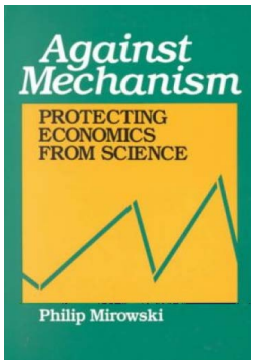

10 *Morishima on Marx*

> It is almost tragic, however, that Walras, who was usually so acute and clearheaded, imagined he had found the rigorous proof, which he had missed in contemporary defenders of free-trade dogma, merely because he clothed in a mathematical formula the very arguments which he considered insufficient when they were expressed in ordinary language.
>
> —Knut Wicksell, *Lectures on Political Economy*

This predilection, to which Wicksell himself was not entirely immune, has continued to bedevil the economics profession down to the present day. It is one thing for neoclassical economists to hawk proto-physics models as "proofs" of the natural order of the market. It is quite another level of hubris to wish also to teach some long-dead economists the error of their ways by nattering at them about lacunae in their educations concerning sums and topology.

The fervent desire to emulate physics has left neoclassical economists with a deep ambivalence concerning their discipline's past. Most, under the mistaken impression that physics has sloughed off its history, would wish economics to do likewise. Others, perhaps more worried about how posterity will treat them, feel impelled to go back to the hallowed texts and reinterpret them, with an eye toward demonstrating that all that is valuable in economics has led up to the current orthodoxy. Combined with a fervid faith that the adoption of mathematical argument has accounted for most of said progress in economic theory, these reinterpretations have been constrained to assume a particularly curious format. Usually they consist of a marshaling of quotes, which are dragooned to justify the casting of some economic relationships in a specific functional form, which are then used to arrive at one of the two alternative conclusions: (a) the esteemed late economist in question merely had anticipated a special case of existing neoclassical theory; or (b) the esteemed late economist in question had tripped himself up in self-contradiction, due to his unfortunate weaknesses in the area of mathematical expertise. (See, for example, Hicks 1972; Barkai 1959; Eagly 1974; Brems 1986). Ricardo seems the all-time favorite butt of this sort of activity, but there are a surfeit of instances of it for Quesnay and Smith as well. As with all other apologetic research agendas in economics, it has become a bit of an academic industry: search for the marginally renowned and as-yet unformalized economist and apply the recipe.

What is wrong with this harmless bit of storytelling? After all, the classical economists are dead and in their graves. Further, the sober and secular arm of the profession thinks it all a tempest in a teapot, since what matters in the hard-nosed world

of economics in the paycheck, and not the *Weltanschaung* . The case of Maupertuis is sometimes mentioned in this respect (Samuelson 1972). His Principle of Least Action was not just a mathematical regularity in his opinion; it was simultaneously evidence for the existence of a benevolent and wise Supreme Being who efficiently minimized all effort. Subsequent physicists found that they could make use of Maupertuis's principle without bothering about questions of Divine Order. Don't neoclassical economists essentially act the same way? In particular, aren't these notions of theoretical progress just a lot of irrelevant excess metaphysical baggage?

Perhaps back in the era of Popper (an era that does evoke a certain modicum of nostalgia among neoclassicals) this stance appeared persuasive, but in the world after Kuhn, Feyerabend, Barnes, Bloor, and Rorty, it does seem a little dowdy and timeworn. The explosion of research in the history of science has taught us quite a bit about how intellectual disciplines work, and one of its primary lessons has been that the potted history of a discipline generated for internal consumption plays an important role as a heuristic in dictating legitimate research methods and topics, as well as fostering a camaraderie of shared perceptions (Graham, Lepenies, and Weingart 1983). Perceptions of progress do matter and are worth fighting over. In the context of a disciplinary matrix, the intellectual history of that discipline assumes a heightened significance; attempts to change the disciplinary reading of the history indicate deeper conflicts over the appropriate topics and methods of research. From this point of view, the neoclassical predilection to recast earlier economists in the mold of their own mathematical formalism is a profound hermeneutical tactic, a move to shift the grounds of the argument over progress onto their own turf.

There are a number of possible responses to this gambit, none of which is guaranteed to score points. One response is simply to denounce all retrospective mathematical models, but that does seem a no-win proposition when one takes into consideration the increased mathematicization of the discipline since World War II. A second response is to try to best the neoclassicals at their own game, in the sense of developing alternative mathematical models of important precursors that support research programs opposed to the neoclassical school. This, for instance, seems to have been the motivation behind Sraffa (1960) and Pasinetti (1973). Much can be said in favor of this gambit, but it does have the drawback that the quarrel appears to degenerate to one over the indifferent acceptability of alternative sets of "assumptions" (the F-twist), and it tends to ignore the larger issues of the impact of the formalization upon the theoretical substance and methodological orientation of the original texts. A third response is to attempt a hermeneutical reading of neoclassical mathematical restatements of important earlier texts, trying to evaluate their success relative to the object text and relative to the (generally unspoken) intentions of the author in rewriting the history of economic thought.

We shall attempt the third mode of response here. To focus our attention on the

tensions inherent in the general practice of mathematical restatement, we shall choose to concentrate on the most criticized and most interpreted text in the entire history of economic thought, Marx's *Capital*. Marx is very important for the self-image and self-esteem of the neoclassicals, and not only because the other half of the world's population claims his text as their primary political inspiration. Marx's *Capital* represents the most advanced development of the classical system of economic theory prior to its demise in the West, and in order to provide a satisfying narrative of the progress of the discipline, that demise must be justified along some very stylized lines. This process was begun by Wicksteed and Bohm-Bawerk, but got seriously under way only when neoclassicals started to restate Marx mathematically, using neoclassical terminology and techniques (Samuelson 1957, 1971), (Georgescu-Roegen 1960). Such crude and openly unsympathetic bids to settle Marx's hash once and for all were not adequate to the underlying motivations of the exercise, which dictated walking a fine line between excessive elevation of Marx's achievements (although some elevation was called for to justify the attention bestowed) and excessive deprecation of Marx's "mathematical errors" (although, again, these had to be substantial to justify the primary message, which was the substantial progress achieved by neoclassicism after Marx).

The first neoclassical economist to aim at this fine balance was Michio Morishima in *Marx's Economics* (1973). It was his avowed intention to "recognize the greatness of Marx from the viewpoint of modern advanced economic theory and, by so doing, to contribute to the development of our science"(M, 5).[1] Now, this notion of a contribution to science is a bit vague. Does it contribute to the development of modern cosmology to praise the mistaken yet fascinating systems of Kepler and Ptolemy? Morishima is actually quite open about the object of his exercise, which is to hasten the day when "the division between valid Marxian economics and orthodox theory has been removed." There have been numerous commentaries and glosses upon Morishima's work, but it seems to the present author that all have missed this, the real point of the work, and consequently they have also passed lightly over many of the mathematical infelicities and theoretical incongruities of his book.

What follows is not another plaintive cry that yet another someone didn't get Marx right, since a hermeneutic perspective teaches that texts are open to contradictory yet legitimate readings. Instead, it is a meditation upon a certain overweening attitude among neoclassical economists that mathematical models are sufficient to brush aside all other considerations and crush competitor research programs under the wheels of their analytical engines. This may be effective from a sociological point of view, but it is simply false from the vantage point of the logic of assessment.

The Labor Theory of Value a Dead Dog?

The first thing that strikes the reader of *Marx's Economics* is that, in return for a substantial investment in mathematical manipulation, there is very little Marxian theory

left unscathed by the end of the book. On page 103 we are informed, “Apart from ideological reasons, values are necessary in Marxian economics, not because they are the first approximation of prices, but because they are more fundamental than prices and enable us to get rid of circularity.” This is a profound possibility (compare Mirowski 1986 and Krause 1982), but Morishima then appears to debase this insight by seeing it as only a problem in the theory of aggregation. The notion of a value theory as a prior prerequisite of a coherent theory of price, which was clearly the notion expressed by Marx in the first six chapters of volume 1 of *Capital*, has gone by the boards without comment or rationale. The reason it is ignored is not one of a lack of susceptibility to mathematical formalization, or even a case of external criticism, say, to the effect that it was a metaphysical residuum of Marx’s unfortunate weakness for Hegelian wordplay. There is for Morishima only one criterion for whether some Marxian concept gets his mathematical attention, and it is stated at the end of the book: “one of the conclusions of this book is that Marx’s economics can acquire citizenship in contemporary economic theory by detaching it from its root, the labor theory of value, and grafting it onto the Von Neumann stock so as to produce the Marx–Von Neumann flower” (M, 194).

If we may be permitted to mix metaphors with the same exhilarating freedom as Morishima, Marx may not be a citizen of the neoclassical nation, but he does not need a passport from Morishima to reside in the land of economics or even to smuggle in some agricultural produce. As for the flora of the land of the neoclassicals, horticulturalists for some time now have been trying to breed away the yeasty bitter juices of classical economics in their cassavas, but every effort seems to result in a root that’s not very nourishing (although admittedly pretty) and a flower that is sterile (although undeniably convex). I suspect that Morishima is aware of this, and some later writings (see Morishima 1984) confirm the suspicion. Nevertheless, this has not deterred Morishima from riding roughshod over both Marx and Walras in order to foster the impression of one nation, one theory, one genetic heritage.[2]

Why should we want to entertain seriously the Marx–Von Neumann hybrid? In the context of *Marx’s Economics*, I suppose there are two justifications: first, to demonstrate the expendability of the labor theory of value, and second, to reveal what new insights might be derived from such a model. Just on the evidence of pages devoted to each, Morishima is much more concerned with the first than the second. Let us then occupy ourselves in this section with Morishima’s reasons for his Marx minus the labor theory, and postpone to the next section some consideration of the novel exercises that he promises.

Reason One

It is evident that consistency is not assured when the age structure of fixed capital

> is no longer stationary. There will not be universal consistency between "the replacement of the wear and tear portion of the value in the form of money" and "the replacement of fixed capital in kind" unless we get rid of the neoclassical [sic] method of depreciation and obey the Von Neumann golden rule in the valuation of capital costs. [M, 173]

While this reason for the rejection of what Marx actually said is somewhat arcane, it is a good place to start because it is fraught with the pitfalls of any attempt to make Marx look bad solely with mathematics. First, to set the stage: Morishima wishes to advocate the adoption of the Von Neumann method of capital accounting, where a one-period-older machine is treated as if it were an economically distinct commodity relative to a newer machine of the same physical identity. Marx does advocate a different method, using straightline bookkeeping methods, which Morishima inexplicably misrepresents as being "neoclassical." For most of the book Morishima just blasts ahead with the Von Neumann method, but relatively late in the book (M, 170-78) he feels the need to confront what Marx actually wrote. Rather than discuss Marx's reasons for such a choice, Morishima decides he will quickly model Marx's alternative and swiftly reveal its internal inconsistencies.

He proceeds as follows. First he defines the current input matrix A_I as composed of individual elements $a_{ij} = (k_{ij}/\tau_{ij})$, letting k_{ij} be the "stock of capital good i" required for the production of one unit of capital good j, and τ_{ij} be the effective lifetime of k_{ij} when used in the production of good j. The matrix A_{II} is defined for the wage and luxury good sectors in the same manner. Note that the entries of A_I are intended to represent *flows* of input per unit time period. Morishima then goes on to write the following cost-price equations:

$$p_I = (1+\pi)(p_I A_I + wL_I)$$

(1)

$$p_{II} = (1+\pi)(p_I A_{II} + wL_{II})$$

(2)

where p_I denotes the price vector of capital goods, p_{II} the price vector of wage and luxury goods, π is the uniform rate of profit, w the given wage, and L the vector of labor requirements in production. The first bit of negligence in the area of textual exegesis is that equations (1) and (2) have the capitalists figuring their profits on flow costs of inputs rather than on stocks. This is an extremely awkward manner of incorporating the notion of fixed capital into mathematical analysis, and in any event Marx never made any such claim in volumes 2 and 3. But more to the point, it is

inconceivable that Morishima can now teach Marx any lessons concerning "the replacement of fixed capital in kind." But there is more, so let us persevere.

On pages 171—173 a few more relatively innocuous assumptions are floated, and then the following relationship is deduced:

$$p_I A_I x_I(t) + p_I A_{II} x_{II}(t) = p_I R(t+1)$$

(3)

where $x(t)$ is the vector of output levels for the respective sectors and $R(t)$ is the vector of input replacements at time t. Now, says Morishima, imagine that all capital goods last just two periods, and all of those available in period t are new. No producer needs any machines in period $t + 1$, hence $p_I R(t + 1) = 0$; but there is value set aside in period $t + 1$ for replacement—that is,

$$\sum_{i=I}^{II} p_I A_i x_i(t) > 0.$$

Hence the objection that value is not equal to replacement in kind. This is the sum total of Morishima's first indictment: the assumptions of *this* model imply an equilibrium condition that will obtain only with a certain pattern of the decay and replacement of stocks, one that exhibits a stationary age structure.

Here are a veritable concatenation of incongruities, so many that it makes one dizzy trying to sort them out. First, the model misrepresents Marx at the simplest possible level in miscalculating profits on flows alone. Second, the basic issue is that depreciation may not equal replacement investment because replacement is often bunched and depreciation is sometimes continuous, although this really is a question of money and accounting, which by no stretch of the imagination are given due consideration anywhere in Morishima's book. Third, the treatment of time is questionable, to say the least. Morishima draws various conclusions in words about the age structure of capital stocks, even though those specific temporal configurations are not seriously modeled.

To make this clearer, let us construct a model in parallel to that of Morishima, confining ourselves only to a state of simple reproduction with no fixed capital. Employing Morishima's notation, we shall now adopt equations (1) and (2) (this time consistently, since there is no divergence of stocks from flows by construction) and assume that the entire output of department II is consumed by the workers:

$$x_{II}(t) = C(t + 1)$$

(4)

$$x_I(t) = R(t + 1)$$

(5)

$$wL_I x_I(t) + wL_{II} x_{II}(t) = C(t + 1)$$

(6)

Substituting and simplifying, we arrive at:

$$p_I A_I x_I(t) + P_I A_{II} x_{II}(t) = P_I R(t + 1)$$

which is identical to equation (3). The fact of the matter is that equation (3) is a flow and not a stock condition and holds even in the case of simple reproduction. It has no real implications for the age structure of physical stocks, says nothing about balanced growth, certainly is incapable of being used to discuss accounting conceptions of depreciation, and assumes away most of Marx's profound concerns with respect to turnover in volume 2. Appeals concerning "money," "fixed capital," and "depreciation" are thoroughly unavailing, and juxtapose an aura of high logical rigor with the practice of low semantic comedy.

Fourth, to complain about Marx's recourse to the stationary state really must be seen as a case of the pot calling the kettle black when it comes to neoclassical theory. It is not even clear that anything exists that might be called a legitimate nonsteady state theory of capital within the neoclassical paradigm. The existence of a nonstationary capital stock implies a process of growth incompatible with the Von Neumann equilibrium path, and therefore Morishima's research program should have been to inquire whether the Von Neumann method of capital accounting materially helps or hinders such analysis. What has not been sufficiently appreciated is that the Von Neumann balanced growth ray is the closest analogue to a one-good economy in the economist's armamentarium, and therefore any analysis that commits itself to that format will discover that it is not capable of even phrasing questions concerning nonstationary capital stocks, because the formalism has effectively ruled them out of court. There still remains the question of the relative attractions of the Von Neumann method of capital accounting, even on the balanced growth ray. We postpone consideration of that issue until Reason Five below.

Reason Two

"In literary terms, Marx assumed that the rate of accumulation of capitalists of department one was an exogenous factor, to which that of the capitalists of department two was adjusted" (M, 145). Morishima objects to Marx's assumption as unrealistic (more pot black-calling?), suggesting that Marx chose it because he could conjure no other adequate way of distributing investment between sectors so that rates of profit could be equalized while the system converged to a state of balanced growth (M, 122). Morishima proposes that we must tie the rate of accumulation to a demand-supply framework to explain it satisfactorily (M, 157). Two classes of reservations might be broached at this point: Does this tactic indeed constitute a consistent explanation? and Does this tactic constitute an explanation that could be coherent in any conceivable Marxian tradition?

The answer to the first reservation is that Morishima chooses an exceptionally awkward method of introducing macroeconomic demand into his model. He postulates a two-equation dynamic input-output system, where the output of department I must cover the flow material input requirements in the next period (but there is no fixed capital, and therefore no long-term investment), while the output of department II must meet the following requirement (M, 147):

$$x_{II}(t) = \omega B[L_I x_I(t+1) + L_{II} x_{II}(t+1)] + F(t)$$

(7)

where B is a column vector of the workers' subsistence wage bundle, and ω is the fraction of B earned during one hour's labor.

In this formulation, "Capitalists' demands for wage and luxury goods $F(t)$ must be fulfilled at every point in time." Now, in what sense is this an improvement over Marx's assumption about the investment behavior of capitalists in department II? Instead of mechanical investment behavior, we have mechanical consumption behavior; recall that equation (7) is a *macroeconomic* condition. Indeed, we can turn Morishima's complaint back upon his own model (M, 146): Why does the general law of capitalist consumption demand forcibly assert itself like a law of Nature, in spite of the fact that all the members of society are ignorant of it (excepting, of course, the neoclassical growth theorists)? Surely this is not at all an adequate representation of the conventional mechanism of supply and demand, nor of the macroeconomic concept of effective demand. Perhaps there is some implied idea of a capitalist power relationship over the consumption of goods, but it does seem even more farfetched to think of a capitalist consumption basket as socially determined. If a power relationship is what Morishima intended, then it is but a poor caricature of the Marxian concept of exploitation, which is not based on the vagaries of consumption but an argument concerning the entire mode

of production.

Regardless of the overall question of the superiority of Morishima's alternative, there are also some technical problems. Again he has a certain devil-may-care attitude evident in his treatment of temporal processes. Morishima describes his model *in words* as asserting that "wages are paid before work and there is no consumption lag" (M, 147). But there is not enough detailed specification in equation (7) to justify any behavioral interpretation, and in fact, the only aspect of equation (7) of any importance for the model is the arbitrary specification that labor consumption requirements are out of phase with capitalist consumption requirements. In this context, complaining about a lead or lag here and there may seem picky and a little trivial, but it is one of the curses of mathematical expression that what at first seems trivial may later turn out to have been decisive. In this particular instance, a thoroughly unmotivated assumption about the phasing of consumption requirements becomes the driving force of a difference equation that is used later in the book to illustrate the stability of the growth path and, in Morishima's strained interpretation, evaluate the Marxian notion of the inherent instability of capitalist accumulation. [3]

In order to see what difference a little difference in the timing of consumption can make to a difference equation, let us temporarily drop capitalist consumption as a distinct category from equation (7), and rearrange the result:

$$x_{II}(t) - \omega BL_{II}x_{II}(t+1) = \omega BL_{I}x_{I}(t+1)$$

(8)

This is a simple first order difference equation whose solution can be expressed as an exponential function of the labor consumption bundle and the growth rate of the output of department I. Recalling that this is only a minor alteration of Morishima's equation (7), one can now state in words that this solution describes a situation in which the rate of accumulation of department I is given exogenously, and the capitalists of department II blithely adjust their investment behavior to it. This is one possible behavioral interpretation that may be imposed upon equation (8), but it is certainly not the only one possible. Remember it is taken from Morishima, where it was justified by some imprecise remarks about when workers got their wages. Morishima, blinded by the mathematics, does not see that what is sauce for the goose is sauce for the gander.

Further, the arbitrary lagging of differential consumption patterns is no way to get at the issues of time-phased production and sales that so preoccupied Marx in volume 2. Confusion has plagued macroeconomic theory because analysts insist on postulating that production and consumption of the same goods occur within the same analytical time period (Kennedy 1969; Bleany 1976). Morishima must have had some awareness of this issue, since he quietly drops the simultaneity assumption later in the book, but does

so with different notation to cover his tracks (M, 171, equation 4).

A final word on the second reservation. Is there anything to be gained from incorporating utilitarian—or, more to the point, energetics metaphors'—formalisms into Marxian economic theory? I will restrain myself from chapter and verse quotations,[4] and try to pose the problem in a manner that avoids the old chestnut "What would have Marx thought of neoclassical economics?" The issue is that the metaphorical inspiration and therefore the respective logical structures of Marxian and neoclassical economic theory are divergent, and therefore incompatible. I do not mean to claim that it is impossible to write down a utility function and call yourself a Marxist; John Roemer, for one, has made a career of it, and Oskar Lange did it from time to time. The problem as I see it is that Marx's world revolved around the primacy of production in all areas of life, but the physics metaphor that provides the core of neoclassical price theory is inherently irreconcilable with any classical notion of production. Now, analysts can ignore this incompatibility, or try to suppress it, but history seems to teach that the problem never really goes away.

Reason Three

> [The existence of alternative technologies] violates the uniqueness of the value system because when there are alternative processes it is possible for the same sorts of commodities to be produced simultaneously by different processes and therefore to have different values. Moreover, when a process is mixed with another equally profitable process the values may depend upon the proportions in which these processes are mixed; and the proportions may easily fluctuate since the processes are indifferent in profitability. [M, 189]

The objection in the first sentence would seem to miss the mark, since modern Marxian analyses generally assume the technique actually in use is itself the product of a search for the optimum technique, given an expected global rate of profit; it is possible to interpret Marx's concept of "socially necessary labor time" in such a light. Morishima himself has demonstrated the convergence properties of one such algorithm in chapter 4 of his *Equilibrium, Stability and Growth* (1964).

The second sentence does present a formidable problem for the labor theory of value *within the class of models that Morishima advocates*. In the jargon of the capital theory debates, it is one aspect of the problem of switch points between techniques. It should be noted that the same weaknesses arise in the neoclassical theory of capital under the same conditions; the neoclassical response to date has been to insist that the problem is an empirical one—whatever that can mean in a theoretical dispute—and to continue using the capital concept whenever convenient. Now, Marxian theory can wave no mathematical wand to make the non-uniqueness of labor values at switch points go away, but I do believe it has the option of a more serious response to the problem than

has so far been exhibited by the neoclassicals. The initial point of the response starts from the fact that neoclassical price theory is predicated upon the physics of the field, and therefore has committed itself to the abolition of hysteresis from any and all explanations: for the neoclassical economist, it is illegitimate to premise any explanation upon dependence on the actual historical date. This time-independence is built into the mathematics through the artifice of conservation principles, as explained above in Chapter 5. Marxian theory, on the other hand, embraces the principle of hysteresis, as evidenced by its connection to historical materialism, the specificity of theory relative to the mode of production, and so forth. The upshot is that a Marxian economist would be willing to entertain a historical solution to a mathematical problem, and that could be one response to Morishima. Labor values at switch points could be conceptualized as a weighted average of the outputs of the plural processes at a particular point in time, and these weights in turn could be the resultants of the historical evolution of the industry. Clearly this is not an "elegant" solution from the viewpoint of a mathematician, and Morishima would probably not admit it to the class of mathematical models he would be willing to entertain. But nonetheless it is a legitimate possibility within the realm of Marxian economic theory. There is more to a school of thought than its isolated algebraic models.

Reason Four

Morishima derives one possible solution to the transformation problem in a dynamic growth model where $\pi = e/(k + 1)$, e being the rate of exploitation and k the value composition of capital for the entire economy. The precondition for this transformation to succeed is that there be no distinctly capitalist consumption; that is, $F(t) = 0$ in our equation (7). From this condition Morishima concludes that "S_c [the savings propensity of the capitalists] is considered being equal to one, at least approximately. Therefore Marx's model of reproduction is reduced to the Von Neumann model" (M, 155). Back under Reason One and Reason Two we had occasion to demonstrate in detail why Morishima's model is not Marx's model. Yet one solution to the transformation problem in Morishima's model, that shares an assumption that is critical to the structure of Von Neumann models, is supposed to be sufficient to demonstrate that Marx is "reduced" to Von Neumann? This is not even an argument.

Reason Five

"The introduction of joint production, alternative processes or heterogeneous labor will conflict with the above four requirements" for labor values to serve as weights for the purposes of value aggregation. These four requirements are: nonnegativity, uniqueness, independence from market phenomena, and the establishment of a uniform rate of exploitation (M, 181).

The problem of alternative processes has been discussed above under Reason Three. The question of the heterogeneity of labor has been dealt with at length in Bowles and

Gintis (1977) and needs no further elaboration here. One ought to note that only Morishima, and not Marxists in general, harbors qualms concerning relinquishing the assumption of a uniform rate of exploitation.

The issue of joint production does deserve serious consideration, if only because it has been inexplicably bound up with the issue of the acceptance of the Von Neumann method (VNM) of capital accounting by most of the well-known Marxian economists, such as Steedman, Schefold, Roemer, and Pasinetti. The argument in favor of the necessity of joint production usually goes as follows: all production processes take time, which implies the ubiquity of fixed capital. The only way legitimately to account for fixed capital is with the VNM, and therefore the joint products are the norm, not a special case. The challenge for Marxian economics is that, once VNM is allowed, all sorts of problems arise with labor values: negative labor values become possible, and any capital-theoretic paradox can be found in the Marxian system.

It is important to understand that this is a theoretical and not an empirical issue. No one would "know" a joint production process, even if it bit them on the foot. "Joint production" is an analytical construct that must be linked to economic phenomena through the use of auxiliary hypotheses and definitions, such as the delineation of the appropriate boundaries of a production process, a precise specification as to what constitutes the product, a specification of the appropriate level of aggregation and appropriate time unit, and so forth. To make this point as sharply as possible, let me make the following claim: if I were to reject VNM and arbitrarily shrink the analytical time unit to an infinitesimal interval, I can effectively *define away* the appearance of joint production in any model of the economy. Take, for instance, Sraffa's own example of mutton and wool (1960, 63). Since the lamb, no matter how meek, cannot be shorn and rendered into chops simultaneously, the mental act of shrinking the analytical time unit cleaves what had initially appeared to be a true joint production process in twain: process one produces woolless sheep, and process two is the production of lamb chops by means of commodities, one of which is shorn sheep.

So, the real problem is not the empirical prevalence of joint production; it is instead the issue of whether VNM makes sense as an analytical technique within the context of Marxian economics. I do not believe it does so for four reasons.

First is the issue of the time unit. Morishima's cavalier treatment of temporal processes is reflected in the lack of discussion of the appropriate time unit in VNM accounts. The unit chosen is not an insignificant matter, because it defines the number of "commodities" in the production sphere: the shorter the unit, the more n-period older machines there are to keep track of. There is an analytical rule for defining the relevant time unit, although I have not seen it discussed by any of the VNM partisans. The basic VNM time unit should be the shortest physical lifetime of any input in any production process in the entire economy: the reason for this stricture is that the "fixity" of capital

must be defined against the most "circulating" of all physical commodities. It therefore goes without saying that the natural time unit for VNM accounts is a remarkably short duration, perhaps approaching our original infinitesimal. This result has three disturbing implications. (a) An infinitesimal time unit means input and output matrices of infinite dimensionality. Needless to say, this is analytically unwieldy. (b) This artificial proliferation of the number of "commodities" increases the burden on the labor theory of value without a commensurate increase in novel analytical propositions. (c) The VNM accounting scheme *assumes* that all commodities are reevaluated by the market at least as frequently as once per infinitesimal time unit. There is something ironic about Marxists finding themselves dependent upon a conception of market efficiency that even the most extreme defender of capitalism would not dream of proposing.

Second, the concept of "prices" as well as that of commodities assumes an awkward and counterintuitive meaning in VNM accounts. One important implication of treating an n-period old machine as a distinct commodity is that it then possesses a distinct price. Most partisans of VNM from Sraffa (1960, 64) onward have not interpreted these prices as the resultants of actual realized trades; this is because in VNM a machine is also identified by its location in a particular production process. By definition, the possibility of trade is thus severely limited for most "machines" to trades within the same industry. Given the incongruity of equilibrium prices without trades, VNM partisans call these prices "book values" or "accounting prices" in order to acknowledge their tenuous existence. If these equilibrium prices had some correspondence to the book values of assets in firms' accounts, then one could interpret the pattern of machine prices over the course of its physical lifetime as representative of the depreciation charges on the machine. This claim is rendered nugatory, however, by the fact that prices of old machines are often negative in VNM accounts, and indeed, negative prices cannot be ruled out by any economically meaningful assumptions. Given the cul-de-sac that this literature has wandered into, it seems plausible to suggest that the endogenous market determination of depreciation is not at all an imperative of Marxian theory (although a case could be made that it is a neoclassical imperative), and that the imposition of depreciation as part of firm accounting practices is a theoretically preferred solution as well as a description of actual empirical practice.

Third, as we have observed, in VNM accounts the physical lifetime of every good must be specified prior to the analysis, in order to identify both the analytical time unit and the number of "commodities" to be analyzed. In practice, all VNM theorists assume that this information is unique, and a given physical datum. But what can be the meaning of a unique exogenous physical lifetime of an input if one allows the option of maintenance? There is no escape from this dilemma: either the VNM accounts must have truck with intractable infinite matrices, or else relinquish claims that depreciation is determined endogenously.

Fourth, the question of the dimensions of the input and output matrices is the ultimate

mathematical Achilles heel of VNM fixed capital. Every VNM partisan, with the possible exception of van Schaik (1976) and Pasinetti (1980), merely assumes that the matrices are square: that is, the number of production processes are exactly equal to the number of "commodities," and that both are finite in number. Table 10.1 reveals that the assumption of square matrices (number rows = number columns) is the special (and unlikely) case in VNM accounts. The reason for this, in simple terms, is that there are many more permutations of time patterns of inputs than there are of commodities themselves, once we leave the matchbook world of a single machine of relatively short lifespan. In complete generality, VNM technologies would never be represented by square matrices, contrary to the impressions promulgated by example in VNM texts (Steedman 1977; Morganstern and Thompson 1976). Again, this has profound consequences for the economic theory, since all VNM theorists—Morishima included—depend heavily upon either the Frobenius theorems for square indecomposable matrices, or theorems that hinge upon the ability to invert the input or output matrices.

Table 10.1. The Maximum Dimension of a Von Neuman Input Matrix

	Then maximum order is	
If matrix contains	**Rows**	**Columns**
Finished goods only	f	$f+s$
Plus: capital goods of identical lifespans	$k+f$	$k+f+s$
Plus: capital goods of differing lifespans	$kp(f+k)$	$[(k!)(p!)(k+f)]+s$
Plus: optional maintenance on fixed capital	$[(vp)!]kp(f+k)$	$\{[(k!)(p!)(k+f)]\cdot[(vp)!]\}+s$
Plus: capital goods which are true joint products	$[(vp)!]kp(f+k)$	$s+\{[(k-y)!]\text{-}[p!]\cdot[(vp)!]\cdot[k-j+f]\}$

s = Number of goods produced by unassisted labor.

f = Number of "final goods"

k = Number of "capital goods"

p = Number of time periods, as defined by the duration in production of the longest-lived capital good divided by the duration in production of the shortest-lived capital good.

v = Number of kinds of maintenance that can be performed on capital goods.

j = Number of jointly produced capital goods.

Note: The rows of the input matrix are generally taken to denote "commodities," whereas the columns represent "processes." The dimensions given in this table are *maximum* sizes because each assumes that all capital goods are used in all processes. Obviously, the dimensions of any actual economy will be *arbitrarily* smaller than the table entries.

To make it appear as if all of economic theory were just leading up to VNM accounts, Morishima sheds a few crocodile tears over Marx's "hard struggles with algebra,

differential calculus and numerical examples" (M, 166); collates a few quotes from volume 2 of *Capital* that are more or less irrelevant for the justification of the VNM method of capital accounting; and insists that "It is almost certain that he could not have solved the mathematical problems of joint production, if he had got the idea of treating capital goods left over for production in the future as by-products of the current production process" (M, 167).

It is a safe bet that Marx would not have stumbled on the fixed point theorem that Von Neumann used to prove existence in the 1920s, given that it was discovered after Marx's death. It is almost as safe a bet that Marx would never have treated fixed capital as a joint product, because it leads to so many incoherencies and intractable problems, problems which Morishima, with all his vaunted mathematical advantages over Marx, is equally unable to resolve. It is misleading to give the impression that Von Neumann capital accounting cuts any Gordian knots in capital theory, and it is just as misleading to foster the impression that there are no alternative methods to deal with the same problems. For what it is worth, there has been an attempt to treat fixed capital as a separate matrix of stock requirements, initiated by Lange and developed by Brody (1970), which at least has the virtue from the vantage of Marxian economics that it is closer to Marx's schemes in volume 2. It is an indication of his disregard for presenting Marxian economics as a viable alternative that Morishima actually cites Brody (M, 167), but without giving any mention of the fact that his book contains a legitimate alternative method of capital accounting.

Reason Six

"If the rate of exploitation is not a maximum, it is clear that Marx's theory will collapse unless we have a theory which can satisfactorily explain the prevailing rate of exploitation" (M, 159). Again, the problem is not the mathematics per se, it is the inability to notice that concepts of explanation can vary drastically across rival research programs. Morishima's ideal situation would be a separate behavioral equation to which could be applied some optimization methods and which would result in a unique determinate rate of exploitation. He does not understand that it is neoclassical theory that dictates that all explanation must assume the format of constrained extrema over conserved fields. Marxist theory is based on another metaphor, that of a Cartesian substance in motion. The two metaphors are irreconcilable: Cartesian motion is not the same as energetic motion. There is no negative heuristic in Marxian thought that says, "Thou shalt not appeal to historically contingent processes in your explanations."

Reason Seven

> In an actual economy, however, capitalists are not aware of the scientific law of reproduction; capitalist decisions are made, as are traders' decisions about

exchange, not in terms of value but in terms of prices. Why does the "general law of capitalist accumulation" forcibly assert itself like a law of Nature, in spite of the fact that all the members of society are ignorant of it?" [M, 146]

Finally, we arrive at the real reason why Morishima dislikes the Marxian system and rejects its theoretical claims. His objection can be summarized thus: it does not propound laws as Morishima understands laws, and it does not conform to methodological individualism. Of course, we did not need to slog through all the mathematics to understand this objection. Neoclassical theory is bowdlerized nineteenth-century physics, and as such it wholeheartedly embraces the mid-nineteenth-century conception of deterministic law. Marx, of course, was an inheritor of the Hegelian tradition, as every volume written about Marx (except that of Morishima) explains in painful detail. If one is bound and determined to construct an economic theory that mimics physical laws (although not *modern* physical laws; see Farjoun and Machover 1983), then there is no escaping it—Marxian theory is going to be a big letdown.

Further, if one insists upon the requirement of methodological individualism, then one is bound to reject most of existing social theory, including much that Morishima favors. For example, why does the law of the marginal propensity to consume forcibly assert itself like a law of Nature, in spite of the fact that all the members of society are ignorant of it? Or, and more pointed, if all the members of society are cognizant of the laws of the economy, why do we have economists? Once one grasps hold of this stick to beat a theory, it can always be turned on the flagellator. Indeed, Boland (1986) has argued that Walrasian general equilibrium does not adequately conform to the dictates of methodological individualism.

The Empty Toolbox

If ends and not means are what really impress economists, then Morishima should be able to make a compelling case for his Marx–Von Neumann hybrid by showing us what we may do with it, regardless of any negative comments about rivals grounded in the labor theory of value. Sadly enough, this strategy is neglected in *Marx's Economics*. We are tendered only a few hints of the types of analysis we might perform; they are sketchy and in substance display a singular lack of novelty.

For instance, in his chapter on extended reproduction, we are instructed how to "graft the modern theory of the trade cycle" (M, 126) onto the proposed hybrid. What we are in fact presented with is a reduced-form linear difference equation that will display cyclical behavior for one of two economic reasons: the disproportionality of production between sectors as determined by the technical coefficients, and the choice of capitalists to consume a proportion of the surplus not commensurate with the optimum balanced growth rate. As in neoclassical economics in general, Morishima posits the only exogenous (causal) factors to be technology and tastes. It is ironic that this brand of

cycle theory is dubbed "modern," since it was repudiated by econometric model builders around that same time (Hickman 1972). Further, Samuelson long ago pointed out that any linear model of the business cycle depended crucially upon the improbable knife-edge configuration of characteristic roots to keep it cycling; otherwise the model would blow up or contract to nothing in a very short time. Neither of those situations described an economy anyone was familiar with. Hicks's subsequent "floors and ceilings" was a not-very-successful attempt to make the mathematics conform a little more closely to the phenomena (Hicks 1950); in the interim most of these models have been quietly dropped.[5] It seems irrational for Marxian economics to emulate a dead end pioneered by its rival research program.

Although this issue of linear difference equations may seem a bit arcane, it is actually quite damaging in the context of Morishima's "positive" suggestions, since he exhibits a predilection for conflating the roots of difference equations with Marx's concerns over fluctuations in the reserve army of the unemployed and his long-run tendency for the rate of profit to fall. Although an entire chapter is devoted to this topic, it does not take a barrage of linear algebra to figure out what is going to happen to a laboring population growing at a fixed exponential rate if national product cycles for technological reasons and fixed proportionate labor requirements are assumed. In some crude sense this captures the Marxian notion that labor must dance to the syncopated expansion of capital, but it is not a very interesting "theory." Things mechanically grind along in much the same manner in Morishima's treatment of a change in the composition of capital (M, 137 et seq.). For ease of mathematical manipulation, "technical change" is defined as an increase in some required material inputs a_{ij} accompanied by an offsetting decrease in the labor requirement l_i, such that the resulting labor value of the commodity stays constant. It becomes immediately apparent that this is not technical progress as much as it is some Marxian equivalent of a neoclassical isoquant. It is interesting but not enlightening that the rate of profit falls along such an isoquant, but unfortunately it has nothing to do with Marx's thesis that the social and technical dynamism of capital accumulation would lead inadvertently and inexorably to a declining rate of profit.

Morishima's "Fundamental Marxian Theorem" (which asserts that exploitation of laborers by capitalists is necessary and sufficient for the existence of a set of prices and wages yielding positive profits) and his solutions to the dynamic transformation problem, while admittedly the most original and substantial achievements of the entire book, do not qualify as positive achievements, since Morishima has maintained that the labor theory of value should be abandoned. Who can be exploited if all trades and transactions are contracted without coercion in a market freely responsive to an individual's demand schedules? And what smacks more of scholasticism than transforming prices into useless labor values?

For all these reasons I think we can conclude that *Marx's Economics* is ultimately a futile exercise born of a self fulfilling picture of progress in economic theory. It is a

book written to prove a point; the real point is that the book need never have been written.

Notes

1 Henceforth, "M" will indicate page references to Morishima 1973.

2 In some respects the critical essay by Jaffe (1983) attempts a parallel hermeneutical reading of Morishima on Walras to what this paper attempts for Morishima on Marx.

3 In fact, Luxemburg's complaint against Tugan-Baranovski that his reproduction schemes allowed only for crises of disproportionality is equally germane here.

4 Except for one: "The quantity of commodities created in masses by capitalist production depends on the scale of production and on the need for constantly expanding this production, and not on any predestined circle of supply and demand, on wants that have to be satisfied." (Marx 1973, vol. 2, 75) Alas, one quotation, or even one hundred quotations, do not in and of themselves constitute a sympathetic reading.

5 In his haste to make Marx respectable—that is, like Hicks—Morishima again interprets the mathematics in a manner inimicable to common sense. On pages 126—27 he derives a second-order difference equation in national income, and then suggests that it is in principle the same as one found in Hicks. The fact that two radically different models can result in very similar reduced-form equations does not mean that one theory has been reduced to the other; they can still have differential theoretical content and be competing theories.