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The Conservation of Value: A Rejoinder to Alan Freeman

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ABSTRACT

This paper is a critique of Alan Freeman's theory of sequential values. In this approach value is conserved from period to period independently of technical change and disequilibrium, contrary to the traditional view that values are reevaluated at each period depending on the existing conditions of production. Our main criticism is that sequential values fail to account for the devaluation of capital, when the economy is considered globally. Devaluation is possible for individual commodities in Freeman's framework, but the loss of value is always compensated by a corresponding gain for another commodity. The paper also points out a number of puzzling properties of sequential values, in particular the compatibility of increasing values with rising labor productivity. The unusual treatment of fixed capital, in which fixed capital is assimilated to an imperishable raw material, also raises serious problems for Freeman. © URPE. All rights

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1. Introduction

Beginning with the early formalizations of the "transformation problem," there has been at least a broad agreement on what was precisely described as Marx's "mistake" in *Capital*. Inputs and outputs should be evaluated consistently in the equations accounting for the determination of prices of production. Concerning the determination of values, it was implicit that the value of inputs must be estimated on the basis of their present conditions of production, independently of the amount of labor actually required for their production in the past. What is transferred to the outputs of the period is the present value of inputs. This view, that we will denote as the "traditional" view, is being challenged by a group of researchers (A. Freeman and G. Carchedi 1996). In their view, inputs transfer to outputs their historical labor content—value is always conserved. Value is only destroyed by final consumption. Values computed along such lines we shall call "sequential values."

The debate over the computation of values has important implications for the "transformation problem." Since inputs are not reevaluated under the present conditions of production in the sequential-value approach, the alleged "contradictions" in Marx's analysis disappear. Consequently, what we will call "the value conservation principle" restores in a straightforward manner Marx's demonstration, allowing in particular for the satisfaction of the famous two equalities on aggregates: the value of gross output is equal to its price, and total surplus-value is equal to total profit. The sequential-value approach contradicts other interpretations of the theory of value, in particular that presented nearly twenty years ago by Gérard Duménil and Duncan Foley (G. Duménil 1980 and D. Foley 1982), still known as the "new" interpretation (see D. Foley 1997). This approach is faithful to the traditional definition of values, and the equalities among aggregates obtain in a different manner.

This paper is a rejoinder to Alan Freeman's demonstration in A.

Freeman 1996. In spite of basic differences in the interpretation of value theory, it must be clear that we share a common recognition of the importance of Marx's labor theory of value, and a similar dedication to the restoration of Marx's framework. The discussion will be conducted at three distinct levels.

(1) All advocates of the traditional approach are under violent attack in Freeman's analysis. The first sentence of his paper is illustrative: "[...] the simultaneous equations approach *[i.e., the traditional definition of values, D.L.]* of General Equilibrium theory [...]" (225). We are familiar with this criticism; it means that all Marxist economists who used in the past, or are still using, the traditional conception of values are actually neoclassicals. We disagree. In our view, it is erroneous to contend that the traditional computation of value assumes that the economy is in an equilibrium and that there is no technical change, and it is specifically incorrect to contend that this computation assumes that the economy is in a *Walrasian* equilibrium.

(2) The main divergence concerns obviously the *value conservation principle* itself. It is based on an inappropriate analogy with physics. Value is not a quantum of energy or an electric charge. The value conservation principle leads to an inapposite treatment of the effects of technical change and of disequilibrium on values.

(3) Another facet of this controversy involves internal criticism. The sequential-value theory is still incomplete (in its treatment of fixed capital). It assumes far more equilibrium than it acknowledges. It leads to properties which contradict basic results and findings, and will certainly not help Marxist economists in their investigation of contemporary capitalism.

The discussion below abstracts from a number of important issues, such as the treatment of money or unemployment. Section 2 introduces Freeman's framework of sequential values, in comparison to that of traditional values, and discusses some of its puzzling properties. The main criticism of the value conservation principle is presented in section 3. Section 4 is devoted to the treatment of fixed capital and some of its deficiencies. Last, section 5 vindicates the traditional approach.

2. Sequential Values

This section recalls the main elements of Freeman's analysis. The definition of sequential values and their relationship to Marx's distinction between individual and market values are introduced in the first section. The second section discusses the common points and differences between the traditional and sequential formalisms. The third section contends that Freeman's equations assume more equilibrium than he acknowledges. The fourth section is devoted to Freeman's simultaneous consideration of labor and market prices in his definition of values. The last section provides several examples of the paradoxical properties of this framework, such as the possible rise of values with a growing labor productivity.

2.1 Creation and Destruction of Value - Individual and Market Values

In many respects, Freeman's view of the creation and destruction of value is traditional:

(1) Circulation does not *per se* create or destroy value, but redistributes it within the economy.
(2) Value is increased in production, by the amount of socially necessary labor time incorporated. The value of inputs is transferred to that of outputs.
(3) Value is destroyed in final consumption.

At this very general level of analysis, there should be a basic agreement.[1] The core of the controversy lies, however, in the notion of "transfer." Freeman endows transfer with a very general meaning, with which we disagree. According to Freeman, value once created can *only* be destroyed in final consumption. One consequence of this value conservation principle is that inputs are estimated, within value equations, at their value as outputs of the previous period. This is the meaning of the term "sequential" as opposed to "simultaneous."

[1] One could, however, question several options in Freeman's analysis. It would, for example, be more appropriate to contend that value is destroyed when commodities are purchased by final consumers, since goods in the hands of consumers can no longer be called commodities in the strict sense. Value is a social relationship, not the property of a good independent of its link to the market.

One problem for the value conservation approach is the possible coexistence on a market of commodities produced at different periods. This issue is not discussed clearly by Freeman, but we can surmise his view from his equations.

Consider, for example, a stock of inventories of unsold commodities transferred to the next market simultaneously with a new round of production. If technology changes, the two categories of the same commodity, according to their distinct origin, will coexist on the market. Following the value conservation principle, the two categories of goods have different "individual values" (since the value of inventories transferred are not reevaluated under the present technique). In this framework, it seems logical to compute the average of individual values, as Freeman does. The same procedure holds in the case of the transmission over time of a stock of raw materials, and of a stock of fixed capital whose service life is larger than one period.

This procedure is consistent with Marx's notions of "market value" and "individual value" (K. Marx 1894 : ch. 10). When various amounts of a commodity are produced by different techniques, the value of the commodity, or its market value, is the weighted average of the individual values. However, in Marx's analysis, the notion of market value applies at a particular point in time when technology is heterogeneous. Freeman uses the same notion in a temporal framework, averaging values inherited from several periods.

2.2 A Comparison of the Formalisms of Traditional and Sequential Values

It is useful to begin with a few remarks concerning the use of formalization in economics in general. It is often hard to actually translate a verbal economic analysis into equations, and the correspondence between the two approaches must be carefully controlled. The problem of the appropriate degree of complexity is also crucial. A formal framework must be simultaneously simple and amenable to generalization.

(1) Excessive complexity must be avoided. Complexity often hides important implicit assumptions. Moreover, it is typically distorted in one specific direction. (One aspect of the model is abusively

developed while others are treated simplistically.[2])

(2) A model must also be susceptible to generalization, *i.e.*, made more concrete in one direction or another.

These principles have straightforward implications concerning value analysis. It is quite appropriate to begin with a simple linear model of single production in discrete time—the simplest manner of modeling production. Most basic problems can be addressed in this framework, the difference between the traditional and sequential computations of values in particular. Then, the analysis will have to pass the test of generalization by, for example, its extension to joint production. As has been shown by several decades of controversies over joint production, most problems are overcome by the discovery that basic concepts have not been correctly defined.[3]

The above principles suggest that we should begin our investigation of sequential values in the simple and natural framework of standard sequential analysis in discrete time, in which production and circulation periods follow one another:

$$\ldots \rightarrow \begin{matrix}\text{Production}\\ \text{Period } t-1\end{matrix} \rightarrow \begin{matrix}\text{Circulation}\\ \text{Period } t-1\end{matrix} \rightarrow \begin{matrix}\text{Production}\\ \text{Period } t\end{matrix} \rightarrow \begin{matrix}\text{Circulation}\\ \text{Period } t\end{matrix} \rightarrow \ldots$$

Freeman favors a continuous time framework, but there is nothing specific to sequential-value analysis which requires the use of continuous time.

Contrary to his claim, Freeman's continuous time framework is not more general than conventional discrete time models.

(1) The conventional discrete time framework assumes that the *period of production* is equal to the *unit time period*, and that the *circulation period* is equal to zero. Any other assumptions, for example the consideration of a production period equal to a

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[2] Marx's analysis in *Capital* provides a clear example of the power of abstraction. A problem is usually treated originally, in the simplest (most abstract) possible framework. The same principles should apply to modeling.

[3] See, *e.g.*, the definition of values (with reference to Marx's concepts of individual and market values) and the discussion of the condition for the existence of positive values (the substitution of the notion of non-reductivity for that of productivity), in G. Duménil, D. Lévy 1988 and 1989.

multiple of the unit time period and a circulation period different from zero, would require the consideration of inventories of goods in process and finished goods. If, for example, the production of a ship requires one year and the unit time period is one day, a stock of goods in process must be considered at each period, the unfinished ship after 1, 2,..., or 364 days.

(2) Freeman criticizes the assumption used in discrete time models of a same production period in each production process, yet his own assumption is even stronger: production periods in his approach are not only of the same duration, but equal to zero. Freeman's model does not solve any of the problems concerning the strictly positive duration of production and circulation periods.[4]

Our description of technology is conventional. n goods exist (with $i = 1,\dots,n$). Each production process is denoted with the same subscript as the good produced. Returns to scale are constant. A production process is represented by a row vector of physical inputs, and an amount of labor (a scalar):

$$A_i,\ L_i \rightarrow 1 \text{ unit of good } i$$

We will adopt the following notation:

A	Matrix of physical input coefficients
L	Column vector of labor input coefficients
Λ	Column vector of traditional values
λ	Column vector of sequential values

These variables all change over time and must be indexed with the superscript t.[5]

The equations for the two definitions of values are:

$$\Lambda^t = A^t \Lambda^t + L^t \tag{1}$$

[4] The consideration of production and circulation periods different from zero in a continuous time framework would be possible (with production and circulation period of a given duration beginning at any instant), in a model in which mixed differential-integral equations are involved.

[5] The notation x^t refers to variable x in period t, whereas $(x)^t$ denotes x power t.

$$\lambda^t = A^t \lambda^{t-1} + L^t \tag{2}$$

The second equation corresponds to Freeman's equation 13 (p. 231).[6]

If a commodity is produced by different techniques, values refer to the average of individual values, and (A^t, L^t) denotes the average, not the best available, technology. There is no significant difference between the two approaches in the treatment of joint production. Both the problems and the solutions are identical (see the appendix).

2.3 The Equations of Sequential Values Assume Market Clearing

Freeman is certainly right to present his views in the simple formalism of his sections 3 and 4, but he should not claim that his equations do not assume equilibrium in opposition to the traditional equation (equation (1) in this paper). Indeed, his equation (13) (or equation (2) in this paper) precisely assumes equilibrium on the market.

What would a disequilibrium sequential-value equation look like? If markets do not clear (one aspect of disequilibrium), it becomes necessary to consider stocks of inventories of unsold commodities. These inventories are transferred to the next period's market, and are part of the supply on the next period's market. If Q_i^{t-1} and D_i^{t-1} denote respectively the supply and demand of good i on market $t-1$, with, for example, $Q_i^{t-1} > D_i^{t-1}$, a stock of inventories $S_i^t = Q_i^{t-1} - D_i^{t-1}$ is held. With the sequential approach, the new output, Y_i^t, and the inventories, S_i^t, inherited from the previous market, do not have the same individual value. The individual value of the commodity produced in period t will be denoted $\lambda_{i,1}^t$ and, as transferred in the stock of inventories, $\lambda_{i,2}^t$. Individual sequential values are determined by:

$$\lambda_{i,1}^t = A_i^t \lambda^{t-1} + L_i^t$$
$$\lambda_{i,2}^t = \lambda_i^{t-1}$$

Weighting these two values by the corresponding quantities (Y_i^t produced and S_i^t inventories transmitted from the previous market $t-1$), one

[6] If technology is constant over time, traditional values are constant, while sequential values vary from period to period, from the initial values to the traditional values.

obtains:

$$\lambda_i^t = \frac{Y_i^t \lambda_{i,1}^t + S_i^t \lambda_{i,2}^t}{Y_i^t + S_i^t} = \frac{Y_i^t (A_i^t \lambda^{t-1} + L_i^t) + S_i^t \lambda_i^{t-1}}{Y_i^t + S_i^t}$$

which is different from equation (2), unless market is in equilibrium in $t-1$, i.e., if S_i^t equals zero.

2.4 A "Labor-Market Price" Theory of Value

An important difference between the traditional equation (1) and the sequential-value equation (2), is the following: in the traditional definition, values, Λ^t, are solely a function of the technology in period t, whereas sequential values, λ^t, depend further upon values manifested during the previous circulation period.

The difference between the two approaches is even more profound. Traditional values are associated with a *production* period, and only depend on the prevailing technology during this period. Sequential values are attached to a *circulation* period, and depend on: (1) the technology of the previous production period; (2) *exchanges* which occurred during the *previous* circulation period. A first problem is that, in a disequilibrium, all commodities produced are not necessarily sold. A second problem that arises is that commodities are sold at market prices that diverge from values. The sequential value in period t is a function of p^{t-1}, the market price in $t-1$ (the "observed input price" in the "economy itself," p. 230), not of the sequential value in *t-1*. Abstracting from any inventories of unsold commodities, equation (2) should, thus, read:

$$\lambda^t = A^t p^{t-1} + L^t$$

This combination of values and market prices within the same equation is puzzling, but it is a basic feature of the sequential-value approach. It is a considerable deviation from Marx's labor theory of value. Sequential values are clearly consubstantial with prices, within a *labor-market price* theory of value. As shown in the next section, paradoxical consequences can result.

2.5 A Productivity Paradox?

The adoption of the formalism of sequential values has unexpected consequences. The progress of labor productivity does not necessarily diminish values, in contrast with the principle of labor embodiment on which the labor theory of value rests. Freeman himself does not question the fact that the rise of labor productivity diminishes values: "These *[sequential values, D.L.]* are lower than the values of previous periods because labor productivity has risen" (229). This is true in the traditional conception, but not always in his.

Consider, for example, the case of "pure" technical progress in which the quantities of physical inputs and living labor both diminish over time. Using the traditional definition of values, one has:

$$\Lambda^t = (I - A^t)^{-1} L^t = \sum_{k=0}^{\infty} (A^t)^k L^t$$

If A^t or L^t diminishes with time, so does Λ^t. But this is not always guaranteed to be true for sequential values.

This puzzling property is already evident in an economy with a single commodity. Suppose the technology is as follows:

a units of good + l units of labor $\rightarrow$ 1 unit of good

Now consider the two cases below, that of two successive periods, and that of an infinite trajectory:

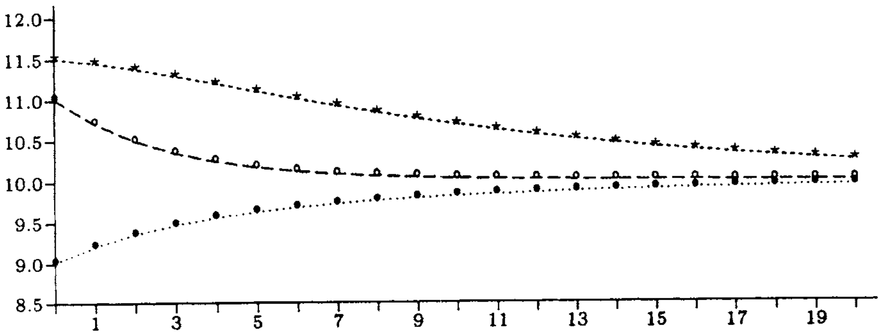
(1) In period 1, technology is $a^1 = 0.5$ and $l^1 = 1$. Both technical coefficients are diminished in period 2: $a^2 = 0.44$ and $l^2 = 0.96$. To compute λ^1 and λ^2 in periods 1 and 2, one needs to know the value or rather the market price in period 0. With $p^0 = 1.2$, it is easy to determine λ^1 and λ^2. One obtains: $\lambda^1 = 1.6$ and $\lambda^2 = 1.664$. Value has been increased: $\lambda^1 < \lambda^2$, and both are larger than p^0.[7]

[7] A different p^0 would yield different results. For example, $p^0 = 1.6$ gives $p^0 < \lambda^2 < \lambda^1$ and $p^0 = 2.4$ gives $\lambda^2 < \lambda^1 < p^0$.

(2) Still within the same economy with a single good, assume now that the amount of physical input is maintained, $a^t = a$, and that of labor reduced at each period: $l^t = \alpha + \beta(\gamma)^t$, with $\gamma < 1$. The sequential values can be determined explicitly:

$$\lambda^t = \frac{\alpha}{1-a} + \left(p^0 - \frac{\alpha}{1-a} + \frac{\beta}{a-\gamma} \right)(a)^t - \frac{\beta}{a-\gamma}(\gamma)^t$$

If $a > \gamma$ and $p^0 + \dfrac{\beta}{a-\gamma} < \dfrac{\alpha}{1-a}$, then λ^t always *increases* with time, instead of declining. Figure 1 illustrates this property for $a = 0.9$, $\alpha = 1$, $\beta = 0.1$, $\gamma = 0.7$, and $p^0 = 9$.



When labor productivity rises, traditional values (∘) always decline. Sequential values may decline (∗), but they can also rise (•), depending on the initial market price : $p^0 = 9$ for (•) and $p^0 = 11.5$ for (∗).

Fig. 1. Sequential, (•) or (∗), and Traditional (∘) Values, with Rising Labor Productivity

In this respect, there is no doubt that the traditional interpretation is closer to Marx's analysis than the sequential-value approach. The origin of the productivity paradox lies in the fact that this approach is not a pure labor theory of value, but rather a *labor-market price* theory of value. When the outcome of the market in period $t = 0$ is modified (when p^0 is changed), the entire sequence of values over time is altered.

3. The Devaluation of Capital

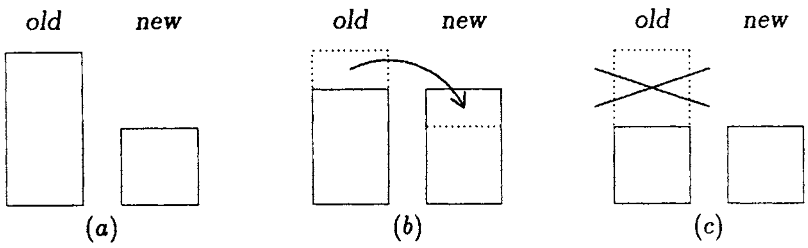
This section is devoted to the criticism of Freeman's value conservation principle, on which the sequential-value approach is based. The first two sections address the issues of the devaluation of capital, in relation to either technical change or disequilibrium. A third section briefly discusses Marx's view of these problems.

3.1 Technical Change

This section focuses on the problems encountered in the determination of the value of a commodity produced in the past, assuming technical progress. In this case, less and less labor is required for the commodity's production (what is known as the progress of labor productivity). The difference between the two perspectives is starkly evident in this respect:

(1) In the sequential-value approach, one must distinguish between the individual values of commodities produced under distinct historical conditions and the market value of this commodity, the average of all individual values (section 2.1). The value conservation principle implies that *all individual values are conserved.* The market value can be smaller or larger than the individual values of these various components. Some capitals may be devalued, other reevaluated. What is clear is that *globally* there is no devaluation (see figure 2).

(2) Within the traditional conception of value, the extra labor embodied in the past is no longer acknowledged as socially necessary labor time, and vanishes. The capital in which such obsolete goods exist as components of commodity or productive capital is devalued. Devaluation is an economywide effect, considered globally.



(a) Individual values of commodities produced with distinct techniques (an *old* and a *new* technique) at two different points in time; (b) transfer of value and equalization, according to the value conservation principle; (c) devaluation of commodities produced in the past, as in the traditional approach.

Fig.2. Two Views of the Effects of Technical Change

3.2 Disequilibrium and Crises

The title of the book in which Freeman's study was published refers explicitly to disequilibrium. The introduction of the chapter lists eight aspects of disequilibrium. We fully applaud the prominence given to disequilibrium. But, unfortunately, some forms of disequilibrium call into question the adequacy of Freeman's approach. The existence of inventories of unsold commodities and the fact that exchanges may be performed at any market prices have already been considered in section 1. This section briefly discusses two examples of more dramatic developments, the destruction of commodity and productive capital, and the problem of the transmission of the value of fixed capital during crises.

Disequilibrium represents a constant threat to commodity or productive capital in capitalism. Each interruption of the circulation of capital risks turning into a crisis. A crisis is a dramatic and general manifestation of disequilibrium. It may lead to the destruction of raw materials, commodities, or fixed capital. In our opinion, this situation translates directly into devaluations, that is, actual losses of values. The values of commodities which have been destroyed are not transferred to those which survive the crisis.

A more difficult issue is that of the capacity utilization rate, a crucial aspect of disequilibrium in capitalism. Productive capacity is not fully used. Even if we accept the existence of a normal capacity utilization rate different from 100 percent, say 80 percent, this level is not continuously maintained. These fluctuations reflect the day to day maladjustments, as well as business-cycle fluctuations.[8] The central problem, however, is that of the conservation of value during crises. If machines lie idle during a considerable period of time, what becomes of their value ? Is it destroyed, conserved, or transferred?

Overall, the value conservation principle certainly suffers from an inability to account for a number of exceptions related to disequilibrium and, in particular, crises. Crisis is an important feature of actual capitalism, which accounts for much of the violence of adjustments in capitalism.

3.3 Marx and the Value Conservation Principle

In point of fact, Freeman is not faithful to Marx's analysis. It is simply not credible to claim that Marx was an advocate of the value conservation principle. Marx never alluded to a compensation (a transfer of value) when capital is devalued. Instead, he repeatedly pointed to the opposite property:

> But in addition to the material wear and tear, a machine also undergoes what we might call a moral depreciation. It looses exchange value, either because machines of the same sort are being produced more cheaply that it was, or because better machines are entering into competition with it. In both cases, however young and full of life the machine may be, its value is no longer determined by the necessary labour-time objectified in it, but by the labour-time necessary to reproduce either it or the better machine. It has therefore been devalued to a greater or lesser extent (K. Marx 1867, ch. 15: 528).

> A commodity represents, say, 6 working hours. If an invention is made by which it can be produced in 3 hours, the value, even of the commodity already produced, falls by half. It now represents 3 hours of socially necessary labor instead

[8] There is no simple manner of formalizing the utilization of productive capacity in Freeman's fixed capital framework. This remark echoes the fact that this important mark of disequilibrium is not listed in his eight points.

> of the 6 formerly required. It is therefore the quantity of labour required to produce it, not the objectified form of that labour, which determines the amount of the value of a commodity (K. Marx 1867, ch. 19: 677).
>
> Apart from all the accidental circumstances, a large part of the existing capital is always being more or less devalued in the course of the reproduction process, since the value of commodities is determined not by the labour-time originally taken by their production, but rather by the labour-time that their reproduction takes, and this steadily decreases as the social productivity of labour develops (K. Marx 1894, ch. 24: 522).

Marx's discussion of the devaluation of capital during crises is also well known:

> The chief disruption *[in a crisis]*, and the one possessing the sharpest character, would occur in connection with capital in so far as it possesses the property of value, i.e. in connection with capital *values*. The portion of capital value that exists simply in the form of future claims on surplus-value and profit, in other words promissory notes on production in their various forms, is devalued simultaneously with the fall in the revenues on which it is reckoned (K. Marx 1894, ch. 15: 362).

4. Fixed Capital

So far we have shown that Freeman's framework diverges from the traditional analysis in only one respect, the *definition of value*. Fixed capital presents even deeper more disturbing problems for Freeman. The *description of technology* is also at issue. In place of the traditional conception of technology in which two machines of different ages are treated as two distinct commodities, Freeman substitutes a view in which they are considered as two distinct quantities of the same good. The purpose of this section is to discuss the relationship between Freeman's modeling of fixed capital and his particular definition of values.

The first section below recalls the standard framework of fixed capital in linear models of production and the traditional computation of values. The next section is devoted to Freeman's analysis, his modeling of fixed capital, and the determination of sequential values. A last section discusses the compatibility of sequential values and the standard modeling of fixed capital. There should be no disagreement that a general theory of values should be compatible with any "reasonable" modeling of fixed capital.

(1) The traditional definition of values is compatible with the two frameworks, the standard model of fixed capital as well as that of Freeman.

(2) The problems faced by the sequential definition of values in the traditional modeling of fixed capital seem insuperable. Freeman has to build an alternative framework to render his interpretation compatible with the existence of fixed capital.

4.1 The Standard Modeling of Fixed Capital and the Traditional Computation of Values

We consider here a simple model, which can be easily generalized. Fixed capital is represented by a machine which can be used over two production periods. Its use-value remains unaltered—the new and one-period old machines produce the same amount, b, of the output—but it must be discarded after two periods. There are no physical inputs other than the machine, and only labor is required. Therefore, technology for the production of each commodity is described by two alternative processes, as follows:

a new machines + l units of labor $\rightarrow$ b units of the good + a old machines

a old machines + l units of labor $\rightarrow$ b units of the good

The traditional values, Λ_1 and Λ_2, of the new and old machines can be easily determined in the subsector producing the machine itself, which can be isolated from the rest of the economy:

$$
\begin{aligned}
a\Lambda_1 + l &= b\Lambda_1 + a\Lambda_2 \\
a\Lambda_2 + l &= b\Lambda_1
\end{aligned}
\tag{3}
$$

It follows that $\Lambda_2 = \frac{1}{2}\Lambda_1$, i.e., half of the value of the machine is transferred during each production period. The determination of $\Lambda_1 = \frac{2l}{2b-a}$ has little interest, except to recall the condition: $b > a/2$.

4.2 Freeman's Treatment of Fixed Capital and the Determination of Sequential Values

Freeman's line of argument in *Age Doesn't Matter* (254–55) is difficult to follow. He first considers the example of an imperishable raw material: copper. We certainly agree that, for this particular category of good, it would be possible to abstract from age. However, Freeman's analysis breaks down when he extends this assumption—not a simplifying assumption, but the establishment of a new approach—to all constant capital, circulating or fixed. In our view, age matters. This seems obvious for some perishable inputs, for example cheese, but is also true for fixed capital.

In Freeman's approach, machines are treated like imperishable raw materials. Old and new machines are different quantities of the same good. During a production process, a "fraction" of the machines is consumed. For example, if we begin with two machines, and if their service life is two periods, one new machine exists after production in Freeman's approach, when the traditional modeling states that two old machines emerge from the production process.

Although we believe Freeman's approach is not the best possible approximation, the treatment is coherent and can be formalized. To come closer to Freeman's analysis, we assume that only one other commodity exists in the economy, a total of two commodities, the machine and a consumption good. Each commodity is produced by a distinct production process using machines and labor. In each process, the fraction of the stock of machines which has not been consumed is conserved (half of the machines if the service life is 2 periods):

a machines + l units of labor $\rightarrow$ b machines + $\frac{a}{2}$ machines

a' machines + l' units of labor $\rightarrow$ b' units of consumption good + $\frac{a'}{2}$ machines

or:

$$(a,0),l \rightarrow \left(\frac{a}{2}+b,0\right)$$
$$(a',0),l' \rightarrow \left(\frac{a'}{2},b'\right)$$

The issue now is the computation of values. There is no problem in determining traditional values in this framework.[9] In contrast, here is how we understand Freeman's computation of sequential values. Since the machine is the output of several processes, the relevant framework is that of individual and market values. The term $\lambda_{1,1}^{1}$ denotes the individual value of the machine in the first process, and $\lambda_{1,2}^{1}$ its individual value in the second process. There is no specific difficulty concerning the first process, which allows for the determination of $\lambda_{1,1}^{1}$ as a function of λ_{1}^{0} which is given:

$$a\lambda_1^0 + l = \left(\frac{a}{2}+b\right)\lambda_{1,1}^1 \tag{4}$$

The second process is similar to a case of joint production: in addition to the output of consumption good, b', we also find $a'/2$ machines. Freeman substitutes for this joint production process two processes of single production. The first one produces the consumption good with half of the stock of machines, and labor. The second half is conserved without alteration:

[9] One finds: $\Lambda_1 = \frac{2l}{2b-a}$ and $\Lambda_2 = \frac{b}{b'}\frac{2l' + \frac{a'l-al'}{b}}{2b-a}$.

$$\left(\frac{a'}{2},0\right),l' \rightarrow (0,b')$$
$$\left(\frac{a'}{2},0\right),0 \rightarrow \left(\frac{a'}{2},0\right)$$

As can be easily checked by summing the two processes, one obtains the original process. For each single production process, there corresponds one value equation:

$$\frac{a'}{2}\lambda_1^0 + l' = b'\lambda_2^1 \tag{5}$$
$$\frac{a'}{2}\lambda_1^0 = \frac{a'}{2}\lambda_{1,2}^1 \tag{6}$$

From equation (5), one can directly determine the value of the consumption good:

$$\lambda_2^1 = \frac{\frac{a'}{2}\lambda_1^0 + l'}{b'}$$

For the production good, one must combine equation (4) with equation (6). The computation of the average of these individual values provides the value of λ_1^1:

$$\lambda_1^1 = \frac{\left(\frac{a}{2}+b\right)\lambda_{1,1}^1 + \frac{a'}{2}\lambda_{1,2}^1}{\frac{a}{2}+b+\frac{a'}{2}} = \frac{a\lambda_1^0 + l + \frac{a'}{2}\lambda_1^0}{\frac{a}{2}+b+\frac{a'}{2}} \tag{8}$$

Equations (8) and (7), giving the values of λ_1^1 and λ_2^1, are identical to Freeman's equations (23) and (24), for $a=70$, $l=300$, $b=50$, $a'=20$, $l'=200$, and $b'=100$.

4.3 Sequential Values in the Standard Modeling of Fixed Capital

The purpose of this section is to discuss the compatibility of Freeman's definition of value with the traditional model of fixed capital: Can the sequential-value approach apply in this framework? In what follows we make two such attempts, but without success. It is difficult to contend that this extension is impossible, but it appears, at best, uneasy. Problems arise in the dynamic properties of sequential values.

The problematic characteristics of sequential values in the traditional modeling of fixed capital are already evident in a very simple model in which only one good is produced, and in the absence of technical change. This good can be used either as a consumption good or a producer good. In this latter case, it can be used during two periods, with the same use-value during the two periods (as assumed in the previous section). However, two "goods" must be distinguished in the formalism. The first good is that which has just been produced, and the second is the production good produced one period earlier. Thus, technology can be described as:

a new machines + l units of labor $\rightarrow$ b new machines + a old machines

a old machines + l units of labor $\rightarrow$ b new machines

or:

$$(a,0), l \rightarrow (b,a)$$
$$(0,a), l \rightarrow (b,0)$$

The sequential-value approach does not provide an ability to consistently treat this case. The value conservation principle is not sufficient in itself without an additional assumption.

A first assumption might be that the individual value of the good is the same in the two processes. This assumption seems quite natural, since the two processes use the same amounts of inputs (a units of consumption good and l units of labor) to obtain the same output (b units of output). The value equations can then be easily written as follows:

$$a\lambda_1^t + l = b\lambda_1^{t+1} + a\lambda_2^{t+1}$$
$$a\lambda_2^t + l = b\lambda_1^{t+1}$$

Subtracting the second equation from the first, one obtains a relation of recursion:

$$\lambda_1^{t+1} = \frac{a}{b}\lambda_2^t + \frac{l}{b} \qquad (9)$$
$$\lambda_2^{t+1} = \lambda_1^t - \lambda_2^t$$

The fixed point can be determined from:

$$\lambda_1^* = \frac{a}{b}\lambda_2^* + \frac{l}{b}$$
$$\lambda_2^* = \lambda_1^* - \lambda_2^*$$

One obtains:

$$\lambda_2^* = \frac{\lambda_1^*}{2} = \frac{l}{2b-a}$$

i.e., traditional values ($\lambda_i^* = \Lambda_i$). The problem is that the recursion is always *unstable*.[10] This instability is manifested in the fact that, beginning with any initial values other than traditional values, one of the two values becomes necessarily negative (see figure 3).

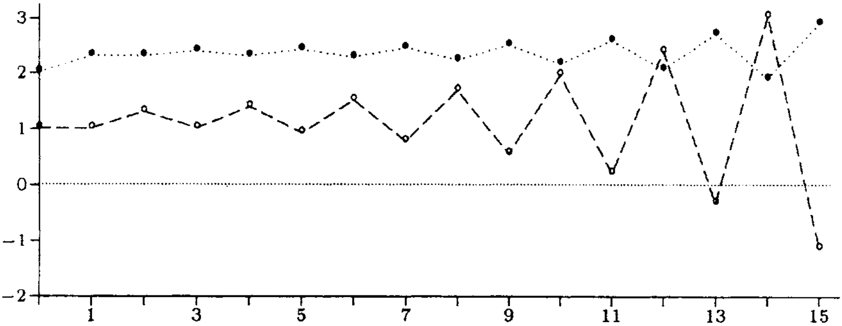
[10] The recursion (9) can be written:

$$\begin{pmatrix} \lambda_1^{t+1} - \lambda_1^* \\ \lambda_2^{t+1} - \lambda_2^* \end{pmatrix} = M \begin{pmatrix} \lambda_1^t - \lambda_1^* \\ \lambda_2^t - \lambda_2^* \end{pmatrix} \text{ with } M = \begin{pmatrix} 0 & \frac{a}{b} \\ 1 & -1 \end{pmatrix}$$

The polynomial characteristic of the Jacobian matrix is:

$$P(\mu) = \det(\mu I - M) = \begin{vmatrix} \mu & -\frac{a}{b} \\ -1 & \mu+1 \end{vmatrix} = \mu(\mu+1) - \frac{a}{b}$$

One root is always smaller than -1.



The determination of sequential value in the usual fixed capital framework raises considerable problems. In the example in this figure, the sequential values of a new machine and a one-period old machine fluctuate over time. Two puzzling properties are observed. The value (∘) of the older machine becomes recurrently larger than that (•) of the new machine, and sometimes negative.

Fig. 3. Negative Sequential Value in a Simple Fixed Capital Model

An alternative to the above assumption might be that the production good transfers its value in proportion to time. If λ_1^t is the value of the new production good, it depreciates by half of its value at each period and the other half remains in the old machine: $\lambda_2^{t+1} = \lambda_1^t / 2$. Under this assumption, the first process can be written:

$$\frac{a}{2}\lambda_1^t + l = b\lambda_1^{t+1}$$

and the second (with $\lambda_2^t = \frac{1}{2}\lambda_1^{t-1}$):

$$\frac{a}{2}\lambda_1^{t-1} + l = b\lambda_1^{t+1}$$

These two equations are incompatible, except if $\lambda_1^t = \lambda_1^{t-1}$, i.e., if the values are constant over time (and equal to traditional values).

The difficulties that are encountered in an extremely simple formalism question whether sequential values can provide a general theory of values. The theory is quite dependent on a specific modeling of fixed capital, and cannot be generalized.

5. The Traditional Approach under Attack

The first section below challenges Freeman's criticisms of the traditional approach concerning technical change and disequilibrium. The second section compares Freeman's view of the explanatory power of the labor theory of value to ours. A last section addresses the issues of equilibrium and dynamics, on which we also diverge.

5.1 Freeman's Criticisms

Freeman's first criticism is that the traditional interpretation is incompatible with the existence of technical change. Consider two successive production periods, with distinct techniques. The technique during the first period is (A^{t-1}, L^{t-1}), and values are Λ^{t-1}. During the second period they become (A^{t}, L^{t}), and Λ^{t}. *A priori*, Λ^{t} differs from Λ^{t-1}. Since the outputs of the first period are precisely the inputs of the second, they have two distinct values. For Freeman, the traditional interpretation is not compatible with technical change. Similarly, the use of simultaneous equations implies, still following Freeman, that equilibrium prevails.

A commodity can have a value when considered in relation to the conditions of production in one period, and another one when considered in relation to the conditions of production of the next period. Semantically, the expression "the value of a commodity" is an abbreviation for "the value of a commodity in the conditions of production prevailing at this particular, present or past, instant." In other words, the reference to value indepen-dently of specific conditions of production is undefined, and a commodity has as many distinct individual values as conditions of production.

5.2 The Use and Abuse of the Labor Theory of Value

We believe we can agree with Freeman on the following. The labor theory of value is an analytical tool to understand the functioning of capitalism. It is fundamental in a sense, since it provides the basis for the development of the theory of *capital*. Capital, following Marx, is "value" in a process of self-expansion. "Value in process" refers to the circulation of value-capital through its various forms of money, commodity, and productive capital, as in Volume II of *Capital*. "Self-expansion" stands here for the theory of surplus value. This is where the labor theory of value is inescapable.

The core of the explanatory power of the labor theory of value lies in the analysis of exploitation. Neither Walrasian equilibrium nor Sraffa's framework allow for our understanding of the origin of profit. Several components must be combined to obtain the theory of surplus value: (1) only labor creates value; (2) value can be created in one point of the productive system and realized somewhere else (and this explains why the notions of prices or physical bundles are not sufficient); (3) through their wages, workers recover purchasing power over a fraction of the total value they have created in one period.

The disagreement with Freeman concerns the *extension* of the explanatory power of the labor theory of value. The analogy with physics is misleading (G. Duménil, D. Lévy 1997). It is true in physics that Maxwell's equations and Newton's gravitation equations provide the foundations on which physics and chemistry are built. But any attempt to ground all economics on the labor theory of value (or any other fundamental principle) is misguided. The labor theory of value is not the necessary foundation for the analysis of every mechanism in capitalism.[11] For example, the gravitation of prices around prices of production must be established independently of the theory of value. Contrary to Freeman, we believe that other theories also exist independently of labor value, such as the theory of crisis or of historical tendencies. In particular, the labor theory of value does not provide the framework to account for disequilibrium and dynamics in capitalism.

The statement that the analysis of disequilibrium in capitalism is separate from the explanatory power of the labor theory of value does not mean that disequilibrium is not a central aspect of capitalism. We have been working for years on this issue. One aspect of this work was, in particular, to translate into equations Marx's description of the behaviors of agents in competition, building disequilibrium microeconomics based on adjustment behaviors, and to provide a framework for the analysis of

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[11] Even if this were the case, additional assumptions would be required. For example, the tendency for the rate of profit to fall cannot be proven independently of assumptions on technical change.

business-cycle fluctuations (G. Duménil, D. Lévy 1993). The labor theory of value was not a necessary part of this investigation.

5.3 Equilibrium, Disequilibrium, and Dynamics

A final disagreement concerns the use of equilibrium in economic theory. This issue touches upon a large number of interesting aspects of economic theory. We will not repeat here basic principles that we presented in other works, and that we used as guidelines in actual theoretical and empirical research, but limit our comments to three points where we obviously diverge with Freeman:

(1) *The usefulness of a theory of equilibrium.* Even if the economy is always in disequilibrium, it does not mean that the theory of equilibrium is irrelevant. A counterexample is the theory of prices of production.[12]

(2) *All theories of equilibrium are not equivalent.* Walrasian equilibrium is apologetic, and does not provide a faithful account of the working of capitalism. Classical (Marx's) theory of long-term equilibrium is an important tool in the analysis of capitalism. One must, in particular, distinguish between an *ex post* conception of equilibrium in which equilibrium is the fixed point of a realistic disequilibrium dynamic system (with stocks and flux relationships and transactions out of equilibrium), and an *ex ante* equilibrium in which no such process can be defined.

(3) *Dynamics and disequilibrium are not synonymous.* The appropriate framework of analysis must combine disequilibrium and dynamics, but a dynamic framework can assume equilibrium as is the case in a sequence of Walrasian temporary equilibria or models with rational expectations (or in the equations of sequential values in the absence of inventories and other disequilibria).

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[12] The existence of structural change does not refute the theory of classical long-term equilibrium. Market prices gravitate around a target (prices of production) moving over time (G. Duménil, D. Lévy 1995).

6. Appendix: Joint Production

Additional notation is required in the modeling of joint production, B_i and B, respectively denoting the vectors and matrix of the amount of the various goods produced. Production in process i can be represented as:

$$A_i, L_i \to B_i$$

In this framework, the conventional neo-Ricardian definition of values is based on the following equation:

$$B^t \Lambda^t = A^t \Lambda^t + L^t \tag{10}$$

A number of technical problems may arise when these equations are solved, in particular negative values may obtain. In Freeman 1996, joint production is not treated, since the matrix X is assumed to be diagonal. The simplest generalization of the sequential approach to joint production is the following:

$$B^t \lambda^t = A^t \lambda^{t-1} + L^t$$

A commonality between the two approaches is that the above difficulties, in particular the possible existence of negative values, exist in both formalisms. Sequential values are no exception in this respect.[13]

In G: Duménil, D. Lévy, 1988 and 1989, we rejected the conventional resolution of equation 10, in reference to the distinction between individual and market values.[14] When the same good is produced in several manners, as in joint production, the relevant framework is that of market value. Each joint production process is disaggregated into as many single production processes as commodities produced. The difficulty lies

[13] Consider, for example, the technology in period 1:

$$A^1 = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}, \; L = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \text{ and } B^1 = \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}$$

the new values, λ^1, during circulation 1 can be derived from those inherited from the previous periods, λ^0. With $\lambda^0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, one obtains $\lambda^1 = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$.

[14] See also P. Flaschel 1983.

in allocating inputs to the various commodities. A problem of indeterminacy is posed, that the theory of value, in the strict sense, cannot solve. Exactly the same procedure is used in A. Freeman, 1991.

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8. References

d'Autume, A., J. Cartelier (eds.). 1997. *Is Economics Becoming a Hard Science?* Aldershot, England: Edward Elgar.

Duménil, G. 1980. *De la valeur aux prix de production*, Paris: Economica.

Duménil, G., D. Lévy. 1988. Linear Joint Production Models: Prelude to a Reassessment of the Classical Legacy (Value, Equilibrium, and Disequilibrium). *Political Economy* 4 (2): 185–212.

______. 1989. Labor Values and the Imputation of Labor Contents, *Metroeconomica* 40 (2): 159–78.

______. 1993. *The Economics of the Profit Rate: Competition, Crises, and Historical Tendencies in Capitalism.* Aldershot, England : Edward Elgar.

______. 1995. Structural Change and Prices of Production, *Structural Change and Economic Dynamics* 6: 397–434.

______. 1997. Should Economics be a "Hard" Science? In J. Cartelier and A. d'Autume (eds.). Pp. 276–303.

Flaschel, P. 1983. Actual Labor Values in a General Model of Production. *Econometrica* 51 (2): 435–54.

Foley, D. 1982 Value of Money, the Value of Labor Power, and the Marxian Transformation Problem. *Review of Radical Political Economics* 14: 37–47.

______. 1997. Recent Developments in the Labor Theory of Value. Department of Economics, Barnard College, Columbia University, New York.

Freeman, A. 1991. Value and the Foundation of Dynamics. Working Paper, University of Sussex.

______. 1996. Price, Value, and Profit: A Continuous, General, Treatment. In A. Freeman and G. Carchedi (eds.). Pp. 225–79.

Freeman, A. and G. Carchedi. 1996. *Marx and Non-Equilibrium Economics*. Aldershot, England: Edward Elgar.

Marx, K. 1867. *Capital, Volume I*. New York: First Vintage Book Edition. 1981.

______. 1894. *Capital, Volume III*. New York: First Vintage Book Edition. 1981.