Cambridge Journal of Economics 2015, 1 of 20
doi:10.1093/cje/bev030

Kaldor's 'technical progress function' and Verdoorn's law revisited

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Kaldor put forward his technical progress function as an alternative to the neoclassical aggregate production function. It is shown that Verdoorn's law is its empirical counterpart, although allowing for increasing returns to scale. However, both may be derived from an aggregate Cobb-Douglas production function. But aggregation problems and the Cambridge capital theory controversies have shown theoretically that aggregate production functions in all probability do not exist. Moreover, the only reason that estimations of 'aggregate production functions' give good results is the existence of an accounting identity. This article reconsiders the technical progress function and Verdoorn's law, especially in the light of these problems. Nevertheless, it is shown that estimates of the law do, in fact, provide insights into the growth process very similar to those of Kaldor, but viewed from another perspective.

Key words: Kaldor, Technical progress function, Verdoorn's law, Aggregate production functions, Accounting identity critique
JEL classifications: B5, E12, O4

1. Introduction

Nicholas Kaldor was highly critical of explaining economic growth in terms of a neoclassical aggregate production function. In particular, he was extremely sceptical of the attempt to dichotomise economic growth into that attributable to the rate of exogenous technical change and that to the growth of factor inputs, in a manner that was first formulated within an analytical model by Solow (1956) and Swan (1956). He was thus equally critical of the use of the linear and homogeneous aggregate production function where the changing state of knowledge and the steady-state growth of productivity are represented by a continuous exogenous shift of the aggregate production function over time (as in the empirical study of Solow, 1957). It made no sense, according to Kaldor, to view capital accumulation as merely the replication of existing capital goods, as the act of investment itself generated new and improved methods of production.

Manuscript received 3 January 2015; final version received 30 March 2015.
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We are grateful for the helpful comments of two anonymous referees and to Jesus Felipe, who has collaborated extensively with John McCombie on the accounting identity critique.

Consequently, in 1957 Kaldor first put forward his alternative approach based on the concept of the technical progress function (although he makes no reference to Solow's growth model). The initial technical progress function is a dynamic production relationship, relating the growth of productivity to the growth of the capital-labour ratio. The specification changed in Kaldor and Mirrlees (1962) to a vintage approach, but the basic insights remained largely the same. In a manner that ironically anticipated the later neoclassical endogenous growth theories, Kaldor argued that the growth of capital per worker induced technical change but at a diminishing rate. Although he nowhere explicitly stated so, it seems that he regarded Verdoorn's law as the empirical counterpart of his initial specification of the technical progress function (Kaldor, 1966). Verdoorn's law, in its simplest form, is the linear relationship between the growth of productivity and output. An estimate of the Verdoorn coefficient (the coefficient of output growth) that is statistically significantly greater than zero implies increasing returns to scale. (See Blankenburg and Harcourt, 2007, on increasing returns.) The original Verdoorn's law was deceptively simple (Kaldor, 1966), but since then more sophisticated specifications and econometric estimations have largely confirmed the original results (see, for example, the references in Angeriz *et al.*, 2008). There is, however, one major difference between Verdoorn's law and the technical progress function. This is the emphasis on increasing returns to scale in the interpretation of the estimates of Verdoorn's law. This is in accord with the greater emphasis Kaldor placed on increasing returns to scale in his later writings, beginning with his seminal 1972 paper, 'The irrelevance of equilibrium economics'. In this, Kaldor was heavily influenced by Young (1928) and he emphasised the process of cumulative causation in economic growth. However, the linear version of technical progress function exhibits constant returns to scale, and the non-linear version has decreasing returns to the growth of the capital-labour ratio.

There are two serious related criticisms of the technical progress function, which may be the reason that it virtually disappeared from the literature without trace after about the early 1960s. The first was that a linear approximation of the technical progress function around, for example, its steady-state growth rate could be integrated to give a Cobb-Douglas production function. Moreover, Kaldor himself used the linear version in his theoretical models. Notwithstanding the fact that the non-linear version could not be derived from a Cobb-Douglas production function (Black, 1962), this considerably reduced its novelty. Second, to the extent that the technical progress function represents an aggregate production relationship (albeit without any recourse to the marginal productivity theory of factor pricing or the other usual neoclassical assumptions), it is subject to the severe criticisms that the neoclassical production function faces from the aggregation problem and the Cambridge capital theory controversies. Ironically, it also suffers from the criticisms Kaldor himself made of the neoclassical production function.

In spite of these problems, estimations of aggregate production functions generally, but not always, give good statistical fits, with plausible estimates of the output elasticities. It is now well established that this results solely from the use of value data, instead of physical magnitudes, for output and the capital stock. All that the estimates of putative aggregative production functions are picking up are a (transformation) of an underlying national income and product accounts accounting identity. This poses a paradox in that the best statistical fits using these data should theoretically give estimates that could be (erroneously) interpreted as indicating constant returns to scale.

(It should be emphasised that this is irrespective of whether the actual production processes, expressed in engineering or physical terms, display diminishing, constant or increasing returns to scale.) However, Verdoorn's law commonly gives estimates that suggest that there are substantial increasing returns to scale.

This article provides an explanation for this conundrum and provides support for Kaldor's insights as to the nature of economic growth, although from a different perspective.

In the next section, we consider the theoretical foundations of the two technical progress functions and discuss their limitations. In Section 3 we show how the original technical progress function is related to Verdoorn's law and the Cobb-Douglas production function. In Section 4 we briefly discuss the aggregation problem and the accounting identity critique. Section 5 explains why estimates of Verdoorn's law may have a statistically significant positive Verdoorn coefficient, whilst the accounting identity suggests that it should be equal to zero. The argument is illustrated by the use of two hypothetical data sets. Section 6 concludes.

2. The technical progress function(s)

There are essentially two versions of the technical progress function. The first (Mark I) was put forward in Kaldor (1957, 1961) and the second (Mark II) in Kaldor and Mirrlees (1962), although we argue the latter was not markedly different from the former. Kaldor's views on modelling growth were in marked contrast to the neoclassical approach as exemplified by Solow (1956) and Swan (1956). In 1957 Solow had, by using the neoclassical production function and the marginal productivity conditions, shown that the rate of exogenous technical progress accounted for over 80% of productivity growth of the USA during the first half of the twentieth century. For Kaldor, this exercise made no sense theoretically.

> The rate of shift of the production function due to the changing state of 'knowledge' cannot be treated as an independent function of (chronological) time, but depends upon the rate of accumulation of capital itself. Since improved knowledge is, largely if not entirely, infused into the economy through the introduction of new equipment, the rate of shift of the curve will depend on the *speed of movement* along the curve, which makes any attempt to isolate the one from the other the more nonsensical. (Kaldor, 1961, p 207, emphasis in original)[1]

Thus in many ways, Kaldor anticipated Arrow's (1962) learning-by-doing model, induced technical change and endogenous growth theory. Kaldor (1961, pp 206–7), for example, discusses 'constant or increasing productivity to capital accumulation'. Consequently, apart from an exogenous component (determined by the dynamism of the economy), productivity growth increases with the growth of the capital-labour ratio.

[1] Kaldor (1961, p 205, emphasis in original) correctly anticipated that Solow's (1957) growth accounting procedure was a classic case of circular reasoning. 'Since the *slope* of the curve [of the production function] . . . is supposed to determine the share of profits in income, the share of profits is taken to be an indication of its slope, and the residual is then attributed to the shift of the curve! There could be no better example of *post hoc ergo propter hoc*'. In 1974, Solow, in response to a criticism of his method by Shaikh (1974), eventually admitted that his procedure was a tautology, although this was not the impression given in his 1957 paper. In that paper, regression analysis was used to estimate various specifications of the aggregate production function, and Solow commented on the very good statistical fits they all gave. Why was this necessary if they were based on a tautology? See also Shaikh's (1980) rejoinder to Solow (1974) and the symposium on Shaikh's (1974) critique in volume 17, issue 1, of *Global and Local Economic Review* (Velupillai, 2013).

However, because new ideas are exploited first and there are limits to the capacity to absorb these, the increase in induced productivity growth will be at a diminishing rate.

The technical progress function, Mark I, takes the form:

$$p_t = f(k_t - l_t), \quad f(0) > 0, f' > 0 \text{ and } f'' < 0 \tag{1}$$

where p, k and l are the rates of growth of productivity, the capital stock and employment. The relationship is shown in Fig. 1, where the technical progress function is given by the curve TT. Kaldor shows that steady-state growth occurs at point A, where the growth of productivity and the capital-labour ratio is equal and there is no growth in the capital-output ratio.

We are concerned only with the technical progress function, rather than Kaldor's full model, but he presents a non-neoclassical explanation about how steady-state growth is achieved. However, as Meade and Hudson pointed out, there were problems with convergence in the 1957 model, although these were solved by respecifying the investment function in Kaldor (1961) (see Harcourt, 1963, on this or the reprint in Harcourt 1963 [1982], p 72).

However, at point A, the non-linear technical progress function may be approximated by a linear function, T′T′, namely:

$$p_t = \lambda + \alpha(k_t - l_t) \tag{2}$$

where λ is exogenous technical progress, determined by the dynamism of the economy, and α is a constant.

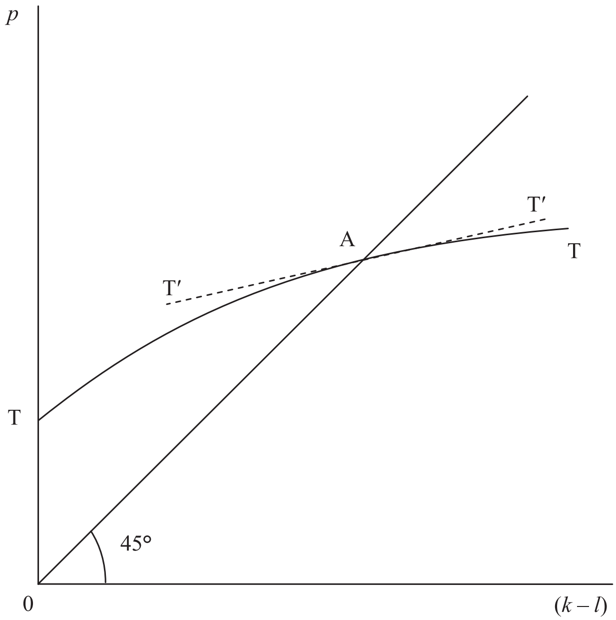

Fig. 1. *The technical progress function (Mark 1)*

Ironically, the steady-state equilibrium growth conditions turn out to be exactly the same as in the Solow neoclassical growth model. The growth of output equals the growth of the capital stock and the growth of productivity is entirely determined by the rate of exogenous technical progress ($p_t = \lambda/(1 - \alpha)$) and, as in the Solow model, is *not* a function of the share of investment in output.

Furthermore, the linear technical progress function, eq. (2) may be integrated to give a conventional Cobb-Douglas production function, that is,

$$P_t = A_0 e^{\lambda t} \left(K_t / L_t\right)^{\alpha} \text{ or } Q_t = A_0 e^{\lambda t} K_t^{\alpha} L_t^{(1-\alpha)} \tag{3}$$

where the uppercase notation denotes levels. The parameters α and $(1 - \alpha)$ are the output elasticities of capital and labour.

This was pointed out, *inter alios*, by Green (1960), Black (1962) and Eltis (1971), although Kaldor (1961, p 215, n 1) was well aware of the problem, having had it pointed out to him by Hahn and Meade.

However, as Black (1962) has shown, the non-linear version cannot be integrated to give a conventional aggregate Cobb-Douglas production function. He demonstrates that the non-linear version of the technical progress function is novel in that it introduces 'path dependence'. Let us take an initial point A on the technical progress function, which shows diminishing returns. The slope of the technical progress function at this point is given by $\alpha(1 + \psi)$ where α is the slope of the linear approximation at A and ψ is the elasticity of $dP/d(K/L)$ with respect to changes in $d(k/l)/dt$. The condition that $\psi < 0$ shows that the slope diminishes as $k - l$ increases. This has the result that if we start with any initial point on the technical progress function (say, at time $t = 0$) then as K/L increases to a value at $t = 1$, there is a whole set of possibilities for the value of Q/L, the level of productivity, depending on the time path of investment over this period. The greatest possible level of productivity is achieved if the rate of investment over all time periods is constant. If investment is bunched in a few of the periods, because of the curvature of the technical progress function, the level of productivity will be less than if the investment had been evenly spread. In these circumstances, it is not possible to integrate the technical progress function into a production function.

Probably because of these problems, Kaldor (with Mirrlees) in 1962 proposed a second version of the technical progress function. This was specified as a vintage growth model, namely the growth of productivity on newly installed equipment and the rate of growth of gross investment. In fact, in the steady-state growth rate this becomes identical to the rate of growth of productivity in the whole economy. The motivation for this change seems to be twofold. First, the function clearly cannot be integrated into a conventional aggregate production function. Second, it sidesteps the problems of the measurement of capital.

> Since, under continuous technical progress and obsolescence, there is no way of measuring the 'stock of capital' (measurement in terms of the historical cost of the surviving capital equipment is irrelevant) . . . the model avoids the notion of a quantity of capital, and its corollary, the rate of capital accumulation, as variables of the system; it operates solely with the value of current gross investment (gross (fixed) capital expenditure per unit of time) and its rate of change in time. (Kaldor and Mirrlees, 1962, p 174)

This has much in common with Salter's (1960) vintage approach and also anticipates Foley and Michl's (1999) formulation of a non-neoclassical 'fossil production function'.

This version of the technical progress function is specified as:

$$p_{jt} = f\left(\frac{dI_{jt}/dt}{I_{jt}}\right),\ f(0) > 0, f' > 0 \text{ and } f'' < 0 \tag{4}$$

where I is the amount of investment per worker and j denotes the latest vintage. The relationship is the same as in Fig. 1, except the growth of investment per worker is now on the horizontal axis. However, in steady-state growth, the investment-output ratio remains constant, so the growth of investment equals the growth of output, which in turn equals the growth of the capital stock, and we are back with the original version of the technical progress function and its limitations.[2] It is difficult not to agree with Scott (1989, p 111) when he comments that it was never Kaldor's intention that his model should turn out to be equivalent to the Cobb-Douglas or that the rate of productivity growth should be independent of the investment-output ratio. 'Hence the proposed technical progress function cannot be regarded as a satisfactory way of giving effect to the relationships that Kaldor had in mind'.

3. Verdoorn's law, the technical progress function and the aggregate production function

According to Verdoorn's law, a faster growth in output increases productivity growth as a result of increasing returns, broadly defined to also include induced technical progress. See the collection of essays in McCombie *et al.* (2002) for a discussion of many aspects of Verdoorn's law.[3]

Whilst Kaldor did not explicitly relate Verdoorn's law to his technical progress function, it is clear that he regarded the former as a production relationship, expressed in growth rates. As Kaldor (1966, emphasis in original) noted in his inaugural lecture where he first drew attention to this relationship, 'it is a dynamic rather than a static relationship—between the rates of change of productivity and of output, rather than between the *level* of productivity and the *scale* of output—primarily because technological progress enters into it, and is not just a reflection of the economies of large-scale production'. He also made reference to Arrow's (1962) learning-by-doing model. Dixon and Thirlwall (1975), in their formalisation of Kaldor's cumulative causation growth model, do make a direct comparison between the technical progress function and Verdoorn's law.

As will be shown, there are many similarities between the (linear) technical progress function and Verdoorn's law. Ironically, the law may also be derived for the ith region (or country) from an aggregate Cobb-Douglas production function (always assuming that it exists) as follows:

$$Q_{it} = A_0 e^{\lambda t} K_{it}^{\alpha} L_{it}^{\beta} \tag{5}$$

with the same notation as before.

[2] Kennedy's (1964) model of induced bias in innovation and the innovation possibility function will generate a relationship similar to the technical progress function without any learning involved.

[3] The estimation of Verdoorn's law has become progressively more sophisticated over the years. Angeriz et al.(2008), for example, estimate the augmented Verdoorn law using spatial econometric methods and including variables to capture the effect of the diffusion of innovations and the density of production together with spatial spillover effects.

It is assumed that $(\alpha+\beta)=\gamma\left(\alpha'+\left(1-\alpha'\right)\right)$, where γ is a measure of the degree of static returns to scale. A key assumption of Verdoorn's law is that the rate of technological progress is largely endogenously determined by growth of the weighted factor inputs and this relationship may be expressed as:

$$\lambda_t = \lambda' + \eta\left[\alpha k_{it} + \beta l_{it}\right] \tag{6}$$

where η is the elasticity of induced technical progress with respect to the weighted growth of the inputs and k and l are again the growth rates of capital and labour, respectively. λ' is the rate of exogenous total factor productivity growth. This 'postulates a relationship between the rate of increase of capital [per worker] and the rate of increase in output [per worker] which embodies the effect of constantly improving knowledge and know-how, as well as the effect of increasing capital per man, without any attempt to isolate one from the other' (Kaldor, 1961, pp 207–8).

Taking logarithms of eq. (5), differentiating with respect to time, using eq. (6) and rearranging gives:

$$q_{it} = \lambda' + \gamma(1+\eta)\left[\alpha' k_{it} + \left(1-\alpha'\right) l_{it}\right] \tag{7}$$

where $\gamma\left(1+\eta\right)=\mu$ is an encompassing measure of the degree of dynamic and static returns to scale. This is assumed to be constant across regions. Re-arranging eq. (7) yields the dynamic Verdoorn's law (i.e. the relationship expressed in terms of growth rates):

$$tfp_{it} = \frac{\lambda'}{\mu} + \left(1-\frac{1}{\mu}\right) q_{it} \tag{8}$$

where *tfp* is the growth of total factor productivity and is defined as:

$$tfp_{it} \equiv q_{it} - \left[\alpha' k_{it} + (1-\alpha') l_{it}\right] \tag{9}$$

For expositional reasons only, let us assume Kaldor's stylised fact that the growth of capital equals the growth of output. This gives Verdoorn's law as:

$$p_{it} = \frac{\lambda'}{\mu\left(1-\alpha'\right)} + \left(\frac{\mu-1}{\mu\left(1-\alpha'\right)}\right) q_{it} \tag{10}$$

A typical value for the Verdoorn coefficient is about 0.5 and this, together with a commonly found value of 0.75 for $(1-\alpha')$, implies an encompassing degree of returns to scale (i.e. the effect of induced technical progress and dynamic and static economies of scale) of 1.6.

There are two important differences between this and Kaldor's technical progress function. First, the (linear) technical progress function, if it is derived from a Cobb-Douglas production function, exhibits constant returns to scale. Verdoorn's law allows for increasing returns to scale, as well as induced technical progress. As we have seen, this reflects Kaldor's shift away from merely emphasising the impossibility of dichotomising productivity growth into that due to the growth of factor inputs and that due to the rate of technical change. It captures his later emphasis on the importance of economies of scale, broadly defined, and the resulting cumulative causation nature of growth (Kaldor, 1970, 1972, 1981).

4. Aggregation problems, the accounting identity and the aggregate production function

Does the aggregate production function, including the technical progress function and Verdoorn's law, theoretically exist? The answer according to Fisher (1987, 1992, 2005), who has done more work on the aggregation problem than most, is emphatically 'no', not even as an approximation.

Further problems arise from the Cambridge capital theory controversies of the 1960s and 1970s, although the issue was first given prominence by Joan Robinson (1953–54). This showed clearly how none of the results of the 'neoclassical parable' held once one moved out of a one-commodity world (Cohen and Harcourt, 2003A). The two critiques are related, although Cohen and Harcourt (2003B, p 232, emphasis in original) agree with Fisher (1971) that 'the aggregation debate is a development *within* neoclassical theory and its applications, whereas much of the Cambridge, England, critique is from *without,* regarding the basic neoclassical intuition, robustness in more general models and appropriate methods'. Nevertheless, both critiques serve to show just how flimsy the foundations of the aggregate production function are. Whilst both criticisms were briefly acknowledged in textbooks and surveys in the 1960s and1970s, any reference to them has now disappeared from the current literature.

Kaldor, of course, was very well aware of the capital theory problems and commented:

> In the absence of any reliable measure of the quantity of capital (in a world where the technical specifications of capital goods is constantly changing, new kinds of goods constantly appear and others disappear) the very notion of 'the amount of capital' loses precision. The terms 'income' or 'capital' no longer have any precise meaning; they are essentially *accounting magnitudes*, which merely serve as the basis for calculations in business planning; the assumption that money has a stable value in terms of some price index enables us to think of 'income' and 'capital' as real magnitudes only in a limited, and not precisely definable, sense. (Kaldor, 1961, p 203, emphasis added)

Nevertheless these reservations did not prevent him from using a measure of capital as a homogeneous physical quantity in his theoretical models.

So why is the aggregate production function so widely and uncritically used? The answer seems to involve a form of Friedman's (1953) methodological instrumentalism. All theories, so the argument goes, involve heroic abstraction and unrealistic assumptions, but what matters is their predictive ability. The aggregate production function passes this test with flying colours, or so it seems. The problem with this defence is that the estimation of a putative aggregate production function using constant-price monetary data cannot provide any inferences about the values of the parameters of the production function (i.e. output elasticities and the aggregate elasticity of substitution) or the rate of technical change. This is because, empirically, constant-price monetary data have to be used as measures for output and capital, instead of physical magnitudes, and an underlying accounting identity precludes any meaningful estimation of an aggregate production function (Felipe and McCombie, 2013).

The implications are far reaching. The existence of the constant-price value accounting identity implies that any estimation of a putative aggregate production can be made through a suitable specification to give a perfect fit to the data. The results must show 'constant returns to scale' and that the estimates of the output elasticities equal their respective factor shares. This will occur even though the aggregate production function undoubtedly does not exist and, for example, individual firms may be subject to substantial returns to scale (Felipe and McCombie, 2006).

There are several ways of presenting the critique and it may be equally applied to cross-sectional (cross-industry) regressions and time-series analysis. As the issues have been fully discussed in the book by Felipe and McCombie (2013) we shall be brief, although it is a deceptively simple argument. It should be stressed that this critique is a matter of logic; the argument is either correct or incorrect. It is not an econometric problem, such as the statistical identification of the aggregate production function.

The application of the critique to time-series data was initially shown by Shaikh (1974). The value-added accounting identity is given from the national and product accounts as:

$$V_t \equiv R_t J_t + W_t L_t \tag{11}$$

where V is value added measured in constant prices, R is the rate of profit, J is the constant-price value of the capital stock, W is the real wage rate and L is employment.[4] Differentiating eq. (11) with respect to time, we obtain:

$$v_t \equiv a_t r_t + \left(1 - a_t\right) w_t + a_t j_t + \left(1 - a_t\right) l_t \tag{12}$$

where v, r, w, j and l denote exponential growth rates of the various variables. The variable $a_t \equiv R_t J_t / V_t$ is capital's share in output and $(1 - a_t) \equiv W_t L_t / V_t$ is labour's share. Assuming that factor shares are constant and integrating eq. (12), we obtain:[5]

$$V_t \equiv B R_t^a W_t^{(1-a)} J_t^a L_t^{(1-a)} \tag{13}$$

where the constant of integration is $B = a^{-a}(1 - a)^{-(1-a)}$. Let us assume that the growth of the wage rate occurs at a roughly constant rate ($w_t = w$) and the rate of profit shows no secular growth ($r = 0$), both of which may be regarded as stylised facts. Consequently, $ar_t + \left(1 - a\right) w \cong (1 - a)w = \lambda$, a constant, and so eq. (13) becomes the familiar Cobb-Douglas with exogenous technical change, namely:

$$V_t = A_0 e^{\lambda t} J_t^a L_t^{(1-a)} \tag{14}$$

[4] We use V and J for the constant price value measures and reserve the notation Q and K for the physical quantities.

[5] Strictly speaking, we do not need explicitly to make this assumption as if we integrate eq. (12) at any one point in time, then the factor shares must be constant. Thus, for, say, any one year, eqs (11) and (13) are exactly equivalent. It is only when we use different observations to estimate the Cobb-Douglas 'production function', using either cross-sectional or time-series data, is it necessary to assume that the 'output elasticities' do not change.

but where the exponents are the factor shares. This is a prediction of the marginal productivity theory of factor pricing which must occur, even though none of the neoclassical assumptions hold.

The use of time-series data sometimes produces implausible estimates of the parameters of the supposed aggregate production function, with, for example, the estimate of the 'output elasticity of capital' often taking a negative value. Sylos Labini (1995, Table 1, p 490) provides a useful summary of a number of time-series studies that give poor statistical results. This may ironically give the impression that the estimated equation is actually a behavioural relationship. However, the failure to obtain plausible estimates of the parameters will occur if (i) either the factor shares are not sufficiently constant or (ii) the approximations $a_t r_t + (1 - a_t) w_t \cong \lambda$ (i.e. a constant) and $a_t lnR_t + (1 - a_t) lnW_t \cong \lambda t$ are not sufficiently accurate, or both. It is usually found that the rate of profit, especially, has a pronounced cyclical component and so proxying the weighted growth rates of *R* and *W* by a constant (or the sum of the weighted logarithms of *R* and *W* by a linear time trend) biases the estimated coefficients of *lnL* and *lnJ* (McCombie, 2000–2001; Felipe and Holz, 2001; Felipe and McCombie, 2013).[6] This requires either a complex non-linear time trend or the capital stock to be adjusted for changes in 'capacity utilisation', which reduces the cyclical fluctuation in *r* so that a linear time trend gives a good statistical fit to the accounting identity.

However, it must be emphasised that the critique does not apply to just the Cobb-Douglas production function but to *any* specification of an aggregate production function.

The accounting identity may be expressed as follows:

$$V_t \equiv R_t \mathcal{J}_t + W_t L_t \Rightarrow v_t \equiv a_t r_t + (1 - a_t) w_t + a_t j_t + (1 - a_t) l_t$$

$$\Rightarrow v_t \equiv \lambda_t + \alpha_t j_t + \beta_t l_t \Rightarrow V_t = f(\mathcal{J}_t, L_t, t) \quad (15)$$

with the arrows showing the 'direction of causation'. This implies that $a_t \equiv \alpha_t$ and $(1 - a_t) \equiv \beta_t \equiv (1 - \alpha_t)$. Attempts are made to fit different functional forms to the underlying data generating eq. (15). In other words, the aim is to find a specific functional form for $V_t = f(\mathcal{J}_t, L_t, t)$. If factor shares change, then either the CES or the translog 'production function' may give a better statistical fit to the underlying accounting identity than the Cobb-Douglas. But these cannot be interpreted as aggregate production functions.

5. Does the accounting identity invalidate Verdoorn's law?

Not surprisingly, the accounting identity also poses problems for estimating the technical progress function. As far as we are aware, there have only been two attempts to test the function empirically, namely, Bairam (1995) and Hansen (1995), and they both use the same methodology. This is the Box-Cox specification. Whilst Bairam found the best estimate gave a convex function, Hansen found a better specification was a

[6] Because we are dealing with an identity, we treat the regressions using either logarithms of the levels or exponential growth rates as equivalent.

linear function. The difference between the two results was due to Bairam proxying the growth of the capital stock by the initial investment-output ratio, whilst Hansen used the more appropriate estimates of the growth of the capital stock, calculated by the perpetual inventory method.

The non-linear technical progress function with diminishing returns to growth in the capital-labour ratio may, for example, be expressed as:

$$p_t = \lambda'_t + \rho(k_t - l_t) + \delta(k_t - l_t)^2 \tag{16}$$

where $\rho > 0$ and $\delta < 0$.

However, when factor shares are constant, the accounting identity may be expressed as:

$$p_t \equiv ar_t + (1-a)w_t + a(j_t - l_t) \tag{17}$$

Consequently, the best statistical fit to the technical progress function will be given by the *linear* function, because the underlying identity expressed as eq. (17) must always hold, by definition. Consequently, $\rho = a$ and $\delta = 0$. In the light of this, Hansen's result is hardly surprising. The technical progress function could, in turn, be erroneously interpreted as being derived from the aggregate Cobb-Douglas production function, which, we have shown, does not exist.

When we consider further the empirical relationship between the technical progress function, Verdoorn's law and the accounting identity, a paradox arises. We have seen that the accounting identity must hold, even though a well-defined aggregate production function does not exist. Furthermore, the best statistical fit will occur when the estimated parameters, or 'output elasticities', equal their respective factor shares. If we assume for expositional ease that these are constant, then estimating the equation

$$v_t = b_1 r_t + b_2 w_t + b_3 j_t + b_4 l_t \tag{18}$$

must give estimates of the coefficients b_1 and b_3 that equal capital's share and those of b_2 and b_4 that equal labour's share. The sum of the estimates of the coefficients of b_3 and b_4 will, by definition, equal unity. But the results of estimating Verdoorn's law implicitly suggests that these will sum to greater than unity, even though value data are used. This is because the Verdoorn coefficient is often statistically greater than zero. Thus, the estimates of Verdoorn's law have generally been interpreted as showing substantial increasing returns to scale.

Consequently, two questions arise. First, why does the estimation of Verdoorn's law using value data suggest increasing returns to scale, when the accounting identity suggests the law should exhibit constant returns to scale? Second, given that we are using value data and an underlying identity, do the statistical results have any economic interpretation?

5.1. A simulation experiment

To answer these two questions, we assume for expositional ease Kaldor's stylised fact that the growth of the capital stock equals the growth of output (i.e. the capital-output ratio is constant). As a consequence of also assuming that factor shares are constant, this implies that the growth in the rate of profit is zero. It should be emphasised that nothing hangs on these assumptions; they just make the exposition easier. We could relax them if necessary without materially affecting the argument.

It is useful to answer the questions using some hypothetical data, as this has the advantage that we know by construct the true underlying relationships. The foregoing assumptions also have the advantage of simplifying the interpretation of the results.

From the accounting identity given by eq. (12) and the two assumptions, or stylised facts, we can derive the relationship that:

$$p_{it} \equiv v_{it} - l_{it} \equiv w_{it} + 0.v_{it} \tag{19}$$

where i denotes the region (or country). In other words, the growth of productivity is definitionally equal to the growth of the real wage, which varies both with time and between regions. It can be seen that if, using average growth rates over a single period (denoted below with the subscript t) and cross-regional (or country) data, we were to estimate Verdoorn's law as:

$$p_{it} = c_i + b_5 v_{it} \tag{20}$$

where the intercept is allowed to vary between the regions, the estimate of the Verdoorn coefficient (b_5) could be not statistically significantly different from zero. (The estimation of eq. (20) could be done either by using regional intercept dummies or by estimating a fixed-effects model.)

In other words, the conventional interpretation would be that the null hypothesis of increasing returns to scale should be rejected. As eqs (19) and (20) are derived from the identity, they will always give the best statistical fit. However, it has been shown that this must always be the case, irrespective of whether the true underlying microeconomic production relationships (measured in physical units) show increasing returns to scale.

But in all studies estimating Verdoorn's law, a common intercept is specified in the model and eq. (21) is estimated.

$$p_{it} = c + b_5 v_{it} \tag{21}$$

The intercept is interpreted as the common rate of exogenous productivity growth. As we have noted already, in most studies using cross-regional or cross-country data, a statistically significant coefficient of about one-half is found. To illustrate why this occurs, we constructed two hypothetical data sets for the above variables, namely, p, w and v, to estimate the equations. It should nevertheless be emphasised that this is merely illustrating the theoretical argument.

There are 15 hypothetical regions, each region with growth rates calculated over 10 periods, giving 150 observations in total. We assumed that the growth rates of each individual region did not vary greatly over the 10 periods. In other words, for region (country) i the growth rate of output was x% per period for each of the 10 periods with a small random term added to prevent perfect multi-collinearity. Consequently, there was little difference in the growth of productivity and output between the different periods for each region. This is what is normally found in much of the regional data: growth rates do not show great differences over time. However, the data were constructed such that there was a significant difference in the productivity and output growth rates between the different regions. Some regions grew persistently faster than others.

The model given by eq. (21) was first estimated using intercept dummies to allow the regional intercepts to vary. The regression results are as follows:

$$p_{it} = c_0 + \text{dummies} + \underset{(0.96)}{0.044} v_{it} \qquad R^2 = 0.989$$

where c_0 is the baseline intercept. The regression is controlled for heteroscedasticity. The values of the regional intercepts range from 0.941% per period (38.42) to 3.481% (11.41). The figures in parentheses are the t-statistics. The coefficient of the growth of output is not significantly different from zero. (Using the fixed-effects estimator gives an identical estimate for the Verdoorn coefficient.)

Consequently, and not surprisingly, the results confirm eqs (19) and (20), namely, because of the identity, there is no relationship between productivity and output growth. Hence, if we were to interpret the equation as a behavioural equation, we would reject the null hypothesis of increasing returns to scale. But as noted already, the traditional specification of Verdoorn's law assumes that all regions have the same rate of exogenous productivity growth. When we impose a common intercept in the regression the following result is obtained:

$$p_{it} = \underset{(35.15)}{1.128} + \underset{(39.44)}{0.445} v_{it} \qquad R^2 = 0.955$$

In other words, a statistically significant Verdoorn coefficient is found with a value of about one-half, suggesting the existence of substantial increasing returns to scale.

The reason for this may be seen in Fig. 2(a), which is a stylised representation of the data. The relationship given by the accounting identity is shown by the three solid lines, AA, BB and CC. Thus, for each of the regions, the Verdoorn coefficient is statistically insignificant and the within-R^2, that is, the correlation provided by the data within each region, is negligible. The overall R^2 is large because the dummies are again explaining nearly all the variation in p_{it}. However, when the intercept is held constant, and we consider the cross-section regression results, a faster growth of output leads to a faster growth of productivity (shown by the dashed line). This may be regarded as an auxiliary relationship of the identity and is a behavioural relationship. There is no theoretical reason arising from the identity as to why we should necessarily find this Verdoorn relationship.

The argument is, in fact, more general than this because in terms of Fig. 2(a) the relationships given by AA, BB and CC could comprise a mixture of different regions, with approximately the same productivity and output growth rates. The best statistical fit will be given when the fixed effects capture these groupings by productivity growth and the regression estimates are of the solid lines, as depicted in the figure. For expositional ease, we assumed above that the observations given by AA, and so on, were simply for one region each.

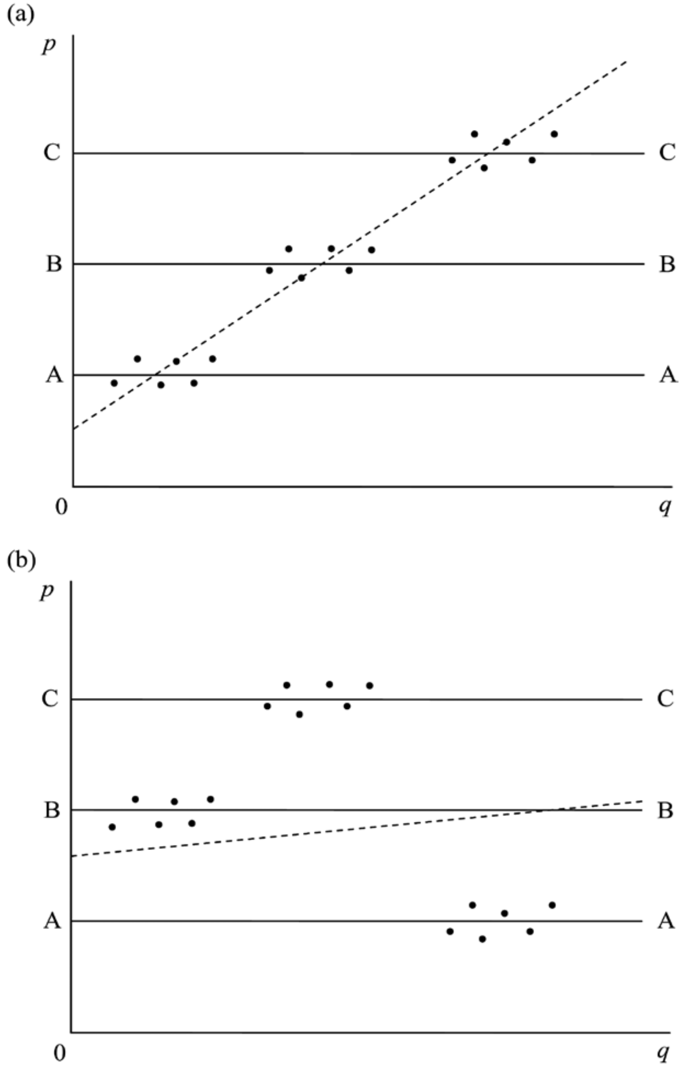

Fig. 2. *The accounting identity and Verdoorn's law*

The fact that there may be no cross-sectional Verdoorn's law is confirmed by a second hypothetical data set. This was constructed similarly to the first data set, with the exception that for any given productivity growth rates of a particular region, the output growth rates were random. The results of estimating Verdoorn's law using regional dummies, not surprisingly, gave similar results to before.

$$p_{it} = c_0 + \text{dummies} - \underset{(-0.16)}{0.008}\, v_{it} \qquad R^2 = 0.980$$

The Verdoorn coefficient is again not statistically significant, whilst the estimates of the regional intercepts (i.e. the regional real wage, or productivity, growth rates) are all highly significant.

The range of the intercepts are from 2.287% per period (26.27) to 7.742% (84.38), with the *t*-statistics again in parentheses. It can be seen that as we should expect, the regional dummies are explaining nearly all the variation in the regional productivity growth rates.

It should be emphasised that the difference in the statistical goodness of fit between the two simulation exercises is simply due to the size of the error term introduced to prevent perfect multi-collinearity and is, to this extent, arbitrary.

However, when using this data set we estimate the law with a common intercept, and Verdoorn's law is now statistically insignificant.

$$p_{it} = \underset{(30.79)}{4.738} + \underset{(0.34)}{0.093} v_{it} \qquad R^2 = 0.0008$$

This is shown in Fig. 2(b). It can be seen that now the slope of the cross-sectional Verdoorn law given by the dashed line is not significantly different from zero. Hence, the cross-sectional Verdoorn law (i.e. with a common intercept) is an empirical relationship and is unaffected by the identity.

There is one further issue with the above regressions. This is that eq. (20) suffers from the problem that p (or w) is itself definitionally related to v. However, this does not invalidate the argument as from the identity we have $p_i \equiv w_i + 0.l_i$. It follows that:

$$p_i = c + b_5 v_i = \frac{1}{(1-b_5)} c + \frac{b_5}{(1-b_5)} l_i \tag{22}$$

Consequently, Verdoorn's law also implies that a faster growth of employment causes a faster growth of the real wage or productivity. Thus, it can be seen that Verdoorn's law with a common intercept imposed does not suffer from the problem of merely reflecting an identity, as w is not definitionally related to l.

However, as Verdoorn's law with a common intercept is a behavioural relationship, econometric issues come into play, such as possible problems of simultaneous equations bias and the need for spatial econometric estimation methods. The use of the growth of factor inputs as the regressors assumes a different error structure to the use of output growth, but we do not pursue such econometric issues here.

What are the implications of these regression results for the interpretation of Verdoorn's law? The first is that the growth of the real wage may be interpreted broadly as the growth of the efficiency in production of each worker. But the Verdoorn's law relationship does *not* reflect a conventional aggregate production function, which in all likelihood does not exist. Consequently, the intercept cannot and should not be interpreted as the separate contribution to economic growth of the rate of exogenous technical change.

The Verdoorn coefficient also should not be interpreted as a measure of increasing returns to scale *per se*. Both these interpretations require the existence of an underlying aggregate production function. All that can be said is that the faster growth of output measured in constant-price monetary terms, for a variety of unspecified reasons, leads to a faster growth of the real wage. This could be because of some combination of increasing returns to scale, induced and exogenous technical change, greater efficiency in the use of resources (such as reduction in X-inefficiency), and the inter-sectoral reallocation of resources. But it is not possible, even in principle, to quantify their effects, such as in the neoclassical growth accounting approach of Denison (1967), which assumes the existence of an aggregate production function.

Thus, growth does occur in a cumulative causation manner. It is ironic that this provides a justification for Kaldor's scepticism in undertaking such an exercise of disaggregating the causes of economic growth in terms of an aggregate production function. Hence, it provides a rationale for his unsuccessful attempt to remedy this through the concept of the technical progress function.

6. Fabricant's law: a further illustration

In this section, we use the detailed real-world (instead of hypothetical) data from Oulton and O'Mahony (1994) to provide a further illustration of the problem that the identity poses. We show how the arguments apply equally to Fabricant's law, which is simply Verdoorn's law estimated using cross-industry data, rather than international or regional data. The conventional interpretation of Fabricant's law is the same as that of Verdoorn's law.

Oulton and O'Mahony (1994) constructed a database for UK manufacturing industries of the growth rates of gross output, capital, labour, materials and the growth of the real wage rate, the rate of profit and the relative price of materials. The data are for individual industries at the three-digit Minimum List Heading for five periods from 1954 to 1986, with 1979–82 being somewhat anomalous due the deep recession during these years. This gives over 1,000 observations.

Assuming an aggregate production function and using the growth accounting approach together with its neoclassical assumptions, Oulton and O'Mahony define the growth of multifactor productivity, *mfp*, as:

$$mfp_{it} \equiv y_{it} - \left(\Theta_{Jit} j_{it} + \Theta_{Lit} l_{it} + \Theta_{Mit} m_{it}\right) \tag{23}$$

where y_{it}, j_{it}, l_{it} and m_{it} are the growth rates of gross output, the constant price value of the capital stock, employment and materials of the *i*th industry in period *t*. θ denotes the factor share of the relevant variable in gross output. The implicit assumptions underlying this approach, in addition to the existence of an aggregate production function, are the usual neoclassical ones of the marginal productivity theory of distribution and perfect competition and the theoretical result that the values of the factor shares equal the respective output elasticities. Consequently, rather than using (labour) productivity growth, they use the growth of the productivity of capital, labour and materials, each weighted by its factor shares. This is because they use gross output, rather than value added.

They estimate Fabricant's law, which as we have noted is the specification of Verdoorn's law but using cross-industry (rather than cross-region or international) data. This is given by:

$$mfp_{it} = c + b_6 y_{it} \tag{24}$$

Oulton and O'Mahony estimate the law for the five individual periods and for longer periods (1954–73 and 1973–86). They introduce a number of industry characteristic variables. Generally speaking, they find a statistically significantly Fabricant (Verdoorn) coefficient ranging from 0.125 (1954–73) to 0.482 (1973–76) with a t-statistic of 4.83 and 3.80, respectively (Oulton and O'Mahony, 1994, Table 7.5, p 169). They discuss the results in terms of the presence of static and dynamic increasing returns to scale and labour hoarding. The fact that value data are used presents the same problem as in Verdoorn's law already discussed.

From the accounting identity, we know that:

$$mfp_{it} \equiv \left(\Theta_{Jit} r_{it} + \Theta_{Lit} w_{it} + \Theta_{Mit} pr_{Mit}\right)$$

$$\equiv y_{it} - \left(\Theta_{Jit} j_{it} + \Theta_{Lit} l_{it} + \Theta_{Mit} m_{it}\right) \tag{25}$$

where pr_M is the growth of the price of the material inputs and *mfp* is definitionally equal to the weighted growth of factor prices, *wgfp*, namely,

$$wgfp_{it} \equiv \left(\Theta_{Jit} r_{it} + \Theta_{Lit} w_{it} + \Theta_{Mit} pr_{Mit}\right) \tag{26}$$

Consequently, it follows that:

$$mfp_{it} \equiv wgfp_{it} + 0.y_{it} \tag{27}$$

Hence, if we were to estimate eq. (27), allowing the intercept to vary using a grouped dummies estimator, then all we would be doing is picking up the underlying identity and the coefficient b_6 in eq. (24) could not be statistically significantly different from zero. This is precisely what happens. The regression results using all periods pooled and intercept dummies with 1,040 observations are:

$$mfp_{it} = c_o + \text{ dummies} + \underset{(1.14)}{0.003}\, y_{it} \qquad R^2 = 0.985$$

The individual intercepts range from −12.204% (−13.84) to 11.604% (14.05); t-statistics in parentheses. (The standard errors are robust, i.e. after heteroscedasticity has been controlled for.)

The above regression was estimated using 19 bands, or groups, of industries, with each covering a range of the growth of multifactor productivity of 1 percentage point. An exception is the two extreme tails of the distribution of output growth rates, which consist of four or five observations and were grouped in two separate bands. These bands turn out to be greater than 1 percentage point in width. (Time period dummies were also included.) The width of 1 percentage point growth rate allows enough variation in the growth of output for the coefficients to be estimated. One band and one time dummy had to be omitted because of multi-collinearity.

If we impose a common intercept on the regression we obtain the result that:

$$mfp_{it} = \underset{(-1.31)}{-0.103} + \underset{(14.07)}{0.230}\, y_{it} \qquad R^2 = 0.248$$

This result is very similar to those of the various different sample sizes and periods that Oulton and O'Mahony use, as noted before.

It can be seen that in this case, the explanatory power in terms of the R^2 is not surprisingly considerably lower in the estimation of Fabricant's law than the identity. Nevertheless, Fabricant's law, like Verdoorn's law, is a behavioural relationship and shows that a faster growth of gross output is associated with a faster growth of the weighted real factor prices.

There are a number of explanations for this. It could be that the fastest growing industries are the ones where their relative prices are growing the slowest, where output growth is the fastest and this reflects a demand-side phenomenon. Oulton and O'Mahony rule this out after statistically testing the hypothesis. Another explanation is that it does reflect dynamic increasing returns to scale. A faster growth of output leads to a faster growth of induced multifactor productivity growth, but like Verdoorn's law it is not possible even in principle to quantify the various components of the growth rate.

7. Conclusions

In this article we examined the relationship between Kaldor's technical progress function and Verdoorn's law. It was shown that whilst the non-linear technical progress function cannot be integrated into a conventional aggregate production function, the linear version can be. The non-linear version has some interesting theoretical properties such as the path dependence of a productivity and capital accumulation. However, whilst the technical progress function was an attempt by Kaldor to remove the dichotomy between growth due to technical change and that due to capital accumulation, it never really succeeds. The steady-state growth was the same as that in Solow's neoclassical growth model. It was shown how Verdoorn's law could be regarded as a specification of the linear technical progress function allowing for the possibility of increasing returns to scale. Both can be derived from an aggregate Cobb-Douglas production function.

There is a conundrum in that theoretically the aggregate production function does not exist and all that estimations of supposed production functions are capturing are an underlying identity. Yet estimates of Verdoorn's law generally find that the coefficient of the growth of output is positive and statistically significantly different from zero, whereas the underlying identity implies that it should not be. We provided an explanation of this paradox theoretically and illustrated it with hypothetical data.

It was shown that when regional growth rates are used in a pooled regression and dummy variables (or a fixed-effects estimator) are used to allow for differences in the intercept, the Verdoorn coefficient will always take a value that is not significantly different from zero. This reflects the influence of the accounting identity. However, when a common intercept is imposed, then the Verdoorn coefficient may take the statistically significant value of one half. However, this is a behavioural result and has nothing to do with the underlying identity, as the use of a second hypothetical data set confirms. We also used actual data for individual UK industries and confirmed the first case for Fabricant's law. Imposing a common intercept does give a statistically significant coefficient on the growth of (gross) output, but the use of industry dummies ensures that the coefficient is not statistically different from zero.

How are these results to be interpreted? The intercept is the growth of the real wage rate that reflects broadly the increase in the efficiency of the economy over time. Thus, in the first case a faster growth of output (measured in value terms) leads to an increase in greater efficiency. But as the relationship is not an aggregate production function in the neoclassical sense of the term, it makes no sense to try to determine the contributions of the various factors that determine growth. Neither does it make any sense to talk about the aggregate elasticity of substitution between capital and labour or to test the marginal productivity theory of distribution.

All this is in accord with Kaldor's vision of the economic system and its growth, although perhaps viewed from a different perspective.

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