proportionate to their unit value (equivalent exchange). Unit values of commodities are determined by the following equations

$$t_1 = a_1 t_1 + \tau_1 \tag{2.4}$$

$$t_2 = a_2 t_1 + \tau_2 \tag{2.5}$$

which assure unique and positive values, provided $a_1 < 1$ (Dmitriev, 1898; May, 1949–50; Okishio, 1955a, 1955b).

Under the assumption of equivalent exchange, we have

$$p_i = \lambda t_i \tag{2.6}$$

where λ is a constant which converts the dimension from hours to, say, dollars. Substituting (2.3) and (2.6) into (2.1) and (2.2) we get

$$t_1 > a_1 t_1 + \tau_1 \frac{B}{T} t_2 \tag{2.7}$$

$$t_2 > a_2 t_1 + \tau_2 \frac{B}{T} t_2 \tag{2.8}$$

By equations (2.4) and (2.5) and the above inequalities, we have

$$\tau_1 \left(1 - \frac{B}{T} t_2 \right) > 0 \tag{2.9}$$

$$\tau_2 \left(1 - \frac{B}{T} t_2 \right) > 0 \tag{2.10}$$

Consequently we arrive at the conclusion

$$T > B t_2. \tag{2.11}$$

This inequality implies the existence of surplus value, because surplus value is the excess of working hours T over the amount of labour necessary to produce commodities which labourers can receive with wages B. If the number of workers employed is n, then total expended labour is

nT and variable capital measured in terms of value is Bt_2n. So the inequality (2.11) can be rewritten as

$$N > V \tag{2.12}$$

This is the reason Marx called capital advanced in wages variable capital.

As shown above, Marx proved the theorem of the source of profit under the assumption of equivalent exchange. Though this is a clear-cut way to show the results, it has induced various critiques. Many critics have said that Marx's theorem would be right if all exchanges were equivalent exchange, but that in reality exchanges are seldom equivalent so his theorem cannot be valid. In order to refute such a criticism we must prove the theorem without the assumption of equivalent exchange (see Okishio 1955a, 1955b, 1963, 1972, 1978; Morishima, 1973). Mathematically, our task is to find necessary and sufficient conditions for inequalities (2.1), (2.2) and (2.3) to have non-negative solutions for p_1, p_2. From (2.1) we know easily that the condition

$$1 - a_1 > 0 \tag{2.13}$$

is necessary for p_1 to be positive. This condition ensures that the society will obtain net output.

Next, substituting (2.3) into (2.1), and from (2.13) we have

$$\frac{p_1}{p_2} > \frac{\tau_1 B}{T(1 - a_1)}. \tag{2.14}$$

On the other hand, from (2.2) and (2.3) we get

$$\frac{p_1}{p_2} > \frac{T - \tau_2 B}{T a_2}. \tag{2.15}$$

We can easily get from (2.14) and (2.15)

$$\frac{a_2 \tau_1 B}{(1 - a_1)} < T - \tau_2 B. \tag{2.16}$$

Inequality (2.16) is rewritten as

$$T > B\left(\frac{a_2 \tau_1}{1 - a_1} + \tau_2\right). \tag{2.17}$$

By (2.17), (2.4) and (2.5) the above becomes

$$T > B t_2. \tag{2.18}$$

Thus we can arrive at Marx's result.

For later convenience we show another expression for the existence of surplus value.

Dividing (2.1) and (2.2) by w, we get

$$\frac{p_1}{w} > a_1 \frac{p_1}{w} + \tau_1 \tag{2.19}$$

$$\frac{p_2}{w} > a_2 \frac{p_1}{w} + \tau_2 \tag{2.20}$$

By comparing (2.19) and (2.20), and (2.4) and (2.5), we get

$$\frac{p_i}{w} > t_i, \qquad (i = 1, 2) \tag{2.21}$$

Equation (2.21) implies that if positive profit exists, then the price–wage ratio (the amount of commanded labour) is greater than the amount of value (necessary labour). In the famous controversy with Ricardo, Malthus pointed out this difference between labour commanded and labour embodied. Though he wrongly thought that this difference injured the validity of the labour theory of value, he had come near to the Marxian theory of the source of profit (see Malthus, 1820, pp. 61–3, 120).

Condition (2.21) is rewritten as

$$1/t_i > w/p_i$$

This condition shows that if positive profit exists, then the productivity of labour $(1/t_i)$ must be greater than the rate of real wages (w/p_i).

3 Organic Composition and Production Price

The concept of organic composition of capital plays an important role in Marx's analysis of prices.

The price of production (Ricardo's 'natural price') that gives every industry the equal rate of profit is determined by the following equations:

$$p_1 = (1+r)(a_1 p_1 + \tau_1 w) \tag{3.1}$$

$$p_2 = (1+r)(a_2 p_1 + \tau_2 w) \tag{3.2}$$

$$w = R p_2 \tag{3.3}$$

where r is the general (equal) rate of profit.

The first problem is to examine the relationship between

$$\frac{t_1}{t_2} \sim \frac{p_1}{p_2}.$$

If they are equal then we have equivalent exchange, if not we have non-equivalent exchange from the point of view of the labour theory of value. The values of the commodities are determined by (2.4) and (2.5). The ratio of the value of production-goods to consumption-goods t_1/t_2 is given as

$$\frac{t_1}{t_2} = \frac{\tau_1\left(\dfrac{a_1 t_1}{\tau_1}+1\right)}{\tau_2\left(\dfrac{a_2 t_1}{\tau_2}+1\right)}. \tag{3.4}$$

The relative price of production-goods to consumption-goods determined by (3.1) and (3.2) is given as

$$\frac{p_1}{p_2} = \frac{\tau_1\left(\dfrac{a_1 p_1}{\tau_1}+w\right)}{\tau_2\left(\dfrac{a_2 p_1}{\tau_2}+w\right)}. \tag{3.5}$$

Comparing (3.4) with (3.5), we obtain

$$\frac{t_1}{t_2} - \frac{p_1}{p_2} = \frac{\tau_1}{\tau_2}\left[\frac{\dfrac{a_1 t_1}{\tau_1}+1}{\dfrac{a_2 t_1}{\tau_2}+1} - \frac{\dfrac{a_1 p_1}{\tau_1}+w}{\dfrac{a_2 p_1}{\tau_2}+w}\right]. \tag{3.6}$$

The expression in brackets on the RHS of (3.6) is given by

$$[\ \ ] = \left(t_1 w - p_1\right)\left(\frac{a_1}{\tau_1} - \frac{a_2}{\tau_2}\right)A, \qquad A > 0. \tag{3.7}$$

If profit is positive, from (2.21) $t_1 w - p_1$ is negative. So we can conclude

$$\frac{t_1}{t_2} \gtreqless \frac{p_1}{p_2} \Leftrightarrow \frac{a_1}{\tau_1} \lesseqgtr \frac{a_2}{\tau_2}. \tag{3.8}$$

The RHS of the above means the comparison of the organic composition of production and also the organic composition of capital, because as shown above the organic composition of production is $a_i t_1/\tau_i$ and the organic composition of capital is $a_i t_1/\tau_i\, R t_2$.

The second problem is to examine the influence of the change in real wage rate on the relative prices determined by (3.1), (3.2) and (3.3):

$$\mathrm{d}\left(\frac{p_1}{p_2}\right)\Big/\mathrm{d}R.$$

Denoting the relative price of production-goods to consumption-goods as p, from (3.1), (3.2) and (3.3) we obtain

$$f\left(p\right) \equiv a_2 p^2 + \left(\tau_2 R - a_1\right) p - \tau_1 R = 0. \tag{3.9}$$

Differentiating (3.9) with respect to R, we have

$$\frac{\mathrm{d}p}{\mathrm{d}R} = \frac{\tau_1 - \tau_{2p}}{2a_2 p + \tau_2 R - a_1}. \tag{3.10}$$

The denominator above is positive, because from (3.9)

$$\text{denominator} \times p = a_2 p^2 + \tau_1 R > 0.$$

We shall show that the sign of the numerator depends on the comparison between the organic composition of capital in both sectors.

The function $f(p)$ in (3.9) is drawn in Figure 1. The meaningful solution of the equation (3.9) is given at p^*. Substituting τ_1/τ_2 into $f(p)$, we get

$$f\left(\frac{\tau_1}{\tau_2}\right) = \tau_1\left(a_2\tau_1 - a_2\tau_2\right).$$

Therefore if $a_2\tau_1 - a_1\tau_2 > 0$ then $f\left(\tau_1/\tau_2\right) > 0$, so considering the graph of $f(p)$ we know that $\tau_1/\tau_2 > p^*$. In the same way we can conclude that if $a_2\tau_1 - a_1\tau_2 \gtreqless 0$, then $\tau_1/\tau_2 \gtreqless p$. Consequently, from (3.10) we can conclude

$$d\left(\frac{p_1}{p_2}\right)\Big/\mathrm{d}R \gtreqless 0 \Leftrightarrow \frac{a_1}{\tau_1} \lesseqgtr \frac{a_2}{\tau_2}.$$

This proposition is first established in Ricardo's *Principles* (1821, p. 43).

4 Organic Composition and the Rate of Profit

The concept of organic composition of capital plays an important role in Marx's analysis of the movement of the rate of profit.

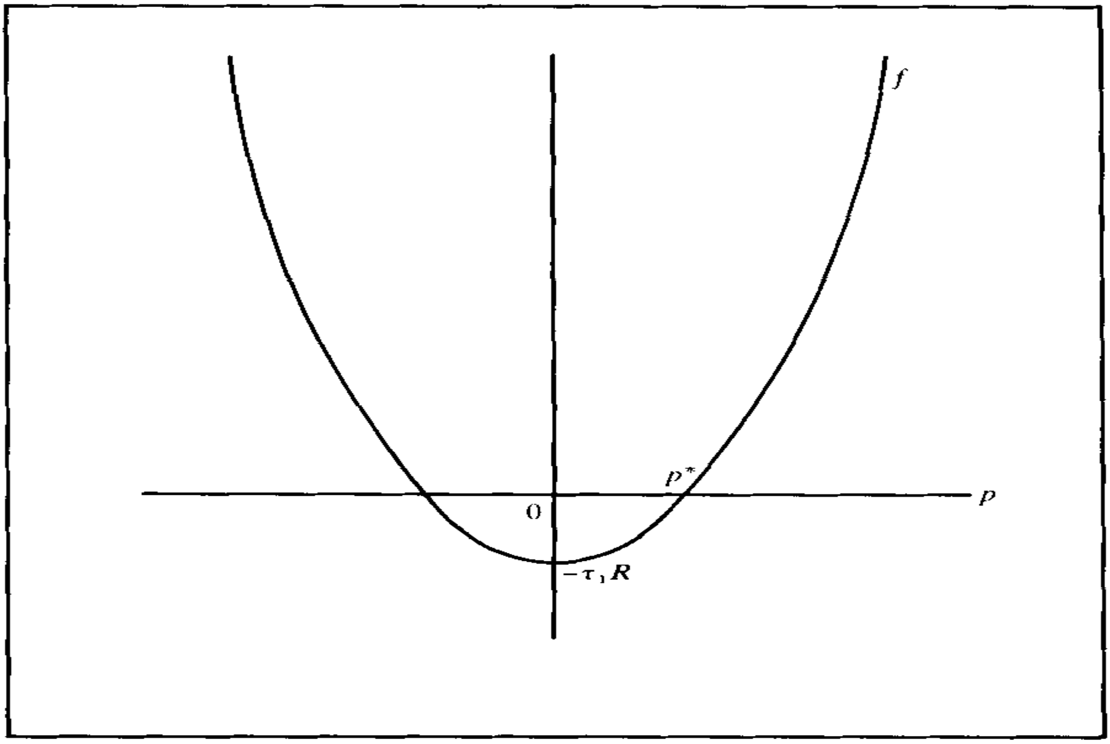

Figure 1

Marx defined the rate of profit as

$$r = \frac{S}{C+V}. \tag{4.1}$$

By (1.1), equation (4.1) is rewritten as

$$r = \frac{e}{\mu + 1}, \qquad e = S/V \tag{4.2}$$

where e is the rate of exploitation.

He asserted that if the organic composition of capital μ increases sufficiently then the rate of profit r must inevitably decrease. This is the famous ‘law of the tendency for the rate of profit to fall’ (*Capital, vol. III, ch. 13*).

Many people have criticized this theorem. They have said that if the rate of exploitation e increases sufficiently, r may increase in spite of the increase of μ. So r does not necessarily decrease, even if μ increases sufficiently (Robinson, 1942; Sweezy, 1942). Such a critique overlooks the logic of Marx’s argument.

Marx stated:

> Since the mass of the employed living labour is continually on the decline as compared to the mass of materialized labour set in motion by it, i.e., to the productively consumed

> means of production, it follows that the portion of living labour, unpaid and congealed in surplus-value, must also be continually on the decrease compared to the amount of value represented by the invested total capital. Since the ratio of the mass of surplus-value to the value of the invested total capital forms the rate of profit, this rate must constantly fall (*Capital*, vol. III, ch. 13, p. 213).

Therefore Marx's true intention is to insist that if the organic composition of production *v* = C/N (the ratio of the mass of materialized labour to the mass of living labour) increases sufficiently, the rate of profit must fall.

This can be proved as follows (Okishio, 1972). From (4.1) and (4.2), and

$$v = C/N \tag{4.3}$$

we have

$$\begin{aligned} r_{t+1} - r_t &= \frac{S_{t+1}}{C_{t+1} + V_{t+1}} - r_t \\ &= \frac{e_{t+1}}{v_{t+1}\left(1 + e_{t+1}\right) + 1} - r_t \\ &= \frac{1}{v_{t+1}\left(1/e_{t+1} + 1\right) + 1/e_{t+1}} - r_t \end{aligned} \tag{4.4}$$

where suffixes *t, t* + 1 denote periods.

The RHS of (4.4) is an increasing function of *e*. If we take the limiting value as *e* tends to infinity, we have

$$r_{t+1} - r_t < \frac{1}{v_{t+1}} - r_t.$$

Therefore we conclude, if $v_{t+1} > 1/r_t$, then $r_{t+1} - r_t < 0$.

The above reasoning can be restated. The reciprocal of the organic composition of production sets an upper limit to the rate of profit, because

$$r = \frac{S}{C+V} < \frac{S+V}{C} = \frac{N}{C} \tag{4.5}$$

If this upper limit decreases sufficiently, the rate of profit must eventually decrease, as shown in Figure 2.

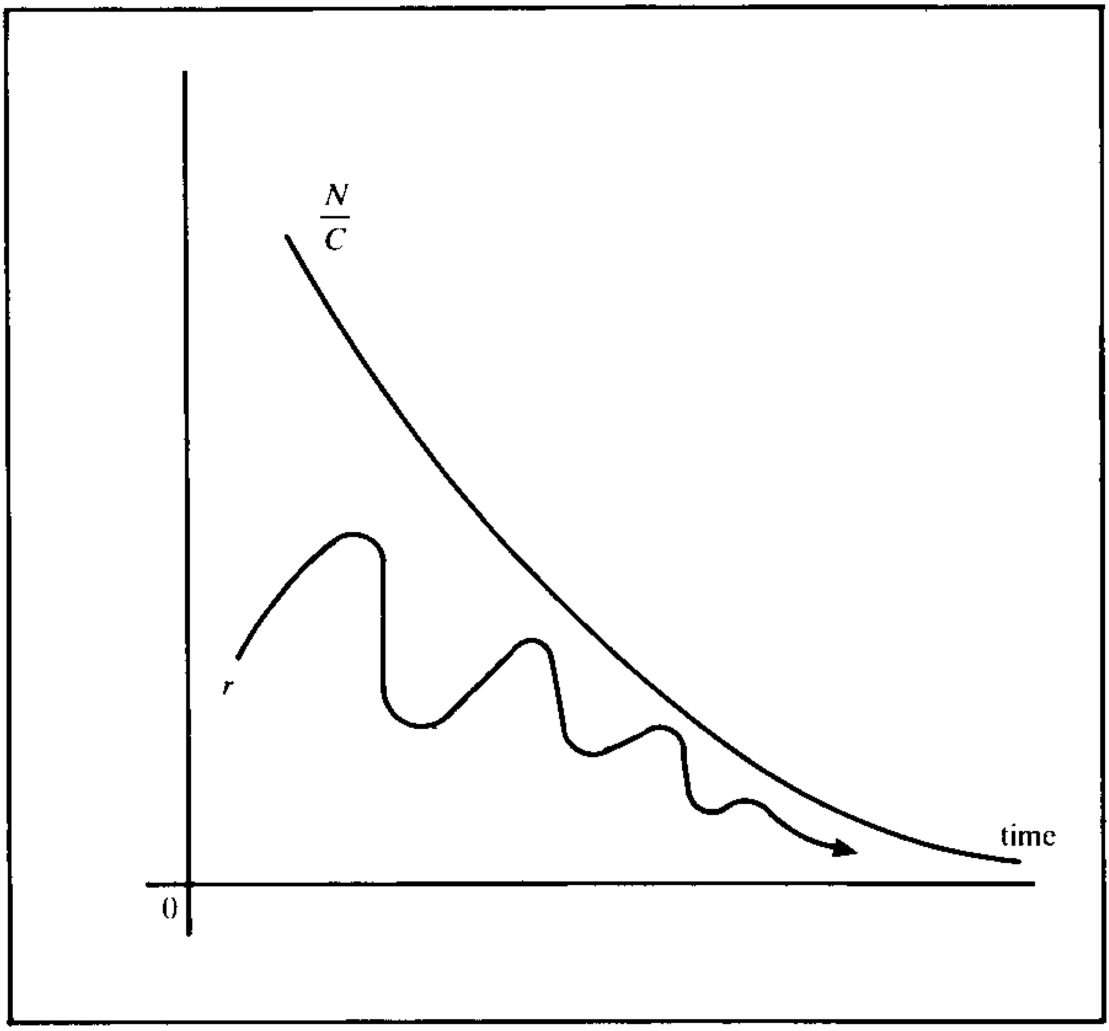

Figure 2

In response to criticisms of this view we must say that as far as we accept Marx's assumption that the inverse of the organic composition (*N/C*) tends toward zero, Marx's conclusion inevitably follows.

So far we have defined the rate of profit as (4.1) and *C, V, S* are all measured in terms of labour value. However, the general rate of profit *r* must be determined by (3.1), (3.2) and (3.3). Can we derive the same conclusions for such a redefined *r*?

Eliminating p_1, p_2, w from (3.1), (3.2) and (3.3) we have

$$f(r,R) \equiv (1+r)^2 R(a_1\tau_2 - a_2\tau_1) - (1+r)(a_1 + \tau_2 R) + 1 = 0 \tag{4.6}$$

Differentiating $f(r, R)$ we have

$$f_r \mathrm{d}r + f_R \mathrm{d}R = 0 \tag{4.7}$$

where

$$f_r = 2(1+r)R(a_1\tau_2 - a_2\tau_1) - (a_1 + \tau_2 R)$$
$$f_R = (1+r)^2 (a_1\tau_2 - a_2\tau_1) - (1+r)\tau_2$$

Considering (4.6)

$$(1+r) f_r = (a_1 + \tau_2 R)(1+r) - 2 \tag{4.8}$$

From (3.1), (3.2), (3.3), we know

$$1-(1+r)a_1 > 0 \qquad 1-(1+r)\tau_2 R > 0 \tag{4.9}$$

From (4.8) $f_r < 0$. f_R is rewritten as

$$f_R = (1+r)\{[(1+r)a_1 - 1]\tau_2 - (1+r)a_2\tau_1\}$$

So by (4.9) $f_R < 0$, from which $\mathrm{d}r/\mathrm{d}R < 0$. As R goes to zero r tends to its upper limit, which is obtained from (4.6)

$$r_{\max} = \frac{1-a_1}{a_1}. \tag{4.10}$$

Since the value of the means of production is determined by (2.4), we have

$$\frac{1-a_1}{a_1} = \frac{(1-a_1)t_1}{a_1 t_1} = \frac{\tau_1}{a_1 t_1} = \frac{N_1}{C_1} \tag{4.11}$$

Thus the upper limit of the general rate of profit is given by the reciprocal of the organic composition of production in the means of production sector. Therefore if the organic composition in that sector rises sufficiently, the general rate of profit must fall.

5 Organic Composition and Unemployment

The concept of organic composition of capital plays an important role in Marx's analysis of the movement of employment (*Capital*, vol. I, ch. 23).

Marx assumed a rise in labour productivity to accompany the rise in the organic composition of production *C/N*. If *C/N* rises then from the definition of organic composition the amount of employment must decrease relative to constant capital.

However, how does the increase in the organic composition influence the absolute level of employment?

Many people thought that even if *C/N* rises sufficiently, still if constant capital *C* also increases then the absolute level of employment can also increase, though less than proportionately to constant capital (Oppenheimer, 1903). But by reasoning similar to that used for ‘the tendency of the rate of profit to fall’, we can prove that if organic composition rises sufficiently, then the absolute level of employment must actually decrease.

The organic composition of production in the *t*th period v_t is defined as

$$v_t = \frac{C_t}{N_t}. \tag{5.1}$$

The accumulation of constant capital $\Delta C = C_{t+1} - C_t$ is financed from surplus value *S*.

$$C_{t+1} - C_t < S_t. \tag{5.2}$$

The surplus value *S* is a part of the amount of living labour which labourers expend

$$S_t < N_t. \tag{5.3}$$

By (5.1), we obtain,

$$\begin{aligned} N_{t+1} - N_t &= \frac{1}{v_{t+1}} C_{t+1} - \frac{1}{v_t} C_t \\ &= \frac{1}{v_{t+1}} \left(C_{t+1} - C_t \right) + C_t \left(\frac{1}{v_{t+1}} - \frac{1}{v_t} \right). \end{aligned}$$

From (5.2) and (5.3) we get

$$\begin{aligned} N_{t+1} - N_t < \frac{1}{v_{t+1}} S_t + C_t \left(\frac{1}{v_{t+1}} - \frac{1}{v_t} \right) < \frac{N_t}{v_{t+1}} + C_t \left(\frac{1}{v_{t+1}} - \frac{1}{v_t} \right) \\ = \frac{C_t}{v_{t+1} v_t} \left(1 + v_t - v_{t+1} \right). \end{aligned}$$

we can say, if $\left(1 + v_t - v_{t+1}\right) < 0$ then $N_{t+1} - N_t < 0$. Therefore, if the organic composition of production in the t + 1th period, v_{t+1}, increases sufficiently so as to exceed $1+v_t$, then the amount of employed labourer, N_{t+1} must inevitably become less than N_t, however high the rate of accumulation of capital may be (Okishio, 1972). The rate of accumulation of capital $\Delta C/C$ itself is bounded by the reciprocal of the organic composition. From (5.2) and (5.3)

$$\frac{\Delta C}{C} < \frac{N}{C} = \frac{1}{v}$$

so that, because it is reasonable to assume that the growth rate of labour supply is non-negative, we can say that if the organic composition rises sufficiently the rate of unemployment inevitably rises. Though Marx did not state this explicitly, we think that this is what he wanted to say.

In analysing Marx's theorem on the movement of the rate of profit and employment, we have accepted his central assumption that the organic composition of production rises sufficiently over time. However, there arises the problem: under what conditions do capitalists choose techniques that have sufficiently high organic compositions of production?

Marx seemed to think that the rise in labour productivity and the rise in the organic composition are two aspects of the same thing. But these two do not always go together. Marx himself knew that if labour productivity in the means of production sector rises very high then even if technical composition rises, still the value composition may remain constant or decrease.

As to the capitalists' introduction of new techniques we have the following propositions:

(1) if the real wage rate remains constant and capitalists introduce new techniques which raise the rate remains of profit (calculated at the current prevailing prices and wage) then the new general rate of profit does not decrease, whatever the organic composition may be.

(2) if the real wage rate rises and capitalists adapt to this situation with the introduction of new techniques, then the new general rate of profit does is higher than the one which would be expected if such a new technique were not introduced.

For the proofs of these propositions, see CHOICE OF TECHNIQUE AND THE RATE OF PROFIT.

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See also

Marxian value analysis; organic composition of capital; surplus value.

Bibliography

Dmitriev, V.K. 1898. The theory of value of David Ricardo. In V.K. Dmitriev, *Economic Essays on Value, Competition and Utility*, ed. D.M. Nuti, Cambridge: Cambridge University Press, 1974.

Malthus, R. 1820. Principles of Political Economy considered with a View to their Practical Application. 1st edn, London.

Marx, K. 1867–94. *Capital*. Translated from the third German edition by Samuel Moore and Edward Aveling, ed. Frederick Engels. New York: International Publishers, 1967.

May, K. 1949. The structure of classical theories. *Review of Economic Studies* 17(1), 60–69.

Morishima, M. 1973. *Marx's Economics: A Dual Theory of Value and Growth*. Cambridge: Cambridge University Press.

Okishio, N. 1955a. Kachi to Kakaku (Value and production price). *Keizaigaku Kenkyu Nempo* (The Annals of Economic Studies), Kobe University, No. 19.

Okishio, N. 1955b. Monopoly and the rates of profit. *Kobe University Economic Review* 1, 71–88.

Okishio, N. 1963. A mathematical note on Marxian theorems. *Weltwirtschaftliches Archiv* 91, pt. 2, 287–98.

Okishio, N. 1972. A formal proof of Marx's two theorems. *Kobe University Economic Review* 18, 1–6.

Okishio, N, et al. 1978. Three topics on Marxian fundamental theorems. *Kobe University Economic Review* 24, 1–18.

Oppenheimer, T. 1903. *Das Grundgesetz der Marxschen Gesellschaftslehre*. Book II, ch. 25. Berlin: Reimer.

Ricardo, D. 1821. *On the Principles of Political Economy and Taxation*. Vol. 1 in *Works and Correspondence of David Ricardo*, ed. P. Sraffa, Cambridge: Cambridge University Press, 1951–73.

Robinson, J. 1942. *An Essay on Marxian Economics*. London: Macmillan.

Sweezy, P.M. 1942. The Theory of Capitalist Development: Principles of Marxian Political Economy. New York: Oxford University Press.