Answers (and Questions) for Sraffians (and Kaleckians)

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Steedman's 'Questions for Kaleckians' is rightly critical of the lack of attention paid by Kaleckian economists to the input-output nature of production. However, his conclusions about Kaleckian mark-up pricing and the irrelevance of dynamics are incorrect, for three reasons. First, his rendition of a Kaleckian model with constant mark-ups is only conditionally stable. Secondly, there is ample evidence that Kaleckian mark-ups should vary inversely with sectoral profitability. Thirdly, given variable mark-ups, and the other factors which must be considered in a general multisectoral dynamical analysis of capitalism, it is almost inevitable that the fixed points of such a system will be unstable. Long-run conditions are therefore not equilibrium ones, and limitations on mark-ups, etc, derived from equilibrium analysis are not applicable. The Kaleckian focus upon the process of price setting is therefore justified in both the short and long term, although this analysis should, as Steedman asserts, be carried out in the context of explicitly multisectoral models.

1. Steedman's Critique

Steedman's (1992) paper 'Questions for Kaleckians' is directed at the conventional Kaleckian practice of ignoring the input-output nature of production. He argues that input-output relations are a 'brute fact about modern industrial economies' (Steedman, 1992 p. 126) and when this is incorporated into a short-run Kaleckian mark-up pricing framework, many of the price setting and income distribution propositions made by Kaleckians can be shown to be invalid. To establish this case, he employs in the main a simple model of an economy with circulating capital only, producing n commodities, with constant returns to scale, unchanging technology and exogenously given labour and imported inputs:

$$p = (u + pA)(I + \hat{m}) \tag{1}$$

where p is a row vector of prices, A is the production matrix, $\hat{m}$ is a diagonal matrix of industry mark-ups and u is a row vector of exogenous input costs (comprising labour costs and the costs of imported inputs required per unit of output; for details see Steedman, 1992).

After using this system to criticise Kaleckian claims about the impact of a change in mark-ups on prices, the concept of an average mark-up, the wages share and mark-ups, and the notion of vertical integration, Steedman anticipates potential dismissals of his critique, saying that he would not be surprised 'to find

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the foregoing analysis giving rise to such responses/rejections as: Kalecki was never interested in long-period equilibrium theory! ...' (Steedman, 1992, p. 145). He then turns to dynamics, by considering first of all an economy where 'production methods, prices and markups have been unchanged for many periods' (Steedman, 1992, p. 145) so that the above equation applies. Next he considers a once-only change in exogenous inputs from u to $u + \mathrm{d}u$, so that the commodity price vector at the end of period 1 will be given by

$$p_1 = (u + \mathrm{d}u + pA)(I + \hat{m})\ldots \qquad (2)$$

At the end of period 2, it will be

$$p_2 = (u + du + p_1 A)(I + \hat{m}) \qquad (3)$$

(Steedman, 1992, pp. 145–146).

Steedman shows that this process leads in the limit to the same situation as shown in his preceding static analysis, and asserts that 'The general point which is illustrated by the above examples is, of course, that our previous "static" analysis does not "ignore" time. To the contrary, that analysis allows enough time for changes in prime costs, mark-ups, etc., to have their *full* effects' (Steedman, 1992, p. 146).

It is this assertion with which I take issue, for three reasons:

(a) The static equilibrium position is the endpoint of a dynamic process only if that process is stable. It is easily shown that the stability of Steedman's equation is conditional, not absolute.
(b) Steedman's equation assumes that mark-ups would not alter in response to a change in relative profit rates. This can be shown to be an inaccurate reading of Kalecki's writings. When variable mark-ups are introduced, the resulting system is necessarily nonlinear, and generally marginally unstable. When a nonlinear mark-up adjustment process is used, the equilibrium ceases to be a fixed point of the system.
(c) Steedman's equation considers only price dynamics, when in a general dynamic model of a capitalist economy, several other dynamics—in particular, those of quantities and finance—would also be present. These additional dynamics all but guarantee that the fixed points of such a system will be unstable, with the consequence that conditions which apply at those points are irrelevant to the short- and long-term behaviour of the model.

These criticisms do not so much invalidate Steedman's critique, as indicate a manner in which the strengths of the two heterodox schools can be combined to produce a multisectoral dynamic analysis of the capitalist economy.

2. The Conditions for Price Instability

The general form of Steedman's dynamical system is

$$p_{t+1} = (u + p_t A)(I + \hat{m}) \qquad (4)$$

Recasting this in the form of an autonomous dynamical system, we have:

$$p_{t+1} = u(I + \hat{m}) + p_t A(I + \hat{m}) \tag{5}$$

The general solution is

$$p_t = c(A(I + \hat{m}))^t + u(I + \hat{m})(I - A - A\hat{m})^{-1} \tag{6}$$

where c is a constant vector (whose elements are determined by initial conditions). The equilibrium value for p of $u\ (I + \hat{m})(I - A - A\hat{m})^{-1}$ is clearly stable only if the modulus of the dominant eigenvalue of $(A(I\ \hat{m}))^t$ is less than one. This is true for the example input–output matrix and mark-ups Steedman uses to illustrate his argument, but it is not true in general for any valid input–output matrix A and arbitrary vector of mark-ups m.

This point can be illustrated by two numerical examples, one identical to Steedman's, the other differing only in having a different input–output matrix. The first model has the following characteristics:

$$A = \begin{bmatrix} 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 \end{bmatrix}, \ m = \left(\tfrac{1}{2}\ \tfrac{1}{2}\ \tfrac{1}{2}\right), \qquad u = \left(\tfrac{1}{2}\ \tfrac{1}{6}\ \tfrac{1}{3}\right) \tag{7}$$

If this system starts with a non-equilibrium price vector—say,

$$p = \left(3\ \tfrac{1}{2}\ \tfrac{3}{2}\right)$$

—then it rapidly converges to equilibrium (see Fig. 1).

The second model has a different input–output matrix:

$$A = \begin{bmatrix} 0 & \frac{3}{5} & 0 \\ 0 & 0 & \frac{4}{5} \\ \frac{7}{10} & 0 & 0 \end{bmatrix} \tag{8}$$

and is quite clearly unstable (see Fig. 2).

There is an unremarkable explanation for this phenomenon: the arbitrary mark-ups chosen are incompatible with positive equilibrium prices for this input–output matrix. From a static point of view, this would automatically rule out such an input–output/mark-up combination as non-viable. However, the combination of a negative equilibrium price vector and a linearly unstable price system is quite viable in a dynamic, non-equilibrium context, since together they ensure that prices will never, in fact, become negative in the long run—because they will forever diverge from that equilibrium point.

This shows that in a dynamic setting, contrary to Steedman's argument, the mark-ups of price over direct unit cost can be fixed without regard to–but not independently of—input prices (although this will result in sustained price inflation). However, this interpretation will not be pursued here, since it is easily shown that the assumption of constant mark-ups over time is not an accurate rendition of Kalecki.

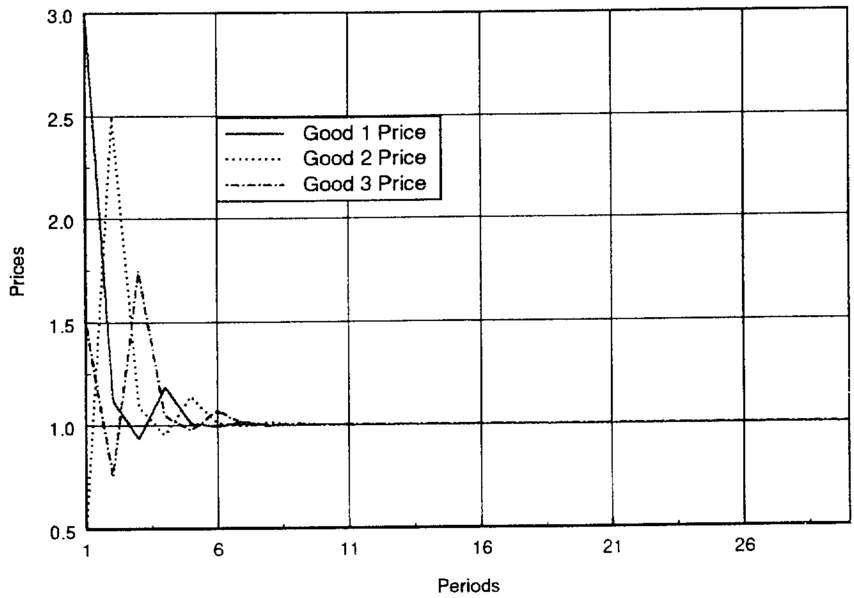

Fig. 1. Convergence to equilibrium for Steedman's numeric example.

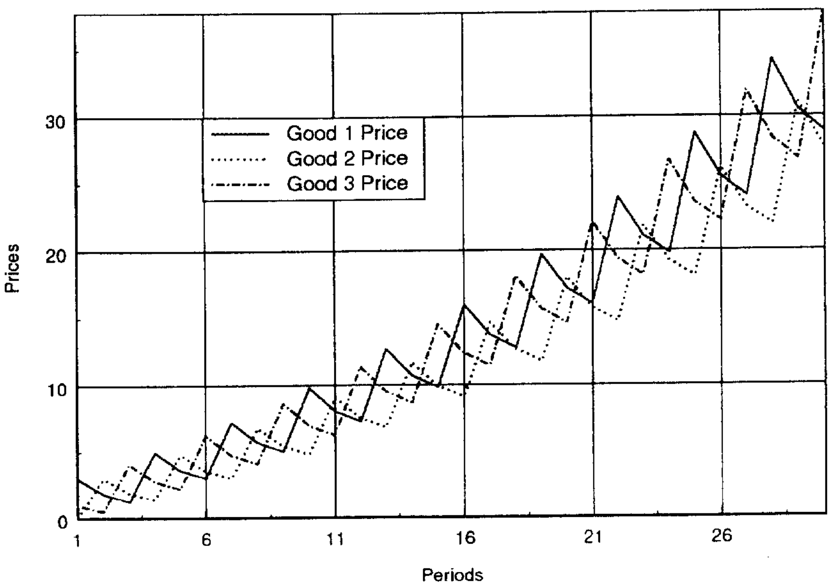

Fig. 2. Divergence from equilibrium for a different input-output matrix.

3. Mark-up Dynamics

There is ample evidence in Kalecki's writings that he would expect industry mark-ups to alter in response to changes in relative rates of profit, as a consequence of competitive pressures in a general setting of monopolistic or oligopolistic competition. These competitive pressures would lead to lower mark-ups in industries experiencing higher rates of profit, and vice versa. Kalecki (1942) considers a supposed counterexample to his mark-up pricing model, with two industries, in one of which the capital is twice, and the other half, annual sales. He argues that 'If the degree of monopoly is the same in both industries, the industry B will certainly earn a much higher rate of profit. This will attract new entries into industry B, and its capital will rise in relation to sales ...' (Kalecki, 1942, p. 122). Later in the same paper, Kalecki argues that gross profit margins and prime costs 'need not change in exactly the same proportion, because "my" degree of monopoly cannot be assumed to remain constant' (Kalecki, 1942, pp. 126–127).

Kalecki's analysis is, in general, cast in terms of monopolistic or differentiated oligopolistic competition (Kalecki, 1971, p. 160), where the degree of competitiveness in each industry reflects partly its own characteristics and partly its relative profitability. Kalecki (1971, pp. 49–52) and Kriesler (1987, p. 76) list the factors which might change the degree of monopoly, and these include 'increase of concentration' which would increase the 'degree of monopoly', leading to higher mark-ups. Conversely, a decrease in concentration should decrease the degree of monopoly and lead to a fall in margins.

This indicates that, to properly construct a Kaleckian partial dynamic model of price formation, the mark-up must be regarded as a variable determined by the degree of competitiveness in each sector, where the degree of competitiveness is directly related to the relative profitability of each sector. Thus, in general terms, Steedman's partial dynamic system must be modified to something of the following form in order to capture properly the impact of input–output relations on Kaleckian mark-up and price dynamics:

$$\begin{aligned} p_t &= (u + p_{t-1}A)(I + \hat{m}_t) \\ m_t &= \mu((u + p_{t-2}A)\hat{m}_{t-1}) \end{aligned} \tag{9}$$

Whatever form is chosen for μ, this will be a nonlinear system, because of the existence of product terms of the general form $p_tA\hat{m}_{t+1}$. Unlike linear systems—such as the ones explored in the previous section—the equilibria of a nonlinear system can be locally unstable but globally stable. Consequently, the conditions which apply at equilibrium can be irrelevant to both the short- and long-term state of the system, without necessarily leading to runaway price inflation.

To illustrate this, I will develop three multisectoral Kaleckian models of price setting in which the adjustment process (a) is partial dynamic, ignoring quantity, technology and finance dynamics; and (b) presumes mark-ups are inversely dependent upon sectoral profits, due to competitive forces. The first two models use a linear form for μ, and demonstrate equilibria which are marginally unstable.[1] The third uses a more realistic nonlinear form for the mark-up adjustment process, and illustrates a novel form of instability.

The mark-up adjustment process in the first model is based simply on changes in gross profits. This apparently contradicts Kalecki's view (Kalecki, 1942, pp. 124–126) that it is not the absolute profit margin which reflects the degree of monopoly, but the rate of profit. However, the gross profit is used here because the partial nature of this model means that the rate of profit collapses simply to the mark-up:

$$\begin{aligned}\pi_t &= (\Pi_t^T \cdot (\hat{C}_{t-1})^{-1})^T \\ &= (u^T + p_{t-1}^T \cdot A) \cdot \hat{m}_t \cdot ((((u^T + p_{t-1}^T \cdot A)^T)^{-1})^T) = m_t\end{aligned} \tag{10}$$

While, in a general model, the rate of profit would depend upon quantity and other dynamics, in this partial model the rate of profit depends only upon the mark-up, which implies a mark-up adjustment mechanism whose dynamics are completely independent of prices:

$$m_t = m_{t-1} - a \cdot (\pi_{t-2} - \pi_{t-3}) = m_{t-1} - a \cdot (m_{t-2} - m_{t-3}) \tag{11}$$

A gross profit adjustment mechanism, on the other hand, incorporates a feedback from prices to mark-ups, while providing a simple illustration of the instability and sustained cyclical behaviour that can emanate from such a dynamic model.

The second model is driven by the comparison of sectoral rates of profit to the economy-wide average. This more closely approximates Kalecki's statements in Kalecki (1942).

The third model makes the more realistic assumption that the mark-up adjustment process will be nonlinear, and its behaviour has some relevance to the debate about the stability of the classical model (Steedman, 1984; Flaschel & Semmler, 1987; Dumenil & Levy, 1987, 1989). The centre of attraction in the classical model is the average rate of profit, not the equilibrium rate. In a linear system, these two necessarily coincide, but in a nonlinear system, this is by no means guaranteed (see Blatt, 1983, pp. 211–216, for an exposition of this as regards Goodwin's 1967 model). If the average and the equilibrium rates of profit do not coincide—as in the third model—then the conditions which pertain at equilibrium are doubly irrelevant to the system, since in this circumstance, the equilibrium is not even a centre of attraction of the model.

4. A Gross Profit Adjustment Mechanism

The mark-up is modelled as a lagged linear function of previous mark-ups and the direction of change of profits (in what follows, all vectors are column vectors):

[1] 'Marginal instability' is the equivalent term in mathematics to the econometric concept of a unit root. It occurs when the Jacobian of a discrete time dynamical system has an eigenvalue of one, and when the Jacobian of a continuous time dynamical system has an eigenvalue of zero, when evaluated at points of equilibrium. In the case of nonlinear models such as those used in this paper, the nonlinear system is first linearised, and the Jacobian of this linear system evaluated. Its characteristics correspond to those of the nonlinear system in the vicinity of the equilibrium points.

$$m_{t+2} = m_{t+1} - a(\Pi_{t+1} - \Pi_t) \tag{12}$$

where a is a constant indicating the degree of responsiveness of mark-ups to changes in profitability[2] $(0 < a_i < 1)$ and profit Π is costs times the mark-up:

$$\Pi_t = ((u^T + p_{t-1}^T A)^T \hat{m}_t)^T \tag{13}$$

This yields a third-order relation for mark-ups in terms of prices, and hence a dynamic system of two third-order vector difference equations:

$$\begin{aligned} p_{t+3} &= ((u^T + p_{t+2}^T A)(I + \hat{m}_{t+3}))^T \\ m_{t+3} &= m_{t+2} - a(((u^T + p_{t+1}^T A)^T \hat{m}_{t+2})^T - ((u^T + p_t^T A)^T \hat{m}_{t+1})^T)^T \end{aligned} \tag{14}$$

The dynamic properties of this system can be quickly indicated with a numerical simulation. If the initial position corresponds to the equilibrium price and mark-up vectors, then these are maintained indefinitely. However, if the initial position differs from equilibrium, the system does not return to it. The following plot has an a value of 0.1 and an initial value for p_1 of 1.01—a 1% divergence from the equilibrium value for one period only (see Fig. 3).

Similarly, a change in the u vector, rather than leading to a new equilibrium, can result in sustained instability. This plot has an a value of 0.965 and equilibrium prices and mark-ups for the initial u vector, but the first element of u is changed from $\frac{1}{2}$ to $\frac{9}{10}$ (see Fig. 4).

While it might appear that the system oscillates around the equilibrium values for prices, and in that sense appears to be determined in the long run by the equilibrium values, in fact the values around which the system oscillates differ substantially from equilibrium. The equilibrium price vector for this model is (1.662, 1.497, 1.248), which differs substantially from the values of (1.6, 1.136, 0.975) around which the prices appear to be oscillating.

These non-equilibrium behaviours occur because the system is marginally unstable: its Jacobian has one or more eigenvalues of one, regardless of the value chosen for a (a sample vector of eigenvalues for $a = 0.2$, is [0, 0, 0, $-0.245 + 0.41i$, $-0.245 - 0.41i$, 0.384, 0.222, $-0.171 + 0.025i$, $-0.171 - 0.025i$, -0.089, -0.085, 1, $+4.8e^{-9}i$, $-4.8e^{-9}i$, $6.544e^{-9}$, $-6.544e^{-9}$, 1, 1]). If the system is disturbed from an equilibrium, then it will almost certainly not return to it, and for some parameter values, the non-chaotic cyclical behaviour shown above may result.

The general conclusions which can be derived from this model are that (a) a truly dynamic input–output analysis necessarily results in nonlinear models; (b) these models may have unstable equilibria; (c) this instability is not dependent upon the equilibrium vector being non-feasible; and (d) the conditions which apply in equilibrium can therefore be irrelevant to both the short- and long-term behaviour of the model. We will now consider a more conventional mark-up adjustment mechanism.

[2] This should be a vector, indicating that the 'barriers to entry' differ between industries, but for the sake of this illustration I will work with a scalar.

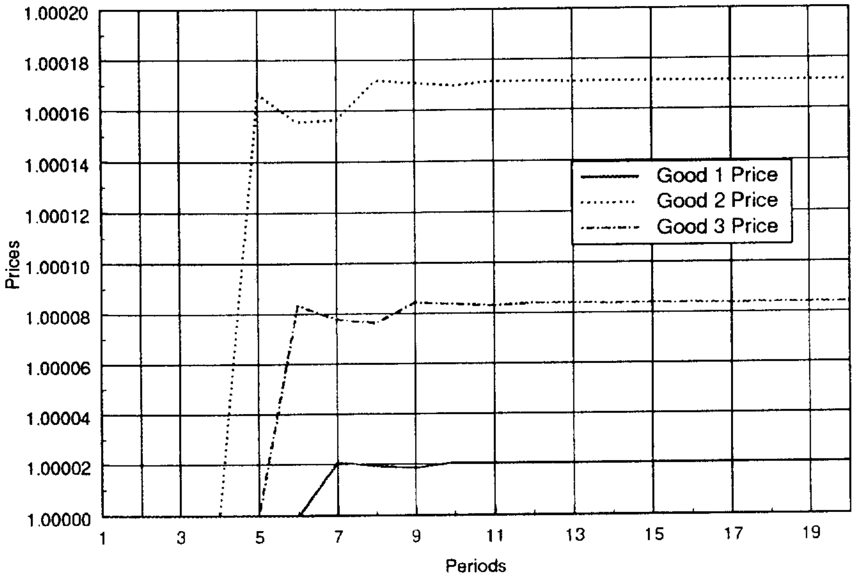

Fig. 3. Marginal instability with a gross profit adjustment mechanism.

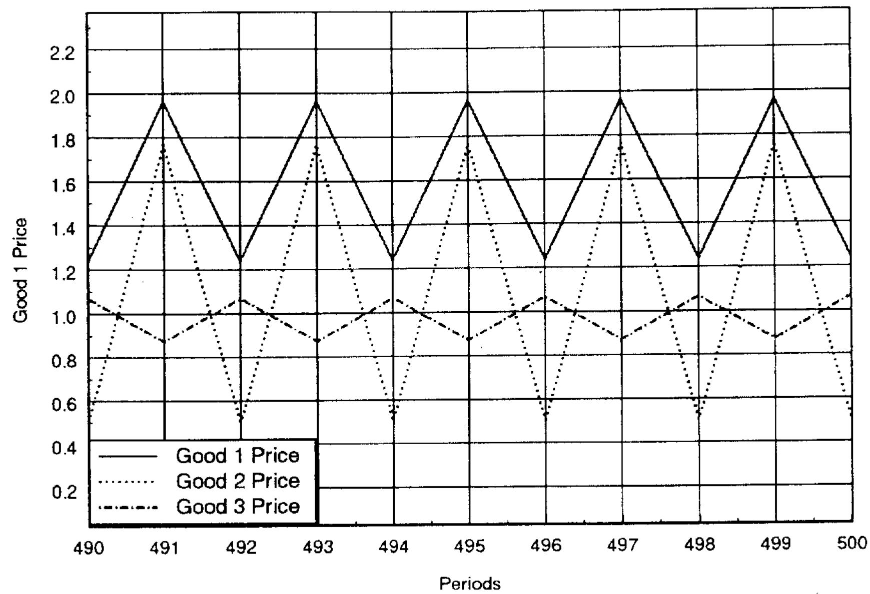

Fig. 4. Marginal instability with a gross profit adjustment mechanism.

5. A More Conventional Mechanism

Traditionally, economists of all persuasions have presumed that competition and investment are driven by the search for the highest possible *rate* of profit. Capitalists are presumed to move capital into industries earning a higher than average rate of profit, and out of industries earning a lower than average rate. In this partial, circulating capital model, the rate of profit for each sector reduces to the mark-up for each sector. The average rate of profit, however, is a weighted sum of sectoral mark-ups. Labelling sectoral costs in period t as c_t, and the average rate of profit as $\bar{\pi}_t$, we have

$$\bar{\pi}_t = \frac{c_t m_t}{\Sigma c_t} = \frac{c_t m_t}{c_t[1]} = \frac{(u^T + p_{t-1}^T A)m_t}{(u^T + p_{t-1}^T A)\hat{m}_t^{-1} m_t} \tag{15}$$

(where [1] represents a n-vector of 1s).

In this model it is presumed that an individual sector will face increasing competition, and hence downward pressure on its mark-ups, if its rate of return exceeds this average. From the perspective of a single sector j, the mark-up adjustment process will be of the general form

$$\frac{m_{j_t} - m_{j_{t-1}}}{m_{j_{t-1}}} = \mu\left(\frac{\pi_{j_{t-n}}}{\bar{\pi}_{t-n}}\right) \tag{16}$$

where n represents a time lag, since this is a competitive process, and whereas we can presume mark-ups are known to each industry as soon as they are set, profits of other sectors are known only with a delay. Presuming a single period lag and a linear form for μ, we get

$$\frac{m_{j_t} - m_{j_{t-1}}}{m_{j_{t-1}}} = a\left(1 - \frac{\pi_{j_{t-2}}}{\bar{\pi}_{t-2}}\right) = a\left(1 - \frac{m_{j_{t-2}}}{\bar{\pi}_{t-2}}\right) \tag{17}$$

The matrix form of this equation is

$$\begin{aligned} m_t &= m_{t-1} + a\left(I - \frac{1}{\bar{\pi}_{t-2}} m_{t-2}\right) m_{t-1} \\ &= m_{t-1} + a\left(I - \frac{(u^T + p_{t-3}^T A)(\hat{m}_{t-2})^{-1} m_{t-2}}{(u^T + p_{t-3}^T A)m_{t-2}} \hat{m}_{t-2}\right) m_{t-1} \end{aligned} \tag{18}$$

This is a third-order equation, so that the resulting multisectoral Kaleckian system is a pair of coupled third-order difference equations:

$$\begin{aligned} p_{t+3} &= (u^T + p_{t-2}^T A)(I + \hat{m}_{t+3}) \\ m_{t+3} &= m_{t+2} + a\left(I - \frac{(u^T + p_t^T A)(\hat{m}_{t+1})^{-1} m_{t+1}}{(u^T + p_{t\,A)m_{t-1}}^T} \hat{m}_{t+1}\right) m_{t+2} \end{aligned} \tag{19}$$

Although this model is replete with nonlinear terms of the form

$$\frac{(u^T + p_t^T)A(\hat{m}_{t+1})^{-1} m_{t+1}}{(u^T + p_t^T A)m_{t+1}} \hat{m}_{t+1}\, m_{t+2},$$

as would be expected, it is not as unstable as the model of the previous section (mark-ups in particular tend to converge to the average value). However, as with the preceding model, a numerical simulation shows that this model is also marginally unstable, since several of the eigenvalues of its Jacobian have a magnitude of *1* for some values of the mark-up adjustment parameter a.[3] In these plots, a has a value of 0.09, while two of the mark-ups are 2% and 4% respectively below the equilibrium value of 0.5 for one of the three initial time periods (see Fig. 5).

Although neither prices nor mark-ups return to the equilibrium values in this model, mark-ups do converge to a single value (see Fig. 6).

6. A Nonlinear Adjustment Mechanism

It is a simplification to assume that the response of capitalists to differences in rates of return in different sectors will be a linear one. A linear reaction implies that a tiny difference in rates of return will engender a tiny response, while a thousand-fold difference will engender precisely a thousand-fold stronger reaction. Two observations can be made against this. First, a tiny divergence from the average is unlikely to generate any response; secondly, there would be a limit to the degree of movement of capital motivated by a large divergence from the average (both these observations are a product of the fact that uncertainty is a fundamental aspect of capitalism). The appropriate model is thus a function which has first derivatives of zero at both zero and maximal divergence from the average rate of profit. However, in order to simplify the presentation, I have chosen to address only the second property.

From the point of view of an individual sector, the mark-up mechanism in this model is

$$\frac{m_{j_t} - m_{j_{t-1}}}{m_{j_{t-1}}} = a \tanh\left(1 - \frac{m_{j_{t-2}}}{\bar{\pi}_{t-2}}\right) \tag{20}$$

where a is a constant, and the average rate of profit $\bar{\pi}_{t-2}$ is as defined above. This results in the third-order system:

$$p_{t+3} = (u^T + p_{t+2}^T A)(I + \hat{m}_{t+3})$$

$$m_{t+3} = \left(m_{t+2}^T\left(I + a\, ITANH\left(\frac{(u^T + p_t^T A)(\hat{m}_{t+1})^{-1}\, m_{t+1}}{(u^T + p_t^T A)\, m_{t+1}}\hat{m}_{t+1}\right)\right)\right)^T \tag{21}$$

where $ITANH(x)$ takes a vector argument and returns a diagonalised matrix whose ith diagonal entry is $\tanh(1 - x_i)$.

This system has the same equilibrium as the preceding models, and if this is the initial state of the system, then it is maintained for all time. However, if the initial conditions diverge from equilibrium, the system cycles away from it towards the average mark-up, where this is a function of initial conditions and the value of a. For a small divergence and a low value of a (corresponding to

[3] The eigenvalues when the parameter a equals 0.09 are (0,0,0,0,0,0,0,0,0, −0.227 + 0.393i, −0.227 − 0.393i, 0.454, 0.1, 0.9, 0, 0.1, 0.9, 1).

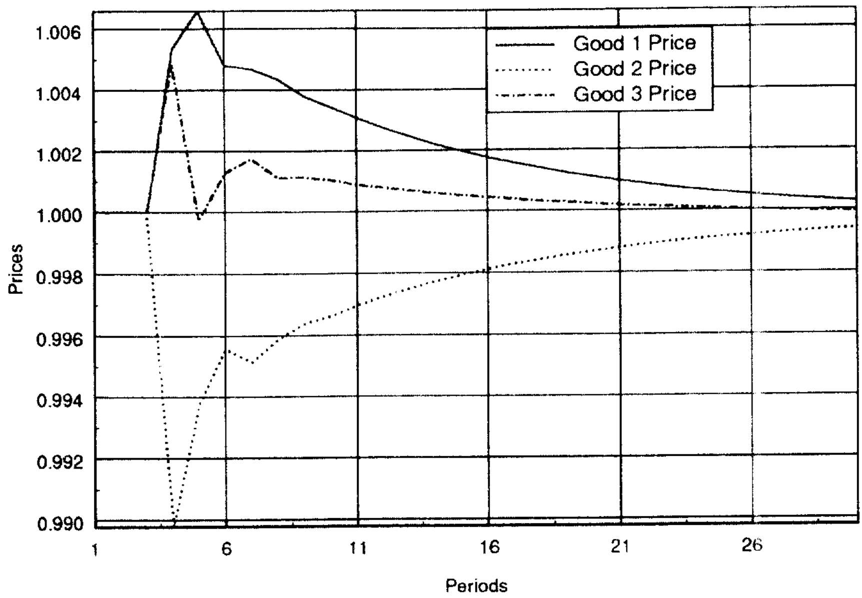

Fig. 5. Marginal instability with an average profit adjustment mechanism.

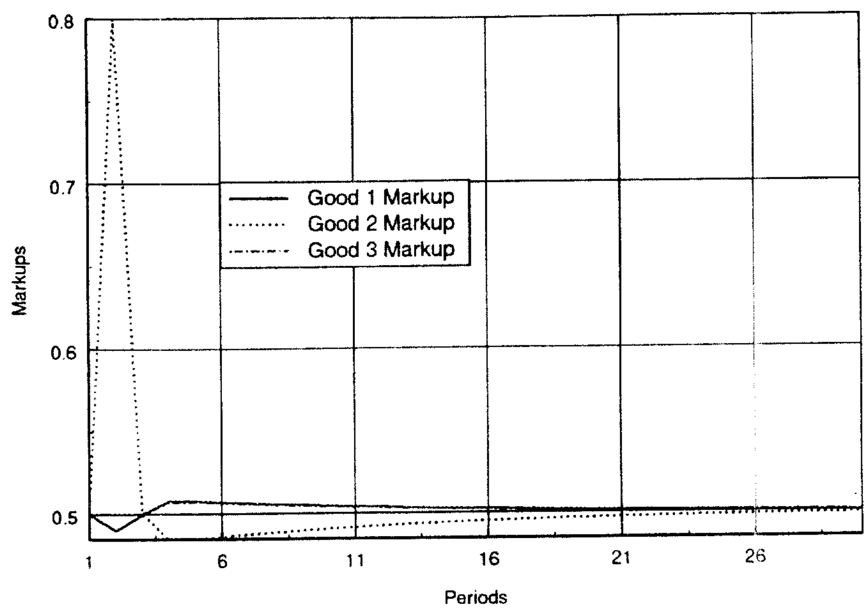

Fig. 6. Convergence of mark-ups with an average profit adjustment mechanism.

an economy with low competitive pressures), the system converges rapidly to an average mark-up (and therefore to prices of production) which is not too different to the initial equilibrium. The following plots were the result of a value of 0.5 for a, and initial mark-ups differing from the equilibrium by 2% for one sector only in each of the three initial time periods (see Fig. 7).

The larger the initial divergence, the more the final average differs (linearly) from the *ex ante* equilibrium, while as a approaches 1 (corresponding to an economy with high competitive pressures), cycles last longer and the final average mark-up approaches zero. The plots in Fig. 8 were the result of a value of 0.98 for a, and a 20% initial divergence for one sector in each of the initial time periods (the uniform mark-up converged to 0.3292 after 600 periods).

7. Conclusions: general dynamics, nonlinearity and price setting

The models considered above are clearly only partial ones, with quantity dynamics ignored, the wage taken as exogenous, and so on. Yet even when the parameters of this partial dynamic system are assumed to be linear, a nonlinear system with a marginally unstable equilibrium results. Consequently, a multisectoral, general dynamical model will be intrinsically nonlinear, due to the dynamic, input–output nature of production.

To this intrinsic nonlinearity, a realistic model of capitalism must add the many nonlinearities which, although extrinsic to the underlying system of production, are intrinsic to the capitalist social system. These nonlinearities include the relations between unemployment and workers' wage demands (Goodwin, 1967), the rate of profit, capitalist investment, and debt (Keen, 1995), in addition to the nonlinear response of mark-ups to changes in relative profits modelled above.

This paper has shown that even when only the last of these is introduced, the resulting model has an unstable equilibrium—although the system does converge to prices of production, at an average rate of profit dependent upon parameter values and initial conditions. A general dynamical model, with quantity, wage, finance and technological dynamics added to the price and mark-up dynamics discussed above would be considerably more nonlinear; first, because of the escalation in the number of product terms and, secondly, through the introduction of many more nonlinear relationships. The equilibria of such a general dynamical system may not be attractors, as in the third model above, with the result that the conditions which apply at equilibrium will be irrelevant to the system in both the short- and long-term. The appropriate style of analysis for this class of systems is far from equilibrium analysis, and is something which is foreign to the analytic traditions of both Sraffian and Kaleckian economics.[4]

[4] This is a field which is far better developed in chemistry and physics than in economics. Some of the concepts involved are outlined in the Santa Fe Institute *Lectures in Complex Systems* series, the latest of which is Nadel & Stein (1995). Ott (1993) is an accessible reference on the general area. Some representative titles from physics and chemistry include *Far From Equilibrium Phase Transitions* (Garrido, L., Ed, 1988), and *Far-from-equilibrium Dynamics Of Chemical Systems: Proc Borki Poland 1993* (Gorecki, J. & Cukrowski, A.S., Eds, 1994).

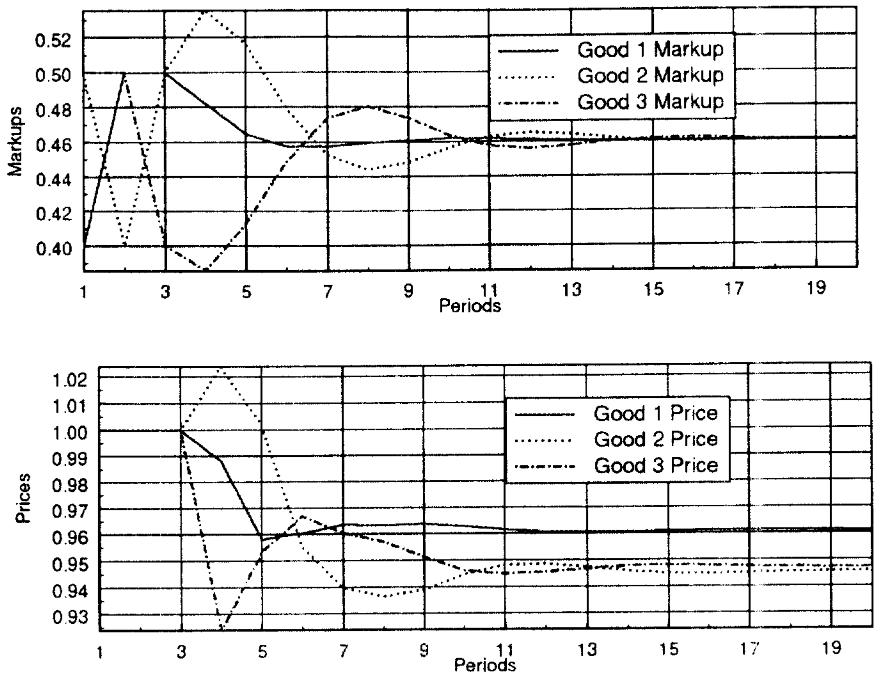

Fig. 7. Convergence with a nonlinear mark-up adjustment mechanism.

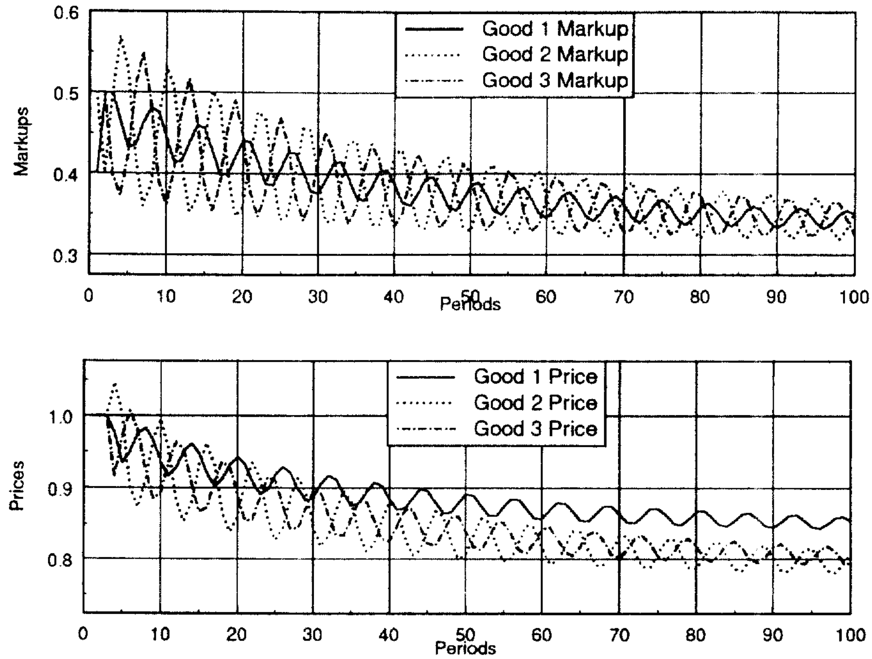

Fig. 8. Non-equilibrium convergence with a nonlinear mark-up adjustment mechanism.

Table 1.

Equations	Linear				Nonlinear	
	One equation	Several equations	Many equations	One equation	Several equations	Many equations
Algebraic	trivial	easy	essentially impossible	very difficult	very difficult	impossible
Ordinary Differential	easy	difficult	essentially impossible	very difficult	impossible	impossible
Partial Differential	difficult	essentially impossible	impossible	impossible	impossible	impossible

Source: Table 3.1, Costanza, (1993, p. 33)[5]

In particular, this style of analysis does not lend itself to analytic conclusions: indeed, as Costanza (1993, p. 33) emphasises, all higher dimensional nonlinear dynamical systems are analytically insoluble (see Table 1)

These models are best analysed via simulation, as in this paper—although techniques such as Lyapunov analysis can be used to characterise the general characteristics of the model, and linear stability analysis can be applied to determine the characteristics of equilibrium points (see Lorenz, 1993). The dynamical route thus imposes many costs, since new techniques must be learnt and some familiar methods be abandoned. However, as Steedman himself argued cogently, while pioneers such as Kalecki can be forgiven for neglecting essential aspects of capitalism, such behaviour is not permissible for those who strive to extend their analysis (Steedman, 1992, p. 127). If our understanding of capitalism is to be extended beyond that developed by Sraffa and by Kalecki, both Sraffians and Kaleckians must work to develop an analysis which acknowledges the dynamical, multisectoral, behavioural foundations of capitalism.

This paper also justifies Sawyer's distinction, in his reply to Steedman, between a theory of pricing, and a theory of prices (Sawyer, 1992, p. 158). The Kaleckian theory of mark-up pricing, with mark-ups set according to the degree of competition within each industry, is only irrelevant if the long-run outcome of the model corresponds to the static equilibrium position. Since this is not the case, a theory of price-setting must be part of a theory of capitalism. Such a theory must be based upon the setting of prices by disaggregated decision units acting with limited information in an environment of uncertainty, and the Kaleckian contribution is thus a valid one. Nonetheless, while Steedman's critique of Kaleckian economics is partially invalid, he is correct to argue that Kaleckian theory must be modified to incorporate the impact of input–output effects. This modification results in a multisectoral dynamical analysis which, in many ways, combines the strengths of these two schools of political economy, at the expense of leading both Kaleckian and Sraffian economists into analytically uncharted waters.

[5] Costanza heads this table with the comment that 'The thick solid line divides the range of problems that are solvable with analytical methods from those that are very difficult or impossible using analytical methods and require numerical methods and computers to solve.'

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