choice of technique and the rate of profit

Capitalistic Criterion

In a capitalistic economy the main production decisions are made by private capitalists. The choice of technique is one of the decisions in their hands, and the criterion for that choice is to maximize the expected profit rate. In order to calculate that rate they must have expectations of the prices of various commodities, and of the wage rate.

Assuming a linear technology, in the *i*th sector capitalists have T_i alternative techniques:

$$a_{i1}(k_i), a_{i2}(k_i), \ldots, a_{in}(k_i), \tau_i(k_i) \qquad k_i = 1, 2, \ldots, T_i$$

where $a_{ij}(K_i)$ is the amount of the *j*th commodity used as input to produce one unit of the *i*th commodity by the k_ith technique and $\tau_i(K_i)$ is the amount of labour necessary to produce one unit of the *i*th commodity by the k_ith technique.

Capitalists have expected prices and the wage rate:

$$p_1^e, p_2^e, \ldots, p_n^e, w^e,$$

where p_i^e is the expected price of the *i*th commodity and w^e is the expected wage rate.
The expected profit rate from the k_i technique, which is denoted as $r_i^e(k_i)$, is calculated as

$$p_i^e = [1 + r_i^e(k_i)]\left[\sum a_{ij}(k_i) p_i^e + \tau_i(k_i) w^e\right].$$

Capitalists choose the technique which yields the highest expected profit rate. If

$$r_i^e(k_i^*) \geqq r_i^e(k_i) \qquad k_i = 1, 2, \ldots, T_i \tag{1}$$

then they choose the k_i^*th technique among T_i alternatives. As is easily seen, (1) can be rewritten as

$$\sum a_{ij}(k_i^*) p_j^e + \tau_i(k_i^*) w^e \leqq \sum a_{ij}(k_i) p_j^e + \tau_i(k_i) w^e \qquad k_i = 1, 2, \ldots, T_i \tag{2}$$

The means that the expected unit cost is smallest in the k_i^*th technique. So in this case *the maximum profit rate criterion* is equivalent to *the minimum unit cost criterion*. However, this equivalence does not hold in general. If we introduce durable equipment the two criteria are not equivalent. But for simplicity here we will ignore durable equipment.

Profit Rate and Techniques

In the *i*th sector by the minimum unit cost criterion capitalists adopt the technique

$$a_{i1}, a_{i2}, \ldots, a_{in}, \tau_i$$

and labourers receive the commodity basket

$$b_1, b_2, \ldots, b_n$$

per unit of labour. Then an equal rate of profit r between n sectors is determined by the following equations:

$$\begin{aligned} p_i &= (1+r)\left(\sum_{j=1}^{n} a_{ij} p_j + \tau_i w\right) \qquad i = 1, 2, \ldots, n \\ w &= \sum_{i=1}^{n} b_i p_i \end{aligned} \tag{3}$$

From these equations it is clear that the profit rate r depends on techniques (a_{ij}, τ_i) and the real wage basket (b_i).

In order to examine the relationship between the profit rate and techniques in various sectors, we must introduce a new concept: *basic sectors*. P. Sraffa has used this terminology, defining basic sectors as those whose outputs are directly or indirectly necessary in the production of every commodity (see ch. 2 in Sraffa, 1960).

However, it is not guaranteed a priori that such basic sectors exist; and even if they do exist, the concept is not useful for our purpose here.

Now we redefine basic sectors as those whose products are wage goods, or whose products are directly or indirectly necessary to produce wage goods. 'Wage goods' means commodities which are included in the real wage basket ($b_1,\ldots,b_n$). If $b_i > 0$ then the *i*th commodity is a wage good. Basic sectors in this sense necessarily exist, for there must be at least one commodity which is a wage good.

Suppose there are m basic sectors, with m ≦; n; after renumbering, let the 1st, 2nd,…,mth sectors be basic sectors. Then the equations

$$p_i = (1+r)\left(\sum_{i=1}^{m} a_{ij} p_j + \tau_i w\right) \qquad i = 1,2,\ldots,m$$
$$w = \sum_{i=1}^{m} b_i p_i \tag{4}$$

are sufficient to determine the profit rate r, where prices ($p_1,\ldots,p_m$) and the wage rate w are both positive. Therefore we can say that the profit rate does not depend on techniques in the non-basic sectors. For example, pure luxury goods are non-basic commodities. Whatever great improvement may occur in the techniques in those sectors, the (equalled) rate of profit is not influenced at all. This conclusion was first found by Ricardo, but Marx did not accept it (Ricardo, 1821, p. 132; Marx, 1867–94, Vol. III, ch.5, pp. 83–4).

Hereafter in this essay we confine ourselves to techniques in the basic sectors only, and we assume that the classification into basics and non-basics remains unaffected by the technical changes considered here.

Technical Progress

Let us suppose the profit rate r to be determined by (4), and that in the kth sector ($1 \leqq k \leqq$; m) a *new* alternative technique

$$a'_{k_1}, a'_{k_2}, \ldots, a'_{k_m}, \tau'_k$$

becomes feasible. Capitalists must then calculate the expected profit rate of this new technique and compare it with those of alternative techniques to decide whether or not to adopt it, so we now need an assumption about how capitalists form their expectations. For simplicity we assume

$$p_i^e = p_i, \qquad w^e = w \qquad\qquad i = 1,2,\ldots,m$$

i.e. capitalists expect that current prices and the wage rate, as given by (4), will remain the same (static expectations).

If the following inequality holds, capitalists adopt the new technique:

$$\sum_{j=1}^{m} a'_{kj} p_j + \tau'_k w < \sum_{j=1}^{m} a_{kj} p_j + \tau_k w \tag{5}$$

Supposing this to be so, the previous technique in the kth sector $(a_{k1}, a_{k2}, \ldots, a_{km}, \tau_k)$ is replaced by the new technique $(a'_{k1}, a'_{k2}, \ldots, a_{km}, \tau'_k)$. How does the profit rate r as given by equation (4) then change, under the requirement that the real wage basket remain unchanged? We can prove that the profit rate r necessarily rises, as follows:

Putting

$$\beta = 1/(1+r), \qquad q_i = p_i/w,$$

equations (4) are rewritten as

$$\beta q_i = \sum a_{ij} q_j + \tau_i, \qquad i = 1, 2, \ldots, m \tag{6}$$

$$1 = \sum b_i q_i \tag{7}$$

Let the solution of (6) and (7) be

$$(\beta, q_1, \ldots, q_m).$$

When $(a_{k1}, a_{k2}, \ldots, a_{k_m}, \tau_k)$ is replaced by $(a'_{k1}, a'_{k2}, \ldots, a'_{k_m}, \tau'_k)$, the profit rate is determined by

$$\beta q_i = \sum a_{ij} q_j + \tau_i, \qquad i = 1, \ldots, k-1, k+1, \ldots, m \tag{8}$$

$$\beta q_k = \sum a'_{kj} q_j + \tau'_k \tag{9}$$

and (7). Let the solution of (8), (9) and (7) be

$$(\beta', q'_1, \ldots, q'_m).$$

As $q'_i > 0$ for all i, the coefficients matrix of q_i (i = 1,2,…,m) satisfies the Hawkins–Simon conditions (see Simon and Hawkins, 1949).
From (6)–(9), we get

$$\begin{gathered} \beta' \Delta q_i = \sum a_{ij} \Delta q_j - q_j \Delta \beta, \\ i = 1, \ldots, k-1, k+1, \ldots, m \end{gathered} \tag{10}$$

$$\beta' \Delta q_k = \sum a'_{kj} \Delta q_j - q_k \Delta \beta + \left\{ \sum q_j \Delta a_{kj} + \Delta \tau_k \right\} \tag{11}$$

$$0 = \sum b_i \Delta q_i \tag{12}$$

where

$$\Delta q_i = q_i' - q_i, \qquad \Delta\beta = \beta' - \beta$$
$$\Delta a_{kj} = a_{kj}' - a_{kj}, \qquad \Delta\tau_k = \tau_k' - \tau_k.$$

The third term on the right side of (11) is negative, by (5). If $\Delta\beta \geqq 0$, then in (10) and (11) $\Delta q_k < 0$ and $\Delta q_i \leqq 0$ for all $i \neq k$, because as shown above the coefficient matrix of Δq_i in (10) and (11) satisfies the Hawkins–Simon conditions. If the kth commodity is a wage good, $\Delta q_i < 0$ contradicts (12). If the kth commodity is a means of production (that is it belongs to the basic sectors), there must be at least one kind of wage good whose $\Delta q_i < 0$; again this contradicts (12). So $\Delta\beta > 0$, or in other words the profit rate r rises.

The proposition that any new technique which satisfies the profit rate criterion (5) and so is introduced into the basic industries necessarily increases the general rate of profit, cannot be compatible with the Marxian law of the tendency for the profit rate to fall. However large the organic composition of production may become, the general rate of profit must increase without exception, provided that the newly introduced technique satisfies the profit rate criterion and the rate of real wage remains constant (see Okishio, 1961, for further discussion).

Joint Production

So far we have disregarded joint production as well as durable equipment. Even if we introduce durable equipment, the conclusion obtained in the former section still holds (see Nakatani, 1984). However, when we consider the joint production it is possible (though not necessary) to find a case in which the proposition does not hold, a perverse conclusion that was originally presented by Salvadori (1981).

	Input			Output	
Technique	1	2	Labour	1	2
1	0.5	0.5	1	1	2
2		0.5	1	1	
3		0.5	1	2	

In order to show such a case we examine the following numerical example. In this economy, shown in table 1, there are two kinds of commodity. The second commodity is a wage good and it is produced jointly with the first commodity. Let the real wage rate be 0.7 unit of the second commodity. At the first stage we assume that techniques 1 and 2 only are feasible.

The profit rates of techniques 1 and 2 are determined by

$$p_1 + 2p_2 = (1 + r_1)(0.5p_1 + 0.5p_2 + w)$$
$$p_1 = (1 + r_2)(0.5p_2 + w)$$
$$w = 0.7p_2$$

where r_1, r_2 are the profit rates of techniques 1 and 2, respectively. Putting $p = p_1/p_2$ these equations are rewritten as

$$p + 2 = (1 + r_1)(0.5p + 1.2)$$
$$p = 1.2(1 + r_2)$$

The profit rates of both technique are drawn on Figure 1.

Now we examine the condition in which both techniques 1 and 2 are used. If technique 1 only is used at activity level x, then the surplus products consist of $(1 - 0.5)x$ units of commodity 1 and $(2 - 0.5 \simeq 0.5)x$ units of commodity 2. Therefore if the capitalists' demand for the surplus products (for their consumption or investment) are 100 units of commodity 1 and 50 units of commodity 2, then there must be excess demand for commodity 1 or excess supply for commodity 2 and the relative price p_1/p_2 increases.

When p rises above p^* the expected profit rate r_2 becomes greater than r_1, so technique 2 is introduced. However, technique 2 cannot replace technique 1 completely because technique 2 cannot produce commodity 2. Therefore both techniques must be used, which requires that the equal rate of profit be determined at r^*.

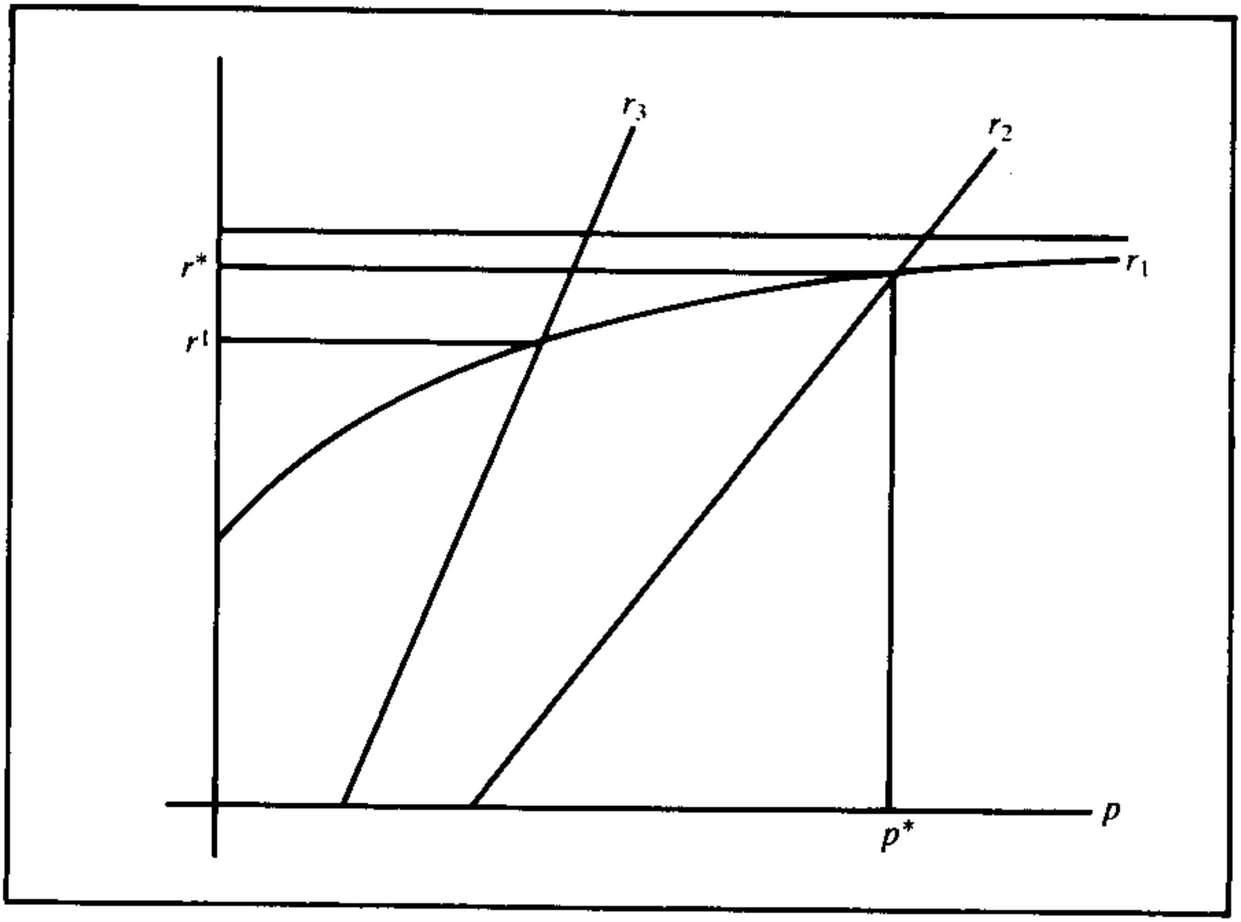

Figure 1

At the next stage we assume that technique 3 becomes feasible. Technique 3 is apparently superior to technique 2, because in the new technique capitalists get more output from the same input, so it replaces technique 2. The profit rate of technique 3 is calculated from

$$2p_1 = (1+r_3)(0.5p_2 + w)$$
$$w = 0.7p_2$$

which can be rewritten as

$$2p = 1.2(1+r_3)$$

This equation is also plotted on figure 1, from which it can be seen that the equalized rate of profit falls to *r*'.

Substitutional technical change

In the previous sections we treated the relationship between the profit rate and technical change under the condition that the real wage basket remain unchanged. The change in the technique adopted was not induced by a change in the real wage rate, but was caused by the introduction of a new process.

The question now is the relationship between the profit rate and technical change that is induced by a change in the real wage rate. We define the level of the real wage rate λ as follows. Assume that each labourer spends his wage income on various wage goods in fixed proportions. Then we have

$$w = \lambda \sum b_i p_i,$$

where the b_i are all constant and λ is the level of the real wage rate.

At the first stage $\lambda = 1$, and the profit rate is determined by equations (6) and (7). At the next stage $\lambda > 1$, which means a rise in the real wage rate. Then the profit rate is determined by the following equations

$$\beta q_i = \sum a'_{ij} q_j + \tau'_i \qquad i = 1, 2, \ldots, m \tag{13}$$

$$1 = \lambda \sum b_i q_i \qquad\qquad \lambda > 1. \tag{14}$$

As shown in (13) the techniques used in every sector may differ from the techniques used at the first stage, because of the rise of the real wage rate and the change of prices which accompanies it. What can we say about the relationship between the newly adopted technique (a'_{ij}, τ'_i) and the old technique (a_{ij}, τ_i)? (Of course we assume that no technique becomes newly feasible for capitalists between the first stage and the next stage.)

Let the solution of (6) and (7) be

$$(\beta, q_i, \ldots, q_m).$$

Then

$$\sum a_{ij} q_j + \tau_i \leqq \sum a'_{ij} q_j + \tau'_i \tag{15}$$

because technique (a_{ij}, τ_i) would not have been adopted at the first stage if inequality (15) had not held; rather they would have adopted $(a_{ij}^{'}, \tau_i^{'})$.

Let the solution of (13) and (14) be

$$\left(\bar{\beta}, \bar{q}_1, \ldots, \bar{q}_m\right).$$

Then, arguing as for (15),

$$\sum a_{ij}\bar{q}_j + \tau_i \geqq \sum a'_{ij}\bar{q}_j + \tau'_i. \tag{16}$$

Using inequalities (15) and (16) we can prove that in going from the first stage to the second the profit rate necessarily falls, as follows:

From (6), (7), (13) and (14) we get

$$\begin{gathered} \bar{\beta}\delta q_i = \sum a'_{ij}\delta q_j + q_i\delta\beta + \left\{\sum\left(a'_{ij} - a_{ij}\right)q_j + \left(\tau'_i - \tau_i\right)\right\} \\ i = 1, 2, \ldots, m \end{gathered} \tag{17}$$

$$0 = \sum b_i\delta q_i + (\lambda - 1)\sum b_i\bar{q}_i \tag{18}$$

where

$$\delta q_i = \bar{q}_i - q_i, \qquad \delta\beta = \bar{\beta} - \beta.$$

From (15) we know that the third term on the r.h.s. of (17) is non-negative. If we assume $\delta\beta \leqq 0$ then all the δq_i become non-negative because the coefficient matrix of δq_i satisfies the Hawkins–Simon conditions. But since $\lambda > 1$ that contradicts (18). So $\delta\beta$ must be positive, or in other words the profit rate must fall.

When the real wage rate rises capitalists cannot avoid a fall in the profit rate, even if they substitute techniques to avoid it; only the introduction of new and superior feasible techniques can prevent the fall. However, we cannot say that the capitalists' efforts to substitute with exciting techniques are of no use to them. Though they cannot avoid a fall in the profit rate, they can mitigate it. We can prove this as follows.

If in spite of the rise in the real wage rate capitalists adhere to the techniques adopted at the first stage, the profit rate is determined by

$$\beta q_i = \sum a_{ij}q_j + \tau_i \qquad i = 1, 2, \ldots, m \tag{19}$$

$$1 = \lambda \sum b_i q_i \qquad \lambda > 1 \tag{20}$$

Let the solution of (19) and (20) be

$$\left(\beta^*, q_1^*, \ldots, q_m^*\right).$$

From (13), (14), (19) and (20) we get

$$\beta^* \mathrm{d}q_i = \sum a_{ij} \mathrm{d}q_j - q_i^* \mathrm{d}\beta + \left\{\sum \left(a'_{ij} - a_{ij}\right)\overline{q}_j + \left(\tau'_i - \tau_i\right)\right\} \tag{21}$$
$$i = 1, 2, \ldots, m$$

$$0 = \lambda \sum b_i \mathrm{d}q_i \tag{22}$$

where

$$\mathrm{d}q_i = \overline{q}_i - q_i^*, \qquad \mathrm{d}\beta = \overline{\beta} - \beta^*.$$

From (16) the third term of the r.h.s. of (21) is non-positive. However, if we now consider the case in which substitution actually occurs, then for some i the third term on the r.h.s. of (21) is negative. If we assume $\mathrm{d}\beta \geqq 0$, then all the $\mathrm{d}q_i$ become non-positive and some actually negative, because the coefficient matrix of $\mathrm{d}q_i$ satisfies the Hawkins–Simon conditions. This contradicts (22) so $\mathrm{d}\beta < 0$. In other words, when the substitution is carried out the profit rate is greater than it would have been if capitalists had adhered to the old optimal technique, which corresponded with the former level of the real wage rate.

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See also

investment decision criteria; investment planning; non-substitution theories.

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