

The Fundamental Marxian Theorem: A Reply to Samuelson

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I AM happy to find that Samuelson has no objection to my Fundamental Marxian Theorem if it is put in the following neutral form: [3, 1974].¹

Theorem: *The competitive rate of profit R^* determined by*

$$a_0 (1 + R^*) [I - a(1 + R^*)]^{-1} m = 1 \quad (1)$$

is positive if and only if

$$a_0 [I - a]^{-1} m < 1. \quad (2)$$

The notation is Samuelson's.² The theorem assumes *inter alia* that wages are fixed at the subsistence level; evidently the subsistence-consumption vector per man-hour m equals the subsistence-consumption vector per day (B in my notation) divided by the working hours per day, my T . Then (1) and (2) can be rewritten, respectively, as

$$a_0 (1 + R^*) [I - a(1 + R^*)]^{-1} B/T = 1, \quad (1')$$

$$T^* < T, \quad (2')$$

where

$$T^* = a_0 [I - a]^{-1} B. \quad (3)$$

Marx regards a_0 and a as technologically given and B as biologically given; hence, T^* can be calculated easily. The theorem then states that R^* which satisfies (1') is positive if and only if T is larger than T^* . No "mysterious" concepts such as "value" or "exploitation" or anything else appear in this form of the theorem. It gives an economically meaningful relationship between T and R^* .

¹ For the Fundamental Marxian Theorem see Morishima [2, 1973, pp. 53-71].

² a_0 is the vector of labor-input coefficients that is my $L = (L_I, L_D)$, a the matrix of physical-input coefficients (my $A = \begin{bmatrix} A_I & A_D \\ 0 & 0 \end{bmatrix}$), R^* the equilibrium rate of profit (my

π) and m the subsistence-consumption vector per man-hour (my B/T).

I.

However, what does the critical value T^* stand for? This question is important, especially to Marx because his contemporaries cannot swallow down the Leontief inverse, $[I - a]^{-1}$. Probably the only way to make them understand T^* is to appeal to the labor theory of value, as Marx does. In fact, $a_0 [I - a]^{-1}$ is nothing else but the solution to the value-determination equation,

$$\Lambda = \Lambda a + a_0, \quad (4)$$

and, therefore, T^* equals the value of the subsistence commodity-bundle, ΛB , that is the labor-time socially necessary for producing B by the techniques (a, a_0) actually prevailing in the economy. It is important to emphasize that there is no element of competitive arbitrage in (4). It is no more than the equation for calculation of the quantities of labor socially necessary for producing goods.

II.

In Marx, competitive arbitrage is exclusively made in terms of prices, the wage rates and the profits rates, not in terms of values and the rates of surplus value at all.³ If the economy is competitive

³ In Marx's economics, value calculation plays a role that is entirely different from the one which price calculation does. Decision of individuals and firms are all made in terms of price calculation, while value calculation gives a technocratic assessment of labor requirement for production. Marx's theory of value should not be considered as a primitive or obsolete price theory. There is no point in comparing translation of the price accounting in the value accounting or *vice versa* with translation of a children's private nursery language into ordinary English. Samuelson's footnote [3, 1974, p. 66, f. 6] is not understandable if this fact is clearly recognized.

Also, in contrast with Samuelson's grouping my theorems into classes A, B, and C, Marx would probably propose to group them into the following A', B', and C'. Class A' includes the theorems concerning competitive arbitrage in the capitalist economy; B' those that relate the

and the subsistence wages prevail, the following two equations must be fulfilled in the state of equilibrium:

$$p = (1 + R^*)(pa + wa_0) \quad (5)$$

$$wT = pB. \quad (6)$$

Equation (5) implies that the rate of profit should be equalized throughout the economy by competitive arbitrage among capitalists, while (6) implies that the hourly wage-rate, w , or the length of the working day, T , should be equalized throughout the economy, by competitive arbitrage among workers, because the wages per day are set at the subsistence level.

Once the length of the working day is equalized among jobs, then a uniform rate of surplus value is established throughout the industries. This is seen in the following way. Let l_i be the labor-input coefficient of industry i , i.e., the i -th element of a_0 , and T_i the working hours per day in industry i . Then by definition

$$v_i = l_i \Lambda B/T_i, \quad (7)$$

$$s_i = l_i - v_i, \quad (8)$$

for each i . Hence,

$$\frac{s_i}{v_i} = \frac{1 - \Lambda B/T_i}{\Lambda B/T_i} \quad \text{for each } i. \quad (9)$$

Therefore the rates of surplus value are equalized if and only if T_i 's are equalized.⁴

price-profit accounting system to the value-surplus-value accounting system; and C' those concerning technocratic calculation of values of commodities. Class A' does not include the theorems of the classical labor theory of value (as a primitive competitive price theory) because they are valid only in the society of "simple commodity production" but not in the capitalist society; on the other hand, class C' is not a null set.

⁴ Marx writes: "This [equalization of the rate of surplus value] would assume competition among labourers and equalization through their continual migration from one sphere of production to another" [1, 1966, p. 175].

In his footnote 2, Samuelson asks whether one can reconcile equal $S_j/V_j = S_i/V_i$ with $V_j/C_j \neq V_i/C_i$ and equal $S_j/(V_j + C_j) = S_i/(V_i + C_i)$ or not [3, 1974, p. 63]. But it is obvious that the two inequalities are incompatible if both are in either the price accounting or in the value accounting. Although Marx is confused sometimes, his problem is not such a trivial one. It is to show that equal positive $s_j/v_j = s_i/v_i$ with $v_j/c_j \neq v_i/c_i$ in the value regime is compatible with equal $S_j/(V_j + C_j) = S_i/(V_i + C_i)$ in the price regime.

III.

Let us now assume that all T_i 's are equalized. Then (9) is written as

$$\frac{s_i}{v_i} = \frac{T - \Lambda B}{\Lambda B} \equiv r^*. \quad (10)$$

In view of $T^* = \Lambda B$, we can say that $T > T^*$ if and only if $r^* > 0$. Hence by the theorem above, the equilibrium rate of profit R^* is positive if and only if the uniform rate of surplus value r^* is positive. Thus r^* is a mirror-image of T , and the latter is considered by Marx to be determined by the relative powerfulness of capitalists and workers. Marx finds that the rate of profit is increased by lengthening the working day—a view that is very consistent with his experience in the Victorian era.

IV.

One of the purposes of Marx's *Capital* is to show the productiveness of the capitalist system or the positiveness of the von Neumann balanced-growth rate. To show this we have to find the necessary and sufficient condition for the augmented input-coefficient matrix $a + a_0 m$ to satisfy the Hawkins-Simon condition. We must, however, remember that when Marx was tackling this problem, Frobenius, Perron and Markov had either not been born or were merely babies so that he could not use their theorems; Marx had to find his own way. For this purpose he assumes that A_I (the submatrix of a concerning the capital-goods sectors) is "productive" (this is Marx's basic and harmless assumption concerning technology); and he finds that $a + a_0 m$ is "productive" if and only if (2') is satisfied. Therefore, the von Neumann equilibrium rate of growth is positive if and only if (2') holds; in fact, it is seen that the growth rate is equal to the positive equilibrium rate of profit.

I take this as a first-rate contribution. To examine whether the crucially important condition (2') is fulfilled or not, we calculate labor values (or $a_0 [I - a]^{-1}$ if one does not like to call it the value vector) and evaluate the commodity-bundle m in terms of values. But this does not mean at all that Marx accepts the theory of value as a theory of prices. In Marx, prices which are determined in the competitive way are distinct from values which are technocratically calculated on the basis of the prevailing production coefficients; on the contrary he insists that the value equations may be taken as the

equations for determining prices only in the simple-commodity-production society but not in the capitalist economy. I am against Samuelson's view: "The algebra of the surplus-value regime is easier to handle. So for purposes of elementary exposition and layman persuasion, there is merit in the Volume I models." It is true that Marx often confuses value and price. But in reinterpreting Marx so that no confusion remains (as we, non-antiquarians, do for Walras) we must distinguish, again as non-antiquarians, value of commodity i (*i.e.*, the quantity of labor congealed in one unit of i) from its exchange value (or price). The former regulates the latter; but they differ from each other in the long-run as well as in the short-run, except in some special cases. According to my interpretation, Marx is concerned with the exceptional cases in Volume I first, because he may begin with macroeconomic analysis of a one-department model by doing so, and then he generalizes it into two-or-three-departmental analysis in Volumes II and III. That is to say, I understand that throughout Volume I it is implicitly assumed as the condition for aggregating sectors into one department that all industries have the same value composition of capital, so that values are strictly proportional to prices and hence surplus values to profits. Then it is very natural for Marx to attack these proportionalities in the transformation problem as soon as he denies the aggregation condition and starts the work of disaggregation in Volumes II and III.

V.

So far so good. However, in the above discussion, as well as in the first thirteen chapters of my book, a number of assumptions (all about technology) are made so as to enable us to calculate values unambiguously and to assure their positiveness. But some of these assumptions turn out to be inappropriate if durable capital goods are allowed for. Therefore, in Chapter 14 of my book I reexamine the theory of value (*i.e.*, the program for technocratic calculation of quantities of labor congealed in commodities) and find that unambiguous and meaningful calculation of values is not necessarily possible once joint production and choice of techniques are admitted. As capital good i of age $t + 1$ appears as a joint output at the end of the process which uses capital good i of age t and capitalists can choose among processes which uses capital goods of age 0, 1, 2, . . ., the existence of durable capital goods is

intrinsically connected with the problems of joint production and choice of techniques. I finally decide to discard the value theory, but I find, at the end of the book, that the concept of "exploitation" may survive.

This conclusion has stronger effects than Samuelson's "erase and replace" conclusion; it should be a serious attack on Marx. I did not know, when I finished the book, whether the Fundamental Marxian Theorem is valid or not in the general model with durable capital goods. But I now know that it does hold true: The long-run equilibrium rate of profit R^* is positive if and only if the rate of "exploitation" is positive. This proposition is completely independent of the concept of value, and the rate of exploitation is defined as I define it at the end of the book. I have not proven this new general theorem here because it was discussed in my Walras Lecture at the 1973 North American meeting of the Econometric Society in New York. Anyway, we may conceive of Marx without the theory of value, as long as we agree that the Fundamental Marxian Theorem is the core of his economic theory.

VI.

There remain a few more points on which I want to reply to Samuelson. First, I accept his criticism about my interpretation of Marx's law of the falling rate of profit. I have made a similar mistake which has been pointed out by Okishio, one of the very best theorists of Marxian economics. It is interesting to see that a mistake which is favorable to Marx has been pointed out by Samuelson and missed by Okishio and a mistake which is unfavorable to Marx has been pointed out by Okishio and missed by Samuelson. However, I do not accept Samuelson's criticism concerning my treatment of the aggregation problem. He says that I claim "that zero-profit embodied-labor contents give better weights for aggregation purposes than do prices calculated at positive interest rates." But I never made such a proposition in my book.⁵ Finally, in relation to Samuelson's footnote 7 [3, 1974, p. 68], I point out that the theorem that prices of commodities are proportional to their values in the simple-commodity-production economy is a proposition concerning the long-run equilibrium. Like the Fundamental

⁵ C. C. von Weizsäcker has made the same misunderstanding. A full detail of my reply on this point is given in my "Marx's Economics: A Comment on C. C. von Weizsäcker," *Econ. J.*, forthcoming.

Marxian Theorem, it should not be applied to profits (or monopoly profits) “which a buyer or seller acquires through an accidental state of supply and demand,” [1, Marx, 1966, pp. 175–178, especially p. 178], so that von Weizsäcker’s corn-wine parable for showing a possibility of existence of a positive profit rate in the classless society is a swing missing the ball. Its existence is transitory when all people are alike.⁶

⁶ Also see my comment on von Weizsäcker.

REFERENCES

1. MARX, K. *Capital*, Vol. III. Moscow: Progress Publishers, 1966.
2. MORISHIMA, M. *Marx's economics*. London: Cambridge University Press, 1973.
3. SAMUELSON, P. A. “Insight and Detour in the Theory of Exploitation: A Reply to Baumol,” *J. Econ. Lit.*, March 1974, 12 (1), pp. 62–70.