Interpretations of Marxian Value Theory in Terms of the Fundamental Marxian Theorem

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Abstract

This paper focuses on the theoretical status of the fundamental Marxian theorem in various interpretations of Marxian value theory, particularly with regard to the logical consistency rather than the analytical implication. It will be shown that each interpretation proves the fundamental Marxian theorem in its own way, and therefore the proof of the theorem is not a decidability criterion for the correct interpretation of Marxian value theory.

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1. Introduction

The well-known fundamental Marxian theorem elaborated by Okishio (1963) and Morishima (1973) asserted that “the exploitation of laborers by capitalists is necessary and sufficient for the existence of a price-wage set yielding positive profits or, in other words, for the possibility of conserving the capitalist economy” (Morishima 1973: 53). At least for a while, this theorem was regarded as a “proof” for Marx’s theory of exploitation. However, it turns out that the fundamental Marxian theorem itself cannot be a safe haven for Marxian economists.

The “New Interpretation” put forward by Duménil (1980) and Foley (1982) in the 1980s approached Marxian value theory from another perspective emphasizing the direct link between labor time and price through the concept of the “monetary expression of labor time.” In the New Interpretation, the so-called transformation problem “becomes a
trivial exercise in definitions” (Duménil 1984: 349). Therefore, the fundamental Marxian theorem was put aside as a marginal issue.

Recently, the issue of the fundamental Marxian theorem was brought back by the “Temporal Single-System Interpretation.” This interpretation regarded the theorem as meaning that “surplus labor is both necessary and sufficient for real profit to exist” and argued that it must be proved “under completely general conditions” (Kliman 2001:106, emphases in original). Therefore, a new light has been shed to the fundamental Marxian theorem with regard to a reconstruction of Marx’s theory of exploitation. Kliman (2001) tried to show that the fundamental Marxian theorem holds without any restrictive conditions in the Temporal Single-System Interpretation. In so doing, he argued that the aggregate price of the net product may become negative, which implies the New Interpretation’s failure in proving the fundamental Marxian theorem in a general context. Although Mohun admitted “that there are some negative net products is undeniable” (Mohun 2003: 98), he argued that Kliman (2001) did not prove the real possibility of aggregate negative net product, and therefore it is impossible to decide between the Temporal Single-System Interpretation and the New Interpretation through an examination of the fundamental Marxian theorem.

The present paper has been motivated by the continuing controversy on this issue (Kliman and Freeman 2006; Mohun and Veneziani 2007; Kliman and Freeman 2008). However, this paper employs a rather indirect way to intervene in the controversy. It focuses on the theoretical status of the fundamental Marxian theorem in various interpretations of Marxian value theory. In particular, it will be emphasized that the theorem only works at a certain level of abstraction with some specific assumptions. One can only check if a certain assumption is more (or less) general than another. Section 2 summarizes the formal structure of the conventional fundamental Marxian theorem for further discussion. Section 3 and section 4 show that the New Interpretation and the Temporal Single-System Interpretation, respectively, prove the fundamental Marxian theorem in their own ways. Section 5 concludes that the proof of the fundamental Marxian theorem is not a crucial criterion for choosing the correct interpretation of Marxian value theory.1

2. The FMT

The FMT shows that “the equilibrium rate of profit is positive if and only if the rate of exploitation is positive” (Lipietz 1982: 59).

The theorem can be proved as follows (Roemer 1981: 16–7). First, prices of production are represented as

\[ p = (1 + r)(pA + wl) = (1 + r)p(A + bl) \]  

where \( p, A, l, w, r, \) and \( b \) denote the vector of prices of production, the input coefficient matrix, the labor input vector, wage rate, rate of profit, and wage basket vector, respectively.

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1. The following is the list of acronyms used in this paper.
FMT: Fundamental Marxian Theorem
MELT: Monetary Expression of Labor Time
NI: New Interpretation
PNP: Price of the Net Product
TSSI: Temporal Single-System Interpretation
According to the Perron–Frobenius theorem, there exists a right eigenvector \( x > 0 \) that satisfies the following relation when \( A + bl \) is an indecomposable matrix. Here \( x \) is to be thought of gross output vector.

\[
x = (1 + r)(A + bl)x
\]

Pre-multiplying (2) by value vector \( \lambda = \lambda A + l \) and using the definition of the rate of exploitation denoted by \( e \equiv (1 – \lambda b)/\lambda b \),

\[
\lambda x = (1 + r)(\lambda A + bl)x = (1 + r)\left( \lambda Ax + \frac{1}{1+e}lx \right)
\]

\[
\therefore 1 + r = \left( \lambda A + \frac{1}{1+e}l \right)x
\]

(3)

In the denominator of (3), using the definition of value, it can be shown that \( r > 0 \) if and only if \( e > 0 \). As \( r \) is the uniform rate of profit, the theoretical setting of the FMT is limited to “positive profits in every industry” of the economy (Morishima 1973: 53).

First of all, it is worth emphasizing that the FMT does not “consider the relation between surplus labor and profit under all positive prices” (Kliman and Freeman 2006: 119). The above proof of the FMT is premised upon the “net production possibility condition” (Okishio 1977) or reproducibility condition, which means that “net outputs, at the socially chosen point, should at least replace workers’ total consumption – where only employed workers consume” (Roemer 1981: 41).

The implication of this assumption can be seen simply in a single-commodity model. Let \( a \) and \( l \) be the quantity of material input and labor input required to produce one unit of the commodity. Then, the value of the commodity is determined as follows where \( \lambda \) is a scalar in this case.

\[
\lambda = \lambda a + l = \frac{1}{1-a}
\]

(4)

As the net production possibility condition means that \( a \) is smaller than one, it guarantees that the value magnitude is positive. If \( a \) is greater than one, the value of the commodity would be negative, which implies that more than one unit of commodity is required to produce one unit of it. If \( a \) is equal to one, the value magnitude would be infinity, which means that the infinite quantity of labor is needed to produce just one unit of the commodity. Therefore, in so far as the value definition such as the equation (4) is maintained, the net production possibility condition is indispensable for economically meaningful analysis.

While the FMT expresses Marx’s basic idea that “there is capitalist exploitation of labor in production when the workers’ productive contribution is higher than their wage” (Scerpanti 2003: 163), it has been criticized for treating the issue of exploitation only at

2. This requires that the economy must be viable in the sense that “the technology has already been developed to such a level that production processes which are ‘productive’ are available to the capital-good industries of the society” (Morishima 1973: 22). If we suppose the general case consisting of capital goods and wage goods, this condition implies positive net outputs in capital good industries and non-negative net outputs in wage good industries.
the physical surplus level, which makes it impossible to analyze a capitalist-specific form of exploitation. Mathematical logic shows that any basic commodity, such as the peanut, can play the same role with labor commodity in the FMT:

…if we define a “basic good” as any which enters directly or indirectly into the production of any element in the wage bundle, then we can show that any basic commodity can be treated as a measure of value. Further, this commodity can be shown to be “exploited” in the sense that profits represent a transformation of surplus-value extracted from this commodity. (Bowles and Gintis 1981: 7)

Based upon this sort of criticism which was called the peanut theory of value in a sarcastic manner, the FMT was thrown away by many non-Marxian (and even Marxian) economists. However, as is clear from the following statement, Okishio himself was already aware that the theorem itself does not completely prove the existence of profit:

If, however, we interpret this theorem from a wider perspective, the problem can be thought of under the assumption that positive profit exists. With regard to this problem, it is clear that the existence of surplus labor is necessary, but not sufficient. (Okishio 1977: 134, translated from Japanese, emphases in original)

The FMT provides an economically meaningful relationship between surplus labor and the competitive rate of profit. However, it presupposes that the real wage is limited to a certain level which satisfies the surplus condition. There must be an actual mechanism keeping the real wage below a certain level (Okishio 1977: 135). This implies that the analysis of the dynamic process of capital accumulation including business cycles and a labor market is required to fully understand the institutional setting of the FMT. Therefore, the FMT only works at the abstract level higher than that of the concrete, real capitalist society where we live.

3. The FMT in the NI

The NI developed by Duménil, Foley, and others tried to connect price and labor time directly through the concept of the MELT. Some references can be made to the NI’s conceptualization of the FMT. The clearest one is Lipietz (1982), who argued for the compatibility between the conventional interpretation and the NI:

I propose to argue here that solutions of Morishima’s kind, once they are filled out and correctly understood, do not contradict any of Marx’s aims in *Capital*, but that there exists another solution, closer to the approach of *Capital*, which exhibits the famous equalities of Volume III, between the sum of prices and the sum of values, and the sum of profits and the sum of surplus value, equalities which are inconsistent with Morishima’s treatment. (Lipietz 1982: 60)

Lipietz’s idea is essentially along the same lines as Duménil and Roy (1982), who provided the relation between the rate of exploitation and the rate of profit, \( r = f (e) \) as a simple

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3. The value-determining equation is changed into \( \lambda = \lambda A + l = \lambda A + \lambda bl + (1 - \lambda b)l \), breaking the newly added value up into variable capital (the value of labor power) and surplus value. With \((1 - \lambda b)l\) representing the vector of surplus labor, it is immediate that \((1 - \lambda b) > 0\) must be fulfilled in order for the surplus value to be positive. This is called the “surplus condition” by Okishio. See Okishio, Nakatani, and Kitano (1978).
relation with “convex, monotonically increasing in $e$ [the rate of exploitation], and bounded by $R$ [the maximum rate of profit]”(Lipietz 1982: 69). It is obvious from this that the NI also considers the relation similar to the FMT.

According to the NI, the MELT ($m$) is defined as the ratio of money value added to aggregate direct labor (Mohun 2004: 72).

$$m = \frac{py}{lx}$$ (5)

Here $y$ denotes the vector of net output.

On the other hand, the value of labor power ($VLP$) is the ratio of money wage to the MELT, which is equal to “the share of wages in money value added” (Mohun 2004: 75).

$$VLP = \frac{w}{m} = \frac{wlx}{py}$$ (6)

Using equations (5) and (6), total surplus value ($S$) is related to total profit ($\Pi$)

$$S = lx - VLP \cdot lx = lx - \frac{w}{m}lx = \frac{py - wlx}{m} = \frac{\Pi}{m}$$ (7)

Thus, it is easily known that the condition $m > 0$ implies the FMT in the NI. In equation (5), as the total labor hour ($lx$) must be positive in any economy, $m > 0$ means $py > 0$. Therefore, the positiveness of the PNP is a necessary condition for the FMT in the NI. This was well recognized by one of the founders of the NI:

As far as the new interpretation of the transformation problem is concerned, the required conditions are the existence of a positive set of prices regardless of the rates of profit, guaranteeing not necessarily uniform positive wages, and a positive aggregate price of the net output. (Duménil 1984: 348, emphases in original)

However, as the MELT represents the ex-post relation between money value added and total labor hours for a certain period of time, the logical possibility for the negative PNP exists if there is any negative element in the vector of net output ($y$). This possibility was originally noted by Stamatis (1998-9):

We point out that $y$ is not necessarily always $\geq 0$ but that $y$ can also contain negative quantities of commodities in addition to positive or semipositive quantities of or zero commodities. It obviously depends on the composition of the gross product $x$ whether $y$ is positive or semipositive ($y \geq 0$) or also contains negative quantities of commodities. (43)

If the net production possibility condition is fulfilled, the PNP must be positive simply because there cannot be a negative price. On the other hand, the positive PNP does not necessarily mean that the net production possibility condition is satisfied. This implies that the NI’s FMT depends on a weaker condition than the original FMT does.\(^4\)

\(^4\) As the PNP is equal to $py = p_1y_1 + p_2y_2 + \ldots + p_my_m$, the net production possibility condition (all $y_i \geq 0$ and at least one $y_i > 0$) implies PNP $> 0$ while the PNP can be positive with some negative $y_i$’s. On the other hand, as Morishima (1973: 21-4) proved, the net production possibility condition is required to get positive solutions for value-determining equations. However, as the equations of the NI are “all specified in terms of aggregates” (Mohun 2004: 77), the NI does not require the existence of positive values in the more realistic conditions such as joint production.
However, taken as a whole, the FMT was regarded as a failure “to motivate the analysis of the embodied labor coefficients system by showing what explanatory power it has over observable phenomenon” (Foley 2000:18). Therefore, the NI does not resort to the FMT to prove the theory of exploitation. As the NI is concerned with the unified conception of money and labor time, it starts from the realm of monetary forms and then goes back to the realm of labor time. In this logical structure, the existence of profit itself is regarded as a symptom of exploitation (Saad-Filho 2002: 45). Therefore, it is highly probable that the proposition that surplus labor is the exclusive source of profit is a given premise in the NI. This interpretation is supported by the fact that the two basic equations of the NI, (5) and (6), are not derived or proved but postulated from the outset. It also dovetails with Foley’s explanation of why the NI is an interpretation, not a solution:

…the New Interpretation proposes to define the relevant categories of the labor theory of value so that what is regarded as the key Marxian insight, the quantitative equivalence between capitalist gross profit and unpaid labor, holds. The “dual” approach to the labor theory of value…, in contrast, considers whether it is possible to deduce Marx’s equivalence from the assumptions (such as the identification of the value of labor power with the labor embodied in the workers’ consumption). (Foley 2000: 22)

The NI does not pursue under which condition the existence of surplus labor leads to positive profit. In a nutshell, the FMT is not the object of a mathematical proof in the NI. If one considers the FMT as “a claim about the real world” (Kliman 2007: 180), the NI cannot explain the sufficiency aspect of the FMT. In response to Kliman’s (2001) “insufficiency critique,” Mohun (2003: 98) stated that “economies in disequilibrium still have some structure; if they are technologically and economically viable they can reproduce themselves, prices are not random, and behavior is not arbitrary. A disequilibrium state is not one in which absolutely anything can happen.” This means that the NI also works at the abstract level higher than that of “the real world.”

Finally, it should be noted that the above discussion is also applicable to the so-called simultaneous single-system interpretation (Wolff, Roberts, and Callari 1982; Lee 1993). This is because the formal difference between the NI and the simultaneous single-system interpretation lies in whether the value of constant capital is defined as its monetary magnitude divided by the MELT or not. However, this difference does not affect the relationship between surplus labor and profit.6

4. The FMT in the TSSI

The TSSI conceives that the FMT must work at the concrete level of “the real world.” Kliman (2007: 175-6) argues that only the TSSI yields the FMT in this sense. It will be useful here to compare the premises of the FMT and the NI with the logical structure of the TSSI.

On the one hand, the TSSI regards Okishio’s net production possibility condition (or reproducibility condition) as the key cause for the failure of the original FMT. This is

5. Kliman (2001: 101) argues that “if the net product valued at end-of-period market prices is negative, then … profit is…negative although surplus labor is positive.” This may be called a “insufficiency critique” in the sense that the causality running from surplus labor to profit does not hold in the negative PNP case.

6. Duménil and Foley (2008) identify the NI with “the single-system labor theory of value.”
because the TSSI thinks that the original FMT “says nothing about the relationship between surplus labor and profit at actual output levels” (Kliman 2007: 190, emphases added) and “the postulate that all net products are positive...is violated in every actual economy” (Kliman 2001:103).

The reason why the TSSI does not require the net production possibility condition is due to its inter-temporal formulation of Marx’s value concept as follows, where subscripts denote time defined discretely (Kliman and McGlone 1999: 37).

\[ \lambda_{t+1} = p_t A + l \] (8)

It will suffice here to show the crux of the argument using the one-commodity example presented by Kliman and Freeman (2006: 121). A single good is produced. Its price, gross output, material input, and labor input, respectively, are denoted as \( p, x, a \) and \( l \). Assuming that \( p = x = l = 1 \) and \( a > 1 \), the TSSI’s value becomes \( \lambda = pa + l > 2 \). This implies that the Okishio-Morishima’s condition is not necessary for getting positive value magnitudes.7

On the other hand, the logical superiority of the TSSI over the NI allegedly lies in the fact that the TSSI’s MELT is always positive even in the negative PNP case. In light of these two points, the FMT of the TSSI seems to work at a more concrete level than that of the NI. However, the TSSI’s proof rests upon the definitional changes in the concepts of inflation and profit which are closely interrelated.

First, the FMT in the TSSI is considered as the statement that “surplus labor is both necessary and sufficient for real profit to exist” (Kliman 2001:106, emphases added). This means that profit should be defined as real profit, not nominal profit. According to Kliman (2001: 107), real profit (\( \pi_R \)) is

\[ \pi_R = \left( \frac{1}{1+i} \right) P - C - V \] (9)

where \( P, C, V \) and \( i \) denote the total price of output, constant capital, variable capital (the wage bill), and the rate of inflation. On the other hand, nominal profit is as follows (Kliman 2007: 186):

\[ \pi_N = P - C - V \] (10)

Furthermore, a new concept of inflation must be introduced. This is because the concept of inflation determines whether the TSSI’s FMT holds or not:

Yet different methods of adjustment for inflation will yield different measures of real profit. It is therefore impossible to prove or disprove analytically that surplus labor is the sole source of profit, even real profit. The answer will depend upon one’s concept of inflation. If the exploitation theory of profit holds under a particular definition of inflation, and one accepts that definition, then one must conclude that surplus labor is the sole source of (real) profit. If one rejects the definition, one must draw the opposite conclusion. (Kliman 2001: 106)8

7. However, it also implies that value and price are not equal even in a single-good economy. This is not consistent with the conventional wisdom of the labor theory of value.

8. The last sentence is true only if it is shown that there must be one concept of inflation which validates the proposition that surplus labor is the sole source of profit. However, this is not guaranteed outside the TSSI framework.
The TSSI’s definition of the rate of inflation is

\[ i = \frac{m_{t+1} - m_t}{m_t} \]  

(11)

where \( m_t \) and \( m_{t+1} \) represent, respectively, input MELT and output MELT (Kliman 2007: 185). This conception results from the TSSI’s value-determining equation (8) which distinguishes the input time and the output time. The \( m_t \) here is different from the NI’s MELT because it is defined temporarily (Kliman and Freeman 2006: 121; Kliman 2007: 191).

\[ m_{t+1} = \frac{P_{t+1}}{C_t/m_t + l_t x_t} \]  

(12)

The TSSI’s FMT can be proved as follows (Kliman 2001: 107-8; Kliman 2007: 185-7). As surplus labor is total labor minus labor time equivalent of money wages:

\[ S = l_t x_t - \frac{1}{m_t} V \]  

(13)

The TSSI defines value added in labor time as

\[ l_t x_t = \frac{P_t}{m_{t+1}} - C_t/m_t \]  

(14)

Multiplying (14) by \( m_t \) and using the definition (11),

\[ \frac{P_t}{1 + i} = C + m_t l_t x_t \]  

(15)

Comparing (15) with the definition of real profit (9) and using (13),

\[ \pi_R = C + m_t l_t x_t - C - V = m_t l_t x_t - V = m_t S \]  

(16)

Equation (16) is the TSSI’s FMT. The possibility of the negative MELT disappears here because \( C, l_t x_t, P_t \) and the initial condition \( m_0 \) are positive and finite (Kliman 2001: 108). With regard to this proof, two issues must be raised.

First, as Kliman acknowledged, this proof is possible only by the definitional change. Equation (11) is transformed into

\[ m_t = m_0 (1 + i)^t \]  

(17)

Therefore, the positiveness of \( m_t \) requires not only the positiveness and the finiteness of \( m_0 \), but also that the rate of inflation must be greater than \(-1\), which implies \( m_{t+1} > 0 \) from equation (11). This means that \( m_t > 0 \) requires \( m_{t+1} > 0 \) for all \( t \)’s, which shows that \( m_t > 0, m_0 > 0, \) and \( i > -1 \) have a tautological structure.

9. This concept of inflation is peculiar in the sense that it includes not only inflation in the ordinary sense of the term, but also the increase in labor productivity. As is well known, the MELT is decomposed into price change and labor productivity (Foley 2005: 4). Even Kliman (2007: 129) stated elsewhere that “the rate of inflation is approximately equal to the growth rate of the MELT plus the growth rate of values.”

10. “The objective has been to make Marx’s exploitation theory of profit make sense (not to prove that it is true). To make it make sense, one needs to find definitions—in other words, an interpretation of his terms and premises—from which his results do indeed follow….To define total surplus labor as the labor-time equivalent of total profit would indeed be trivial, tautological, and require no proof, but to find the definitions that lead to this result is a quite different matter” (emphases in original, Kliman 2007: 189).
Second, the claim that the TSSI can include the negative PNP case still needs further clarification. Kliman (2007: 188-9) illustrated a numerical example which shows that real profit is positive even when the PNP (and therefore nominal profit) is negative. However, from the definitions of real and nominal profit, (9) and (10) and the TSSI result (16), the following is obtained.11

\[ \pi_N = \pi_R + \frac{i}{1+i} P = m_s \frac{i}{1+i} P \]  

In order for Kliman’s (2007) case to hold, \( \pi_N < 0 \) and \( \pi_R > 0 \) should happen simultaneously. Therefore, from equation (18), \( \frac{i}{1+i} \) must be negative. As the denominator \( (1+i) \) is required to be positive for the positiveness of \( m_s \), the numerator \( i \) must be negative. Therefore, the TSSI needs a declining MELT to solve the NI’s dilemma of the negative PNP.12

### 5. Conclusion

Table 1 summarizes the relationship among the three interpretations of Marxian value theory and the FMT elaborated in this paper. Each interpretation starts with some assumptions and attempts to prove the FMT in its own way. In this sense, the alleged proof of the FMT itself cannot decide which interpretation is better than the others.

In the conventional interpretation, the FMT is proved under the assumption of reproducibility. As its task is to show the bi-directional relationship between the rate of exploitation and the uniform rate of profit, the FMT does not prove the existence of exploitation in the capitalist society where we actually live. Therefore, it is outside the scope of the FMT to give an additional explanation of the institutional background in which exploitation exists.

In the NI, positive PNP (and therefore, positive MELT) is a given premise. As reproducibility implies positive PNP and not vice versa, the NI’s FMT depends on a weaker assumption than the conventional interpretation does. However, the sufficiency aspect of the FMT must be complemented by the NI’s explanatory power.

The TSSI tries to lower the FMT’s level of abstraction to that of an actually working economy. Although the TSSI does not require reproducibility or positive PNP, the concepts of inflation and profit are defined at the outset to make the FMT hold. Furthermore, it also has to assume a declining MELT.

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11. One obvious impact of this definitional change is that the proportionality between gross profit and surplus labor (or surplus value) is not preserved. This is because gross profit here reflects the effect of inflation.

12. The author is indebted to Fred Moseley for noting this point. Without a doubt, there is no guarantee for a declining MELT. See also footnote 9.
As this paper is mainly concerned with the theoretical status of the FMT in interpretations of Marxian value theory, it has not explored other important issues such as the distinction between labor and labor power or the logical link between exploitation and appropriation. This will be the task of another paper.

References


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