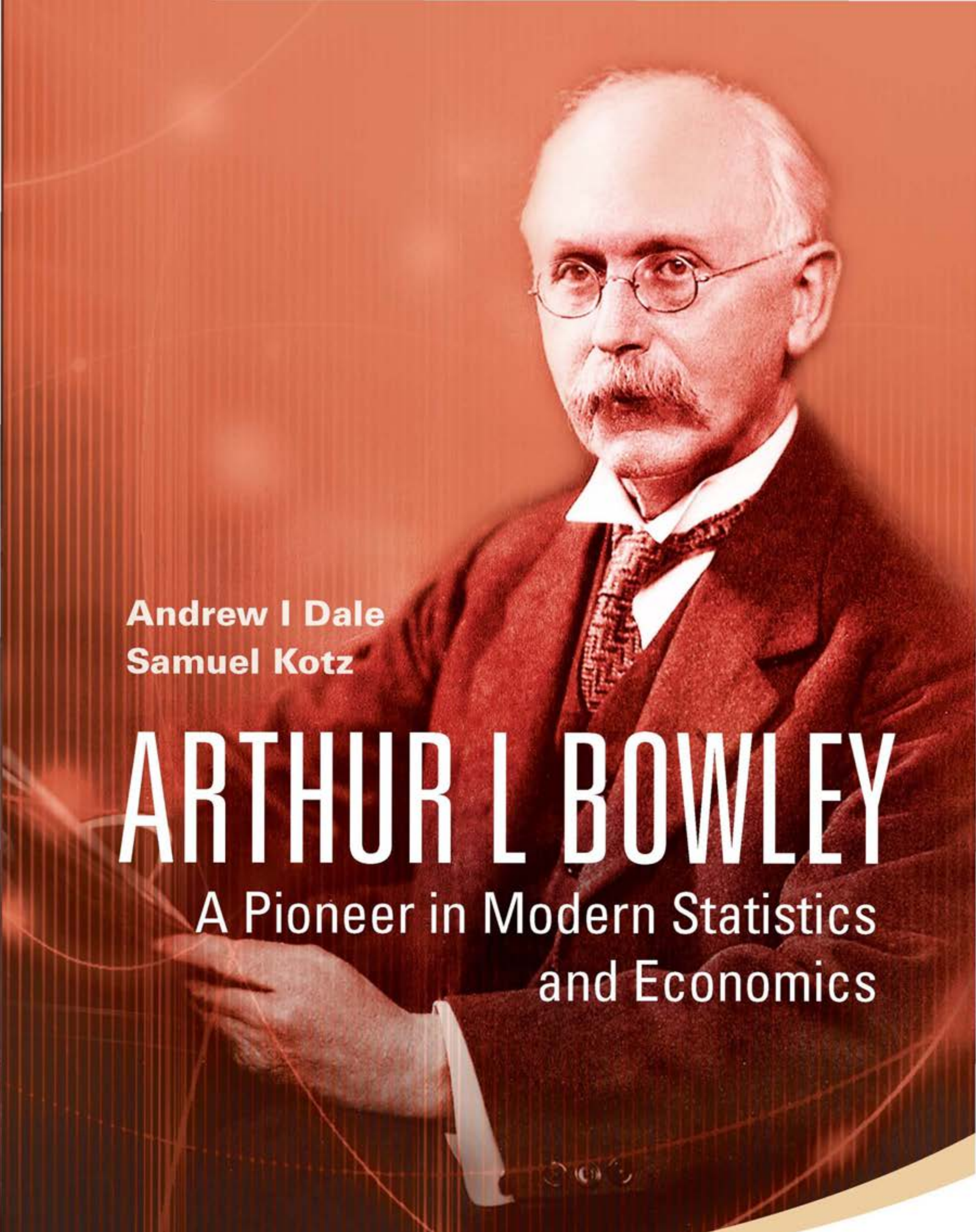


Andrew I Dale
Samuel Kotz

ARTHUR L BOWLEY

A Pioneer in Modern Statistics and Economics

World Scientific

ARTHUR L BOWLEY

A Pioneer in Modern Statistics and Economics

Andrew I Dale
University of KwaZulu-Natal, South Africa

Samuel Kotz
George Washington University, USA

World Scientific

NEW JERSEY • LONDON • SINGAPORE • BEIJING • SHANGHAI • HONG KONG • TAIPEI • CHENNAI

Published by

World Scientific Publishing Co. Pte. Ltd.
5 Toh Tuck Link, Singapore 596224
USA office: 27 Warren Street, Suite 401-402, Hackensack, NJ 07601
UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

British Library Cataloguing-in-Publication Data
A catalogue record for this book is available from the British Library.

Portrait of Arthur L Bowley on the cover – courtesy of Royal Statistical Society

ARTHUR L BOWLEY
A Pioneer in Modern Statistics and Economics

ISBN-13 978-981-283-550-5
ISBN-10 981-283-550-4

Printed in Singapore.

Sir Arthur Lyon Bowley

Preface

It is given to few in this world to make an outstanding contribution to a single discipline, and even fewer are capable of contributing in a noteworthy way to more than one. There are more, though they are also rarely found, who can make a significant and meaningful, though perhaps not outstanding, addition to two disciplines, and one of these was Arthur Lyon Bowley, whose work in the late 19th and the 20th centuries is remembered in both economic and statistical circles today.

In 2002 Stephen Stigler attempted a classification of a group of economist-statisticians. The *Great Middle Class* contained the names of those who were of importance in both economics and statistics but who lacked the importance of a *Master* in at least one of these disciplines, and here Stigler placed Bowley together with William Stanley Jevons, Ladislaus von Bortkiewicz, Wilhelm Lexis, Vilfredo Pareto and Frank Plumpton Ramsey.

From another point of view one may say that Francis Ysidro Edgeworth, Ronald Aylmer Fisher, Karl Pearson and Bowley dominated the development of statistics and economics in the United Kingdom from the late nineteenth century through to the first third of the twentieth. While the first three scholars dealt successfully with other scientific fields, Bowley devoted his work essentially to statistics and statistical economics.

The first issue of the *Journal of the Statistical Society of London* (a society that was to become the Royal Statistical Society in 1887) clearly set out the interpretation of the word *Statistics* as it was to be understood by the Society [Anon., 1838, p. 1]:

> Statistics ... may be said ... to be the ascertaining and bringing together of those "facts which are calculated to illustrate the condition and prospect of society;" and the object of Statistical Science is to consider the results which they produce, with the view to determine those principles upon which the well-being of society depends.

Distinction is made between Statistics as just defined and Political Economy, for while both have the same ends in view, the former neither discusses causes nor does it 'reason upon probable effects' (loc. cit.). Despite this sentiment, however, and perhaps more in the line of statistics as a deductive than an inductive science, it is further said here that 'Statistics seeks to deduce from well-established facts certain general principles which interest and affect mankind' [Anon., 1838, p. 3]. To a large extent these early statements were endorsed by Bowley in his statistical work, and in his *Elements of Statistics* we find the explicit definition of Statistics as 'the science of the measurement of the social organism, regarded as a whole, in all its manifestations' [1901a, p. 7].

We believe that Allen [1968] was correct in describing Bowley 'first and foremost' as an applied statistician. His field of practice was wide, as we shall see, but one may well view it as the social sciences, perhaps with emphasis on economics. Bowley's major contribution to statistics lay in his discussion of sample precision and in his development of sampling techniques—not in the agricultural field where analysis of variance and experimental design are paramount, but rather in the application of such techniques to economic and social studies (see Allen [1968]).

Fisher wrote 'The science of statistics is essentially a branch of Applied Mathematics, and may be regarded as mathematics applied to observational data' [1925, p. 1], and later he stated that Alfred North Whitehead used to say in one of his courses 'The essence of applied mathematics is to know what to ignore' [1938, p. 16]. Elsewhere we read that Lord Kelvin is supposed to have said that the essence of applied mathematics is to know when to approximate (Smith [1997, p. 41]). Both sentiments, we shall see, were characteristic of Bowley's work.

While his knowledge of, and ability in, mathematics was not inconsiderable—he had, after all, been joint tenth Wrangler in the Mathematical Tripos examination at Cambridge, and had published *A General Course of Pure Mathematics from Indices to Solid Analytical Geometry* in 1913—Bowley did not shine in this field, publishing little in mathematical statistics and mathematical economics. Mathematics to him was in the main a tool, and incidental to his chief concern.

Bowley's interest in social welfare naturally required his use of data obtained from official sources and censuses. He was a severe critic of such data when the need arose, and had he been called upon by the government as an advisor Britain might have enjoyed considerably improved social and economic statistics in the early twentieth century. Nevertheless, describing Bowley as 'the most innovative, policy-oriented social and economic statistician of his generation', Szreter and Smith said

> It was substantially Bowley's considerable authority and influence that resulted both in the important innovation of a parallel classification of the employment information, by personal occupations and by industrial function, which was adopted at the 1921 census; and also in the attempted family dependency analysis at that census. [1996, p. 275]

Bowley undertook several studies of working-class households in England, carefully describing in print how the sample was obtained. His use of representative and purposive sampling, later endorsed by the International Statistical Institute, ensured that data obtained from these studies were both reliable and useful. In the early 1930s he contributed significantly to Hubert Llewellyn Smith's *New Survey of London Life and Labour*, a large study of conditions in the capital. Here again Bowley may be seen as following the principles of the founders of the London Statistical Society, who stated that 'its [i.e. statistics'] peculiarity is that it proceeds wholly by the accumulation and comparison of facts, and does not admit of any kind of speculation' [Anon., 1838, p. 3]. Bowley's surveys, we shall see, also followed the further stipulated requirement that they be based on well-attested data and that they 'admit of mathematical demonstration' (loc. cit.).

A list of Bowley's published work runs into hundreds, including not only papers—Bowley lectured for many years at the London School of Economics and Political Science—but also numerous books (many of which have become classics in the corpus of English statistical writings of that period and remain unsurpassed in quality, coverage and exposition) and many thoughtful reviews.

Bowley was a diligent member of the International Statistical Institute and Fellow of the Royal Statistical Society, and often featured as one of the discussants of papers read before the latter.

Are sins of omission as bad as sins of commission? The reader will no doubt wonder not only why we have chosen to include certain of Bowley's writings but also (and this might prove more exasperating) why we have omitted certain others. In mitigation we can only say that we have tried our best to consider those that we thought important and representative of Bowley's varied interests.

No commentary on or discussion of Bowley's work can do justice to the clarity, carefulness and conciseness of his writing. In his discussion of one of Bowley's papers Rew said 'to praise Dr. Bowley's statistical work was almost an impertinence, and to criticise it almost an impossibility' [Bowley, 1914c, p. 646], and as a general observation one can only echo William Hazlitt, who wrote

> *If we wish to know the force of human genius, we should read Shakespear. If we wish to see the insignificance of human learning, we may study his commentators.*
>
> [*Table Talk*, 1908, p. 77]

Acknowledgements

Acknowledgement is made of the help received in the writing of this book from the following (in random order): Anthony Edwards, for his painstaking search for the correct reference to G.H. Hardy's review of Bowley's *A General Course of Pure Mathematics from Indices to Solid Analytical Geometry* (1913), printed in the *Cambridge Review* XXXV, No. 879 (1914), and for sending us a copy; John Aldrich, for a copy of Bowley's 'The precision of measurements estimated from samples' *Metron* 2 (1923); Alexandra Saunders, for her sterling research, particularly in the Archives of the London School of Economics and Political Science; Canon Malcolm Widdecombe, Vicar of SS Philip and Jacob in Bristol, for details of the memorial in that church to James Bowley; the London School of Economics and Political Science, for permission to reprint excerpts from the College manuscripts Coll Misc 0772 and Coll Misc 0773; the Royal Statistical Society, for the provision of the photograph of Arthur Bowley included in this book.

Considerable use has been made of Agatha H. Bowley's *A Memoir of Professor Sir Arthur Bowley (1869-1957) and his Family.* This book was printed privately in 1972, and despite considerable effort, we have regrettably been unable to trace the present copyright holder. Quotation from this book has done much to shed light on the Bowley family and to enliven the present work.

This page is intentionally left blank

Contents

Chapter 1

Biography

1.1 Introduction

Arthur Lyon Bowley was born on the 6th November 1869 at No. 12 King's Square, Bristol, in England, and was baptised on the 2nd of February 1870. His daughter Agatha in her biography of her parents (derived in part from autobiographical notes made by her father) records that it was her paternal grandmother's appreciation of Alfred, Lord Tennyson's depiction of King Arthur, that led to the choice of first name, the 'Lyon' being a family name.

Five years before Arthur's birth his parents had moved to Bristol, where his father, James William Lyon Bowley, to give him his 'sponsorial and patronymic appellations', as Dr Pangloss would have put it[1], had become vicar of SS Philip and Jacob (first mentioned in 1174 as St Jacobus-in-the-market, and popularly known today as 'Pip 'n' Jay'). James was born in 1826, and after starting, at the tender age of fourteen, in a smithy, he became a clerk in a drapery when seventeen. At twenty he became an assistant master at a school in Totteridge[2], qualification for this appointment coming about as a consequence of assiduous private reading during his time among the leathers and the laces. A spell at Durham University resulted in the award of a Masters degree in 1854. Two years later he married Ann

Elizabeth Jackson, a marriage that was cut short after five years and the birth of three children (James, Mary and Florence) by Ann's untimely death. By this time James occupied a curacy at St Barnabas's in South Lambeth, a post he left for various tutorships after a difference of opinion with his vicar on doctrinal matters (thus exhibiting a strength of character that was passed on to Arthur).

In 1863 James married Maria Johnson, whom he had met while chaplain at Isleworth Naval College. Four children blessed the union before James died of colitis, leaving Maria with seven children ranging from James[3], aged thirteen, to Arthur, aged one year. A memorial tablet was erected at the West end of the South Aisle of the church with the following wording[4]:

> To the beloved memory of the Rev. James William Lyon Bowley Vicar of this parish who entered into rest on Sunday January 1st 1871 aged 44 years. This tablet was erected by teachers and scholars in the day, night, Sunday and ragged[5] schools amongst whom he laboured with untiring energy for more than six years.
> "There remaineth therefore a rest to the people of God."
> Hebrews, IV. 9
> "Be not slothful, but followers of them who through faith and patience inherit the promises."
> Hebrews V. 12

The Mayor and a number of businessmen of Bristol contributed generously to a fund that raised £2,100 for the relief of the Bowley family. Careful investment of this sum provided some £200 annually, which ensured that the large family was adequately provided for. To supplement her income Maria took in two paying guests[6], the result being that when all the children were at home during the school holidays, the household consisted of twelve people, including two servants. Maria's brothers, Agatha suggests, might also have helped their sister and her family.

After her husband's death Maria retained her religious interests: she was superintendent of the Sunday School, and one would expect her children to have been correctly instructed in spiritual matters — though Wynne Maunder [1972, p. 8] claims that 'the formalities of religion seem to have been little to his [i.e. Arthur's] taste'.

Arthur recorded later [A.H. Bowley, 1972, p. 10] that the parish 'consisted mainly of poor people'. Helen Meller indeed notes that, at that time, 'all denominations would have agreed that the challenge of their time was to bring Religion, Cleanliness and Temperance to the urban masses' [1976, p. 85]. Certainly noble sentiments, though they would no doubt be seen as somewhat patronising by some today.

1.2 Education

At about seven years of age Arthur started at Cotham Park School, situated in 'one of the highly respectable areas' of Bristol [Meller, 1976, p. 24], a scant ten-minute walk from his home. Here he was exposed to arithmetic, Latin and English grammar, handwriting and some basic science. After three years and as a result of the intervention of his aunt Margaret Johnson (sanatorium matron at Epsom College), Arthur was admitted as a pupil at Christ's Hospital (following in the footsteps of his paternal grandfather James Browning Bowley [A.H. Bowley, 1972, p. 5]) in Newgate Street, London[7]. Clearly Arthur met the prime condition for admission, viz., that his widowed mother was in need of assistance towards his education.

The dissolution of the monasteries during the reign of Henry VIII (1491-1547) resulted in an evident and a worrying increase in the number of the impoverished in the streets of London, and three Royal Hospitals[8] were established for their relief. On his accession to the throne in 1547 Edward VI (1537-1553), son of Henry VIII, confirmed his father's gift to the City of London of the Grey (Mendicant or Franciscan) Friars' Monastery site at Newgate[9]. Christ's was established by Royal Charter in 1553, the wording in the letters patent issued ten days before Edward's death directing that the three hospitals should be called 'the hospitals of King Edward the Sixth, of Christ, Bridewell, and Saint Thomas the Apostle' [Nichols, 1852].

According to records of Christ's Hospital in the Guildhall Library, London, a branch of the school[10] was opened in Hertford (some twenty miles to the north of London) in the mid-1600s, these premises being used as the girls' school and the boys' preparatory school from 1778. The boys' school was moved from Newgate to Horsham (forty-odd miles to the south of London) in 1897, the girls' school remaining at Hertford until the sexes were re-united at Horsham in the mid-1980s.

Garbed in shoes and yellow stockings, yellow knickerbockers, red belt, white shirt and bands and long blue coat, the young Arthur began his nine years at Christ's[11]. Someone coyly known as 'Uncle Jonathan' noted in 1895 that a worsted cap was also part of the dress but was seldom worn by the boys 'who in consequence often suffer from deafness'. In company with other new boys of preparatory school age, Athur was sent to Hertford, it being thought by those in authority that the country air would be more salubrious to a growing child than that of London.

Agatha Bowley provides a fascinating account of the life of a bluecoat boy; playing fields, bathing in the river Lea[12], cricket on the village green and visits from family bearing gifts of cake and jam were all to be looked forward to and enjoyed in their season. The weekly bath administered on Saturdays by 'stalwart women' accompanied by strong soap, was succeeded by the distribution of threepence pocket money, a halfpenny of which was 'for charity'.

Delicate as a child, Arthur was 'bucked up' by being earnestly encouraged to drink porter (a dark-brown malt liquor), a beverage he apparently hated[13]. When, slightly older, he moved from Hertford back to London, he was put on a special diet, supplemented by fish. While the ordinary meals were considered adequate for most of the boys, Agatha relates that her father was given extra meals by relations he visited in London.

Writing towards the end of the nineteenth century, Thornbury notes that the food at Christ's had not always been satisfactory:

> The boys ... still eat their meat off wooden trenchers, and ladle their soup with wooden spoons from wooden bowls. The beer is brought up in leather jacks, and retailed in small piggins. Charles Lamb ... does not speak highly of the food. The small beer was of the smallest, and tasted of its leather receptacle. The milk-porridge was blue and tasteless; the pea-soup coarse and choking. The mutton was roasted to shreds; the boiled beef was poisoned with marigolds. [1881, p. 377]

The menus today show considerable improvement, from full English or continental breakfasts to 'healthy' diets.

The original great dining-hall itself was also an object of Thornbury's attention. At one time, he writes, it was notorious for the hundreds of rats that foraged nightly for crumbs, and old Blues (as the boys were known) took particular pride in being able to catch the vermin in their bare hands. He describes the two famous paintings in the dining-hall (one of Edward VI, falsely ascribed to the famous painter Holbein, and the other Verrio's painting of James II receiving an audience of Christ's children[14]), 'neither of them of much real merit' [Thornbury, 1881, p. 368].

Perhaps Arthur's physical weakness owed something to his birthplace: in *The Posthumous Papers of the Pickwick Club* of 1837 Charles Dickens wrote of Bristol that 'the city ... struck him [Mr Winkle] as being a shade more dirty than any place he had ever seen' [Chap. XXXVIII]. Meller [1976, p. 25] recorded that 'In 1850 Bristol was the third most unhealthy city in England with a death rate of one in twenty-eight'. William Farr[15], in a detailed discussion in 1859 of the construction of life-tables, described sixty-four districts in England as *healthy*, that is, with an annual death rate of seventeen to 1,000 living—a rate about half that in Bristol.) Yet Szreter and Hardy [2000, p. 635] found both London and Bristol to be 'relatively salubrious' compared to other urban areas.

Some reason for the unhealthiness of British cities even at the end of the nineteenth century may be seen in Rowntree's important work *Poverty: a Study of Town Life*[16] of 1901. Basing his remarks on the 1891 census Rowntree noted that in Bristol 8.0% of the population of 221,665 lived more than two persons per room[17], such occupancy being described by Bowley, in the second of his two Chadwick Public Lectures[18] of 1921, as 'overcrowding'. Note, however, that this word had neither a clear nor a fixed meaning—see, for example, Rowntree [1941, p. 265].

Data on such matters might well have come from the Bristol Statistical Society, whose members had from the outset determined to investigate social conditions in the city [Meller, 1976, p. 45]. This society had been established in November 1836 and was the fourth such society to be formed in the United Kingdom, having been preceded by those of Manchester, London and Glasgow[19].

By a second Royal Charter, this time of Charles II in 1673, and apparently at the suggestion of Samuel Pepys[20], the Royal Mathematical School at Christ's was founded with the express aim of training boys in navigation so that they might serve the King at Sea. Agatha makes no mention of this in her book, merely noting that in Arthur's time tuition was carried out in the 'Writing School' (mathematics, writing, history and geography) and the 'Grammar School' (Latin and English language and literature). One week half the form went to the Writing School in the morning and the Grammar School in the afternoon, with a switch the following week.

Nor were the arts neglected. Arthur did not excel at drawing (partly, and perhaps initially, because of his poor eyesight, which was only discovered when he was thirteen), but he learned the piano and later the organ. At these instruments he must have become proficient, for Allen and George [1957] note in their obituary the pleasure Bowley gained in later life from playing Bach and Haydn.

Arthur moved from Hertford to London as top of both the Writing and the Grammar schools, and the recipient of prizes for English (a 'handsome calf-bound copy of Tennyson's poems', which would no doubt have given his mother considerable pleasure), for Latin and for 'diligence and good conduct' [A.H. Bowley, 1972, p. 19]. He was a pupil at Christ's from 1879 to 1888, his scholarship being evinced by his winning the Tyson Gold Medal for mathematics in 1886, and, in his last year, the Montefiore Prize for mathematics and classics and the Thompson Gold Medal for mathematics.

Bowley maintained his interest in the school throughout his life, and from 1936 to 1947 he represented the University of London on the Council of Almoners[21], by virtue of which representation he was, for more than a decade, a Governor of Christ's.

Arthur left Christ's Hospital as top Grecian (the senior scholars at Christ's were at one time either *Grecians* or *King's boys*, depending on whether they were in the classical or the mathematical 'stream'). On Speech Day it was the custom for Grecians to line up holding white kid gloves and take a collection for their expenses at university. Arthur's share of this 'glove money' was £15. 10s. On returning his school uniform he received a £10, and a further £30 was given to him in his first term at university.

Arthur went to Trinity College, Cambridge, on a mathematics scholarship. In his second year, Agatha relates, he was awarded a Christ's Hospital Exhibition Scholarship of £287, and while at Trinity College he received a Trinity Scholarship (£265), a Bell Scholarship (£52, and awarded only to the sons of clergymen) and the Cobden Prize (£60). From records that Arthur kept it appears that his termly expenses came to a minimum of £44.

The ill-health Arthur had suffered as a child returned, and he experienced bouts of illness (anæmia and general lassitude) while at Cambridge[22]. Doctors prescribed lime-water, a spoonful of whisky in milk before retiring (no doubt an improvement on Christ's porter!) and a sea-voyage. The financial generosity of dons and tutors allowed Arthur to take a trip to Egypt in the hope that it might strengthen his constitution, but soon after his return to England he had to spend some time in Bournemouth[23] for further recuperation.

While at the seaside Arthur earned some money (£54) by tutoring. A short spell as a teacher of arithmetic at a boys' school was cut short by the headmaster who found Arthur's religious views unsound. In all he was away from Cambridge for eight months, though he earned £37 while absent. These breaks from his studies resulted in his being only joint tenth Wrangler in the Mathematical Tripos[24], no doubt disappointing his school mathematics master, James Barnard, who thus had his hopes (and perhaps expectations) of producing a Senior Wrangler dashed; for, after all, Arthur had come first in the Trinity scholarship examination.

Arthur's illness resulted in his having to spend an extra term at university before being admitted to the degree. From October 1891 to March 1892, the Michaelmas (or Autumn) and Lent (Spring) terms, he therefore studied physics and chemistry (working in the Cavendish Laboratory) and economics (following a reading course under Alfred Marshall).

With the growth of a general interest in socialism[25] in the 1880s, the concern Arthur had shown for his father's parishioners gained an added impetus. At one time he and some of his fellow students started 'a waiters' Club', with the dining-hall waiters being invited to their rooms for refreshments and conversation; one wonders whether these occasions were anything but awkward and unprofitable. At another time he helped organise a boycott of a Commemoration Dinner, such a feast being regarded as wasteful of College moneys. (Agatha relates that by the time her father was elected an honorary Fellow, fifty years later, this scruple seemed to have been overcome!)

His interest in socialism and social reform perhaps contributed to Bowley's growing interest in Economics. However he did not blindly embrace the whole socialist package; in his own words,

> I was doubtful of the socialist's statement that the rich were getting richer and the poor poorer. If the contrary were true, poverty would diminish without revolution. This question led to a great part of my statistical work after 1892. [A.H. Bowley, 1972, p. 33]

When it comes to connecting Bowley's interest in socialism to his later work, one would however do well to bear in mind the necessity for distinguishing between inferences motivated by scientific considerations and those made as a result of the political or even social inclinations of the scientist (see [Hilts, 1978, p. 21]).

At some point—perhaps after the births of his children—Bowley became more practical and less interested in socialism *per se*. One must however be quite clear about his political views: in her review of Lord Robbins's[26] autobiography Marian Bowley took exception to Robbins's description of her father as 'almost certainly a conservative'. She explained that Arthur, even in later life, was 'a genuine liberal of pre-1914 vintage after an early period of interest in Fabian Socialism[27]' [1972, p. 808]

Despite what was perhaps a concealed yet strong passion for socialism, one that is sometimes carried to extremes, Bowley may be seen as exhibiting a trait that Helen Meller found common among certain groups of people in the 1860s and '70s, viz., the vision 'of a civilisation based on morality and culture' [1976, p. 237].

In 1896, or thereabouts, while he was teaching mathematics at St John's School, Leatherhead, and also lecturing statistics at the London School of Economics (better known as the LSE), Bowley gave a lecture on Socialism[28] to the masters and (perhaps) the senior boys at St John's. He described what were perhaps sometimes perceived as different kinds of socialist, working from anarchists through the French communists, the German Social Democrats, Collectivists, 'Socialists of the Chair' (i.e. theorists rather than men of action, or arm-chair socialists) to Christian Socialists. He outlined not only the evils that socialism wished to cure (viz. 'poverty, luxury and the stigma of manual employment'), but also the means that socialists hoped to employ to achieve their ends.

Bowley's aim in presenting this paper was clearly set out:

> My intention has not been to argue for or against any particular form of socialism, but to endeavour to explain

> the aims of modern socialists (with perhaps some bias in their favour), to emphasise the necessity of some reform in economic conditions, and to indicate the lines which the controversy on the subject is now taking. [p. 8]

Bowley did not allow the seriousness of his topic to stifle a momentary touch of humour. In presenting a picture of an imaginary socialist State he noted that to prevent there being a rush for employment in a popular trade, either the hours of employment could be lengthened or the wages reduced[29].

> Thus such occupations as coal-mining, scavenging, manufacture of chemicals or explosives, monotonous machine-work and teaching Euclid would be rewarded with short hours or high pay; while artists, cricket-professionals, those in healthy out-door occupations, students of abstruse mathematics, and others engaged in enjoyable work would have longer hours or lower pay. [p. 7]

Was Bowley's view on occupations expressed above original to him? One cannot be sure, but it is certain that something similar appears in *Looking Backward: from 2000 to 1887*, a book published by Edward Bellamy in 1888. In Chapter 7, writing of the business of the administration in the year 2000 to make all trades equally attractive, Bellamy writes

> This is done by making the hours of labor in different trades to differ according to their arduousness. The lighter trades, prosecuted under the most agreeable circumstances, have in this way the longest hours, while an arduous trade, such as mining, has very short hours.

Bowley referred to Bellamy (though with no indication of whether he meant Edward or his great-grandfather Joseph) in his [1939b], and when we recall that Bowley had been giving a lecture on socialism (in which he mentioned *Looking Backward*), and that Edward's book was widely seen as launching a world-wide movement of National Socialism, the connexion is certainly not impossible[30].

Bowley's tutor at Cambridge, the Rev. Richard Appleton (an Old Grecian), had introduced Arthur to Alfred Marshall[31], thus catalysing not only a long and firm friendship, but also the start of a noteworthy career in economics, econometrics and statistics. Bowley soon became intimate with the Marshalls, visiting them at their home in Madingley Road, and Agatha records that 'He got an insight into the body of economic theory known then as the Theory of Value and an idea of the use of statistics' [A.H. Bowley, 1972, p. 35].

In 1892 Bowley (as we shall start to call him) won the Cobden Prize[32] for his essay *Changes in the Volume, Character and Geographical Distribution of England's Foreign Trade in the XIXth Century and their Causes*, a revised version of which was later published. Maunder notes that a result drawn from this study was that profitable production was not necessarily unfavourably affected by labour that was highly paid, and nor was poorly paid foreign labour disadvantageously competitive.

In 1894 Bowley competed for, and won, the Adam Smith Prize (the 'Adam' is important, for Cambridge also has prizes named for *Robert* Smith[33]). An elaborated version of this prize work, *Changes in Average Wages (Nominal and Real) in the United Kingdom between 1860 and 1891*, was read before the Royal Statistical Society in 1895, and in the ensuing discussion [Bowley, 1895a, p. 279] Marshall commented that, as one of the examiners of the original work,

> he ... had been struck by the brilliancy of the plan by which Mr. Bowley proposed to extract some information from the great mass of wage statistics which had hitherto been almost useless because of its fragmentary character.

Maunder [1972, p. 9] suggests that the methods advanced by Bowley, viz. the study of wages by ratios of movements (or index numbers) rather than the actual wages, are today regarded as commonplace precisely because of that work. Maunder also notes that while the paper was of extreme importance because of its treatment of error, it had ‘singularly failed to influence subsequent practice in this respect’ [loc. cit.], probably because randomness was of importance in respect of probability statements, whereas the sampled items were ‘purposefully selected’.

1.3 Early career, family & friends

On leaving Cambridge Bowley took up a post as an assistant master at Brighton College. Moral turpitude involving a senior master resulted in the appointment of a new headmaster and the consequent ‘cutting-back’ in staff, and Arthur, as the most recent appointee, was asked to leave. Soon thereafter, however, he became mathematics master at St John’s School, Leatherhead (on the river Mole[34]), remaining there from 1893 to 1899. This school was founded by the Rev. Ashby Haslewood in the 1850s with the dual aim of catering for the sons of financially distressed clergymen and of providing choristers for his church. The school’s home page states that the school, settled in Leatherhead in 1872 after various moves, rejoiced in the ‘significant headmastership’ of Arthur Forster Rutty[35] (1883-1909), which perhaps contrasts with Agatha’s assertion that, during her father’s time there, ‘the school began to deteriorate’ [1972, p. 40]. Bowley eventually left Leatherhead after telling his headmaster that ‘the boys’ religion was only perfunctory attendance at chapel, that the services were vain repetition and that I refuse to attend chapel in the future’ [A.H. Bowley, 1972, p. 40]. The school-mastering continued with a temporary post at Clifton College in Bristol, but after two terms Bowley’s appointment was terminated.

In 1900 Bowley became a mathematics lecturer at University College, Reading (at that time an extension college of Christ Church College, Oxford), and seven years later he was promoted to Professor of both Mathematics and Economics. In 1913 he resigned from this dual position, but stayed on as lecturer in Economics until 1919. In all likelihood it was in his early years at Reading that he met Julia Hilliam, the woman who would become his wife.

Julia was born at Spalding, Lincolnshire, on the 15th October 1871. Her father, Captain Thomas Hilliam (1823-1901), was agent to the Marquis of Huntley, while her mother, Catherine Roberts, was distantly connected to Oliver Cromwell on the distaff side. With the agricultural depression Catherine found herself with eleven children and a bankrupt husband. The latter became a bank manager, and Julia was sent to live with two aunts. The three ladies later moved to London, and Julia trained as a wood-carver at the South Kensington Art School. Occupation as a teacher of wood-carving followed, and in 1897 she opened a studio in Reading. In 1899 she took up the post of instructress[36] in wood-carving at Reading College [A.H. Bowley, 1972, p. 54], rapidly becoming known as one of the best women in her field in England.

Arthur and Julia were married on the 25th March 1904 in St George's Church in Reading, and in that city their daughters Ruth, Agatha and Marian were born, in 1907, 1909 and 1911 respectively. Julia had probably left her post at Reading College by March 1908, for Agatha records that in that month a presentation of books was made to Julia from the Reading Staff Common Room, signed by fifty-four members. Arthur was now lecturing at both Reading and the London School of Economics[37], with mathematics having been added to his statistical duties at the latter. Agatha relates (perhaps with a touch of 'how inconsiderate!') that their peaceful existence 'was to some extent disrupted by the 1914-1918 war' [1972, p. 63]. Arthur worked for a time for the Ministry of Munitions, while Julia helped to organise a Station Canteen and nursed at a Voluntary Hospital. After the war the Bowleys moved to Harpenden (some five miles from St Albans in Hertfordshire and some twenty-five miles from London), Arthur by now having resigned his post at Reading.

Allen [1972, p. 629] relates that the Bowley's garden at Harpenden had been part of the Rothamstead Experimental Station, where

Ronald Aylmer Fisher (1890-1962) carried out a great deal of his experimental work. Joan Fisher Box [1978, p. 97] notes that her father and Bowley played bridge together, with William Roach (1895-1984) and William Sealy Gosset[38] (1876-1937) sometimes making up the four. (Incidentally, Roach, a colleague of Fisher's at Rothamstead, married Blanche Muriel Bristol, who achieved fame outside her field of algology as Fisher's 'lady taking tea'.)

Bridge is not a game that always brings out the friendliest feelings among the players, and what may start out as support for one's partner can rapidly change to sharp words. One does not know whether Bowley and Fisher managed to maintain cordiality at the bridge table, but professionally their relations seem to have been slightly taut at times. For example, in 1927 Fisher supported Bowley at the World Population Conference, pointing out that Bowley's demographic method for the prediction of population growth was more useful than the logistic method inasmuch as it allowed for a period of population increase followed by one of decrease (see de Gans [2002, p. 101]). In 1934, however, Fisher read a paper, 'The logic of inductive inference', to the Royal Statistical Society [Fisher, 1935]. Bowley proposed the vote of thanks, and while he thanked Fisher sincerely for his contributions to statistics in general, he stated he 'found the treatment [in the present paper] to be very obscure'. Fisher was no less reserved in his response, stating that the acerbity expressed by both the proposer and the seconder (Leon Isserlis) of the vote of thanks did not surprise him. Noting that Bowley had expressed his puzzlement at why he had been chosen to propose the vote of thanks, Fisher said 'The choice of order in speaking ... seems to me admirably suited to give a cumulative impression of diminishing animosity' [1935, p. 77].

The family's move to Harpenden resulted in Julia's having to give up much of her carving. She shared her husband's interest in social welfare, however, and in addition to doing good works in their new home town, addressed the Association of Liberal Women on the subject of the extension of The State Insurance Scheme (less charitably the dole) to fifteen-year olds (she was against it, believing that a better answer would be the provision of work)[39]. She did still manage to find time for artistic expression, however, and sculpted a pulpit for Bradley Church, near Basingstoke in Hampshire, forty-eight miles southwest of London.

Julia's opinion, incidentally, would have been heartily endorsed by Maimonides, who wrote in his *Charity's Eight Degrees*

> Anticipate charity by preventing poverty; assist the reduced fellowman, either by a considerable gift, or a sum of money, or by teaching him a trade, or by putting him in the way of business, so that he may earn an honest livelihood, and not be forced to the dreadful alternative of holding out his hand for charity.

One does not know Bowley's opinion on the dole though it is possible that he did not share his wife's view[40]. For in 1922 he contributed to *The Third Winter of Unemployment* the report of an inquiry from which it emerged that the dole did not necessarily have a demoralising influence and that most of the unemployed were eager to obtain almost any sort of work (see Burns [1923, p. 248]). In 1912 Bowley had published a careful investigation of employment (less euphemistically, *un*employment), pointing out that an index of employment should depend not only on the number of persons employed, but also on factors such as (i) the amount of employment, (ii) the inclusion or exclusion of unions of different industries and (iii) the average number of days worked in a specific time period[41].

Allen and George portray Bowley as 'somewhat shy and retiring' [1957, p. 238]. He was also described by Allen [1971, p. 134] as 'rather dour', at least when serving on committees, an example of which trait is perhaps seen in his comment on returning from the 1929 Session of the International Statistical Institute[42] in Warsaw: 'Scenery monotonous, currency varied'. On the other hand, someone who could write 'The density of population involves further conceptions' [Bowley, 1915b, p. 40] must have a certain sense of humour[43].

Bowley did not form friends easily, but he did maintain a very close relationship with the economist Edwin Cannan for many years. Cannan (1861-1935), incidentally, according to the *Oxford Dictionary of National Biography*, was, like Bowley, 'of a delicate constitution', though *his* cure was attempted by a tour around the world (see Bowley [1935e])—somewhat more extravagant than Bowley's trips to Egypt and Bournemouth in search of relief! He too was one of the original lecturers at the London School of Economics[44]. He was as keen a cyclist as Bowley, the two occasionally being joined on their excursions by Francis Ysidro Edgeworth (1845-1926). Bowley records in his obituary of Cannan, that when the latter, who lived in Oxford, failed to find a book he needed in the Bodleian[45], he calmly said 'then I must go on to the British Museum'.

In a memorial to Edgeworth in 1934 Bowley noted that, on his appointment to the London School of Economics and at Marshall's instigation, he wrote to Edgeworth for information about suitable material. Such advice was warmly tendered, the recommended texts being principally Venn's *The Logic of Chance*, Todhunter's *A History of the Mathematical Theory of Probability: from the Time of Pascal to that of Laplace* and Lexis's *Zur Theorie Der Massenerscheinungen in der menschlichen Gesellschaft.* Bowley related later [1934a, p. 119]

> From that time till his [Edgeworth's] death I constantly learned from him, worked with him, and met him frequently in London and Oxford. It was with difficulty that I could turn the conversation from the nature of probabilities and the applications of the Law of Error.

Edgeworth's fixation on these topics is further proved by noting that while a party of economists were out cycling Cannan said 'Put on the pace, Bowley, he [i.e. Edgeworth] can't talk mathematics at more than 12 miles an hour' [Bowley, 1934a, p. 119]. (Arthur kept up this form of transport—neither he nor his wife could drive a car—in later life.) This fixation on cycling, by the by, was a remnant of the athletic stoicism (following Moore [2005, p. 91]) shown by the

Victorian Cambridge economists who both worked and played hard.

Edgeworth is not regarded as having established a 'school' of followers, either in economics or in statistics. It does appear, though, that Bowley admired Edgeworth's work and came as near to being his disciple and heir without morigeration as anyone.

1.4 Later career

Although Bowley joined Reading University College in 1900, his career as a university lecturer had in fact started while he was still at Leatherhead. In 1895 the London School of Economics and Political Science was founded, to a large degree because of the work of Sidney (1859-1947) and Beatrice Webb[46] (1858-1943) and also as a result of a bequest to the Fabian Society of nearly £10,000 from Henry Hunt Hutchinson[47]. The money was to be used for the furtherance of socialism and the Fabian Society. At Marshall's urging, Bowley was appointed part-time lecturer in statistics (Agatha Bowley records that Arthur marked this occasion by giving his mother a punnet of strawberries), thus starting a forty-year service with the institution.

The LSE, Hayek noted, was intended, right from its founding,

> to provide, not a general course for young beginners, but an introduction to independent research work for maturer people with some knowledge of the world. [1946, p. 5]

Sydney Webb's requirement as far as statistics was concerned, was not for statistical theory but 'statistics for junior civil servants' [Hayek, 1946, p. 7].

Bowley fulfilled his London duties by travelling from and back to Leatherhead (some eighteen miles) by bicycle and train, or by bicycle alone if the weather was fine, on Wednesday half-holidays, the weekly visits continuing even after he moved to Bristol. (We shall see later that he used train timetables and cycling in a number of examples in his statistical writings.) Indeed, Bowley related in a lecture to the

Students' Union of the LSE in Cambridge[48] in 1945 that he lectured at 5.45 or 6.00 p.m. on Wednesdays for some 38 years[49] [Hayek, 1946, p. 27] (he missed his first lecture because of influenza). In 1908 he became Reader in the School, and promotion to the first professorship of statistics in the University of London followed in 1919. This position he was to occupy until his retirement[50] in 1936.

Part-time lectures in Statistics in London were not new when Bowley started at the LSE. In 1859 Thorold Rogers held the Tooke Professorship of Economic Science and Statistics at King's College (a position later held by Edgeworth), and Bibby [1986, p. 481] relates that one of the requirements of the post was that

> at least Ten of the (twenty) lectures shall take place in the evening so as to admit the attendance of young men and others engaged in business during the day.

Bowley took considerable interest in official statistics, and was occasionally[51] called on to appear before Commissions or Inquiries. For example, in 1920 he gave evidence in an official inquiry into the regulation of the dock labourers' wages and conditions of employment, publishing a report of the inquiry in his [1920c].

In 1927 Bowley, once again dogged by ill-health (a recurrence of his earlier problem?), took the family on a six-month holiday to Italy. The rest-cure seemed effective, and Arthur returned to England and resumed his customary work.

Agatha Bowley records that, while at Leatherhead, her father taught Chemistry up to Matriculation standard. One of his pupils is reported to have said that, while Arthur did not know very much chemistry, he could teach what he knew. Yet the situation seems to have become reversed once Arthur got into university lecturing. No one would disparage Bowley's knowledge of statistics, but from what one reads in obituaries it would seem that, like Edgeworth, he was not at his best as an undergraduate lecturer. His obituary in *The Times* described him as 'a diligent teacher but in his care as an expositor he made little or no concession to the student mind'

[Bibby, 1986, p. 486]. And Allen and George note that Bowley's lectures often seemed to be nothing but a 'confidential monologue addressed to the blackboard' [1957, p. 237]. Advanced students, however, came to realise the importance of supplementing Bowley's lectures with their own work, and thus benefitted greatly from the formal instruction. Those doing research under Bowley, once they were sufficiently prepared, soon came to appreciate his methods.

Bibby [1986] records that copies of handouts used by Bowley are to be found in the Huddersfield Polytechnic Library. These documents show that Bowley's 1897 lectures covered (a) the collection of statistics, (b) the tabulation of statistics, (c) the criticism of results and (d) the absence of information.

It was perhaps in sample survey techniques, at least inasmuch as they are applicable to social investigation, that Bowley shone. For almost the whole of his career the social question interested him deeply, and Allen and George record that Bowley viewed his share in the mammoth *New Survey of London Life and Labour* [1930-1935] as the high point of this concern. (This work might be seen as a continuation of the social investigator Charles Booth's seventeen volume work *Life and Labour of the People in London* [1891-1903] and Rowntree's more modest *Poverty: a Study of Town Life* [1901].) Indeed, one might describe Bowley as Clara E. Collet[52] had described herself, i.e. as 'a student of social conditions' [Bowley, 1950, p. 408].

Bowley's interest in social welfare was evinced in papers that he published throughout his working life[53]. There were his writings on the national income and also his work as editor[54] from 1923 to 1945 of the London and Cambridge Economic Service. His contributions to the Service continued until 1953.

But perhaps his major contribution, at least to statistics, was the development of sampling techniques with especial reference to the social sciences, though he was perhaps pre-empted by the distinguished Norwegian statistician Anders Nicolai Kiaer[55] (1838-1919). Eschewing complete enumeration, Bowley first justified his methodology with a sample survey of Reading in 1912, and the results were published, together with similar studies on Northampton, Warrington and Stanley (conducted by A.R. Burnett-Hurst under Bowley's supervision) as the pioneering work *Livelihood and Poverty* in 1915. In 1924 the question raised in the title of the follow-up study, *Has Poverty Diminished?*, was emphatically positively answered.

Bowley's interest in and obvious knowledge of the 'representative method' led to his being appointed a member of a committee set up in 1924 by the International Statistical Institute to examine the use and methods of such sampling. A result of this report was a demonstration of the superiority of stratified over simple random sampling, though Bowley did not always clearly distinguish between the latter and systematic, or cluster, sampling. Continuing in this vein, Bowley presented the culmination of his research in this field in his contribution to the *New Survey of London Life and Labour*[56].

Although Bowley spent his early working years as a mathematics schoolmaster, the direction of his future career was established by his prize-winning Cambridge essays and the start, in 1899, of a series of highly articulate and informative papers on changes in wages and prices, five of the fourteen being written jointly with another Bristol man, George Henry Wood[57]. In his entry on Bowley in the *International Encyclopedia of the Social Sciences* [vol. 2, 1968, p. 134], Allen wrote: 'Bowley approached the subject with statistical and historical care verging on the pedantic, yet at the same time with a deep and sympathetic appreciation of the human problems involved.'

The Cobden Prize essay, published in book form in 1893, examined the events and causes, together with their importance, that affected the growth of trade in the nineteenth century, with a consideration of the social importance of this growth. Tables were to a large extent replaced by diagrams. The essay for which the Adam Smith Prize was awarded had as special features the following: '(a) statements of wages were never compared unless they were given by the same authority and (b) ratios, rather than amounts, were considered' [Dale, 2001, p. 280].

Two text-books[58] emanated from Bowley's lectures at the London School of Economics: the first, *Elements of Statistics*, was intended for those wanting to understand official statistics. With this aim in mind Bowley covered topics from the collection and tabulation of data, graphical methods, correlation and index numbers to the application of mathematics to statistics, with special reference to sociological and economic matters and a novel discussion of skewness and kurtosis. The *Elementary Manual of Statistics* was less taxing on the reader, being devoted less to mathematical than to official statistics. The index includes topics as different as Abatements, income-tax; Barley, prices; Death-rates; Labour exchanges; Railway statistics; Tailoring industry, unemployed; and Worsted manufacture, exports.

As regards Index Numbers, Maunder [1972, p. 15] notes that

> The most widely known of his [i.e. Bowley's] contributions on index numbers is perhaps the formula to which his name was given but this has been a source of puzzlement to some since it is known that both Edgeworth and Marshall had made a similar suggestion previously.

In his paper on the history of index numbers M.G. Kendall says that the method of averaging weights was proposed by Edgeworth and Marshall 'and endorsed by Bowley' [1969, p. 11]. The index is

$$\sum p_1(q_1 + q_0)/\sum p_0(q_1 + q_0),$$

where p_0, p_1, q_0 and q_1 denote the prices of a commodity in the base and the current years and weights in the same years respectively, and the sum is taken over all the commodities included.

Bowley did not however altogether forsake his earlier interest in mathematics. In 1913 he published *A General Course of Pure Mathematics from Indices to Solid Analytical Geometry*, a work, as we shall see later, that had a mixed reception from professional mathematicians. A second book more in the mathematic than the economic line was the 1924 *Mathematical Groundwork of Economics; an introductory treatise*. Intended for the economic practitioner rather than the neophyte, this work attempted a unification of the mathematical methods used by Bowley's economic predecessors.

Roy Harrod, in his biography of John Maynard Keynes[59], relates that he had asked Keynes how much mathematics an economist needed to know. The latter replied that 'Johnson[60] in his article in the *Economic Journal* had carried the application of mathematical analysis to economic theory about as far as it was likely to be useful to carry it' [Harrod, 1951, p. 8]. In his obituary to Marshall in 1924 Keynes commented on Bowley's *Groundwork* saying that it 'runs somewhat counter to Marshall's precepts by preferring, on the whole, algebraical to diagrammatic methods' [1924, p. 335].

Marshall offered some good and sympathetic advice to Bowley[61]:

> (1) Use mathematics as a short hand language, rather than as an engine of inquiry. (2) Keep to them till you have done. (3) Translate into English. (4) Then illustrate by examples that are important in real life. (5) Burn the mathematics. (6) If you can't succeed in (4), burn (3). [Marshall, 1961, p. 775]

Marshall's own mathematical ability had been considerable[62], but, especially in later years, he resented the mathematisation of economics theory.

Papers on international housing statistics by 'Members of Dr. Bowley's Seminar, 1923-4', with an introduction by Dorothy S. Thomas and preserved in the Bowley Collection in the LSE, exemplify the style used by Bowley for the presentation of such data. One must work one's way from the total population of a country through the total number of family groups, the number of occupied and unoccupied dwellings, a description of such dwellings, the number and types of the rooms, their area and whether (and how) they are ventilated, water supply and sanitation, details (and appropriate definition) of overcrowding, information about rents, rates etc., and the variability of these factors over the country, together with a comparison of preand post-war conditions. A mammoth task (tailor-made for Bowley), but one that Bowley and his fellow investigators had not shirked in earlier work.

As part of his welfare work Bowley developed ‘a most ingenious method’ [Maunder, 1972, p. 12] for measuring unemployment. The qualitative data (‘measurement by adjectives’, in Bowley’s own words) given by the Labour Department on a five-point scale from ‘very good’ to ‘very bad’, were first examined to see if they were internally consistent, monthly changes in a given year being linked to those of the preceding year. Then a graphical analysis was undertaken under the assumption that the four changes (e.g. from ‘very good’ in one month to ‘good’ in the same month in the subsequent year) could be assumed equally numerically significant with an arbitrary scale. Finally the results for more than twenty industries were combined into a single measure. Satisfactory agreement with the index used by the Labour Department suggested that Bowley’s method was reliable, and it could thus not only be used for years lacking numerical data but also be extended to a wider range of industries.

One must not however think that Bowley made no contribution to *mathematical* statistics. The statistical stage during the first half of the twentieth century was of course dominated in England by Karl Pearson and Ronald Aylmer Fisher, with Pearson *fils* (Egon Sharpe, 1885-1980) and Jerzy Neyman (1894-1981) making clear ‘noises off’. Yet as we shall see later in this work Bowley contributed to both the theory and the practice of statistics.

At the age of sixty-three Bowley, in conjunction with Roy George Douglas Allen[63] published the innovative *Family Expenditure*, a work that may perhaps be seen as heralding Bowley’s move from ‘pure’ economics to econometrics. Here the expenditures of individuals within families were compared with those of the families themselves, a χ^2 goodness-of-fit test being used to examine the appropriateness of an empirically-derived linear relationship.

1.5 Professional activities

In later chapters particular attention will be paid to Bowley's work in economics, econometrics and statistics. A few words about other professional activities might not come amiss here.

In 1905, when he was thirty-five years old and discharging his joint duties at Reading and the London School of Economics, Arthur visited the five British colonies (the Cape, Natal, Orange Free State, Transvaal and British South Africa) of Southern Africa for a meeting of Section F, the Economic Science and Statistics Section, of the British Association for the Advancement of Science[64]. Bowley's paper was entitled 'Changes in the sources of the world's wheat supply since 1880' [Herbertson, 1905, p. 641]. A report on the visit was printed in 1905 in *The Economic Journal*, and Bowley, in this report, was full of praise for the excellence of the talks delivered and the interest shown in the discussions.

Section F of the British Association for the Advancement of Science was the precursor of the Royal Statistical Society. A report in the *Journal* of the latter in 1935, based heavily on a Notebook by John Eliot Drinkwater[65] (1801-1851), records that 'a meeting of Gentlemen desirous of forming a Statistical Section of the British Association' was held at Cambridge, Thursday morning, June 27th, 1833. Exactly what such a society would study was set out quite clearly on the 28th of June at a subsequent gathering:

> In its narrowest sense considered as subordinate to the enquiries of the political economist alone, the science of statistics would have for its subject-matter such phenomena only as bear directly or indirectly upon the production or distribution of public wealth.
>
> It is with wider views that such an Association as the present would approach the subject. It may be presumed that they would think foreign to the objects of their enquiries no classes of facts relating to communities of men which promise when sufficiently multiplied to indicate

> general laws. To repress, however, to some extent the spirit of premature speculation which is too apt to mingle itself with such researches, perhaps it might be prudent to limit as far as possible their reception of such matter to facts capable of being expressed by numbers.
> [Anon., 1935, p. 143.]

The Royal Statistical Society to an extent kept up the preciseness of the earlier body with the adoption of the motto *Aliis Exterendum*[66].

At the invitation of the Secretary of State for India Bowley, in company with (Sir) Dennis Holme Robertson (1890-1963), visited India in 1934 to carry out a survey on the provision of economic and statistical data for the Indian Government[67]. In a sense this was a peculiar pair of investigators: Bowley's interest in statistics and economics is well known, but Gordon Fletcher, in the *Oxford Dictionary of National Biography* records that Robertson, a literary economist and (eventually) Professor of Political Economy at Cambridge, was mathematically lacking, regarded economics as 'aesthetically dull', and was averse to 'statistical modelling of economic phenomena'. No doubt the two pundits managed to pull it off.

In addition to his acting directorship of the Oxford University Institute of Statistics from 1940 to 1944 (i.e. after his 'early' retirement), Bowley held a number of important positions. He was, at one time or another, a Fellow of the Royal Statistical Society (elected in 1894), a member of its council (first elected in 1898), vice-president (from 1907 to 1909 and again from 1912 to 1914), and, from 1938 to 1940, its president. He was elected a Fellow of the British Academy in 1922. He was recorder, secretary and, in 1906 (the year after his visit to Southern Africa), president of Section F of the British Association for the Advancement of Science.

Elected to membership of the International Statistical Institute in 1903, Bowley attended no fewer than twelve sessions of that body[68], serving twice as treasurer[69] (from 1929 to 1934, and again from 1947 to 1949) and being elected one of the honorary presidents in 1949.

At the Paris meetings in 1909 Bowley was elected a member of a committee established 'to examine the methods of statistical comparison, with a view to standardisation' [Rew, 1909, p. 597]. Here too Bowley delivered a short note, making an 'interesting suggestion' (according to Rew) which was published in English as Bowley [1909a]. Here Bowley, perhaps in line with the newly-established committee's terms of reference, proposed that the median be used as a standard of comparison for wages between different countries. In 1946 Bowley served on a committee established by the then President Armand Julin (1865-1953) to advise on the I.S.I. statutes.

In 1934 the I.S.I. held its meetings in London on the occasion of the centenary of the Royal Statistical Society [Anon., 1934]. Julia served on the committee that arranged entertainment for the ladies.

Bowley's merits were also recognised in the economic and econometric communities. He was a Fellow and council member of the Royal Economic Society[70] (elected in 1893), and a foundation member and president, in 1938, of the Econometric Society. From 1923 he was not only a member of the executive Committee but also Editor of the London and Cambridge Economic Service[71]. Bowley's influence extended to the United States of America, where he served on the Advisory Council of the Cowles Commission[72] from 1935 to 1938. Bowley, Cannan and Graham Wallas (1858-1932) all edited *Economica* in its early years.

Bowley was Newmarch Lecturer[73] at University College, London, in 1897 and 1898, and again in 1927 and 1928. On the first occasion he gave a series of lectures in 1897 on 'The accuracy of averages' and, in the following year, on 'Wages in the United Kingdom in the nineteenth century'; on the second occasion he lectured on 'Tests of trustworthiness of public statistics'. Although the second series of lectures drew members of Karl Pearson's department at University College, numbers were generally small—like those in the early days of Statistics at the London School of Economics—and Agatha [1972, p. 43] records that on one Whit Monday only Bowley's mother and a friend were present!

1.6 Retirement

On Bowley's somewhat premature retirement from the London School of Economics[74] in 1936, the family moved to a house off Marley Common just south of Haslemere in Surrey, some forty-four miles from London. Important in the early twentieth century in the Peasant Art Movement, the town also boasted the instrument-making workshop of Arnold Dolmetsch, whose replicas of original instruments did so much to advance the cause of ancient music.

The newly-built house, Agatha Bowley recorded, 'was a trifle inaccessible and hardly suitable in its situation for an elderly couple' [A.H. Bowley, 1972, p. 73]. She also notes the difficulty she experienced in trying to reach the house by car in winter, though Arthur and Julia did not seem particularly worried by the weather or the steep, muddy footpath uphill from the village to their house. The 'remarkably infertile' garden was assiduously attended to by Julia and her youngest daughter Marian.

The London School of Economics commemorated Bowley's career with the founding of the Bowley Prize to celebrate Bowley's services to Economics and Statistics. The first award was made in 1939, to H.S. Booker for his essay on 'Aspects of food consumption with special reference to milk'.

While Julia continued with her handicrafts, retirement did not come easily to Arthur. He read a lot and kept on with his scientific work. For relaxation, he still played the piano, did jigsaws with success and attempted (though not very successfully) knitting and rug-making. Allen and George [1957, p. 238] relate that Bowley's statistical interest intruded even into his attempts at weaving: a skein of yarn was cut into more or less similar lengths, and Bowley and a visitor discussed the importance of the standard deviation of these lengths.

The second World War must have come as somewhat of a relief to Arthur: he was asked to act as Director of the Oxford University Institute of Statistics (a post he accepted and filled with his usual vigour), and Julia found herself even busier as Billeting Officer for the district, her duties being the requisitioning of quarters for soldiers.

'Success,' wrote Jane Austen in *Emma*, 'supposes endeavour', and Bowley's labours were accompanied by Honours throughout his life. The Royal Statistical Society awarded him the Guy Medal[75] in Silver in 1895, very early in his career, for a revised version of his Adam Smith Prize essay, and the Guy Medal in Gold followed on the eve of his retirement in 1935. In 1913 he was awarded an Sc.D. by Cambridge University and honorary D.Litt. (Oxford) and D.Sc. (Manchester) degrees followed in 1943 and 1949 respectively. He was made a C.B.E. in 1937, and a knighthood followed in 1950. His membership of and numerous activities on behalf of the International Statistical Institute were not formally acknowledged, however, and he was not made an honorary member of the Institute on his retirement.

Arthur and Julia were no doubt chuffed that all of their daughters manifested an interest in social and welfare problems. About the eldest daughter, Ruth, we have found little: indeed, nothing beyond a 1934 paper 'The cost of living of girls professionally employed in the County of London'. Agatha, who died early in the twenty-first century, had a high level career in a social work agency, and published a number of books concerned with children's problems[76]. Marian, having obtained her B.Sc.(Econ.) and her Ph.D. at the London School of Economics, became Professor of Political Economy there[77]. She died in 2002. One of us was told by a 'still living' retired professor of the London School of Economics that she was dedicated, bright, quiet and shy. For some time Agatha and Marian, both unmarried (Ruth was married[78]), lived together in London.

Haslemere, the town chosen for Arthur and Julia's retirement, is situated between the ridges of Hindhead and Blackdown, on the latter of which is Aldworth, where Tennyson died in 1892. Near here, at the Otara Nursing Home, Fernhurst, the man named in 1869 after one of the poet's most celebrated works died on the 21st January 1957. Julia died two years later.

Chapter 2

Social Statistics

2.1 Introduction

In his review of Žižek's *Soziologie und Statistik* Bowley wrote

> statistics have been and should be in an increasing measure inspired by the achievements and lines of investigation of sociology, which in turn is continually depending for modern and measurable natural and racial relations on statistical methods and results[1] [1913e, p. 325]

Within sociology Bowley would include economic statistics, the latter relating to group activities and to indivduals in relation to groups. Society is to be regarded as an organic whole, and the investigator's task is to 'give a reasoned quantitative description of all its parts' [Bowley, 1915b, p. 7].

Charles Booth's survey, initially of East London and extended to the rest of London, and Benjamin Seebohm Rowntree's later survey of York were to a large extent surveys of poverty. Bowley's survey of Reading [1913k] continued in this vein, and his innovative use of random sampling extended the usefulness of the methods of the earlier work by allowing the results to be put in a comparative context[2], and permitted the connexion between the results for Reading and the closely-following surveys of Northampton, Warrington, Stanley and (later) Bolton.

> There is, perhaps, no better test of the progress of a nation than that which shows what proportion are in poverty; and for watching the progress the exact standard selected as critical is not of great importance, if it is kept rigidly unchanged from time to time.
> [Bowley, 1915b, p. 213]

2.2 *Livelihood and Poverty*

In 1915 Bowley and Burnett-Hurst, under the auspices of the Ratan Tata Foundation[3], published *Livelihood and Poverty*. This book, a milestone in Bowley's career, was devoted to a detailed sample survey of poverty in Northampton, Warrington, Stanley and Reading[4]. Of the six chapters, II (Northampton), III (Warrington) and IV (Stanley) were written by Burnett-Hurst, Chapter V on Reading and the conclusion were by Bowley, while the first chapter was a joint effort.

It was suggested in the first chapter that Warrington, Northampton, Stanley and Reading could be taken, in respect of population, as representative of towns ranging in population from 40,000 to 150,000, though not of course as typical of large cities like London, Liverpool, Birmingham, Manchester or Glasgow. While the towns covered all major English industries, they were also very different from each other, one relying solely on one industry and the others having a variety of industries. A sample of roughly 1 in 20 working-class households was taken: more precisely, one house in 23 in Northampton, one in 19 in Warrington, one in 17 in Stanley and one 21 in Reading.

Chapter I sets out in summary form some of the main conclusions reached in the studies of the various towns, and the results were compared with those given by Rowntree in his 1901 study of York, though in this latter investigation *every* working-class household was examined. It was found, for instance, that in Stanley about a half of the working-class houses were overcrowded (overcrowding being defined as more than two persons per room, where by 'room' was meant either a living-room or one with floor space for sleeping accommodation). The corresponding figures for Northampton, Warrington and Reading were 8.7%, 19.7% and 13.5% respectively.

The study allowed an estimate of the proportion of working-class families in the four towns who lived in 'primary' poverty and beneath the minimum standard necessary to physical health: the results for Warrington and Reading were 'shocking'. Here 'Primary Poverty' was defined as a situation in which

> the actual earnings (including pensions) of the family, when pooled together, are insufficient to give all members the food and clothing of the New Standard, after paying for rent, food and household sundries.
> [Bowley & Burnett-Hurst, 1915, p. 45]

Use of a modified standard of the minimum cost of living—Rowntree's standard had been based on 'the cheapest rations authorised for use in workhouses and is mainly vegetarian' [op. cit. p. 79], while the new standard allowed the purchase of about two pounds of meat per week—showed that 5.9%, 10.9% and 15.1% of all households were below the poverty line in Northampton, Warrington and Reading respectively (in Stanley 11 of 203 working-class families were below the poverty line and 2 were on it[5]). While it appeared that the principal immediate cause of primary poverty was low wages, factors such as (a) whether the chief wage-earner in a family was dead, ill, old or out of work, (b) irregular employment of the chief wage-earner, and (c) whether the wages were sufficient to support the number of children in the family were also extremely relevant.

In his *Poverty and Progress: A Second Social Survey of York* Rowntree emphasised that the poverty line 'was a standard of bare *subsistence* rather than *living*' [1941, p. 102]. Putting it more bluntly, 'How to live' on X shillings a week was frequently necessarily implemented as 'How to avoid dying' on the same amount[6].

The studies of rents and income showed that a working-class household living on 20*s.* to 25*s.* a week spent well over one-sixth on rent. Other depressing results were (1) in certain towns the scale of poverty that existed was appalling and (2) of the 3,287 children in the study some 27% were part of families 'which fail to reach the low standard taken as necessary for healthy existence' [Bowley & Burnett-Hurst, 1915, p. 47].

The study of Reading was essentially one undertaken by Bowley in 1912 and published as his [1913k], and a brief discussion of this paper[7] is pertinent at this point. Bowley made good use here of his sampling methods (a sample of 1 in 20 was taken, resulting in the examination of 677 working-class households. Of these 55 were identified as having occupiers above working-class, and only the rest were examined in detail), and he noted that

> The results are of much more than local interest, since they prove that an inquiry adequate for many purposes can be made rapidly and inexpensively by a proper method of samples. [1913k, p. 672]

The survey yielded information that was analysed in this paper under the headings housing and rents, family income and rent, earners and dependants, earnings and needs, rates of wages and expenditure. By comparison with other figures the results were found to be very good as regards rent, the number of persons in a house, ages and occupations.

Even the classification of households in the Reading study was difficult. Of some 600 households there were 260 different groupings (depending on the wage-earners—man only, man and girl, man and lad, man and daughter, etc.—and the dependants—wife, sons, daughters, etc.), in which 'the statistician's normal family of man (at work), wife (not working) and 3 dependent children only occurs 33 times' [1913k, p. 681].

In Reading, in the working-class, practically all able-bodied men, all boys over school-going age, and a great majority of young women and girls worked (this was common in industrial towns). Prices in Reading were some 16% higher there in 1912 than they were in York in 1901 (but less coal was used in the poorer households).

The detailed reports presented in *Livelihood and Poverty* are followed by a concluding chapter, written by Bowley, in which the accuracy of the results is criticised. Bowley considers four possible sources of uncertainty or error as being possible in such investigations:

> The information obtained may be incorrect; the definitions and standards used may be loose, unsuitable, or wrongly conceived; the households actually visited may not contain a fair sample of the whole population; and there are also calculable possibilities of error arising from the process of estimating the whole by measuring a part. [Bowley & Burnett-Hurst, 1915, p. 174]

Each of these points is investigated separately, it being found that all possible steps have been taken in the study to avoid these pitfalls. Bowley notes, for instance, the difficulty of defining 'working class' and 'minimum standard', and also justifies the choice of a 'one in twenty' sampling scheme.

Finally, not only is it noted that systematic sampling results in a smaller margin of error than randomness does, but in some cases the study allowed the comparison with official statistics, and it was possible in two instances to show that the latter were incorrect.

Rowntree [1915] reviewed Bowley and Burnett-Hurst's work favourably. He viewed as particularly disquieting the findings that (1) a large percentage of children lived in primary poverty and (2) housing was extremely poor in certain places.

In 1921, and in some sense as a development of *Livelihood and Poverty*, Bowley published a paper on working-class families with respect to age, sex and numbers of earners and dependants—information that was not obtainable from the 1911 Census. For the Census did not even allow the answering of simple questions[8] like 'In how many families does a man have more than three children to support?' Bowley notes here that a major difference between Rowntree's work and

Livelihood and Poverty was that the former dealt with completed families, tracing their history since marriage, while the latter presented an *instantaneous* picture at a specific date.

The data for Bowley's present study, figures not published before because of the war, were obtained from the Census Office, the clerks drawing a sample of one in fifty from the schedules for Bristol, Newcastle-on-Tyne, Leeds, Bradford, Bethnal Green, Shoreditch and Stepney. Data on the age and sex of every person in a household, whether the persons were occupied or unoccupied and the occupation of the householder, were obtained, and the husband and wife distinguished by special marks in the case of married couples. Theoretical checks showed that these sample results agreed with the published Census figures where these were obtainable. Combination of these results with figures for Warrington, Northampton, Reading, Stanley and Bolton resulted in a population of nearly 1,900,000 in 1911.

Removal of households classified as 'middle-class' (identified as such by for example, the occupation of the householder, the presence of servants or the number of rooms in the tenement) resulted in consideration of working-class households only. Difficulty was caused by the facts that the Census extracts did not show the relationship between the people in a house (apart from husband and wife) and also that people who were permanently incapable of working owing to accident, age or illness were shown as 'occupied' in the Census. The figures that were eventually available for use were therefore rough but capable of use, though Bowley noted here that the Census would be far more informative if it had paid more attention to households and less to individuals.

Analysis of the number of occupied and unoccupied people showed that the number of unmarried unoccupied women was very small (for instance, per thousand households where there were children under 14 years 489 women were unoccupied, the number per thousand where there were no such children was 353). 'The popular idea that there was a large reserve of women who were drawn into industry during the war is erroneous' [Bowley, 1921g, p. 104].

No great differences were found between the towns in respect of proportions of skilled and unskilled workers. When earners and dependants were considered, in the twelve towns together there were 41 households per 100 that depended on one male earner (over 20 years), 28 per 100 in which the main earner was aided by children, and $8\frac{1}{2}$ in which he was helped by his wife. While it was generally accepted that minimum wage was based on the idea that a man on his own had to support a wife and three children under 14 years, it emerged from this study that such a family occurred in 56 households per 1,000 in the skilled group and in 52 in the unskilled. The most common case was that of husband and wife with no dependants, this accounting for roughly a tenth of all households.

As a final thought on wages Bowley [1921g, p. 111] writes

> In a rational system of wages, in which they increased with needs, there would be increments with age and service up to the age of 40 years; and in a rational organisation of training and work a man would be progressively worth more as he approached the prime of life ... The suggestion from the tables is that the minimum wage for men over 20 should allow for the support of one child, and that every industry should be organised so that promotion to higher grades of wages should come in the ordinary course to all capable men in the first ten years of work.

2.3 *Measurement of Social Phenomena*

The Nature and Purpose of the Measurement of Social Phenomena contained 'the substance of five lectures given in the Faculty of Economics in the University of London in April and May, 1914' [Bowley, 1915b, p. v]. The study had two aims:

> First there is the purely scientific end of the description, of classification and of investigation of causes. Secondly, there is the utilitarian end of obtaining such knowledge

> of conditions and their relations, that we may be able to modify them with a view to constructing a society more in accordance with some ideal. [Bowley, 1915b, p. 7]

In Chapter I Bowley notes the growth of interest in the economic conditions of different classes of society, and asserts his intent

> to consider from the beginning the general objects and methods of social investigation, and to inquire how far these objects have been or are in the way of being attained. [Bowley, 1915b, p. 4]

From the first recorded census the collection of statistics was for administrative reasons, and Bowley asserts that 'now most of the purely administrative statistics are published in such a way that their meaning and content can be grasped by very careful readers' [1915b, pp. 5-6]. This was an opinion that he was to change in later years.

The second chapter is entitled 'The nation or society'. Bowley gives the result of various enumerations made in the United Kingdom from 1901 to 1911, but is forced to conclude that terms like 'British subjects' and 'British and Irish nations' are capable neither of exact definition nor of exact enumeration.

In Chapter III Bowley notes the difficulty of associating a person with an area, because of the intermingling of town and country. He adopts the 'inorganic' definition of a nation as[9]

> a group of persons occupying, or residing for a considerable time on, an area under a single government, the group not being entirely homogeneous because of the presence of a relatively small number of foreigners, some of whom are visitors, others of whom will stay permanently, and because of the absence of a relatively small number of natives who may or may not return.
> [Bowley, 1915b, p. 31]

Yet even when one restricts oneself to permanent residents further subdivision is necessary. In the case of independent persons one has to consider

> (1) resident workers, (2) non-resident workers, (3) residents whose work is elsewhere, (4) resident owners, (5) non-resident owners, (6) owners whose property is elsewhere, [Bowley, 1915b, p. 33]

and dependants should be allotted to the class on which they depend or should be given as being State dependent.

Once the areas have been appropriately identified and their populations enumerated three methods of comparison may be envisaged. Firstly one may tabulate the areas having specific characteristics (e.g. contrasting urban with rural), secondly tabulate the populations of these areas and 'give a reasoned account of the distribution of the population in regions' [Bowley, 1915b, pp. 38-39], or thirdly combine measurements and consider the density of the people.

The use of an average may itself suggest fields for investigation, yet one must bear in mind that 'Of itself an arithmetic average is more likely to conceal than to disclose important facts; it is of the nature of an abbreviation, and is often an excuse for laziness' [Bowley, 1915b, p. 46].

Having arrived at an adequate conception of a nation and other territorial groups one can next proceed to the matter of Chapter 4, 'Classification of the members of a nation'. This may be attempted in two ways: either (1) determine *à priori* the nature of the classes (occupied or unoccupied) engaged in various pursuits (e.g. commercial), or (2) examine the people and see whether they fall into distinct groups with recognisable characteristics. As examples of suitable division Bowley instances sex and age, marital status, whether the people are dependants or independants, occupied or unoccupied, social standing and income. No matter what class one chooses, though, 'we should always place it in relation to a general scheme of classification which embraces the nation (or other major group) as a whole' [Bowley, 1915b, p. 54].

Bowley takes considerable pains with a classification scheme for industrial classification, a precise definition of occupation allowing clean lines of division. He considers (1) classification by degree of occupation (e.g. completely occupied in domestic work at home, occupied in production of utilities), with an analysis of the nature of the occupation (employed versus employers and an analysis of the condition of employment), and (2) classification in relation to dependence (entirely or partly dependent, with or without dependants).

The classification adopted by the Census authorities, however, 'lacks system and purpose, and it necessarily leads to curious results' [1915b, p. 61]—for instance, a private postman is in Domestic Service, while a library messenger is among the Professions. In the 1911 Census classification of persons is by the end product of their work rather than by the materials they handle. Thus those given as employed in the cotton industry may include clerks, fitters, porters and messengers as well as those who actually handle cotton. A detailed classification will allow not only the measurement of a nation's engagement in or dependence on particular industries, but also the classification of people by economic function.

When it comes to the examination of the degree of dependence Bowley cites his earlier investigations of the households of Reading, Warrington and Northampton as showing how classes shade into one another (e.g. lodgers may hire unfurnished rooms, take care of themselves and thus be seen as separate households). Here he discusses carefully the classification of lodgers, domestic servants, etc. and examines the mutual dependence of household members.

Classification by social class is explored, such a class being defined as 'a group of persons and their dependants ... who have intercourse on equal terms so far as sex and age allows' [Bowley, 1915b, p. 85]. As part of a possible classification by social position Bowley considers the status of children compared with that of their father (a very detailed and interesting table is given in the Appendix to his tract, and the investigation was continued in his [1935d])[10]).

When neither distinct types nor clear lines of division can be determined, classification by order may yet be possible, and this is the topic of Chapter 5.

'The only order available for anything that corresponds to social grading is that by amount of income or of expenditure (e.g. expenditure on house rent' [Bowley, 1915b, p. 97]), and to this end classification by percentiles (or in some cases deciles) may be possible, all families being arranged in order of income and the numbers in each division being determined. Ascertaining the income dividing two grades would then allow the examination of the expenditure and general habits of families with that income, and comparison with similarly selected data from other countries would be possible.

As a formula 'which tends to express the distribution of individual incomes in a nation' [1915b, p. 106] Bowley instances Pareto's distribution[11] $N = A/x^a$, where N is the number of persons 'whose individual incomes are greater than x units'. From this it follows (though Bowley's formulation is a little involved) that 'the average of all incomes above £x varies directly as x' [1915b, p. 106], this average being found to be £$ax/(a-1)$. It is further noted that this formula is merely approximate: it seems to express well individual incomes from £160 to £700 but is less accurate in regions where super-tax is applicable and fails completely at the lower end of the scale. Separation of classes by occupations or industries results in data that are better represented by Karl Pearson's skew curves[12].

One of the reasons for the variation of income between occupational classes is the difference that exists between people of the same age and standing in an occupation caused by difference in ability and industry. As an illustration Bowley considers the cotton industry, and states 'it is very interesting to see how closely the earnings of women weavers and men mule-spinners conform in their grouping to the normal curve of error' [1915b, p. 112].

We have stated this passage precisely in Bowley's words because Major Greenwood writes in his review of the book that 'It appears to us that the agreement is very poor[13]' [1915, p. 617]. A goodness-of-fit test conducted by Greenwood resulted in a χ^2 value of nearly 130 for men spinners, and in the case of the women weavers the group with wages under 15*s.* alone contributed 325.54 to the value of χ^2.

Discussion of the classification of family, rather than individual, incomes then follows. A detailed investigation shows not only 'the extreme complexity of any division into classes allied to social classes' [1915b, p. 118] but also the importance of the consideration of age and sex in grading family income.

Chapter 6 is concerned with the nature of family income. Bowley points out that one should not assume that when the incomes are pooled 'the money or goods and services purchased with the total are distributed by some mutual or matriarchal arrangement' [1915b, p. 126]. Difficulties arise when lodgers are involved, for this term covers a range of people from 'paying guests' to those who rent unfurnished rooms. A further problem arises in deciding how far work done around the house by the inhabitants should be viewed as income.

Bowley turns in the next chapter to production and consumption. Each stage in the complication of manufacture, he notes, causes more difficulties, and it is only in the simplest cases that personal productive efficiency can be compared. The matter of consumption perhaps holds out more promise. Needed here are 'detailed accounts of expenditure of households or groups who consume in common' [Bowley, 1915b, p. 138]: while these may sometimes be available such availability depends on the readiness and ability of a person to keep such records, and these in turn perhaps show that such a person has unusual characteristics that may well be reflected in his expenditure.

Further problems arise in considering the nature of goods purchased. For instance, a simple thing like a loaf of bread may vary in constituents, digestibility etc. and from town to town and even shop to shop. Clothes and amusements are even more difficult. Consumption is thus patent of only general ideas, and things become even more nebulous when one tries to compare one country with another. Finally, while acknowledging that most information is available for working-class budgets, Bowley suggests that attention needs also to be given to other classes.

A long chapter (some forty pages) is then devoted to the standard of living. In an attempt to provide some definite meaning to and measurement of this concept Bowley suggests that one should take a family that is typical of its class in as many aspects as possible (including not only measurable characteristics but also possessing the mode of characteristics).

While a large mass of data is available finer results are still needed. Standards, Bowley writes, are arbitrary and conventional, and the standard of living of an artisan in 1850 was perhaps reached by an unskilled labourer by the end of the nineteenth century. A considerable amount of possible improvement had been wasted,

> by a considerable straining after what I may call conventional uselessnesses ... No one can estimate how much is spent on the trappings of respectability or on the desire for show. [Bowley, 1915b, p. 159]

Not only family budgets should be used in describing standards of living but consideration should also be given to things like lighting, street cleaning and educational facilities. Clothing is also a distinctive characteristic of a class, but 'it is very difficult to suggest any systematic way of measuring or describing clothing' [1915b, p. 161].

Bowley finally ends up with a definition of 'standard of living':

> By the standard of living, then, I understand a composite of the goods and services obtained in nearly the same quantities by normal families whose general mode of life is similar, and by an economic class I understand a group with a definite standard differing in respect of recognizable and measurable characteristics from other groups in the same society or nation. [Bowley, 1915b, pp. 164-165]

Some discussion of 'minimum standard' follows, with Bowley noting that this should not be confused with the notion of standard of minimum subsistence. And in the case of the latter concept one should distinguish between minimum subsistence for a family or minimum subsistence for an individual. Attention is also given to the slightly different idea of 'minimum subsistence necessary for maximum productivity'. If there is to be a minimum standard where food is concerned, Bowley suggests that there should be a similar standard for other necessary expenditure (e.g. fuel and clothes). Such a standard would to a large extent be based on convention: for instance, clothes are necessary, but second-hand garments are cheaply available (though hard to value) while there may be doubt about the necessity for boots.

The question of the Poverty Line is then considered—that is, the minimum wage on which an adult workman can support himself, a wife and three children[14]. In writing of the minimum annual expenditure needed by an ordinary growing family Bowley considers whether a man's parents should not be expected to provide for their own old age and also whether 'incapable adults should only be supported by their brothers and sisters if they have already fulfilled their obligations to their own wives and children' [1915b, p. 178]. Further, children who start work should not be required to support younger siblings. (Note that it is not asserted that family members should not help one another, but only that such help should not be necessary.) The wife should also help with the family income by working before the children are born and after the youngest child has started school.

There are also estimates as to the minimum on which a single worker can live—a woman lodging in a household and contributing her share of the rent needs about 6*s.* per week in a town of moderate rentals, though this may vary with the need, for instance, of expensive dressing for work.

It is doubtful though whether such comparison could be carried out between two countries. The only way this might be possible is if one could rank all families in each nation in order of income and then pick out those at the first, second, ... decile in each nation for comparison. This is something that Bowley had examined before: in his [1909a] he suggested that it might be sufficiently accurate to determine the median rather than the arithmetic average of wages of, say, all adult male wage-earners in a nation. To determine the median wage, it is enough to know that the great majority of wage-earners in certain occupations are above or below the median, knowing roughly how many people are employed in the various occupations. It then follows, for example, that the median wage most probably lies among the wages of town labourers or partially skilled workmen.

The last chapter is concerned with economic progress, with Bowley building on his previous work to consider the measurement of the progress of the whole nation or society.

The first point to be considered is the matter of population, there being three categories, viz.

> (1) those who are legally subjects of a monarch or citizens of a state and resident in its territory, (2) subjects or citizens resident outside the territory, (3) foreigners resident in the country. [Bowley, 1915b, p. 192]

One must not assume that population growth necessarily involves progress. The size and growth of a population should be regarded 'as a framework into which other measurements can be fitted' [Bowley, 1915b, p. 195].

The economic progress of an individual may be studied by tabulating his income year by year. When it comes to a society, one should consider not only the income distribution but also 'the total income, the amount of work devoted to obtaining it, and the number of dependants or non-effectives to be supported' (loc. cit.).

The total income of a nation may be estimated in two ways: (1) ascertain the net product of all firms producing material commodities, and add estimates of the value of transport, dealing, goods, services etc., and (2) take various groups of individuals and corporations, find their income and add them together. There is, of course, a difficulty with the estimation of earned income that is not subject to tax, which, at the time Bowley wrote, was an income of less than £160. Nevertheless he believed that some estimate might be made though there would be difficulties of estimation and a margin of error of 10% on the estimated total of incomes. Once again more accurate estimation is possible of the *movement* of total income than of the income itself.

Estimates of total income have however to be related to goods and services. Index numbers are not useful here: firstly, they refer to commodities and not to directly rendered services, and secondly they include neither rent nor domestic and professional services. The purchasing power of money changes not only with time but also with place (within a country) and with social class. Further

> The numerical measurement of total national income is thus dependent on the distribution of income and would alter with it. It is not an aggregate of goods and services, nor of a number of pieces of money, but of exchange values. [Bowley, 1915b, p. 208]

Once again a meaning can be more easily attached to *changes* in aggregate income over shortish periods, and with extra care one may even be able to compare the total incomes of two nations. A common difficulty is the estimation of any limit to the margin of error.

The measurement of the distribution of total income of individuals and its change would be most efficiently done by the finding of an appropriate mathematical formula and the study of the change in its constants. In addition to Pareto's Law Bowley suggests as other conceivable methods: (1) division by equal divisions of income and (2) division by equal numerical divisions of income receivers. Bowley plumps for (2), the method of deciles, quartiles and medians,

> since the incomes at the dividing points can be determined with more accuracy than average and total incomes. It has the further advantage that it is readily applicable for describing changes. [1915b, pp. 211-212]

Perhaps somewhat curiously Bowley remarks that attention should also be paid to change in the disutility of earning an income. While happiness and satisfaction cannot be measured, allied quantities like decrease in hours of work, the decrease in muscular activity needed in the light of the increase of mechanisation, and the compensation awarded in the event of accident or illness are to some extent measurable, although no numerical result is obtainable.

There remain some few points still to be made. The change in the areal distribution of the population is of great interest and importance[15]. Attention must also be paid to birth, marriage and death-rates, due account being taken of changes in age and sex constitution. These points will in turn have an impact on the changes in the relative numbers of persons in various industries.

> There are no data as regards the economic categories of landlord, capitalist, entrepreneur and earner. Our failure to make adequate delimitation of industrial or social classes involves equal failure in measuring their change. [Bowley, 1915b, p. 223]

In his concluding section Bowley sees the need for both official and private collections of statistics. The former are better suited to administrative purposes, while the latter are perhaps better for the study of isolated phenomena or of a special and ill-defined class. Further, they are most suited to filling in the gaps in official collections. It should however be borne in mind that 'measurement is a means to an end; it is only a childish mind that delights in numbers for their own sake' [Bowley, 1915b, p. 225].

Contemporary reviews of this book were generally complimentary. Major Greenwood described it as 'a succinct presentation of [Bowley's] ideas' [1915, p. 616] and congratulated him 'on the production of another useful textbook for students of sociological phenomena' [1915, p. 617]. Henry Furniss found the book to be

> full of suggestions, both as to the best methods of obtaining statistics in various fields of inquiry, and as to the uses to which they may legitimately be put when obtained. [Furniss, 1915, p. 430]

James Field commented on 'Its brevity, its pleasant lecture style, its skilful epitomising of statistical rules and principles' [1916, p. 408] and said that while it was no epoch-making venture, 'it represents a more than usually successful attempt to occupy the difficult middle ground of statistical procedure where fact and theory meet in scientific interpretation' [op. cit., p. 409].

In a somewhat cooler review Robert Chaddock wrote

> The chief merit of this book appears not so much in the author's statement of what it is desirable to measure in precise terms, as in his scientific limitations of the things which can be measured with sufficient accuracy to warrant conclusions of value. [Chaddock, 1916, p. 335]

He also notes Bowley's insistence on the correct definition of the things measured and on the need for comprehensive and correct classification.

2.4 *Has Poverty Diminished?*

Bowley and Margaret Hogg followed up *Livelihood and Poverty* with a study, published in 1925, of Northampton, Warrington, Reading, Bolton[16] and Stanley from (roughly) 1914 to 1924. Considering simply the number of persons per house or per room, they found that there had been little change since 1915, although the majority of the population under consideration had benefited by the Rent Restriction Acts[17] (while rents and rates had increased by somewhat less than 50% wages had increased by much more).

The primary data were the following: the persons included in the survey (i.e. those who normally slept in the house); the relationship to the head of the household, a man or a widow; earners and non-earners; ages; lodgers; wages and other income (e.g. unemployment insurance); ownership of the houses; rent; lodger's payments; occupation and name of employer; description of house.

The results are summarised in the introductory chapter under the headings Housing, Rent, Constitution of the Family, Responsibility for Dependants, Poverty, Unemployment[18] and Pensions. The declining birth-rate in the United Kingdom, with a consequent change in the age distribution, resulted in a change in the earning strength of a family. These changes, it was noted, 'are sufficient to cause a considerable reduction of the proportion of families, and of children below the minimum standard of living, even if no other cause had been at work' [Bowley & Hogg, 1925, p. 11].

A most striking improvement had occurred in the amount of poverty since 1913.

> Even on the assumption that all the families suffering from unemployment in a particular week had no adequate reserves and that their unemployment was chronic, the proportion in poverty[19] in 1924 was little more than half that in 1913. [Bowley & Hogg, 1925, p. 16]

The intent was to have a sample of some 800 to 1,000 working-class households from each town, and to achieve this it was decided to choose sampling factors that were expected to yield about 1,200 addresses. The factors chosen were as follows: 1 in 17 in Northampton, 1 in 13 in Warrington, 1 in 18 in Reading, 1 in 36 in Bolton and 1 in 8 in Stanley. Institutions and purely business premises were excluded, and if a house was found to be empty the investigator was instructed to take the left-hand neighbour[20]. Precise definitions were adopted: thus 'A kitchen is distinguished from a scullery by having a coal range[21] and being usable as a living-room' [Bowley & Hogg, 1925, p. 31], and even 'pantry' had to be carefully defined since in Warrington it was used for a cupboard under the stairs for coal.

Despite a rapid rise in prices workers, if employed, were better off in 1924 than in 1913, and in most of the towns surveyed unemployment was down since the previous study had been conducted.

The survey of Northampton, conducted by simple or random sampling, was marked on the one hand by the kindness and notable co-operation of the town officials and, on the other hand, by the 'extreme reluctance' of most of the householders who were chosen to be in the sample to give the information required. Bowley and Hogg noted that the essential quality of an investigator was 'tact bordering on unscrupulousness' [1925, p. 63].

In his review of this work Cannan wrote 'The answer given to the question in the title is in the affirmative' [1926, p. 222]. And while he queried the use of the five towns chosen for examination, because of the absence of occupations like ship-building and agriculture, he noted in mitigation that the changes shown in *Has Poverty Diminished?* could in general be traced to causes that were not specific to the five towns. Cannan also noted that Bowley and Hogg had recalled the statement from *Livelihood and Poverty* to the effect that it had taken a war to raise the wages of the worst paid workers: this, 'as a man of peace and economist' he found 'somewhat disconcerting' [Cannan, 1926, p. 222].

2.5 *New Survey of London Life and Labour*

In 1886 Charles Booth began the monumental survey that resulted in the publication in 1902-3 of the last of the seventeen volumes of the *Life and Labour of the People in London*[22]. Focussing on poverty, industry and religious influences of the people, the inquiry grouped people in eight classes, from savages and criminals to the upper middle-class. A major finding was that approximately one third of the inhabitants of London (in a broad sense) lived in poverty (as defined), either because of drink or because of factors such as low wages or industrial depression. Whether the survey had explanatory validity, as opposed to its mass of data, is still debated.

After the first World War a number of those well acquainted with Booth's work considered that there was need of a similar survey that would allow comparison with the earlier inquiry and give a picture of the London folk at that time. Under the directorship of Hubert Llewellyn Smith (who had himself worked on Booth's survey), and with the enthusiastic support of the London School of Economics, a similar survey was begun, with Bowley contributing a number of chapters to the seven volumes of the *New Survey of London Life and Labour* that arose. Llewellyn Smith read a paper before the Royal Statistical Society in June 1929 in which he presented a general and detailed introduction to the *New Survey*, comparing the methods to be used with those used by Booth, linking the old and the new surveys and mentioning topics (e.g. leisure) that were not considered by Booth but were now to be covered [Llewellyn Smith, 1929]).

Volume I: *Forty Years of Change* appeared in 1930, carrying two chapters by Bowley. In the first of these, Chapter II, 'Area and Population', Bowley, as a general introduction, notes the impossibility of defining an area that is strictly comparable with that studied by Booth. The area chosen here was made up of four 'sections': (a) the County of London, (b) outside the County to 10 miles from Charing Cross, (c) 10-12 miles from Charing Cross, and (d) 12-15 miles from the same. 'When, however, we depend on published statistics and not on the results of new investigation, we can extend our area as far as we please' [Llewellyn Smith, I, p. 59][23].

In reviewing this volume in 1931 Carr-Saunders finds this Chapter to be 'admirable', and humorously concludes that as according to the Survey most Londoners were still poor at that time, 'if many of them are to possess this volume, it can only be in a cheap edition' [p. 84]. At the time the book cost 17*s.* 6*d.* (in his 1912 study of Reading Bowley showed that the minimum expenditure needed by a family at marriage would be 16*s.* per week).

In Chapter X, 'London Occupations and Industries', Bowley examines things like the increase in the proportion of occupied persons, proportions occupied by sexes and various age-groups, the constitution of the 'average family', with a comparison of London and England and Wales, changes in individual industries in three steps—1891-1911; 1911-1921; and 1921-1929. He concludes that

> In the County of London the number of occupied males

> increased $10\frac{1}{2}$ per cent. in the 30 years [1891 to 1921], in the adjacent boroughs nearly 140 per cent., and in the Survey Area as a whole 25 per cent. Of occupied females the corresponding increases were 16 per cent., 135 per cent. and 28 per cent. [Llewellyn Smith, I, p. 315]

Bowley gives the numbers working in London and the Home Counties as Males: 2,960,000 ± 50,000 and Females: 1,445,000 ± 20,000.

Bowley's next contribution was to Volume III: *Survey of Social Conditions. 1 The Eastern Area (Text)* his responsibility being the whole of Part I, 'The House Sample Analysis'.

In Chapter I, 'Method', Bowley notes that in conducting an investigation on the scale of that attempted here

> two alternative methods are possible; we may either obtain some information about every unit in the area concerned, or more information about some selected units. The first is the method of the Population Census; ... Nothing is learnt from the Census of the economic position of a household ... Little is learnt, and nothing is tabulated, relating to social position.
>
> [Llewellyn Smith, III, p. 29]

The Sample method was thus used, and Bowley outlines precisely how this was to be done. He points out that 'So far as there is a choice, we get a more complete view by increasing the number of details studied than by increasing the number of families included in the sample' [Llewellyn Smith, III, p. 30]. The Survey, which took account of about 30,000 families, had to be done in such a way as to reflect in due proportion every aspect of the universe from which the sample was taken. For this two conditions were necessary:

> the universe shall be accurately defined, and ... every unit of the universe shall have an equal chance of being selected. The precision or accuracy of the measurements

> found in the aggregated sample can then be determined on known mathematical principles.
> [Llewellyn Smith, III, p. 30]

The units selected for the sample could be obtained in either of two scientifically validated ways: the first is to number all units in a district and use a table of random numbers to select the sample. The second is to choose numbers 1, 51, 101, 151, ... or 2, 52, 102, ..., if a sample of one in fifty is desired. (One can also choose the starting number at random if bias in that unit is to be avoided.)

One problem with the Survey was that there were cases in which there was refusal of information (ranging from zero in Woolwich and Shoreditch to 11 per cent. in Stoke Newington and 20 per cent. in East Ham). Interestingly, 'These wide variations were to some extent connected with differences of competence among the interviewers' [Llewellyn Smith, III, p. 33]. This was comparatively easily handled:

> Since the main purpose of the inquiry was to study the relative amount of actual poverty and sufficiency in wage-earning households, it was not necessary to do more than ascertain the number of households whose régime was superior to that of the wage-earning class.
> [Llewellyn Smith, III, p. 34]

Bowley begins 'The Family', the second chapter in this part, by stating that in the Survey a family is defined by occupation of a room or tenement. In the Eastern Area some 31% of families consisted of only one or two persons, although three-person families were also very common. While the difficulty in defining a family and the variety in its constitution adds a fictitious touch to the definition of a working-class family, Bowley manages to consider things like the earning strength of families, the proportion of wage earners to dependants, the sex and age of dependants and the change in Inner North-Eastern Boroughs since 1911.

In Chapter III, 'Rent', Bowley considers the rents paid for different types of housing: separate houses, divided houses (more than one family in the same house), flats, lessors and sub-tenants. Partial correlation is used to investigate 'how far rent depends on wages and how far on the number of persons in the family' [Llewellyn Smith, III, p. 56].

The next chapter is concerned with wages and family income[24]. Here a useful distinction (which is invoked time after time in the *New Survey*) is made between full-time income and income in the week of investigation (the figures may well be different, owing to illness or incapacitation, say, in the week preceding the inquiry). The distribution of families according to the size of their income is studied and the average working-class family income estimated.

Chapter V, 'The Poverty Line', opens with a careful discussion:

> There is no universal definition of poverty, and the meaning attached to the term varies from place to place and from time to time. It has not the same significance as destitution, which is a much stronger word ... It does not correspond closely with pauperism which signifies relief from public resources ... We must relate poverty to some minimum standard of economic welfare, which is based on provision of the primary needs of food, clothing, shelter, warmth, etc. ... Though we cannot reach any absolute definition of poverty, we can define a standard which corresponds to the ordinary conception of the term at one time and place, and use 'poverty' as a technical term related to this standard.
> [Llewellyn Smith, III, p. 70]

This definition differs from that used by Booth, according to which 'poor' was a description of those who had a sufficiently regular though meager income, while 'want' meant an aggravated form of poverty and 'distress' an aggravated form of want. Nevertheless Bowley believed that the two standards were much the same[25].

Bowley also gives here the calorie requirements of people of different sex and ages, and also supplies some hypothetical budgets. He shows, for instance, that at prices prevailing in East London in 1929, a family of man, women and two children (aged 10 and 4) needed a minimum of 39*s.* a week, 19*s.* of which was for food (bread, beef sausages, potatoes, sugar, tea, butter, etc.).

'The Extent, Distribution and Causes of Poverty' is the title of Chapter VI. It emerges that

> in a "full-time" week 6.3 per cent. of the families in the Eastern Survey Area were in poverty, the proportion varying from 14.5 per cent. in Poplar to 1.0 per cent. in Leyton. In the week of investigation the general percentage is raised to 11 per cent., varying from 19.5 per cent. in Poplar to 2.5 per cent. in Leyton.
> [Llewellyn Smith, III, p. 78]

When one turns from the proportion of *families* in poverty to the proportion of *persons* the figures are somewhat lower, on account of the large number of older persons, classified as poor, living on their own on old age pensions, a number that outweighs the percentage of families with children that are also regarded as poor.

An important result is the following:

> the reduction of the proportion of persons in poverty in the forty years [since Booth's survey] is enormous, whichever figures we take, and there is no doubt that the measurements are approximately comparable.
> [Llewellyn Smith, III, p. 80]

There is no room for complacency, however, since the figures are still very high in certain districts and age groups.

A number of causes of poverty are mentioned as being significant, including age (a primary factor in the case of those living on their own who are older than 65), permanent illness or incapacitation of the natural male head of the family, and insufficient wages.

Bowley and Llewellyn Smith were together responsible for Chapter XI, 'Overcrowding'. Here the suitability of considering results based on *averages* rather than actual *percentages* is shown:

> We may therefore infer with some confidence that at all events in the more crowded working-class areas the *relative* conditions of overcrowding derived from average density figures agree substantially with those based on percentages of population living under overcrowded conditions. [Llewellyn Smith, III, p. 218]

Things are however different in boroughs of low density,

> since this condition may result either from absence of overcrowding or from the presence in the borough of a considerable well-to-do population with practically no crowding, alongside an overcrowded slum.
> [Llewellyn Smith, III, p. 218]

The question of slum clearance, it was noted, was still the dominant social question facing London.

There are six appendices to this Volume, the first four to Part I, the fifth to Part II and the sixth to Part III. These are respectively entitled 'Instructions and definitions', 'The minimum standard', 'The sampling factor', 'Measurement of the precision of the results of the house sample', 'Statistical notes to the street survey[26]' (with Notes 'On relation of family income to wages', 'On the number and proportion of adults to families in "red" streets', 'Resident domestic servants in private families', 'Persons of the "M" class in non-red streets[27]', 'Persons in poverty not living in private families' and a specimen working sheet for certain boroughs) and 'Interviews with old people'. There is no indication who wrote these appendices: the titles, however, suggest that Bowley might have been responsible for Appendices III and IV, and we shall therefore consider them briefly.

Appendix III is devoted to a discussion of the sampling factor[28]. For large boroughs the sampling factor was one household in 60, reduced for smaller boroughs to enable enough results to be obtained. Nine different estimates of the sampling factor were made for each of the seventeen boroughs, e,g.: method (a): the 1921 Census gave the whole population as 162,600, the number in private families as 156,300, the number of families as 37,300 and the number of houses as 23,100. Sampling factor: 60 $\times$ number of houses $\div$ number of families = 37.1. Method (d): using School Attendance Officers' returns. 20,059 families with one or more children between ages of 3 and 14. The working-class cards showed 421 such families. If this proportion was the same in the middle class, we should add 37, and the sampling factor is $458 : 20,059 = 1 : 43.8$. It is noted that some of the sampling factors may be too low if there are relatively fewer children among the middle class. For the *New Survey* sample factors like 21 (Bethnal Green), 33.5 (Shoreditch), 46.5 (Hackney), 37 (Stoke Newington), Woolwich (50), Leyton (49.5), Poplar (42.5) and West Ham (48.5) were eventually adopted.

Appendix IV, 'Measurement of the precision of the results of the house sample', while containing results obtaining only for the working class, is certainly devoted to a topic dear to Bowley's heart.

> Errors may be due to two sets of causes. Those in the first set result from omission of part of the sphere defined, owing to refusal of information, inexact information, departure from the rules laid down, and errors in extracting and tabulating the data. ... The second set of errors is that due to measuring a sample instead of including every house and every person in the defined sphere.
> [Llewellyn Smith, III, p. 439]

The importance of the standard deviation is emphasised, it being useful in estimating the roughness of the approximation, and in deciding whether numbers should be given as per cent. or per thousand, or as correct to a shilling, etc. In order that the formulae used should indeed be applicable it is essential that every household in the universe should have an equal chance of being included in the sample. In the Survey this was ensured by the selection of the houses and the directions given beforehand to the investigators. On the method 'choose 1 in 40' (say) we find

> The latter method [the former consists in the use of a table of random numbers], which is termed "stratification"... gives an obvious security that the characteristics of every street and every district shall be reflected in due proportion. [Llewellyn Smith, III, p. 440]

Some calculations of standard deviations, depending on the methods used, are given. For Method A(1): n the number of people in the sample, r the sampling factor, pn the number in the sample which always have a particular characteristic, and $q = 1 - p$. The standard deviation of pn is then $\sqrt{[pqn(1-(1/r))]}$. The number in the sample can then be written as $pn \pm \sqrt{[pqn(1-(1/r))]}$. The number in the borough is $pN \pm \sqrt{pqN(r-1)}$, where $N = n \times r$. The proportion in the sample is $p \pm \sqrt{[pq(1-(1/r)) \div n]}$, and the percentage $X = 100p$ is $X \pm \sqrt{[X(100-X)(1-(1/r)) \div n]}$. Method A(2): determine the proportion in an aggregate of boroughs 'by multiplying the whole number of families and the number which have a particular characteristic by the sampling factor for each borough, and adding the latter for the numerator and the former for the denominator'. Thus

$$P = \sum r_i p_i n_i / \sum r_i n_i.$$

A few examples are given. In working out the standard deviation for average numbers of persons per family or average income per family the usual formula $\overline{x} \pm (s/\sqrt{n})$, though it is noted that this more accurately has $n(1-(1/r))$ rather than n. This latter correction seems to have no effect here. Detailed analysis of the formulae for standard deviations of groups of boroughs are also presented.

For a few moments of light relief[29] let us look at some comments recorded by the enumerators and given in Appendix VI, 'Interviews with old people'. For instance: 'Miss S—lives in some very nasty flats in a very poor street ... What I saw of the bedroom looked very untidy and not too well kept ... She seemed to be quite a cheerful old woman and to have nothing much to grumble at' [p. 459]; 'Mrs. K—was spotlessly clean both times I saw her and very neatly dressed, although she was expecting no one to call' [p. 458]; 'Miss R—says she does not get enough to eat, and that she would be much fitter if she had more nourishing food' [p. 464]; 'Mrs. S—goes twice a week to afternoon meetings at the chapel and always goes there on Sunday evenings' [p. 466]; 'Mr. Pi—was very cheerful and tried to make the best of things, although he did complain of being very lonely and said that his children did not come to see him very often. He seemed very grateful for the Old Age Pension, although he did not think it was enough to live on' [p. 468].

Volume VI: *Survey of Social Conditions (2) The Western Area (Text)* of 1934, was the last to contain a contribution by Bowley[30], and the method of investigation used here was the same as that in the study of the Eastern Region, but carried out a few months later. The Introduction to this volume, by Llewellyn Smith, provided a useful summary of the overall findings on poverty reached in the survey:

> The broad effect of the New Survey is to show that in 1929 the proportion of the London population who were below the poverty line, in the sense of the term referred to above [i.e. as in Vol. III], was somewhere between a third and a quarter of the proportion recorded by Charles Booth forty years earlier. In this Conclusion both the Street Survey and the House Sample Inquiry agree, though ... the latter method yields a rather lower percentage of poverty than the former, and consequently makes the reduction of Poverty since Charles Booth's day appear slightly greater.
>
> That two independent estimates of the proportionate reduction of London "poverty" in the forty years 1889-1929, viz. by 69 per cent. (Street Survey) and 71 per cent. (House Sample), should differ so slightly is very remarkable considering the roughness of the data.
>
> Both inquiries, moreover, agree in finding that the reduction of poverty has been somewhat greater in the Western Area than in the East. [Llewellyn Smith, VI, p. 3]

The Street Survey estimated that in 1929 there were 490,000 people beneath the poverty line in the London Survey Area, somewhat more that 8.7% of the total population of 5,653,000.

It is worth noting that Linsley and Linsley [1993] found the assertion in the *New Survey* that the aim had been to undertake the investigation so that 'so far as is possible the standard of minimum needs which marks the poverty line shall be directly comparable with that employed by Charles Booth' [Llewellyn Smith, III, p. 3] to be untrue. They adapted Rowntree's standard used in the 1937 edition of *The Human Needs of Labour*, his second social survey of York[31], to obtain a poverty line for the London of 1928-30, and concluded that 21% of working-class households in London lived in poverty, as compared to the estimate of 9.8% given in the *New Survey.* Linsley and Linsley also show that if Llewellyn Smith *had* measured poverty using Booth's standard the percentage living in poverty would probably have been between 3.7% and 5.9%

Bowley himself was perhaps not altogether satisfied with the method used for the poverty line in the *New Survey*, for in 1936 he published a paper in which he considered the effect of modifying this measure. Here he noted that while the definition of poverty adopted in the *New Survey* was chosen to allow comparison with Booth's results, opinion as to what constituted 'poverty' had changed since Booth started his survey, and accordingly it seemed advisable to formulate a new definition for future use[32].

Of the many reviews of the *New Survey*[33] we shall look in particular at those by Bradford Hill, who reviewed all nine volumes.

In his review of Volume I Bradford Hill perhaps picked out something not noticed by other commentators:

> Analysis of their vices suggests that the people of London have become much less inclined to acts of personal violence, are less addicted to drunkenness and are perhaps a little more honest, but that the standard of sexual morality is lower. The decline in number of burglaries is, at least in part, explicable by the use of the motor-car,

> which enables the London burglar readily to transfer the scene of his operations from London to the surrounding country. [1931, p. 447]

His 1932 review of the second volume, while generally enthusiastic, expressed the regret that there was no reference to official statistics when it came to data relating to occupational mortality and morbidity.

Bowley is congratulated in the review of Volume III on being in charge of the House Sample Inquiry, it being noted that the volume 'adds to its valuable collection of statistical material an invariably interesting account of so many aspects of London life' [1933, p. 109].

Certain points of interest are noted in Bradford Hill's review of Volume V: for instance,

> Londoners who find little or no intellectual satisfaction in the statistical analysis of wage rates or of hours of labour will find a great deal in this and the previous volume, which cannot fail to interest them, about the way in which their neighbours live and work. [1934a, p. 164]

It was noted too that while detailed inquiries about the health of seamen, the printing industry and laundries had been published in the three years preceding the appearance of this volume no mention was made of these reports.

But Bradford Hill was not above sharper criticism:

> It is a pity, where so much is well done, that facile conclusions on various aspects of industrial health should have been included. The collection of adequate data would, admittedly, have been an impossible task, but without them the subject were better left alone [1934a, p. 164]

Several instances of such 'facile conclusions' are mentioned: for instance

> it is difficult to see why it should be decided that the rather warm and artificially humidified atmosphere of parts of a tobacco factory "obviously has a beneficial result on the health of the workers" [1934a, p. 165]

And finally there is a warning to investigators '[to] be specially on their guard against deductions which cannot be substantiated' [1934a, p. 165].

While noting the valuable studies in Volume VI (Volume VII contains the relevant maps) of specific problems of London life (e.g. the housing problem, the migration of population and the Jewish element in East London) Bradford Hill also airs a slight criticism:

> the measure of health used in the Borough Summaries, at the end of the volume,—namely, the crude death-rate—is not very helpful. [1934b, p. 491]

(He notes that the crude death-rates 'cannot give a true aspect either of fertility or of the hygienic advantages or disadvantages of an area' [1934b, pp. 491-492].)

Volume VIII, Bradford Hill notes, contains in general respects much of interest and value. Further, it contains, as usual,

> an interesting store of the minor details of other people's lives—for example, that omnibus drivers and conductors receive a daily bonus of 1*d.* per working day for each day free from accidents—and, as usual, one or two unsubstantiated statements on problems of industrial health.
> [1935a, p. 153]

The pursuits of leisure treated in Volume IX, must of necessity, Bradford Hill notes, be susceptible of only slight statistical treatment, for no official figures were available for comparison. Nevertheless this volume 'well maintains the interest and value of its predecessors' [Bradford Hill, 1935b, p. 725].

2.6 *Consequences of the Great War*

In 1930 Bowley published *Some Economic Consequences of the Great War*, a book that perhaps sits squarely in both the sociological and the economic camps, and of which he himself said ‘The title of this study is more pretentious than its contents’ [1930d, p. 16].

Devoted mainly to a study of various aspects of British society before, during and after the first World War, the book also contains some details of changes in France, Germany and the United States.

The postwar uneasiness in Great Britain manifested itself in three fundamental aspects: economic (adaptations of industries and the passage from war to peace economy), political (parliamentary difficulties and misconception of the policies of other countries) and social (post-war indolence, discontents and revolutionary agitations).

Bowley states in the Introduction that the work is ‘based chiefly on statistics’ [1930d, p. 17]. Yet the statistical work is relatively minimal, being derived from official reports and returns and contained mainly in comparative tables. The imperfection of national and international official statistics in general is noted, but Bowley once again remarks that tendencies and movements can often be observed in the absence of precise measurement.

Among his general remarks Bowley suggests that developments may be classified as (a) mainly unconnected with the war (e.g. increase of petrol-driven traffic, the development of wireless transmission and the fall in the birth- and death-rates), (b) accelerated or retarded by the war (e.g. aviation and the economic position of women) and (c) apparently arising out of it (e.g. the destruction of life, the increase in taxation and the new economic relation between Europe and the United States).

Some economic lessons could certainly be learned from the war. For one thing, no matter who wins on the field both sides lose in wealth. Vast numbers of fighting forces could be mobilised, while civilian services and industry could be redirected to the production of appropriate goods. ‘In brief, economic conditions and requirements do not prevent war' [1930d, p. 32]. While the expenses of a war could be postponed (e.g. borrowing of money from neutral countries), these expenses had to be paid back, and on a wider front goodwill, credit and trade-relations had to be restored.

While use is made of statistics in this work, Bowley notes that it would be inappropriate to view the work as a statistical report, and although care had been taken to use only authoritative and carefully collected data, such are of very varying precision.

The second chapter is concerned with population, and Bowley notes that the destruction of people, property, trade and the general organisation of commerce are the most evident effects of war on economics. The consequences are not only instantly obvious but also of long duration. Bowley regards it as more important to consider, rather than deaths, the change in the sex and age distributions of the survivors, and he concludes that

> Though the general fall in the birth-rate is not mainly attributable to the war, the specific fall in 1915 to 1919 and the increase in 1920 and 1921 are directly due to the absence and return of the men. [1930d, p. 60]

As far as migration is concerned it seems that pre-war emigration was greater than immediately post-war—to a large degree because of restrictions on movement, but also because of a decrease in the need for such emigration.

Chapter III, 'Currency and prices', deals with 'the wide-reaching effects of the changes in the amount and nature of currency, and of prices' [1930d, p. 66] (note: *effects* rather than *causes*).

On looking at the industrial countries affected by the war one finds that pensioners and the older members of the middle-class were worse off in terms of purchasing power in 1922 than they were in 1913, and in Great Britain 'there has been a visible fall in the standard of living of the professional classes' [1930d, p. 77].

When it came to capital it appeared certain that there had been a considerable redistribution. For example, the treatment of housing had resulted in a transfer of capital from owners to the wage-earning class as a whole. Building of new houses had slowed down in Britain during the war, and rents for working-class houses were fixed at pre-war levels, with tenants being given security of tenure.

Demobilisation resulted in a serious shortage of housing, and the promised 'Homes fit for heroes[34]' was cynically replaced by those physically and psychologically damaged by the war by 'heroes fit for homes'. The movement of labour was seriously affected by the shortage of houses, rent control and the marked difference between the rents of new houses and of old. For tenure of a house at a moderate rental meant that the ordinary household could not move to housing at the same rent in another district where work was available.

Chapter IV is concerned with a deeper examination of capital. Consideration of research by Keynes and Stamp suggested to Bowley that some two, three or four years normal growth of the property of the belligerent countries was lost by the war (for instance Great Britain lost ships, while France lost mines and agricultural land). The world was poorer in 1919 than in 1914; nevertheless people in England seemed to think that 'plenty was also restored with peace' [1930d, p. 91], and an orgy of spending ensued. The problem now was not a shortage of natural resources but rather the difficulty of the full utilisation of the existing productive capacity.

Trade also suffered a considerable set-back. Trading connexions, credit, confidence and goodwill all received severe damage, and were only partially restored, among the belligerents, nine years after the end of the war. New boundaries were established between a number of European countries, which entailed a recasting of trade connexions. New trading connexions had been established during the war, and old ones severed. All this resulted in a slow and painful re-establishment of commerce.

Chapter V, 'National debts and taxation', starts with the depressing statement that the total national debt in the United Kingdom increased from before the war to after by about £7,000 million. The aim of this chapter is to examine 'how this debt arose, to whom the interest is payable, and how it is provided' [1930d, p. 106], and Bowley takes pains to point out that it would be wrong to suppose that the debt arose 'principally out of the ill-gained wealth of profiteers' [1930d, p. 108]. The interest on the debt, an interest that amounted to some 10*s.* per week per family in the United Kingdom, was met by an increase in income-tax and death- or estate-duties. It was therefore clear that both the increase in income-tax and supertax were mainly a result of the war.

Things became more complicated when an attempt was made to look at similar things in other countries. Bowley frankly admits that 'it is almost impossible to master the taxation system of other countries' [1930d, p. 115], though he makes a manful attempt to compare the taxation, expressed in pounds Sterling, per capita of population in 1913-14 and 1923-4 in the United Kingdom, Germany, France, Italy and the United States.

A long chapter on 'Changes in distribution of income in the United Kingdom' then follows. Bowley notes that while the total income of individuals in the United Kingdom had more than doubled from 1913 to 1924, it should be remembered that in that period (a) the population had increased by 5%, (b) the value of money had changed—due to rise in retail prices of about 80%—and (c) considerable sums of money had been transferred from one group of people to another by taxation and rates. It followed that the average income in 1924 was something like £86, which was practically the same value as in 1914 taking account of population growth and changing prices.

Some popular misconceptions are discussed: for instance

> Casual observers and readers of newspapers, especially in the South of England, are impressed by the apparently lavish expenditure on luxuries and pleasure especially in London and at seaside resorts. [1930d, p. 134]

This Bowley explains by noting that the luxurious expenditure in the capital is often that of 'colonial and foreign visitors', and also that the poor and unsuccessful do not make the society pages in the press.

Examination of other areas, South Wales for example, might lead to completely different conclusions. The landed proprietors had had their wealth cut considerably by rising prices, income-tax, super-tax, death duties and the steady but unchanging income from land, with some former tenants becoming owners.

The number of salaried women had doubled while the number of salaried men increased by 50%: 'The detail and the causes of the change are a little obscure' [1930d, p. 141], and it was noted in general that the increase in salaried persons was partly due to an increase in government employees (bureaucracy it seems never changes!).

When it came to the case of wage earners it was found that 'the real wages of skilled workmen were little, if at all, greater in 1926 or in 1929 than before the war' [1930d, p. 148]. Unskilled men, however, had benefitted considerably. On the downside, though, it was noted that 'skilled women's wages only equalled unskilled men's wages' [1930d, p. 150].

Bowley presents tables of the wages in less skilled occupations as percentages of those in skilled occupations in the building, mining, cotton, engineering and railways industries. A significant factor was the matter of foreign competition, especially noticeable in the mining, cotton and engineering industries.

> The rise in wages of all operatives in the sheltered industries and of unskilled labourers in all industries is no doubt partly the result of a determined effort to raise the standard of living, at almost any cost in unemployment, and is partly due to the increased power of the workers' organisations, helped by the existence of unemployment insurance. [1930d, p. 153]

It had also been found that the relation of women's wages to men's wages had changed similarly, though for different reasons (for instance, women who had been employed in munitions factories during the war earned more thereafter as typists).

Wage-earners in general did not pay income-tax, and when it came to the effect taxes had on such workers Bowley noted that

> The only increases that affect the working class are taxes on tobacco, alcohol and entertainment. The first two are very considerable, and for the ordinary fairly careful man, who drinks and smokes, may amount to 10 per cent. of his wages. [1930d, p. 159]

Such workers of course also benefitted from the Rent Control Acts.

Bowley now devotes a few pages to the study of poverty, and as an illustration of the general improvement of the lot of the poor he refers to his studies of Northampton, Warrington, Reading, Stanley and Bolton. As a conclusion he notes that

> great progress has been made towards the extinction of remediable poverty, considerable inroads have been made on excessive wealth, and generally income is less unequally distributed than it was ten years ago. The changes are due to many factors, some of which are directly traceable to the war, while others, such as the fall of the birth-rate and the extension of social services, are the continuation of processes that began before the war, which, however, cannot have been without influence on the manner and date of their development. [1930d, p. 165]

'Displacement of labour. Production' is the title of Chapter VII. Bowley notes some of the difficulties caused by demobilisation. For instance, those who had essential or definite 'civilian' work to return to were released first, in some cases returning to jobs that had been filled for the duration by the elderly or retired who, in turn, were happy to leave. Inflation in 1919 and a failure, or unwillingness, to recognise that the world in general was poorer after the war than before contributed in no small measure to the collapse of prices in 1920 and the great increase in unemployment. Bowley considers the changes in the coal, iron and steel, ship-building and cotton and wool industries in particular: we shall look only at the last two here.

The difficulties experienced by the Lancashire cotton industry could only be partly attributable to the war. The trade depended heavily on shipping both for the import into Britain of raw material and for the export of finished goods, and while this was a temporary hindrance during the war the impetus that eastern countries had received during that period was of more lasting effect. The woollen industry was subject to uncertainty and price variability, partly because of the large stock of wool that had accumulated in Australia during the war and partly because of fluctuating tariffs.

In the case of mercantile shipping Bowley notices that more than a quarter of the world's tonnage afloat in 1913 had been lost during the war. Efforts to redress this by a hastily improvised programme of ship-building were almost too successful: for instance, a large number of the ships built in the United States were never used, enough freight not being able to be found to pay their running expenses. Further, there were a number of ships in other countries that were now too old to be effective, since even obsolete ships that would have gone to breakers under normal circumstances had been used during the war.

In Chapter VIII foreign trade and unemployment are examined. To the impoverishment of nations, the disturbed currencies and broken relations that were widespread after the war were added difficulties resulting from the re-drawing of the map of Europe and the growth of national spirit. Additional tariffs were established and customs regulations were set and changed frequently.

The United Kingdom was of course affected, though things were not worse there than in other countries: in fact, it seemed that by the time Bowley wrote trade had settled down back into pre-war patterns. There were however important details in changes. Decrease in exports to Germany, Austria and Russia had been compensated for by increase to European allies and neutrals, while a loss to India was balanced by an increase to Australia. Further, the decline in exports extended over all principal manufactures. Some competitive nations paid lower wages than were paid in Britain, and this advantaged the former in inflationary times. Further, the faulty location of labour in both occupations and districts played a part, as did the scarcity and dearness of capital.

> Whatever the relative importance to be attached to these different factors, the result has been that in the United Kingdom a million or more persons have been out of work in nearly every month since the beginning of 1921. [1930d, p. 209]

This unemployment was not that of 'a standing army of a million unemployed' [1930d, pp. 209-210], but rather made up of a continual movement into and out of work, though a number of aged and infirm workmen would only find work in times of good trade. Unemployment was widespread over industries, but women were generally not seriously affected except in the cotton and woollen industries.

The final chapter is concerned with nationalism and internationalism. Alterations of territorial boundaries as a result of the war had occurred in many European countries, with sixteen former countries becoming twenty-three[35]. Purely economic questions were settled to a degree after the war by the League of Nations and the International Labour Organisation.

The Geneva Conference of 1927, viewing not only the complexity of tariffs but also their severity, declared that it was time to end the increase in tariffs and move in the opposite direction. The League of Nations set up machinery to implement the decisions of this conference, but progress was painfully slow.

The International Labour Organisation was set up under the Treaty of Versailles (signed 28th June 1919).

> The considerable support that the I.L.O. has received is due to a curious blend of egoism and altruism, of national selfishness and international sympathy, and it is often mistaken economic theories that lead the former to give play to the latter. [1930d, p. 229]

Bowley has something to say as a general remark about the 'widest spread fallacy' [1930d, p. 229] that a country in which wages are low, standards of safety are not upheld and working conditions are bad, competes unfairly with a country with higher standards, in which there may then result unemployment or a fall in standards.

> As a matter of fact it has been the case in the whole epoch of international trade, that exchange of goods has continued between nations whose economic conditions have differed enormously without injury to either; ... A plentiful supply of cheap labour in a country from which goods are obtained is as great an advantage to the purchaser as are favourable natural conditions. [1930d, pp. 229-230]

While this is a general principle, two exceptions arise. Firstly, if Japan and England both sell cotton goods to India, if the Japanese are equally (why not 'more'?) efficient and work longer hours for less pay, Japan will obtain the Indian market and unemployment will occur in Lancashire. The second exception occurs when it is desired to improve labour conditions but that improvement does not result in immediate greater efficiency (for instance, suppose several nations produce matches and some abolish the use of phosphorus, while others do not. The former may lose trade to their less scrupulous rivals).

> But whether the participators are selfish, illogical and shortsighted or not, the result of international efforts to improve working conditions is from a wide aspect beneficial. [1930d, p. 232]

Attention is also paid to the question of working hours. Bowley notes that while employers 'were more willing to concede advantages in working conditions than to raise the nominal rates of wages, since it would be difficult to force a reduction when present prices fell' [1930d, p. 235], nevertheless 'it is contended that reduction of working-hours does not necessarily mean an increased cost of output, at any rate after a period of adjustment' (loc. cit). He warns against the comparison of output before and after the introduction of the eight hours working-day, pointing out that other circumstances may also have changed in the same period.

The general conclusion, perhaps summarising the whole book, is

> The sense of common interests is more wide-spread, the injury of unrestricted competition is realised; and the organisation and co-ordination necessary for common action on a national scale have nearly equal power in compelling or restricting individual action, whether they are based on legislation or voluntary combination.
>
> [1930d, pp. 242-243]

2.7 Aspects of population tendency

In 1927 the first World Population Congress was held in Geneva. This led to the formation of the International Union for the Scientific Investigation of Population Problems. At the 1931 meeting Bowley read a paper ‘Economic aspects of the tendency of population in Great Britain’, in which he presented some general lines of investigation into ‘the interaction of the changes in the economic position of various classes of the population and the changes in the economic constitution of the population itself’ [Bowley, 1932d, p. 47].

While the data considered relate to working-class families, Bowley notes the difficulty of defining such a family from Census returns and in deciding how the income of such a family should be pooled. Over the period from before World War I to the mid 1920s ‘the real wages of nearly all grades of unskilled town labour have increased markedly’ [1932d, p. 48]. As an example,

> in Warrington—to take a typical industrial provincial town—the ordinary unskilled wage was 22*s.* in 1913 and 44*s.* in 1924. In 1924 this wage met the standard of bare sufficiency of man, wife, and four children; in 1913 it hardly sufficed for three children. [1932d, p. 48]

He considers too the changes in the population as a whole, noting the decrease in the number of women of child-bearing age as a consequence of the falling birth-rate since 1914, and estimates the population distribution in 1951. Here he calls on results of the surveys published in his *Has Poverty Diminished?* of 1925.

A decline in large families had been noticeable, and in conjunction with an increase in social and official care of children this had led to an environment more suitable for satisfactory development. It was still necessary to consider the effect on household economy of the increase in the number of people over 65.

> The broad result of this analysis is that (apart from unemployment) the earning strength of the population is increasing rapidly in relation to the number of dependent children, and that therefore some of the main causes of poverty and mal-development and insufficient training are being removed. But it is evident that these general averages need interpretation by detailed study of separate classes. [1932d, p. 51]

It is interesting to note that Bowley concludes here that, at that time, poverty was due not to lowness of wage-rates but rather of things like illness, age, unemployment, incapacitation or death.

In our Introduction to this chapter we mentioned the important surveys conducted by Charles Booth and Seebohm Rowntree, and it is fitting to close with the just opinion of Ernest Hennock, Emeritus Professor of Modern History in the University of Liverpool:

> Their [i.e. Booth and Rowntree] understanding of statistical methods was elementary. Bowley's training made him someone very different and much rarer, namely a person concerned with the application of academic mathematical studies to social studies. He was not himself a creative mathematician; his originality lay in recognising how matters familiar to mathematicians could be used in social investigation, a field in which high-powered mathematicians were usually not interested. [1987, p. 220]

This page is intentionally left blank

Chapter 3

National and Official Statistics

3.1 Introduction

As a collector and examiner of statistics Bowley is perhaps mainly seen as a gatherer of (sociological) data on various English cities and on the wages in a number of industries. However he was also concerned with official statistics, as collected and analysed by governmental and quasi-governmental bodies, and it is this latter aspect of his work that will be examined in this chapter.

3.2 Early papers

In September of 1903 Bowley published a paper in which he categorically stated that the inconsistent and confusing conclusions drawn by many in considering the fiscal controversy[1] were more often occasioned by faulty logic than by faulty statistics. He notes that a published theory based on the author's own view of history and political economy and fitted with incomplete or even irrelevant figures is, within a short time, regarded as 'proved by statistics', a most unsatisfactory state of affairs. Bowley suggests, however, that

> if the *a priori* proof is complete, statistics are not necessary to confirm it, though they may give a useful complementary quantitative measurement; if it is incomplete, and therefore an appeal is made to facts, the statistics must be examined as strictly as if the whole burden of proof rested on them. [1903c, p. 303]

Bowley discussed errors made in the assessment of national prosperity under six headings: (1) Criticism of change in a fluctuating series (it is important to consider series rather than the results for a single year (say), and to examine the trend and the fluctuation); (2) The addition of unlike quantities (the commonly used formula

$$[\text{Value of Imports} + \text{Value of Exports}]\,/\,\text{Population}$$

for the index of a nation's trade prosperity is unsatisfactory, as 'value of imports' and 'value of exports' are *not* like quantities); (3) Value and quantity (the suggestion is that no estimate of the *value* should be used without examination of the corresponding *quantity*); (4) The precision of estimates (both the error—defined here as the ratio of the difference between an incorrect estimate and the true value, to the estimate—and the precision—the reciprocal of the error—should be examined); (5) Incomplete measurements (complete data may not be at hand to allow the determination of some specific quantity) and (6) 'Statements are often fathered on statistics which have no logical connection with them' [Bowley, 1903c, p. 311] (for instance, some might say that when it comes to exports foreign countries should not be penalised since they take so much of the exports, while others would claim that since the colonies take twice as much as foreign countries, *they* should be favoured). The rules of criticism—for example, always consider a statistical estimate in the light of similar estimates in previous years; consider the effect of replacing values by quantities—emerging from this investigation suggest, writes Bowley, 'useful tests of the truth of current arguments' [1903c, p. 312].

Following in the vein of Bowley's [1903c] Robert Palgrave[2], in 1904, read a paper entitled 'An enquiry into the economic condition of the country' at the Banker's Institute. Bowley took exception to some of Palgrave's opinions, and later in the same year he analysed the differences between his approach and Palgrave's.

Bowley begins by recalling that in his 1903 paper he had suggested that the best available tests of national prosperity were the measurements of (1) average wages, (2) average tax-paying income and (3) amount of unemployment. He now sets down as a rule that

> statistical evidence can be admitted only when it extends year by year over the *whole* period; and in every series both the fluctuations and the trend must be considered. [1904b, pp. 457-458]

In his examination of the changes in the rates of wages Bowley used index-numbers he had published often before for changes of rates of wages, taking the Wage Census of 1886, the only one available at the time. The third point, unemployment, is examined by referring to figures given by George Wood and the Board of Trade.

The matter of the second point, income, perhaps receives the greatest attention in this paper. Estimates were made with great help from the officials of Somerset House, with the addition of (1) a traditional estimate of one-sixth for trade profits escaping assessment and (2) an estimate of returns from foreign investments (suggested by Robert Giffen). Bowley warns of the danger of using income tax returns, noting 'that they are full of concealed pitfalls and that it is unsafe to travel in those regions without a guide' [1904b, p. 461].

Bowley presents a detailed table (given in abbreviated form here as Table 3A) for the years 1860 to 1901. Even a cursory examination of this table (particularly Column 4) indicates an accelerated increase during the decade 1890-1901. The percentage out of work at the end of the nineteenth century is slightly disturbing.

In Bowley's opinion the disagreement between his and Palgrave's results was caused by at least three factors. The first of these was

Table 3A. (Wages & income in £million.)

	1	2	3	4	5	6	7	Wages+income subject to tax 8	9	10	11	12
Years	Average rate of wages. Index number.	Percentage out of work.	Corrected wage. Index number.	Total paid in wages.	Total income above exemption limit.	Total income adjusted for change in exemption limit.	Income relative to populations. Index number.	Total	Index number relative to population.	Sauerbeck's index numbers of prices.	Wages+income corrected by Sauerbeck's numbers. Index number.	Consumption. Index number.
1860	60	1.9	61	300	285	290	53	590	57	141	40	67
1865	64	1.4	66	340	385	385	67	725	67	144	46	71
1870	68	2.7	69	365	465	460	76	825	73	137	53	76
1875	82	1.9	83	465	540	560	89	1025	86	137	63	84
1880	75	3.3	75	440	545	560	85	1000	80	126	64	83
1885	76	9.0	72	440	565	580	83	1015	78	103	76	86
1890	84	2.1	86	550	635	640	85	1185	87	103	84	93
1895	87	5.8	85	580	650	660	87	1240	87	89	98	96
1901	100	3.8	100	705	800	800	100	1500	100	100	100	100

the use of different estimates of income tax returns. The second factor was the fact that deposits in savings banks had not increased since 1897 as fast as they had before. The third point of difference concerned the fact that the import of raw materials, as measured by value, had dropped off.

Another point of dispute was Palgrave's contention that the proportion of the population employed in the fifteen principal trades was less in 1901 than 1851. Palgrave's remarks dealt entirely with manufactures, excluding agriculture. 'The question,' Bowley stated, 'is one of the shifting of occupations' [1904b, p. 464].

In 1908 Bowley read, before the Royal Statistical Society, a 'somewhat unusual' paper, 'for it has none of the paraphernalia of statistical tables, and it does not contribute to statistical knowledge' [1908b, p. 459]. Despite this, as a contribution to the improvement of official statistics (both with respect to what had occurred in the past and in the future[3] tense), it was not without importance.

Bowley begins by noting the increased public interest in statistics that had occurred in the preceding five years, particularly with respect to the decrease in the birth rate in the United Kingdom, national finance and foreign trade. While attention is paid here to the way official publications can satisfy the interest, Bowley opens with a discussion of the nature and conditions of social measurement under seven headings. Most official statistics, he notes, fail to meet at least one of these requirements.

The first is the *unit* of measurement. This is illustrated by referring to the Census, in which a unit would be a person who has all of the attributes (a) humanity, (b) living, (c) at midnight, March 31st 1901, G.M.T., (d) in the United Kingdom and (e) being entered on a census schedule. But even these attributes may be further divided, showing just how complex the statistical unit is.

The second heading is *homogeneity*. For instance, the addition of imports to exports results in a heterogeneous total, and still more heterogeneous is the result on division by the total population, to give the value of the amount of foreign trade per head of the population.

Universality is the next topic. The point is made here that an investigation 'must deal impartially with the whole district, the whole class, or the whole period in question' [Bowley, 1908b, p. 465]. Achievement of this is in general attempted by counting all that is practicable and ignoring the rest, and this introduces an error of unknown magnitude. Two methods are suggested for the improvement of this situation. The first is the estimation of the maximum or minimum differences that would have arisen if the missing information had been included. The second method is sampling, the essence of which is '*every unit in the district or class dealt with must have approximately the same chance of inclusion*' [Bowley, 1908b, p. 466].

The fluctuation, possibly extremely rapid and irregular, possibly of regular periodicity, calls for the consideration of *stability.* To ensure this frequent measurements may be required (in the case of rapid or irregular fluctuation), these measurements should cover an entire period (where periodicity is concerned), or only occasional measurements may be needed when the changes are slow.

The question of *comparability* is one that Bowley often mentioned. An isolated statistical total is almost useless, changes or differences being actually required. When similar totals are to be compared, the unit must remain the same and changes made infrequently but permanently.

Relativity is about 'the logical relation of two numbers which are brought together as numerator and denominator, or as factors' [Bowley, 1908b, p. 467]. This differs from comparability in being concerned with the relation of one group of phenomena to a dissimilar group (e.g. value of exports divided by population). Further, in the computation of a quotient not only should both numerator and denominator be homogeneous but 'each unit in the denominator should bear the same potential relation to the attributes of the units in the numerator' [1908b, p. 468].

Bowley's final heading concerns *accuracy.* He notes that the accuracy of official statistics is only superficial, and that their universality is limited by the collection methods.

In the third section of the paper Bowley discusses some examples of official statistics that can be faulted for being inadequate in one or other aspect. Unsuitable units, for instance, are things like the person 'engaged in the conveyance of men, goods, and messages' which includes a post-office clerk but not a postman. In the matter of homogeneity both headmasters and pupil-teachers are lumped together in the occupation census. And in the case of universality Bowley notes with wonderment the fact that in 1906 £9,179,333 worth of diamonds were exported from the Cape of Good Hope to the United Kingdom, but only £61,966 were declared to British Customs[4].

Bowley found only one reasoned statement of accuracy in all the United Kingdom Official Statistics, and that was 'the test of the significance of a difference between the rates of mortality in different professions in a recent report of the Registrar General for Scotland' [1908b, p. 475]. Nevertheless 'Every statistician knows that the true meaning of published official statistics is quite different from their face meaning' (loc. cit.)[5].

In the fourth section of the paper Bowley considers past and future improvements in official statistics. While he commends the Labour Department and the Board of Agriculture for their work, he is critical of the Census, saying that

> It is essential that no statistics should be issued without a clear statement, *bound with them*, of what they mean, how they were obtained, what are their limitations, and what cautions are necessary in using them. [Bowley, 1908b, p. 476]

On the other hand, some statistics whose publication was suppressed, because they did not satisfy the false ideal of accuracy that Bowley had described, could well be given in round numbers 'with cautions that even a newspaper editor could not ignore' [1908b, p. 476].

The Occupation Census had however shown a great improvement, although there was still room for further improvement. Compulsory powers should perhaps be exercised (for example in the taking of the Population Census) to allow the obtaining, through sampling, of unbiassed information. Much could also be done by using well-trained, intelligent, interested and well-paid enumerators, and better organisation would result in a quicker release of pertinent statistics.

The plea here is not for *more* statistics, but rather for *better presented and organised* statistics, and Bowley quotes with strong approbation the suggestion implicit in Julius Mandello's 1905 paper[6] to the effect that statisticians should 'think more and publish less'.

Bowley concludes his 1908 paper by urging, once again, the establishment of a 'Central Thinking Office of Statistics[7]', and he discusses a number of things such an office should do: e.g. (1) take cognizance of all published statistics and not just those of departmental significance, and (2) edit, where necessary, statistical publications, providing careful definitions, analysis and criticism. This could best be accomplished, Bowley notes, by establishing a new office rather than trying to expand an office already in existence, and he hoped that Parliament would take steps to carry out any recommendations that the Royal Statistical Society might make.

In the discussion following the reading of this paper the President of the Society, Charles Dilke, added his support to Bowley's views on official statistics by saying that

> This so-called census [1886 Wage Census], quoted by a Minister in the House of Commons, was a document not only misleading, but of such a nature that it was impossible to read a single page of it without seeing that mere guess work would lead to better results; it was absolutely delusive in the highest degree. [Bowley, 1908b, p. 481]

The word 'girls', for instance, was undefined, and it in fact meant different things in every trade and industry—nor was any distinct age limit set between 'girls' and 'women'.

Chiozza Money, while beginning his comments somewhat coolly (in the same sentence he found the paper 'a most valuable contribution' and expressed some disappointment with the treatment) soon became more critical, finding that Bowley's methods were destructive rather than constructive. Bowley had to a large degree discussed *methods* of statistics, rather than the improvement of official statistics. As a newspaper editor[8] (he sarcastically spoke of them as 'those despised persons') Money had himself been responsible for the improvement, noted by Bowley, in the Board of Trade Returns since 1901. Money also objected to Bowley's complaint that members of parliament asked too many questions, and pointed out that their motives in doing so were twofold: the first was 'the amiable desire to advertise the Member who asked the question' [Bowley, 1908b, p. 483], and the other to direct the attention of parliament and the public to something the member knew very well. In Money's opinion these were perfectly legitimate motives. He concluded by hoping that Bowley would continue his work in a more constructive manner.

Timothy A. Coghlan had reservations as to the effect Bowley's paper would have in official statistics circles themselves, and somewhat pessimistically he said 'it must be a very sanguine person who would suppose that Mr. Bowley's paper would have any material effect on the compilation and presentation of official statistics' [Bowley, 1908b, p. 484]. Coghlan further bemoaned the fact that the Census Report did not allow the determination of exact ages, and that in the case of occupations dealer and maker were mixed up: for instance, a seller of American boots in London and a Northampton boot-maker were treated as if their occupations were the same, or at least similar.

Exception was also taken to Bowley's advocacy of round numbers, Coghlan believing that the statistician's job was to present the figures just as they were gathered. 'Until evidence was produced,' he said, 'it was not justifiable for anyone to alter the figures in an arbitrary manner for the sake of appearance' [Bowley, 1908b, p. 485].

George Udny Yule agreed that Bowley had not strictly stuck to his topic, and took exception to Bowley's insistence on *random* samples, saying that the taking of such was almost impossible in practice and, more conservatively, that he himself 'could never trust a sample as representing a universal investigation' [Bowley, 1908b, p. 488].

In his oral reply Bowley declared that the 1886 Wage Census could be useful if interpreted with care. While he agreed with Coghlan about the usefulness of inaccurate statistics, but pleaded for the recognition of a measure of their inaccuracy, he disagreed entirely with him about the collection of statistics for comparison with the past. For such comparison it was essential that the present statistics be compiled as they had been in the past.

In 1921 Bowley discussed the possibility of organising public statistics[9] and 'the chaotic state of information on many of the most pressing world problems of the day' [1921a, p. 303]. He noted that a petition had been presented to the Government from the Royal Statistical Society in 1919 'asking for a public enquiry into the existing methods of the collection and presentation of public statistics' [1921a, p. 302]. The petition was referred to an official committee, the members of which (many of whom were heads of Government Departments) seemed to view the petition as a personal attack. The petitioners' remark that 'official statistics, as at present collected and published, are inadequate in arrangement and scope for the purposes they should serve' [Bowley, 1921a, p. 302] was met by the committee's response that it had never been accepted in the United Kingdom that the Government had responsibility for statistics covering the whole life and activities of its people and that the national administration would therefore not discharge any such obligation.

The committee's *Report on the Collection and Presentation of Official Statistics* of 1921 proceeded to show that

> the establishment of a Central Statistical Office is impracticable, and finally recommends the setting up of a permanent Consultative Committee of statistical officers for the purpose of ensuring more effective co-operation and co-ordination between the different departments.
> [Bowley, 1921a, p. 302]

Noble though that may sound, it transpired later that any one with a comment or query should, as in the past, approach the relevant

Government Department and not the Committee. Not only was the Department under no obligation to pay any heed to such comment or query, but it was under no obligation to present such comment or query to the Committee. 'Here's a how-de-do!', to quote W.S. Gilbert[10].

The first report of the Permanent Consultative Committee was issued as the *Guide to Current Official Statistics* in 1922. In his review of the report Bowley [1923h] noted that in all seventy-six different Departments, Boards, Committees, etc. were listed there as issuing statistical publications. He commented too on the usefulness of the Guide when used in conjunction with a good library, and noted that the Index, welcome though it was, presented certain problems: thus to find out how many sailors there were one had to follow a chain from Shipping to Employment, there to find a heading Shipping and a sub-heading Engagement of Seamen and Persons.

In the summer of 1919 the Council of the Royal Statistical Society appointed a committee, with Bowley as chairman, to suggest improvements in the Census of 1921. While the committee acknowledged with gratification that many of the recommendations of the similar committee appointed in 1908 had been carried out in the Census of 1911, there was still no uniformity between the censuses of England and Wales, Scotland, and Ireland.

A number of recommendations were submitted to the Council by the Committee [Anon., 1920], including the following: (1) the schedule should be completed by special enumerators rather than the householders themselves, (2) a distinction should be made between the *de facto* and the *de jure* population (e.g. visitors should specify their usual place of residence), (3) there should be no questions about infirmities, (4) date-of-birth should be requested rather than age at last birthday and (5) more details of the manual working class should be obtained (e.g. workers or dependants). There were three General Recommendations: a quinquennial census should be taken; a common census for the whole of the United Kingdom was required; and there should be an Imperial Census.

3.3 *Statistical Studies*

In 1904 Bowley published his *Statistical Studies relating to National Progress in Wealth and Trade since 1882*, much of the material in which had been used in his LSE lectures. This book—or 'pamphlet' as Bowley called it—has an Introduction, three chapters headed 'The progress of the nation', 'Production, trade and commerce' and 'The progress of foreign nations' and a conclusion.

Bowley begins by writing that, in addition to putting some of the main statistics of the past twenty years into a 'simple and intelligible form' his aim here is

> to establish the following statements:—(i.) *Our information is not sufficient to allow us to form an absolutely certain judgment as to our recent progress*; but (ii.) *Our information, so far as it goes, suggests that very remarkable and stable progress has been made in recent years in those aspects of national well-being which are generally considered in measuring prosperity.* [1904a, p. ix]

Five fundamental rules of statistical investigation are stressed:

> (a) Statistics must cover the whole field of enquiry ... (b) All statistics used must be closely related to the quantity whose change we wish to examine ... (c) When we are dealing with a quantity which fluctuates year by year, no judgment can be formed without examination of the records of a long series of years ... (d) The purchasing power of gold is continually changing, and it is always necessary to examine statistics relating to the value of goods to tell whether an argument will be affected if we express the facts in quantities instead of in value ... (e) Before the change in a total is used as significant, it must be considered whether the various groups which compose it are of the same nature, or whether a change in one part has masked a change in the opposite direction in the other. [1904a, pp. x-xii]

The first section of Chapter I, 'The shifting of occupation', is compiled from Census Reports for England and Wales in 1881, 1891 and 1901. Here figures are presented of the number of youths (15 to 20 years of age) and men (over 20) in various occupations, the separation of figures for youths and men allowing an investigation of whether changes are due to a change in the available supply of youths or to a change in adults' occupations.

In his table, an abbreviated version of which is given in our Table 3B, Bowley lists 36 occupations, 'so far as the heart-breaking difficulties of the census tabulation seem to allow them to be grouped' [1904a, p. 3]. Indeed, the information that can be drawn from Census data is so scanty that one cannot be sure of the broad changes in occupation, to say nothing of detailed changes[11].

The next section, 'The progress of wages', uses figures provided by the Labour Department, the Board of Trade, the Fiscal Blue-Book, some of Bowley's papers and his *Wages in the United Kingdom*.

The actual wages are shown in Table 3C. Bowley bemoans the fact that little is known of the wages of railway workers[12], that there is no information about the movement from one grade of skill to another (say in the engineering trades) and that many workmen in fact shift from one trade to another depending on work and money.

There then follows a short section on changes in national income. This income he divides into four groups: (1) the part received as wages, (2) the part under the review of the Inland Revenue Department, (3) money received as salaries, but too small to be taxed and (4) profits from investments abroad and businesses at home that may (illegally) escape the tax-collector. Comparison of the period 1898-1902 with 1883-1887 suggests that 'the total income of the nation has increased not less than 38 per cent., the population about 15 per cent., the average income per head not less than 20 per cent' [Bowley, 1904a, p. 16].

Bowley's interest in unemployment and poverty is brought to the fore in the next section. He notes that from 1884 to 1903 there were

Table 3B. Shifting of occupation.

Occupations	Numbers employed (000's omitted)					
	1881		1891		1901	
	Age 15-20	Age over 20	Age 15-20	Age over 20	Age 15-20	Age over 20
Agriculture & gardens	202	1,033	193	986	168	960
Fishermen	5	24	3	22	2	22
Sailors & watermen	15	120	17	122	12	115
Builders	90	584	75	612	129	802
Furniture & woodwork	19	133	23	146	36	192
Road labourers	1	14	1	20	2	48
In coal mines	65	291	95	389	102	312
Textiles	75	274	75	291	57	294
Tailors	12	94	17	100	17	118
Boots	22	161	29	164	26	165
Vehicles	9	53	13	65	20	85
Shipbuilding	6	47	9	60	13	72
Police	0	33	0	40	0	45
Post & messages	7	20	12	29	16	47
Commercial	52	250	69	318	88	432
Professional		229		265		312

Table 3C. The increase of wages.

	Average weekly wages								Numbers to which the Rates of Wages may be estimated to apply		
	1881		1886		1891		1902		1881	1891	1901
	s.	d.	s.	d.	s.	d.	s.	d.			
Agriculture	15	0	—		15	0	16	6	103	99	96
Building	27	0	27	0	28	0	31	0	58	61	80
Cotton	24	9	25	3	26	0	27	3	15	16	17
Wool	24	3	23	3	23	3	23	3	8	8	8
Machinery	25	6	25	9	26	9	28	6	23	28	44
Shipbuilding	—		29	3	33	6	36	6	5	6	7
Coal	23	0	23	0	30	0	33	6	29	39	51
Sailors	25	0	25	0	31	0	30	0	12	12	11
Printers	31	0	—		32	0	33	0	4	5	7

periods of bad employment and periods of good, and, relying on figures from Trade Unions, he concludes that, at least as far as members of such unions were concerned, ‘the circumstances of trade in the six years ending with 1901 were not unfavourable to regularity of work’ [1904a, p. 20].

Bowley was less enthusiastic about the reports on pauperism, finding that the figures available were essentially useless in point of accuracy, for the following reasons: (1) the frequent changes in the system of relief adopted, and (2) the lack of means to distinguish between the able-bodied who were capable of work but could not or would not get it, and the *so-called* ‘able-bodied’, who were incapacitated by reason of temporary accident or illness (the latter *could* not have worked at the time relief was provided).

Bowley’s investigations showed that in the year ending 25th March 1892 about 1 in 30 males of working age received some sort of relief, vagrants and lunatics excluded. Things seem to have improved by 1900, for Bowley deduced that in that year only 11 or 12 truly able-bodied men received relief per 10,000 males over 15 years. The statistics available showed a rapid decrease in the number of female paupers from 1884 to 1903, though those for the so-called able-bodied men (including vagrants) showed an increase nearly as fast as the increase in population (there was however an improvement in the period 1899-1903).

There was clearly considerable (Bowley’s own phrase is ‘an appalling amount of’) poverty in cities, and while he commends Booth’s and Rowntree’s investigations of London and York, Bowley concludes that one can only say that ‘we have no *comparative* statement whatever’ [1904a, p. 25].

Section 5, compiled using Board of Trade figures, is devoted to the change of prices. Considering a budget ‘of the goods most usually purchased by the working-classes’ [Bowley, 1904a, p. 25], Bowley deduces that in the period from 1883 to 1903 the purchasing power of money had increased about 8%, or, equivalently, that prices had fallen by the same amount. The forty items entering into this budget included bread, meat, sugar, tea, oil, coal, beer and tobacco, while items such as rent and 'the innumerable small and rapidly-cheapening articles of modern manufacture ... and furniture, bicycles, travelling, newspapers, ... ' [1904a, p. 26] were omitted. Bowley suggests that for the artisan the increase in rent is probably counterbalanced by the decrease in the price of items in the omitted group. This was likely not to be the case for the urban poor, who might well find rent to be an increasing burden.

'Consumption of necessaries and common luxuries' is the title of the next section. Records were sufficient to allow investigation of the consumption of the following: wheat and wheat flour, meat, sugar, tea, coffee, cocoa, rice, raisins and currants, tobacco, beer, spirits and wine. On averaging the appropriate figures Bowley concludes that the consumption per head of these consumables in the United Kingdom had increased by 20% comparing the period 1898-1902 with the base period 1883-1887. The fall in the price of bread had not produced increased consumption, a larger proportion of the working class could get more meat, and there was more money available for buying beer and tobacco. However it was not possible to say whether the increase was uniformly distributed over the whole nation.

Again drawing on data from the Labour Department Bowley next investigates the topic of savings. The figures available, drawn from trade unions, workmen's co-operative societies, principal friendly societies, the post office savings bank, trustees' savings banks and incorporated building societies, indicated that there had been a great and continuous increase in savings nationwide.

In his summarising section Bowley deduces from Table 3D that

> all [these estimates] show considerable progress when the second period is compared with the first, slackening or retrogression between the second and third periods, and great progress (except that prices rose a little) from the third to the most recent period. [1904a, p. 32]

Similarly, Bowley's study of real wages (i.e. wages expressed in

Table 3D. Estimates of price & other indexes.

	1883-87	1888-92	1893-97	1898-1902
Average money wages	100	110	115	130
Average prices	100	95	90	92
Average money income per head of the population	100	113	108	120
Consumption of commodities	100	108	112	120
Percentage out of work	7.2	3.8	5.4	3.5
Number of adult male paupers per 1,000 adult males	35	33	38	31
Number of adult female paupers per 1,000 adult females	42	36	35	29

terms of what can be bought) from 1830 indicates

> that the rate of increase in the last twenty years has been greater than in any previous period of equal length; and that the progress in the last decade has only been equalled in that immediately preceding. [1904a, p. 33]

In view of the fact that much information was lacking (e.g. what changes had there been in the numbers of men who did not have regular work, what were the wages of men in unorganised trades or agriculture), Bowley found it conceivable that these missing data might lead to a reversal of his conclusions, but the burden of proof of this was to be placed on the backs of those who acted on such an assumption.

The first section of Chapter II is concerned with the consumption of raw materials in manufacture, and is drawn up using the Statistical Abstracts and the Fiscal Blue Book. Comparing the period 1898-1902 with 1883-1887 Bowley finds that Britain had used more cotton (15%), more wool (40%), more iron (18%), more lead (60%), more zinc (80%), more leather (85%), more imported wood and timber (60%), more coal (28%), the same amount of tin, and 25% less silk in the latter period (viz. 1898-1902) than the former, a period during which the population had grown by 14%.

'Production for the home market, and for the foreign market' is the subject of the second section of this chapter. Bowley finds the available information to be less than adequate, and concludes that 'no general conclusions can be drawn from such incomplete statistics' [1904a, p. 42]. The only way of judging Britain's productive power is by her consumption of raw materials, a matter already discussed.

The main changes in the character of exports are discussed in the fourth section (not the *causes* of such changes, but only the quantities and values of the exports themselves). Bowley claims that 'It is not possible to base general arguments on periods shorter than a decade, because of the fluctuating character of the statistics and especially of prices' [1904a, p. 43].

Extensive lists are provided of manufactured products whose exports have (1) increased perceptibly in twenty years, (2) remained almost stationary in value and (3) decreased perceptibly in value. It is noted that

> It is clearly quite impossible to base on these facts any general statement that our exports tend to be the product of low-paid and unskilled labour, or the reverse. Our exports have consisted and do consist of a great miscellany of goods of every description. The guarantee of their continuance is their variety. [1904a, p. 45]

Specific attention is paid to the exports of cotton, woollen and metal goods in turn, these three accounting for 57% of the whole of the exports of 1903.

Section 4 is devoted to imports and foreign competition in the home market. Bowley notes the difficulty of separating imports into categories of raw materials, food, and manufactured goods, or of separating imported goods that compete with home-made products from others. Yet there are commodities that may, 'with sufficient accuracy', be labelled 'raw material' or 'finished products'.

Table 3E. Imports of finished manufactures.

Quinquennial Averages (000,000's omitted)					(000,000's omitted)
Imports of	1883-87	1888-92	1893-97	1898-1902	1903
Cotton goods	£1.8	2.1	2.7	4.3	5.1
Woollen goods	6.8	8.5	9.5	9.1	8.1
Silk goods	10.0	10.4	14.0	13.9	12.4
Iron & steel, wrought or manufactured	1.9	2.5	3.3	7.1	8.2

Among imports are grain and flour (competing with home products), tea, coffee, tobacco, wine, cotton, leather, unwrought iron and steel, oils, textile manufactures and chemical products. Our Table 3E shows the import details for finished manufactures.

When it comes to foreign competition in the home market Bowley begins by saying 'There are no means of measuring the stress of competition, and statistics for the most part only add confusion to the problem' [1904a, p. 57]. On the one hand there is the gain to the consumer and the manufacturer who uses foreign goods, and on the other there is the loss to producers who have to turn from one occupation to another. Bowley's investigations lead him to conclude that the value of imported items is not necessarily related to the gain or loss of any person or community.

Section 5, 'The balance of imports and exports', begins with an almost obvious statement: 'Imports of bullion and merchandise must be paid for by present, past, or future services' [1904a, p. 61]. The amount of money accounted for by shipping and the capital and interest account had almost doubled in the period under consideration, from £95,000,000 in 1883-87 to £178,000,000 in 1898-1902. Bowley notes the difficulty of getting any reliable estimate of capital passing out of the country.

The next section is concerned with the relative changes of prices

Table 3F. Relative changes of prices of imports and exports.

	At the prices of		
	1893-97	1888-92	1883-87
	£	£	£
A budget of goods in general would have cost*	91	103	106
Exports would have sold for*	95	101	102
Imports would have cost*	97	111	116

*At a price of £100 in 1898-1902

Table 3G. Values that imports & exports would have had if 1883-87 price levels had been maintained.

	Quinquennial averages (000,000's omitted)			
	1883-87	1888-92	1893-97	1898-1902
Exports	222	243	246	270
Imports	317	377	443	519

of imports and exports. From Table 3F it may be deduced that while exports fetched approximately the same price throughout the twenty-year period considered, imports were some one-seventh cheaper at the end than at the beginning.

If *quantities* (using index numbers) rather than *values* are considered, it appears that both imports and exports have increased almost without interruption (see Table 3G).

Section 7 is concerned with the study of trade with the Empire and with foreign countries. Since exports shipped to Country A may very well have another country as their final destination, and since imports into Britain from Country B may well have originated in some other country, it is difficult to deal adequately with the distribution of British exports and the source of her imports.

Table 3H. Exports of British manufactures.

	Quinquennial averages (000,000's omitted)		
	1890-92	1893-97	1898-1902
	£	£	£
To Germany, Belgium & Holland	30	32	34
To France & Italy	16	13.4	16
To U.S.A.	26	20	16

Table 3I. Value of imports received by the United Kingdom.

	Quinquennial averages (000,000's omitted)			
	1883-87	1888-92	1893-97	1898-1902
	£	£	£	£
From British Possessions	89	95	94	106
From Foreign Countries	311	324	331	400

However Bowley manages to produce some figures relating to this question, showing, among other things, that the British Empire exported as much to Germany, Holland and Belgium as it imported from them, and that imports from Canada were greater than exports to that country, while the reverse was true in the case of India and Australia. The exports and imports from and to Britain are shown in Tables 3H and 3I.

This chapter is concluded as follows:

> The total output of our manufactures has increased greatly in recent years; the part exported to foreign nations has increased a little, that sent to colonies considerably, but that retained for home consumption most. [1904a, p. 75]

Bowley begins Chapter III, 'The progress of foreign nations', by noting that little accessible information is available about the

Table 3J. Estimation of population and consumption of coal.

		(000,000's omitted)			
		U.K.	U.S.A.	Germany	France
Coal consumed:	1883	134	102	49	31
million tons	1900	167	235	99	36
Population	1883	35	54	38	46
	1900	41	75	39	56

progress of foreign countries apart from the United States of America, France, Belgium and Germany—and what is available for these named countries is scanty. While some estimates of wages and prices, some statistics of the consumption of a few raw materials and for production of external markets exist, there are no comparative statistics[13]

> of incomes, pauperism, employment, consumption of ordinary necessaries, consumption of many important raw materials in manufacture, or production for the home market, for any foreign nation whatever. [1904a, p. 77]

Thus 'Under these circumstances it is quite futile to attempt any scientific measurement of foreign progress' [1904a, p. 77].

Some attention is also paid to the question of production and export of the countries named, and here things are a little better. It is clear, for instance, that Germany and the United States both increased the amount of coal and iron used in manufacture and also produced more textiles. See, as an illustration, the data pertaining to coal in Table 3J.

Bowley begins his concluding chapter as follows:

> It must be admitted that our statistics and those of most other nations are insufficient to bear the strain of any general enquiry. It is true that in many of the points,

> where exact knowledge halts most, opinions could be given by trained and experienced observers, which in ordinary times would carry conviction; but in the present turmoil of argument and misuse of imperfect statistical data, every statement is rightly challenged, and no personal authority accepted till the evidence is unimpeachable. [1904a, p. 84.]

If accurate and urgent data are needed Bowley suggests the appointment of a Royal Commission, which could probably supply results in a few months time. If such haste is not deemed necessary he suggests that longer statistical studies, which could perhaps be concluded in a year, be undertaken into Britain's economic system.

The pamphlet is concluded with a plea Bowley was to reiterate time and again:

> we need men whose business it is to know the facts, digest them, and explain them to the public, and whose time is not occupied with serving tables, and initialing memoranda. [1904a, p. 88]

In conclusion let us note the appreciation for this work expresseed by Edgeworth in his review:

> Mr. Bowley's work belongs to a small class . . . characterised by impartial statements, which command the respect of disputants on either side of a heated economic controversy. The modesty of the true statistician contrasts favourably with the hectoring tone of political partisans. [1904, pp. 268-9]

3.4 *Official Statistics*

In 1921 Bowley published another 'pamphlet', entitled *Official Statistics: what they contain and how to use them.* Here he addressed a number of the matters considered in the *Statistical Studies*, and added further data that had been obtained since its publication[14].

In the Introduction Bowley outlined Government's needs for the performance of its duties (things like the size and local distribution of the population, the national income and taxable capacity), and distinguished between *national* and *official* statistics:

> The measurable aspects of these subjects [health, social conditions, etc.] form the material of national statistics, which are called official when the information is collected and tabulated by Government departments. [1921b, p. 7]

As relatively suitable Abstracts of public statistics Bowley recommended the *Annual Abstract of Statistics of the United Kingdom* and the *Annual Abstract of Labour Statistics*[15]. However he made the important point that

> The first thing to realise about official, and indeed all, statistics, is that their meaning is always technical and generally not precisely that which might at first be expected. [1921b, p. 9]

Very often the meaning of the words used in official reports would be known only to the officials themselves, the method of computation could well have been changed from the time of the previous report, or estimates might have been made by methods that were not to be divulged. It was thus expedient to use a number of different sources in order to get a comprehensive view of a topic. Further,

> No one should attempt to use statistics unless he is prepared to devote considerable time and thought to ascertain the exact meaning, nature and limitations of the particular reports which relate to the subject in question. [1921b, p. 13]

The first chapter, 'Population', is begun with a general discussion of the decennial Census, the next of which was due in April 1921 (details in Bowley's book were essentially from the 1911 Census). The unit of the Census was the Civil Parish, the area and population of each being aggregated into various Urban and Rural Districts.

In the first section of this chapter Bowley follows the information for two small towns in Leicestershire—Hinckley (one Civil Parish) and Coalville (three Civil Parishes). He gives details from the two-volume official Report of area and population, age and marital status, buildings and tenements, numbers of families of various sizes and age in relation to occupation.

The second section is devoted to an examination of occupations in general. Some 20,000 different occupations were catalogued and classified under 23 Orders, 79 Sub-Orders and 475 Headings. There is also an 'industrial' classification, 'in which persons are credited according to the industry or service in which they are employed, whether their actual occupation is peculiar to that industry or not' [1921b, p. 21]. Bowley notes that this point is of particular importance in connexion with Government Service. Here the *occupational* listing showed 290,000 people employed by Central or Local Governments in England and Wales, whereas the classification by *service* listed 838,000. Bowley also examines results from the Census of Production of 1907 (showing the numbers of people employed in selected weeks) and the Home Office returns for Textile Factories, and he finds that the totals are roughly the same.

The third section of Chapter I is concerned with births, marriages and deaths. A distinction is made between the 'crude' death-rate (based on the total number of deaths and the entire estimated population) and the 'corrected' death-rate (found by applying the ascertained or estimated death-rates age-group by age-group to the age distribution of England and Wales at the 1911 Census).

In the fourth and final section Bowley discusses migration. Here information obtained from the Masters of ships carrying passengers to or from the United Kingdom is used, though in the case of travellers to non-European countries the country listed is that of disembarkation rather than final destination.

Table 3K. Value of production and services in the United Kingdom.

Product of industry, mining and agriculture	£ 940 Mn.
Carriage, merchanting and retailing home and imported goods	570
Custom and excise duties	75
Services, professional incomes, etc.	230
Income from ownership of houses, etc.	150
Income from abroad	235
Total	£ 2,200
Less depreciation of buildings, plant, etc.	190
Total income in 1907	£ 2,010 Mn.

Chapter II is devoted to industry, trade and prices. In the first section, 'Production', Bowley notes the impossibility of getting a common unit of production for the whole nation (desirable though this would be)

> for there is no common unit by which the result of the efforts of miners, shipbuilders, railwaymen and others can be measured except the unit of value of the goods produced, and the statement of the total value involves the changing factor of price as well as that of quantity.
> [1921b, p. 31]

Bowley especially commends the 1907 Census of Production for the calculation of 'a very interesting and difficult estimate' [1921b, p. 32], that is, the relation of the value of material products to the total income of the inhabitants of the United Kingdom (see Table 3K). When it comes to consideration of Consumption in the second section there is again a problem, the necessary statistics being unavailable because of the lack of adequate machinery for their collection. However Bowley does manage to give figures for the consumption of butter, margarine, cheese, eggs, wheat, rice, meat, sugar, tea, coffee, cocoa, currants & raisins, wine, spirits, beer and tobacco for home and imported goods. 'These should be multiplied by $4\frac{1}{2}$ to give the annual consumption for an average household' [1921b, p. 34].

Foreign trade forms the subject of Section 3. As an example Bowley considers the statistics of cotton in 1907, detailing both imports and exports, quantity and value, for raw cotton (imports, re-exports) and manufactures (imports of waste from worked cotton, from yarn and from piece goods, re-exports of waste and piece goods and exports of waste, yarn and piece goods).

A fairly short section on transport then follows. The figures were compiled for a number of companies operating in the United Kingdom for the four weeks ending 20th June 1920. Bowley gives the figures for tons carried and ton-miles for General Merchandise, for coal etc. and for other minerals. Details are also provided of freight train loads, and of the miles travelled and the average load per wagon. Some statistics of shipping are also discussed.

The last section of the second chapter is 'Wholesale prices'. Bowley gives the wholesale price (British Gazette Average and imported), the quantities (United Kingdom harvest and imported) and the population of the United Kingdom in ten-year intervals from 1871 to 1911. He points out that care should be taken in regarding the wholesale price as anything more than an indicator of the retail price, since things such as the cost of manufacture and distribution should be considered in the determination of the latter.

'Income and wages' is the title of Chapter III. The first section is concerned with income and capital. Using income-tax reports for 1913-14 Bowley gives figures for gross income and taxable income.

Bowley was clearly not greatly enamoured of the Inland Revenue Commissioners' report. He writes 'there are few bodies of statistics so difficult to use or so liable to suggest erroneous inferences' [1921b, p. 43], and further 'The reports do not show the number of tax-payers ... and in very many cases the same individual is assessed more than once' [1921b, p. 45].

'Wages, earnings and hours' are presented in Section 2. Bowley, it would appear, does not hold out much hope of doing anything about wages:

> There are many publications dealing with wages, but they are so imperfectly summarised, especially during and since the war, as to make it very difficult to follow either the wages in a particular occupation or the general movement of the average of wages or earnings.
> [1921b, p. 46]

Section 3, 'Working-class budgets', allows a more explicit and satisfactory investigation. In view of Bowley's evidence before the Dock Labour Inquiry of 1920 on a suitable budget for food we present here Tables 3L(a) and (b). Comparison of these figures with those given elsewhere by Bowley for 1912 shows that before the war a considerable amount of meat was not accounted for (partly because sausages, offal and tinned meat are not included in Tables 3L(a) and (b)). A further disparity is that the amount of sugar in the 1912 statistics included that used in the making of jam and confectionery.

The final section is concerned with prices and cost of living. While, as might be expected, the cost of rent and coal varies from city to city, there are also remarkable differences between prices of food (for instance, a four-pound loaf of bread cost 5d. in Manchester, 6d. in Liverpool, and 6d. to 7d. in Newcastle). However the retail price of food as a whole did not vary much from place to place. Adequate information on prices of clothes and boots is difficult to obtain, though fairly adequate information may be found for rent and fuel[16].

The last chapter in the book is devoted to social conditions, beginning with unemployment. Figures are obtained from the *Labour Gazette*[17], published monthly by the Labour Department. In September 1920 the Union membership was 1,670,000, of whom over 600,000 were in the Engineering and Shipbuilding Trades.

Also useful were records arising from the operation of the Insurance Act, which gave details of the numbers of men and women

Table 3L(a). Weekly budget of a standard family (Principal foods only).

	1914	1918
Bread & flour	33.5 lb.	34.5 lb.
Rice, tapioca, oatmeal	2.7 lb.	2.7 lb.
Meat & bacon	8.0 lb.	7.0 lb.
Lard, suet, butter, margarine	3.1 lb.	2.5 lb.
Fresh milk	9.2 pints	11.7 pints
Cheese	0.84 lb.	0.41 lb.
Sugar	5.9 lb.	2.8 lb.
Eggs	13 no.	9 no.
Tea	0.68 lb.	0.57 lb.

Table 3L(b). General average of weekly expenditure of families who made returns of food consumption.

	1914		1918	
	s.	d.	s.	d.
Food	24	11	47	3
Sundries	1	2	2	6
Fuel and light	2	4	4	2
Rent	6	7	6	9
Fares		10	1	0
Insurance	3	0	3	0
Clothing	5	6	10	9

insured. Details of the number of unemployed were provided by the Labour Exchanges: here again the Engineering and Shipbuilding industries dominate. The percentages of unemployed obtained from the Insurance records and those given by the Trade Unions were very different.

> It should be remembered that the trades coming under the Insurance Acts are in some measure those where unemployment is prevalent, while the trade union returns exclude occupations where the burden of unemployment is too considerable for their funds. [1921b, p. 54]

Nevertheless Bowley concludes from the two reports that 'at least 106,000 men belonging to unions or insured were professedly willing and unable to get work in August, 1920' [1921b, p. 55].

The second section of this chapter is concerned with old age pensions, the official account of which was in the Reports of the Commission of Customs and Excise(!), and summarised in the Abstract of Labour Statistics. Bowley presents a summary table for the old age pensioners on the 28th March 1919. Figures are given, with various subdivisions, for England, Wales, Scotland and Ireland, pensioners (70 years of age or older) being counted on a particular day on which pensions were drawn. Each pensioner, some 920,000 in the United Kingdom in all, received on average 4s. 11d. per week, or, with additional allowances, 7s. 5d. Bowley noted that perhaps as many as 50,000 people over 70 were in Poor Law Institutions, most of them receiving no pensions at all.

Pauperism forms the subject of the third section. Information is mainly supplied by the count made in all institutions each year on the 1st of January and the 1st of June. There is however no record of whether the relief be indoor or outdoor, nor of the ages of those between 16 and 70 who received such relief, and the classification into ill and well is done in many apparently contradictory ways. Further, there is little if any information about those in hospitals, orphanages, schools and almshouses supported by private subscription or charity.

Nevertheless Bowley manages to provide some figures of men, women and children under the age of 16 who received either indoor or outdoor relief (and in 140 cases apparently both simultaneously!) together with some details of their health (well, insane, etc.) and the reason for such relief (e.g. personal sickness or accident). Statistics are also provided of the 68,039 indoor children, not insane, who were in various institutions (workhouses, infirmaries, group cottages, training ships etc.) on the 1st of January 1914. The Report from which this latter set of figures was extracted seemed once again to contain inconsistencies, and, to make matters worse, it was 'full of technical terms that are no doubt clear to the officials but whose different shades of meaning elude the uninitiated' [1921b, p. 61].

This chapter—and therewith the book—is concluded with a miscellaneous section. Here Bowley mentions those people who have made provision for themselves through Friendly Societies, Co-operative Societies and Trade Unions. For instance, in 1911 Trade Union membership in the United Kingdom was recorded as 3,010,954, that of Friendly Societies was nearly 4,000,000 and that of co-operative, distributive and productive societies 2,778,000.

In the field of education Bowley relied on the reports of the Board of Education for England and Wales and of the similar authorities for Scotland and Ireland. Unfortunately, 'The statistics are not very useful except for administrative purposes and for the study of special questions' [1921b, p. 62]. Similarly the data for sickness and health were uncertain, since there seemed to be no general statistics on hospitals and no report on the effects of the National Insurance Act. Finally, 'The only other important official accounts bearing on general social questions are those of crime and convictions, which need expert study before use' [1921b, p. 63].

3.5 National Income

The subject of the national income was one to which Bowley paid no little attention, writing some dozen books and papers on the topic, in addition to reviewing a number of books by others[18]. Three of his books, *The Division of the Product of Industry; an analysis of* *national income before the war*, [Bowley, 1919a], *The Change in the Distribution of the National Income, 1880-1913*, [1920f] and *The National Income, 1924; a comparative study of the income of the United Kingdom in 1911 and 1924* [Bowley & Stamp, 1927] were reprinted together as *Three Studies on the National Income* in 1938. A further book edited by Bowley, *Studies in the National Income, 1924-1938*, appeared in 1942.

Bowley begins *The Division of the Product of Industry* with a clear statement of his purpose in undertaking this work:

> to examine closely the statistics on which they [i.e. apparently inconsistent estimates] are based, and to restate them in such a way as to show the amount and the origins of the aggregate incomes of the people of the United Kingdom and the proportions of the aggregate that go to various economic classes. [1919a, p. 5]

The work is set out in five sections. In the first, the population itself is analysed, while in the second the incomes are aggregated and the aggregate examined. The national wages bill is computed in the third section, the fourth containing the decomposition of the product of the main industries between 'wage-earners, salaried employees, employers and property-owners' [1919a, p. 6]. The investigation is then summarised in the last section. Bowley concludes with the perhaps obvious statement, but nevertheless one that is worth stressing,

> the most important task ... incumbent on employers and workmen alike, is to increase the national product, and that without sacrificing leisure and the amenities of life. [1919a, p. 58]

In his discursive review of this work Cannan makes the delightful observation that Bowley's conclusions 'appear to be eminently sound, though they have excited some indignation in minds of undue optimism which find the atmosphere under his wet blanket somewhat suffocating' [1919, p. 207]. Cannan also comments on the amusement felt by the bourgeois economist when his charwoman says that the Great War produced many happy homes, but he wonders whether the economist will also laugh when the charwoman's son, returning from the war, expects to be financially supported by the government as he had been while away in the trenches.

Willford King, in his review, finds Bowley's investigation of the National Income incidental to the main aim of the book, i.e., 'apparently to answer the queries of socialists and others as to how much income might safely or unsafely be diverted from the existing share of the rich and added to the wages of the laboring class' [1919, p. 618]. Bowley, King further claims, 'considers what might be done under the existence of the present competitive régime and also what might be accomplished if a socialistic state were substituted for the present order' (loc. cit.) and he concludes that

> This little book in short makes the fact clear that the most fundamental reason that the British people, on the whole, are far from affluence is due to failure to produce a sufficient quantity of goods per capita rather than to a failure to distribute the goods produced in the best possible way. Although the author does not make the point, he has nevertheless forged another link in the growing chain of evidence that poverty is a problem of eugenics and population rather than the distribution of the products of national effort. [King, 1919, p. 620]

Not all commentators were as approving of *The Division of the Product of Industry* as were Cannan and King. The anonymous writer of 'Notes of the week' in *The New Age: a weekly review of politics, literature, and art* (Vol. 24, No. 20, 20th March 1919) begins his comments with the words

> It was said of somebody that when he was original he was wrong. In the case of professors as a class, we can usually say that when they defend the governing classes they are suborned. [p. 319]

'The case of Professor Bowley, the eminent statistician, is really tragic' (loc. cit.) The tragedy was allegedly caused by the fact that Bowley appeared to think that his figures were more important than the facts: for instance, it emerged from his research that workers received nearer two-thirds than one-third of the product of their industry: 'it is a matter of indifference to us, as moral economists, whether the amount received by Labour is two-thirds or four-fifths. Anything less than the whole is unjust' [p. 319].

Written after the First World War, Bowley's *The Change in the Distribution of the National Income, 1890-1913*, a brochure of 27 pages, was a study of the changes that took place in the period before the war and that resulted in the distribution of income examined in his *Division of the Product of Industry.* The study starts about 1880, by which time the the depression of 1878-9 was past, the inflation of prices in the early '70s had disappeared, and when 'the price-level differed very little from that of 1913' [Bowley, 1920f, p. 6].

The distribution of incomes could be considered from three different points of view: (1) economic categories (the aggregate of income from land ownership, from enterprise and management), (2) the rate of remuneration (per week, per acre of land, per £100), (3) the income of individuals (whether from ownership or from effort). However only an empirical rather than a logical classification seemed to be possible, viz. (1) income subject to tax, (2) wages and (3) intermediate income.

In the first case, that of income-tax income, Bowley declares that things like allowances for repairs and wear and tear must be omitted, the rest, excluding abatements, being termed 'taxable income'. The average income of the 1880 income-tax payers (income over £160 per annum) was about £855, while that of the 860,000 tax payers with incomes above £225 in 1913 was about £1,120 (the figure of £225 was chosen to ensure that the same proportion of the occupied population paid tax in 1913 as had paid in 1880).

In the case of the intermediate incomes, and allowing for a margin of error of some 20% in the estimates in the number of persons in this category and their income in 1880, Bowley gives the following estimates for that year (£160 taken as the limit): number of persons: 1,500,000 to 2,000,000; average income: £70, and aggregate income: £100 Mn. to £155 Mn. In 1913 there were 4,310,000 persons with incomes below £160, 380,000 between £160 and £225, and the aggregate income was £445 Mn.

The third case, that of wages, is more complicated. Three different estimates were needed: (1) the change in the aggregate of wages, (2) the change in the average per wage-earner and (3) the change in the average of the rates in different industries. Available figures for 1913 allowed estimation of the wage-bill for 1880 as shown in Table 3M. Thus

$$x = (123/152) \times (100/134) \times \pounds 770\,\text{Mn.} = \pounds 465\,\text{Mn.}$$

The preceding calculations now permit the calculation of the national income. It is found that, for all three categories together, the total number of incomes (in 000's) in 1880 was 14,770, the total income being £1,125 Mn. The corresponding figures for 1913 were 20,700 and £2,165 Mn. With certain adjustments being made for the change in the income-tax limit from £160 to £225, it is found that

> tax-paying income was very nearly the same proportion of the whole in 1913 as in 1880; but the income over £225 in 1913 was a smaller proportion of the whole than was the income over £160 in 1880 which accrued to an equal fraction of the occupied population. [1920f, pp. 16-17]

Using Sauerbeck's wholesale price index and the estimated real wages Bowley finds that average real wages were very nearly stationary from the late 1890's to 1913. While accepting that workers are more inclined to criticise luxurious living in good times than in bad, Bowley writes

> I think, however, that the increase of luxury and the abundance of wealth which many people believe they ob-

Table 3M.

	1880	1913
Number of earners	12,300,000	15,200,000
Relative wages	100	134
Aggregate wages	x	£770 Mn.

> served before the war were illusions, fostered by the newspapers. I can find no statistical evidence that the rich as a class were getting rapidly richer in real income (money measured by its purchasing power) in the years preceding the war, though the actual amount of money spent was the greater the higher prices rose. [Bowley, 1920f, p. 20]

This illusion, he also suggests, was furthered by the diversion of expenditure from various items to motor-cars: ‘A few motor-cars can in a week give evidence of wasteful and arrogant expenditure over several counties’ [1920f, p. 20].

Among the conclusions reached by this stage, for the period 1880 to 1913, were the following: (1) the proportion of the national income received as wages decreased from about $41\frac{1}{2}\%$ to $35\frac{1}{2}\%$, (2) the proportion received by those assessed to income-tax increased slightly, (3) the proportion received by the intermediate class increased from 14% of the whole to 17%, and (4) the average of all incomes was about £76 in 1880 and about £104 in 1913.

When one considers income arising from ownership and income arising from labour, exertion or services, analysis shows that the proportions to property and to labour are $37\frac{1}{2}\%$ and $62\frac{1}{2}\%$ respectively in both 1880 and 1913. (Estimation at intermediate years suggests that the proportion to property fell to 36% or 35% in the late 1890’s and increased again from 1900.)

While the intermediate class seemed to have grown, the class of manual labourers had decreased in proportion: ‘More of the whole effort of the population has turned to direction, distribution and exchange, and relatively less to production' [1920f, p. 26]. The bulk of the population had not yet achieved a satisfactory livelihood, and from what one was able to deduce from pre-war data there seemed to be no promise of rapid improvement. With a socialistic tinge to his writing (perhaps as an olive branch to *The New Age*?) Bowley suggests that 'the methods of production and of sharing the product needed re-examination' [1920f, p. 27].

Concluding with the observation that he had restricted himself in this tract only to the consideration of problems that had been his interest for many years, Bowley remarks

> Whether the radical changes that have been brought about in opinion, in resolution to effect improvements, in outlook generally and in methods of production, can produce a new equilibrium radically different from the old is a question which a student of statistics is not specially qualified to answer. [Bowley, 1920f, p. 27]

Would that other 'specialists' could exercise similar restraint!

In his review Edgeworth stresses the importance of Bowley's having shown that the observed proportions of the distribution of the national dividend tend to be the same from one year to another. This suggests that, under normal circumstances, the proportions will change very slowly from year to year in the future. The similarity Edgeworth finds between the stability exhibited by Bowley and that shown in physical nature lies, he suggests, in neither being likely

> to be benefitted by unscientific practices, remedies suggested by association of ideas and first appearances. Such was the medieval practice of blood-letting, such the depletion of profits prescribed by the modern socialist. [1920, p. 483]

In 1922 Bowley published a paper, perhaps more in the economic than the statistical line, on the definition of the National Income, in which he considered a number of difficulties that arise in trying to define (or evaluate) the National Income.

He notes, for instance, that the effect of the Rent Restriction Acts and the housing shortage has been to keep people in their houses and to make them travel considerable distances to work (saving on rent and spending more on travel). Another matter of interest is the value of women's domestic service. During the war many well-to-do women did their own housework while their former domestic servants went into factories. The total of goods and services would then have been increased by the value of the munitions products, but decreased by the fact that the domestic service was no longer being paid for.

An important question is the following: 'To what extent do taxation and rates lead to duplicate reckoning?' [Bowley, 1922a, p. 5]. This topic is introduced (loc. cit.) as follows:

> If the rates I pay go in policemen's or sanitary inspectors' services, I get the benefit of their services, and their incomes are additive to mine before rates are deducted just as much as my gardener's would be. If the rates, however, go to the support of paupers, to the education of other people's children, to the upkeep of parks and libraries I do not use, or in aid of the rent of houses built without my consent, I get no services,—at least no direct services. I might pay for these things out of my free-will, heaping up treasure in heaven; but when I am forced to pay for them, I doubt the realisation of that treasure. I may regard it as ransom, as insurance against discontent; and though Dr. Cannan excludes the proceeds of robbery recognised as such from income, those of robbery not recognised but actually enforced by the State are no doubt income to the beneficiaries, but are they also income to me?

Further, are things like old age and soldiers' pensions income? And what about the question of the payment of interest on war loans?

Quoting Hugh Dalton's[19] view of the National Income Bowley notes that, in making a statistical aggregate,

> we should lose part of income by his method—all such

> items as the income enjoyed by children in school hours, by the populace talking during a band performance in a public park, by the workman sleeping over a newspaper in a public library, by students dosing in the Museum Library. [1922a, pp. 6-7]

Also excluded would be the incomes of all civil servants, the King, salaries of teachers, judges, and bandsmen, 'on the ground that they were included already in the incomes of those who paid them compulsorily. But this would assume that because we must pay for a service whether we wanted it or not, it was therefore valueless to us. A private scavenger renders us a service, a public authority does not, on this reckoning. In a completely socialistic state there would be no aggregate income' [Bowley, 1922a, p. 7].

Offered as 'the result of a preliminary examination' Bowley defines Social Income as follows:

> Social income then = consumption and saving in a year = aggregate of individuals' incomes (as ordinarily reckoned, say, for income tax), less incomes received from compulsory reductions for no services or for services not rendered in the year in question (old age pensions, soldiers' pensions, interest on National Debt).
> [1922a, p. 10]

Studies in the National Income, 1924-1938 was written in a novel vein. In an attempt to elicit the main considerations in the definition and measurement of national income, a questionnaire, consisting of thirty-one questions and thirty-six subdivisions, was drawn up and 'hypothetically addressed to some dozen principal writers on the subject'. The works of these authors were then studied to see how each would answer the questions. 'This method,' writes Donald MacGregor in his review, 'produces the most compact and comprehensive survey of national-income methods which has been prepared thus far' [1945, p. 272].

The four chapters of this book are entitled ‘The definition of national income’, ‘Some constituents of the national income’, ‘The census of production’ and ‘Price movements: index of real income’.

Exactly what the phrase ‘national income’ means is uncertain. In the Introduction Bowley writes

> A rough computation suggests that the possible alternative definitions would number over 200 milliards [a milliard = an American billion], i.e. more than half the number of sixpences in the national income, [1942c, p. viii]

and in the first chapter Moritz Elsas discusses some sixty-three points that various authorities had used in defining national income.

The second chapter, in four parts, contains a reprint of Bowley [1940b], the reprint benefitting by a reconciliation of the national income aggregate for 1938 with that given in the Government's White Papers on war finance. The second part is concerned with salaries in Great Britain (1924-1939), and Part 3 is on income from agriculture. In Part 4, in which the number of income earners is investigated, margins of error in the constituents of income estimation are examined.

The third chapter deals with the estimation of national income from production data. Sufficient quantities of data are not available for all products, and two methods of interpolation are therefore proposed. In the first it is assumed that the unknown increase in quantity is in the same proportion as the known increase in other products in the same industry. In the second it is supposed that the unknown change in average price is in the same proportion as the known change in price of other products.

This chapter has four parts in which the following are considered: (a) statistics of output and employment, (b) the relation between the census results and and the total national income, (c) quantities, prices and efficiency in the census years (reports for 1924, 1930 and 1935) and (d) the compilation of an index of production for each year from 1924 to 1938. Among other conclusions was the observation of the drastic decrease in the number (in 000s) employed in coal-mining from 1,202 in 1924 to 934 in 1930 and 764 in 1935. There had been a moderate increase in the iron and steel trades from 539 to 533 and 572 in the same years, with food trade employees moving from 387 to 422 and 477. All in all the results were interpreted by some as showing a moderate industrial boom during a period of eleven years of stability.

The most novel (Rothbarth [1943, p. 58] calls it the most 'revolutionary') part of the book lies in the index of real income in Chapter 4. Departing from the customary method of finding such an index by deflating index totals by a price index, Bowley offers a quantity index computed directly from various quantity series using values merely as weights.

In his review of 1943 Milton Gilbert commented that in the second and third chapters 'the technical excellence which one associates with Professor Bowley's name is everywhere in evidence' [1943, p. 474], and Willford King wrote: 'Like all of Bowley's books, this study has all the earmarks of a very scholarly piece of work' [1943, p. 613]. Solomon Fabricant, on the other hand, was far more critical. He found that the text 'bears the marks of haste. It is hard reading' [1943, p. 637]. He also found the work to be somewhat patchy (as a result of the interruption in the investigation caused by the war) and that it consisted 'essentially of a series of more or less rough notes' and that it was 'uneven and incomplete' (loc. cit.). The extensive bibliography is also found to be unsatisfactory, chiefly because of its neglect of references to work emanating from the United States of America.

More glowing was the review by Erwin Rothbarth, who says

> What better representative could there be of the peculiar combination of qualities required in a pioneer statistician—his judgement, his boldness, where required, and his cautious meticulousness where possible—than the author of this book? [1943, p. 55]

Once again one sees the suggestion that a task can be *too* well done: Rothbarth suggests that henceforth progress in the accuracy of estimates of the total national income will be slow: 'For that the pioneers of have done their work much too well' (loc. cit.). What may be hoped for, however, is a greater linking of economic reasoning and statistical procedure. He is far more flattering when it comes to Elsas's chapter on definitions, finding it to be 'staggering in the comprehensiveness of its treatment and invaluable as a sort of dictionary of the single term national income' [1943, pp. 55-56].

As a last review of *Studies in the National Income* let us mention that by Richard Stone. Among a number of criticisms he finds that Bowley's index seems to regard the falling quantity of consumption as a measure of real income, and suggests that this defect could be avoided by recognising that there are two generally differing indices that could be constructed.

> First we may use market prices as weights, in which case we shall obtain information about the situation from the consumers' point of view and our estimate of quantity change may be called an index of real expenditure. On the other hand, we may compile an index for which the weights are not market prices but factory costs, i.e. market prices *plus* subsidies *less* indirect taxes. [1943, p. 315]

Stone completes his review by dismissing the bibliography as 'a disappointing piece of work' [1943, p. 315]. Although it is long, many important works are omitted and there are too many references to old and obscure sources and to 'the semi-learned periodical literature of the last century' (loc. cit.) (MacGregor was somewhat less critical in his review, writing that 'Although it leaves much to be desired, the bibliography is a contribution of first importance' [1945, p. 273]). In conclusion Stone writes

> It is not an easy book to read. Its authors do not appear to have been at great pains to smooth their reader's path by providing a clear and orderly presentation. Rather

> they have flung their wares before him and bidden him unravel them to discover which are to his liking.
> [1943, p. 315]

The National Income, 1924 was written to provide an adequate estimate of the change in the national income from 1911 to 1924. There were three difficulties in the way of finding such an estimate, viz. (1) rapid changes in the value of money until 1922, (2) the instability of industry and (3) an important part of the report of the Income Tax Commissioners was formed by the average of the trading profits of the preceding three years, so that the figures for 1921 could not be disentangled until 1924-1925.

The book has six chapters. In the first of these the number of incomes is examined. Using figures from censuses of Great Britain and of Northern Ireland and details of age, sex and occupation for Great Britain in 1921, Bowley and Stamp were able to estimate the occupied population[20] of Great Britain and Northern Ireland in 1924 and the United Kingdom[21] in 1911. Workers were classified by sex (men and boys, women and girls) and occupation (wage-earners, salaried, independent workers and employers, farmers, professionals). Technically women's activities in their own homes were not regarded as occupations. Three groups were considered for the estimates of the National Income[22]: (1) Incomes assessed to income-tax, (2) wages and (3) intermediate income (i.e. income below the exemption level of taxation). Chapters II, IV and III respectively are devoted to these three classes.

For the first class the actual income of residents (other than wage-earners) in the United Kingdom whose income was over £150 in the fiscal year 1924-1925 was £2,183 million. Things become more complicated when one moves on to the third class, and the method of procedure is laid out as follows:

> The method of estimating the numbers and incomes of non-manual workers who are not assessed to income-tax is

> to divide all occupied persons other than manual workers into classes, and estimate the proportion in each class whose earned incomes are less than £150.
> [Bowley and Stamp, 1927, p. 19]

(The exemption taxation level at the time was £150.) The classes chosen here range from agriculture and fishing, industry, army and navy, central government, teaching (under Local Authorities) to personal service. To ascertain the incomes questionnaires were sent to a number of private firms and to statutory authorities that employed clerks.

In Chapter IV, 'Wages', Bowley and Stamp write

> We have to estimate not only the change in rates of wages but also the change in actual earnings; we have to allow for the differing growths of the numbers occupied in various industries, as well as the general increase in the number of wage-earners; we must estimate any variation in unemployment, sickness or holidays; and, finally, we must exclude South Ireland from the total. [1927, p. 28]

Remarking that studies that were then recent had usually used 1914 wages as a base, Bowley and Stamp shifted their own starting point of 1911 to this date and noted that 'The increase of population and the rise in wages during these three years is estimated to add 8 per cent. to aggregate earnings, all other factors being assumed unchanged' [1927, p. 29]. The total earnings of the manual working class is estimated, in round numbers, as '£1,600 millions, with a possible margin ±£80' [p. 30], but the authors believe that a margin of ±£40 might well suffice.

Chapter V is devoted to the consideration of various notions of 'total income'. As the aggregate one wishes to mention Bowley and Stamp suggest one might take as the definition of Social Income to be measured

> [the] aggregate of U.K. individual and collective incomes, less incomes received by compulsory reductions from other

> incomes in return for no services or services not rendered in the year in question. [1927, p. 41]

However the addition of the estimates of the preceding three chapters might result in a certain amount of duplication. For example, the interest on the National Debt was ‘paid by tax-payers to holders in return for no current services’ [1927, p. 42] and had essentially been counted twice. Further, certain pensions, poor relief and unemployment insurance had not been counted.

The results of the analysis are given in Chapter VI. The figures for 1911 came from Bowley’s *The Division of the Product of Industry.* The total number (in thousands) of males (females) in the United Kingdom, including South Ireland, in 1911 was 14,300 (5,850) and in 1924, excluding South Ireland, was 14,300 (6,000). The figures for 1911 were estimates.

> The reduction in the number of female wage-earners [from 4,650 in 1911 to 4,400 in 1924 (in thousands)] is definitely due to the diminution of domestic service and the increase of clerical work. [1927, p. 48]

Attention is paid to the question ‘what has been the change in the National Income when the fall in the value of money is eliminated?’ [1927, p. 53]. Examination of the index numbers of prices shows that from 1911 to 1924 there was

> a range of price changes from 47 per cent. increase in controlled rent and 57 per cent. in eleven years (60 per cent. in the thirteen years) in the price of imported food to over 100 per cent. in the price of an hour’s labour. It is evident that under these conditions no accurate comparison is possible. [1927, p. 53]

Bowley and Stamp suggest a way of resolving the problem: first divide the 1911 income into tenths of the total with 4 portions for manufacture, 1 for agriculture, 1 for houses, 3 for services, including transport, and 1 for income from abroad. Then for manufacture (the other groups are similarly treated) an increase of price of 105% is taken for the change from 1911 to 1924, this being arrived at by considering the rise from 1911 to 1914, the index numbers for exported manufacturers and the retail price of clothing.

The final conclusion is

> the real home-produced income per head (when duplication is eliminated) was very nearly the same in 1911 and 1924; it is improbable that it was any greater in the latter year, and it may have been 4 per cent. less. [1927, p. 56]

Among the conclusions relating to the comparison between 1911 and 1924 reached were: (1) while the aggregate income of the inhabitants of Great Britain and Northern Ireland had rather more than doubled, the exclusion of duplication resulted in the remaining income (the Social Income) being increased by 90%, (2) since the effective price increase was also about 90%, the real Social Income was much the same at the two dates, (3) examination of the distribution of income between wage-earners, other earners and unearned income showed a slight change in favour of the earning class[23].

> When the full effects of taxation are taken into account the real income available for saving or expenditure in the hands of the rich is definitely less than before the war. The sum devoted to luxurious expenditure is (allowing for the rise of prices) definitely less than in 1911, but it is still sufficient to bulk large in the eyes of the public, since it is concentrated in small areas, enlarged by the spending of visitors from overseas, and advertised by the newspapers. It is, however, equally evident, if we are to depend on appearances instead of on statistics, that there is a great deal of income available for cheaper amusements.
> [1927, p. 59]

As is often the case, reviews of Bowley and Stamp's *The National Income, 1924* were varied. Macrosty [1927] considered some of the estimates made in detail, finding that the social incomes of 1911 and 1924 were respectively under- and over-estimated, when compared with his own estimates.

In his review Flux accused the authors of having relied on 'treacherous memories' in their quoting of figures from the First Census of Production of 1906. However he commented favourably on the positive results reached, especially the conclusion that 'the real home-produced income per head (when duplication is eliminated) was very nearly the same in 1911 and 1924' [Flux, 1927, p. 257].

Watkins was more lavish in his praise, describing the tract as 'an excellent example of skillful statistical work' [1927, p. 404], the authors having shown the requisite knowledge of available data, being conversant with underlying facts and possessing 'rare powers of statistical analysis' (loc. cit.).

In 1910 a Committee, under the chairmanship of Cannan and with Bowley as secretary, appointed by the British Association at its Dublin meeting in 1908, presented a report on the distribution of income, other than wages, below the income tax exemption level in the United Kingdom.

The study classified 'occupied classes, other than manual labourers working for employers (over 10 years old)' [Cannan et al., 1910, p. 40] into 31 groups, ranging from civil service (officers and clerks), army officers, clergy, teachers, through merchants, salesmen and commercial travellers, farmers to railway clerks, shopkeepers, costermongers and sweeps. The number of persons (in 000's) were 3,375 males and 1,145 females. The information obtained was found to be 'adequate' for the Civil Service, Local Government, the Army, Navy, the clergy, elementary teachers, banks and shop assistants, and 'sufficient for an estimate' in the case of clerks, farmers and shop assistants. In the remaining cases the committee had to do the best possible with the finding of limits of aggregate income.

> In each case we have endeavoured to assign, in the light of all the information available, whether published in this Report or omitted as too confidential or for want of space,

> limits within which it seems highly probable that the true measurement must lie. [Cannan et al., 1910, p. 46]

(The estimates were made by Bowley, who was the only member of the Committee to have access to all the information.)

Considerable difficulties were involved in some of the categories—it is noted, for instance, that the estimation of the income of farmers was 'a task which has always proved beyond the powers of statisticians' [1910, p. 57]. Civil Service information was also difficult to deal with, some being only available as '39 second division clerks, minimum 70*l.*, annual increment 10*l.*, maximum 250*l.*, total 5,400*l.*' Nevertheless it was possible to give the estimate for the men in this group as 36,000 $\pm$ 2,000 and 95*l.* $\pm$ 5*l.* and for the women 15,000 $\pm$ 2,000 and 57*l.* $\pm$ 5*l.* For the clergy (all men, of course) the figures were 21,000 $\pm$ 2,000 and 120*l.* $\pm$ 20*l*, while missionaries (both male and female, and including monks, sisters and nurses), who were among the worst paid, the figures were 24,000 $\pm$ 2,000 and 40*l.* $\pm$ 20*l.*

The Committee made several recommendations, including the following: (1) the Irish and the England and Wales Censuses should be harmonised, (2) clerks should also be given according to the main occupations to which they were attached, (3) the Census office should publish estimates of the number of manual workers and (4) the Inland Revenue Commissioners should publish more detailed information.

This page is intentionally left blank

Chapter 4

Wages and Incomes

4.1 Introduction

In *Wages and Income in the United Kingdom since 1860*, the last of his major works on such matters, Bowley lists in the Bibliography 91 pertinent items on wages, prices and incomes. Of the 65 listed as 'Articles by A.L. Bowley' a few were jointly written with George Wood or by Bowley as a committee member.

Clearly we cannot hope in this book to consider all of these works in detail. In Chapter 3 we looked at some of Bowley's work on wages and national income: here we shall examine some of his books and papers in which wages and incomes in specific trades were investigated, though there will of necessity be some reference to national income and index numbers[1].

4.2 Early Papers

An amplification of the results of Bowley's Adam Smith prize-winning essay at Cambridge was read before the Royal Statistical Society in March 1895, and published in the *Journal* of that society in the same year. At that time, Bowley suggested, the most important question to be considered was the following: 'Who are benefitting most by the development of industry; those who obtain profits or interest, or those who receive wages?' [Bowley, 1895a, p. 224].

While several steps *towards* answering this question are set out, Bowley devoted this paper simply to an examination of the total amount paid in wages and the average money wage, and to finding the actual changes in these quantities; further, to investigating the gross receipts resulting from both profits and interest, and to discovering the average income of the whole nation.

Important though this topic might be, Bowley notes that a statistical analysis is almost impossible: many of the problems do not lend themselves to numerical measurement, and in many cases sufficient data are lacking. He further points out that, in order to ensure accuracy of conclusions, the following basic principle had been adhered to:

> I have never compared two statements of wages, except when they have been given by the same authority; and that I have dealt entirely with the ratios, not with the amount, of wages. [Bowley, 1895a, p. 227]

Bowley proceeds by dwelling briefly on changes in national income other than wages, and presents a comparison of the changes of income and wages, noting the changes in the purchasing power of money and emphasising the limitations of the results (e.g. new trades will have appeared on the scene between 1860 and 1891, index numbers take no account of rent or personal services, and the number of the unemployed).

The final conclusion drawn is the following:

> the inadequate information extant indicates that average income and average wages have increased at nearly equal average rates, and that both have nearly doubled during the period under review. [Bowley, 1895a, p. 251]

Six months after his [1895a] Bowley gave a talk before Section F of the British Association comparing the rates of wage increases in the United States and Great Britain. The study was based on the report ordered by the United States Senate in March 1891 to investigate the effect of tariff laws upon trade, prices and wages, the greater part of this report being devoted to estimates of average change in wages since 1860.

While acknowledging the 'care and conscientiousness' with which the data were collected [1895b, p. 370], Bowley rejects the conclusions of the U.S. Senate Report, his quarrel being 'not with the facts exhibited, but with the deductions drawn from them' (loc. cit.). He questions not only the assumption that the establishments examined (nearly one hundred in twenty-two industries) are typical of the trades to which they belong, but also the assumption that the subdivisions for which data are given (which very often do not cover the entire establishment) are indeed typical of the establishment.

A final major point of dissent was that no account had been taken of the different level of wages in the different industries in computing the averages. Such averages of course bore no relation to concrete wages, and Bowley therefore introduced the following definition:

> By the average wage of an industrial community I understand that amount which, multiplied by the number employed, will give the total paid in wages in that community; and by *changes in average wages*, I understand changes in the amount thus defined. [1895b, p. 372]

For these (and other) reasons Bowley preferred here to work with the actual rather than the 'average' wages given in the report.

Using the data in the Report Bowley reworked the percentage increase from the table of exhibits. The great amount of work involved, and the tediousness of the process, resulted in the reworking of the data, in four different ways, for the years 1860 and 1891 alone. He found the third method to be the most reliable and therefore employed it in his paper. This method he summarised as follows:

> The number actually employed and the wages actually earned per diem (according to the pay-sheets) are tabu-

> lated, and the average wage found for each industry in each year. Each of these averages is weighted by the corresponding number from the census. The average wage is then found for each year for the whole group of industries. These averages are reduced to percentages of that for 1860. The numbers so found give the most probable course of average wages through the period considered, as indicated by the whole body of figures. [1895b, p. 374]

The U.S. Senate Report contains figures for twenty-one industries. For example, the figures for *dry goods* depend on the wages of one porter, four salesmen and five saleswomen, and those for *groceries* on those for two salesmen and one teamster. Bowley comments (perhaps with a touch of 'tell it to the marines!') that

> we are expected from these scanty data to accept the increase of wages of 353,444 clerks in stores, more than one-sixth of the total number of wage-earners considered! [Bowley, 1895b, p. 374]

Turning next to the comparison with English (sic) figures Bowley used data obtained from various sources: the Board of Trade, Trade Union Reports, 'and in other ways' [Bowley, 1895b, p. 378]. Referring to his [1895a] Bowley records here that in the main the figures from these disparate sources all yield the same results when *only* relative changes are considered. The final results are shown in Table 4A.

The results are analysed, the following conclusion being reached:

> In both countries real wages rose some 20 per cent. between 1860 and the maximum period, 1871-4; money wages rose 50 per cent. in the United States, and between 30 and 40 per cent. in the United Kingdom in the same period. The rise in real wages was checked in 1879-80 in the States, but continued with little interruption in England; money wages fell to a minimum in 1879-80 in both countries. [Bowley, 1895b, p. 382]

Table 4A. Relative money wages in the United Kingdom and the United States in selected trades, 1860 = 100. (*All U.S. wages in gold — not currency.*)

		1860	1871-73	1874	1879	1880	1883	1886	1891
Building	U.S.	100	161	—	—	—	155	157	165
„	U.K.	100	—	126	—	125	125	126	128
Metals	U.S.	100	145	—	—	—	139	133	141
„	U.K.	100	—	128	—	118	124	121	125
Cotton	U.S.	100	148	—	—	—	140	139	152
„	U.K.	100	—	148	—	134	146	155	176
Wool	U.S.	100	135	—	—	—	158	154	161
„	U.K.	100	—	121	—	126	120	115	115
Textiles	U.S.	100	143	—	—	—	147	145	156
„	U.K.	100	—	141	—	132	140	143	160
Result for the whole group	U.S.	100	148	—	120	124	139	139	148
„	U.K.	100	—	134	—	128	133	131	138

In 1898 Bowley read a paper before Section F of the British Association in which he compared the changes in wages in France, the United States and the United Kingdom for 1840 to 1891. Noting that a comparison between average wages in the three countries could quite easily lead to wrong conclusions, Bowley chose rather to consider an equally interesting problem, one less subject to error, 'namely, the comparison of the rates of increase of wages in these three countries during the period for which statistics are to be found' [1898b, p. 474].

The results are presented in three tables: the first two pertaining to wages in the United Kingdom, the next to a comparison of nominal wages in the three countries, and the last two to a comparison of real wages. The new work presented here consisted in the use of newly-collected material to carry the evaluation of English wages back to 1840 (the comparison with the American and French wages was of course also new)[2].

The English wage statistics are characterised by their miscellaneous nature (the figures for Scotland and Ireland were found to be deficient).

The reports for the United States and for France (Bowley relies on his [1895b] in the case of the former) are found to be equally reliable, though the American Report contains far more information than the French. The data are summarised in Table 4B: note that the American and French wages have 'marched together', except during the Civil War and the Franco-Prussian War, and that they increased faster than the English[3].

Bowley's final conclusion ran as follows:

> the average real wages of regularly employed workmen and women in France, the United States and England, had doubled in the half-century ending 1891, and increased by one-half in a period of less than 20 years ending at the same date. [Bowley, 1898b, p. 489]

Following on the 1898 paper Bowley published in March of 1899

Table 4B. Average *nominal* wages in the United Kingdom, France, and the United States, as percentages of those in 1891.

Years	1840	1850	1860	1866	1870	1874	1877	1880	1883	1886	1891
United Kingdom	61	61	73	81	83	97	94	89	92	90	100
France	52	52	65	70	75	80	83	86	91	90	100
United States	49	54	59	66	81	87	80	85	95	92	100

a paper [1899e] on wages in the United States and Europe. His major source here was the *Bulletin of the Department of Labor* in Washington, which contained an account of wages paid in twelve major cities of the U.S.A. and in Paris, in Liége, and in London, Manchester and Glasgow. These figures were compared by Bowley with those estimated by him in the December issue of *The Economic Journal* in 1898.

The *Bulletin* wages are taken mostly directly from the payrolls of two firms (in continuous operation from 1870) and they thus arise from different sources. Of the twenty-five specific occupations listed, relating to *town* rates of wages, only thirteen appear throughout the period from Britain and Liége and twenty-one from Paris. Bearing this in mind one cannot expect the figures to provide a close representation of the change in average wages in the relevant countries or cities, nor will they shed much light on results for any specific year.

Table 4C shows the general course of wages, while further tables (not given here) give wages in the building and iron trades and wages for compositors. There is a marked divergence between the figures for the years 1870-1877.

In 1896, when he was twenty-six years old, Bowley published a review of the second issue of the Board of Trade's *Abstract of Labour Statistics*. In this article he stated that the purpose of the figures published in the *Abstract* should presumably be to serve the understanding of amateurs and to be of the greatest use to those who lacked either the time or the inclination to study the figures at source. Therefore

> An exact understanding of a list of figures is only to be obtained when the method of their collection and compilation is completely known, and for even a reasonably clear conception of their meaning a lengthy explanation is often necessary. [Bowley, 1896a, p. 465]

Following the above requirements, Bowley provides a discussion of the limitations of labour statistics in terms of incomplete data

Table 4C. General course of wages. Thick type, average given in the *Bulletin. Small type, from The Economic Journal, December 1898.*

Years	1870	1874	1877	1880	1883	1886	1891	1896
U.S.A.	**87**	**91**	**88**	**92**	**97**	**97**	**100**	**97**
”	81	87	80	85	85	92	100	
Paris	**81**	**82**	**88**	**92**	**94**	**96**	**100**	**101**
France	75	80	73	86	91	90	100	
London, Glasgow, and Manchester	**90**	**95**	**98**	**96**	**98**	**97**	**100**	**104**
Great Britain			94	89	92	90	100	
Liége	**93**	**101**	**96**	**96**	**100**	**98**	**100**	**103**

on minimum wages, incompatibility of different wage types, and the omission of price changes. He expresses concern that the way data are collected may menace other important information[4].

With all the limitations mentioned above, Bowley covers Trade Unions, Co-operatives and Friendly Societies for single estimates of how the working-class position was changing (or whether it was remaining stationary). Bowley found that Trade Unionism seemed to show a diminishing trend, having reached its peak in 1890. On the other hand, Co-operative and Friendly Societies showed a steady, healthy and regular growth.

While general progress was perhaps indicated, Bowley notes that the overall conditions of workers in the United Kingdom had not improved in terms of employment, pauperism and working accidents. Moreover trade disputes were still common, though arbitration seemed to be of increasing importance.

In his [1905c], a paper on the statistics of the woollen industries, Bowley noted the importance of relating developments of industries at that time to the state of things ten, twenty or thirty years before, if a true understanding of the present state were to be arrived at. This was particularly true at that time of industries in the West Riding of Yorkshire, especially in the woollen and worsted trades. Bowley's rapid view was based on Board of Trade figures.

Regrettably, though, serious qualifications of these figures were needed before the progress of the industry could be ascertained. Firstly, the value of the wool used in exported goods was unclear, since there was no estimate of what proportion of the wool retained for use was to be credited to exported manufactured items. Secondly, values could not be ignored in favour of consideration of quantities.

A further difficulty arose from the fact that one could not take a yard as an equally common measure for all fourteen categories, while prices varied from 9*d.* for mixed stuffs to 14*s.* 6*d.* for broad, heavy, pure woollen cloth.

Using estimates made by William Acworth[5] Bowley estimated the value of wages in the export of woollen products annually from

1900 to 1904. In this period he found (in $\pounds 0,000$'s) an increase from 12 to 20 (for tops and noils)[6], a decrease from 84 to 81 (for yarn) and an increase from 3,72 to 4,20 (for cloths and stuffs).

Bowley concluded that ‘till we have statistics of production for the home market ... we can go but a very little way in the statistical history of the West Riding’ [1905c, p. 590].

4.3 Wages in the United Kingdom

From 1898 to 1906 Bowley published a series of papers (six were co-authored by George Wood) in which he examined the wages of workers in various occupations in the United Kingdom during a period of a hundred years[7]. The various parts of the memoir were concerned with wages in the following trades: agriculture (the first four parts), printing, building, worsted and woollen manufactures, and engineering and shipbuilding. The last two trades required five parts, being written by Bowley and Wood and filling some 160 pages. Apart from Part XIV, we shall not consider these parts in any detail (the method of investigation was very much the same for other trades).

Part XIV, read before the Royal Statistical Society in February 1906, began with an introduction by Bowley in which he drew his listeners’ attention not only to wage statistics in general but also to wages in the engineering and shipbuilding trades in particular. While commenting on the availability of historic data on wages for some occupations, he notes that ‘it is for want of adequate definition that so much of the existing evidence is valueless’ [Bowley & Wood, 1906a, p. 149]. Despite the amount that had been done towards the compiling of suitable tables Bowley found that in the early twentieth century the wage statistician lacked sufficient data.

In this part of the memoir Bowley and Wood attempted to condense, into an intelligible result, the material gathered in the preceding four parts. To this end they

> first set aside all the information as to numbers employed, and as to the relation of the average, the standard rate, the mode, and earnings, to each other, and aim at following for each occupation in each locality what we will call in this paper "the normal rate". [1906a, p. 154]

Areas considered, which appeared to be of most importance and for which most information existed, covered twenty-six occupations and eighteen localities, and it is claimed that, by doing so,

> we increase our precision four or five-fold, and we obtain the result that we are unlikely to be more than $\frac{1}{2}$ per cent. wrong, that is, that the index number is correct within the limits of arithmetic working! [1906a, p. 156]

If one considers that this paper was read in 1906—long before statistical concepts related to time series analysis were crystallised—one sees that Bowley and Wood definitely grasped and appreciated the subtleties involved in a rigorous statistical treatment of the data that, as they discovered, involved serial inter-connected variables.

The first table in this paper (abridged in Tables[8] 4D(i) and 4D(ii)) provides index-numbers for normal rates for twenty-two occupations (we have omitted the data for boiler-makers), the index for the year 1880 being taken as 100. Table 4D(iii) here lists index-numbers of normal rates in occupations common to shipyards and engineering shops in the eight maritime districts mentioned before[9].

There is detailed discussion of the question of weights. Separate index-numbers are to averaged either by (a) taking all the occupations to be of equal importance (resulting in an unweighted average), or (b) grouping the original under 19 localities, thus determining an index number for each. (See Tables 4E(i) & 4E(ii).)

The detailed tables provide very valuable information. Concentrating, only for illustrative purposes, on fitters in engineering shops, we see a gradual mild increase from 1850 $\equiv$ 92 to 1904 $\equiv$ 131. Moderate yearly increases of three or four points were observed for the

Table 4D(i). Index-numbers of Normal rates in 22 Occupations. Engineering Shops.

	1850	'55	'60	'65	'70	'75	'80	'85	'90	'95	1900	'04
Patternmakers	90	99	100	103	106	121	115	120	124	125	139	139
Ironmoulders	96	103	100	107	105	114	110	112	119	117	131	130
Fitters	92	100	100	103	106	117	111	116	123	120	132	131
Turners	92	98	100	102	104	117	110	116	122	120	132	131
Smiths	89	101	100	103	105	117	112	116	123	121	133	132
Strikers	92	100	100	104	105	120	115	120	125	125	138	137
Brassfinishers	—	100	100	103	105	119	110	113	122	120	134	133
Brassmoulders	—	—	100	102	107	115	109	111	120	118	132	131
Planers	—	104	100	104	108	119	113	119	126	123	135	135
Drillers	—	—	100	105?	106	118	115	120	128	128	141	139
Dressers	—	—	100	100?	100	114	110	117	126	131	138	138

Table 4D(ii). Index-numbers of Normal rates in 22 Occupations. Ship Yards.

	1850	'55	'60	'65	'70	'75	'80	'85	'90	'95	1900	'04
Ship-joiners	94	106	100	111	110	127	122	122	139	137	154	151
Shipwrights	93	114	100	114	109	122	118	124	135	130	141	139
Ship-painters	—	—	100	114	*126*	133	134	134	143	145	152	151
Ship-plumbers	—	—	—	—	—	118	117	118	125	123	136	136
Sailmakers & riggers	100	102	100	*103*	*109*	133	129	129	132	134	138	141
Angle-smiths	—	100	100	—	*110*	119	119	118	136	128	142	140
Platers	—	—	100	—	—	119	115	120	135	127	141	139
Rivetters	—	—	100	—	—	128	124	128	148	139	157	155
Caulkers	—	—	100	—	—	127	126	131	154	143	161	158
Holders-up	—	—	100	*108*	104	119	119	123	147	137	157	154
Labourers	—	—	100	103	105	121	114	120	124	124	132	132

Table 4D(iii). Index-numbers of Normal Rates in Occupations common to Shipyards & Engineering Shops in 8 Maritime Districts.

	1850	'55	'60	'65	'70	'75	'80	'85	'90	'95	1900	'04
Fitters	88	100	100	104	106	120	111	116	131	121	140	137
Turners	87	98	100	103	103	119	111	115	129	120	138	135
Smiths	87	101	100	106	106	122	115	118	135	127	144	141
Strikers	89	100	100	104	104	124	121	122	134	131	144	142
Drillers	—	—	—	—	—	120	115	117	122	126	142	137
Labourers	—	—	100	103	105	123	117	121	127	126	136	135
UNWEIGHTED AVERAGES												
Engine shops &c.	92	101	100	104	106	119	113	118	127	125	137	136
Shipyards	95	105	100	111	111	125	122	125	140	134	148	146

Table 4E(i). Comparative levels of Average Rates of Various Classes of Workpeople in Different Years. Engine Shops.

	1850	'55	'60	'65	'70	'75	'80	'85	'90	'95	1900	'04
Patternmakers	133	146	147	152	156	178	170	177	189	184	205	205
Ironmoulders	149	160	155	166	163	177	171	174	185	182	203	202
Fitters	133	145	145	149	154	170	161	169	178	174	192	190
Turners	134	143	146	149	152	171	161	170	178	175	193	191
Smiths	126	144	142	146	149	166	159	165	175	172	189	188
Strikers	84	91	91	95	96	110	105	110	114	114	126	125
Brassfinishers	—	148	148	152	155	176	163	168	181	178	199	197
Brassmoulders	—	—	157	162	169	181	171	175	189	186	208	206
Machinists	108	118	114	118	123	135	129	135	143	140	154	154
Drillers & Screwers	—	—	98	100	103	113	110	117	125	125	137	135
Dressers	—	—	101	101	101	115	111	118	127	127	139	139

Table 4E(ii). Comparative levels of Average Rates of Various Classes of Workpeople in Different Years. Shipyards.

	1850	'55	'60	'65	'70	'75	'80	'85	'90	'95	1900	'04
Ship-joiners	131	148	140	156	154	178	171	171	195	192	216	212
Shipwrights	141	173	152	173	166	185	179	189	205	198	214	212
Ship-painters	—	—	127	144	*160*	169	170	170	181	184	193	193
Ship-plumbers	—	—	—	—	—	177	175	177	187	184	204	204
Sailmakers & riggers	122	124	122	*130*	*133*	162	157	157	160	163	168	171
Angle-smiths	—	153	153	—	—	183	183	181	209	196	218	215
Platers	—	—	155	—	*167*	184	178	186	209	196	218	215
Rivetters	—	—	127	—	—	163	158	164	191	178	201	198
Caulkers	—	—	124	—	—	157	156	162	191	177	199	195
Holders-up	—	—	*112*	—	—	133	133	137	164	153	175	172
Labourers & Helpers	—	—	85	87	89	103	97	102	105	105	112	112

fitters in the years 1882, 1889, 1890, 1898 and 1899. Moderate oscillations were similarly noticed in the years 1883, 1889, 1895 and 1899 for ship-plumbers.

General observations include the following:

1. The wages in maritime towns and in the shipbuilding trade increased from 1855 to 1904 more quickly than wages in inland towns and engineering.
2. Considerable divergence was evident among the rates of increase in different towns and for different occupations.
3. Errors in the choice of weights are of less effect than apparently equal errors in quantity.

On the matter of ship-building the authors were glad to have the opinions of 'employers' representatives specially qualified to know' [Bowley & Wood, 1906a, p. 171]. Among these (sometimes divergent) views were the following:

I. One representative stated that little change had been seen in the relative numbers in different grades in the industry, and the increased use of machinery had increased the demand for highly skilled labour as much in one grade as another. Another representative, on the other hand, speaking of a large shipyard among those covered by the previous assertion, said that

> the number of partly skilled workmen in proportion to that of skilled workmen is unquestionably increasing, one reason being the continuous introduction of new and labour-saving appliances worked by partly skilled men. [Bowley & Wood, 1906a, p. 171]

II. The apprentice system seemed to be breaking down.

III. Surprisingly, the transition from wood to iron (steel) ships did not affect the distribution of wages.

The authors finally provided a cautious summary:

> both with shipbuilding and engineering no definite statistical evidence has come to light which shows any general effect on the rate of change of the average wage from the shifting of occupations, [Bowley & Wood, 1906a, p. 178]

and they recommended that their index-numbers be left unchanged.

In the discussion of this paper after its having been read to the Royal Statistical Society two discussants, Sir Edward Bradbrook and Mr Macrosty, said that the paper, like others in the series, was clearly destined to become a classic. Sir Edward also noted that one thing shown in this paper, perhaps above all,

> was the great advance in the application of a scientific method to statistics which had taken place of late years. Such a paper could not have been written at the beginning of the period to which the paper related, for at that time neither the facts nor the methods would have been available to statisticians. [Bowley & Wood, 1906a, p. 193]

4.4 *Prices and Wages, 1914-1920*

We have already examined Bowley's [1930d], *Some Economic Consequences of the Great War*. Some ten years earlier, under the banner of the Carnegie Endowment for International Peace[10], *Prices and Wages in the United Kingdom, 1914-1920* appeared, in which details were given of the movements of prices and wages in the United Kingdom from the start of the War to 1920. The book, one that 'deals with results, not causes' [Bowley, 1921h, p. xvii], is divided into two parts: Prices, and Wages.

The five chapters in the first part are: I. General movement of wholesale prices, II. Wholesale prices of groups of commodities, III. Retail food prices, IV. Other retail prices. Change in cost of living, and V. Comparison of wholesale and retail food prices.

After stressing the difficulty of measurement under War circumstances, Bowley explains in the first chapter the method of index numbers and presents those used by the *Statist* (formerly Sauerbeck's), the *Economist* and the Board of Trade.

The calculation of the index numbers presented certain problems. For instance, both the *Economist* and the *Statist* gave numbers for beet-sugar, those for German sugar being computed by some interpolated method whose details were not given. As another example it is noted that flour as obtainable before the War was replaced by Government Regulation flour which was of variable quality (the quality of other items had also changed).

The commodities considered in Chapter II are divided into six groups: (1) cereals, (2) meat, (3) cotton, yarn and cloth, (4) wool, tops and yarn, (5) iron, steel, coal and other minerals, (6) miscellaneous (rubber, timber, leather, etc.). It appears, for example, that cereals in general showed a regular increase up to the end of June 1917, at which time the price dropped and stayed constant for about eighteen months before rising again. Further, textiles dropped for six months after the start of the War and then rose, only to fall after the Armistice and then rise rapidly to March 1920. Bowley points out yet again here that in the computation of index numbers 'a great deal appears to depend on the choice of and the relative importance given to the commodities' [1921h, p. 19].

The next two chapters are concerned with retail price changes. The first part of Chapter III considers some general changes, specifically as given in the *Labour Gazette*, while the second is devoted to the prices of specific foods in detail. In the first part Bowley notes that one's concern now is 'to find a factor by which money wages may be converted into real wages and the movement of wages expressed in terms not of money but of the goods they purchase' [1921h, p. 32]. The method used is the estimation of a budget of the amount the average working-class family spends, at some specific date, on food, clothing, rent etc., and to use this to compute the cost of exactly the same items at other dates at prices prevailing at those dates. The averages at the two dates are then compared.

During the War the Ministry of Food introduced a new method of measuring the effective change in the cost of living. For instance, people were eating more bacon[11] and margarine, fewer eggs and less butter, meat, sugar and tea. Further, a shortage of sugar led to an increase in the consumption of things like jam and honey, while a shortage of butcher's meat might result in an increase in demand for rabbit and poultry. It was found that by the end of the war the average nutritive value of the budget, in calories, had fallen by some 3% from its value in 1914.

The second part of this chapter begins with a detailed investigation of the changes in prices of beef and mutton, both British and imported. Here Bowley's investigation leads to the conclusion that

> It is clear that the retail index-number does not give an adequate measurement of the change in the cost of living, and it is equally clear that it cannot be replaced by an index of expenditure. [Bowley, 1921h, p. 58]

Other retail prices and the cost of living are the subject of Chapter IV. Retail prices of items other than those considered in the previous chapter (e.g. motor cars, furniture, tobacco, beer, whisky, railway travel and postage) are extremely difficult to account for. Attention is therefore especially directed to things necessary to the working-class family: food, clothing, fuel and light, rent and 'miscellaneous'. By law working-class house rents were kept at 1914 levels until July 1920, though landlords were allowed to raise rents to recover increased local rates.

Changes in the cost of living are also investigated here.

> The most important practical question to which an answer ought to be attempted is whether the increase of wages has kept pace with or exceeded the increase in the cost of living, or (in different words) whether the working classes have preserved or improved their standard of living. [Bowley, 1921h, p. 72]

What ought to be investigated is whether the purchasing power of money has decreased. Taking as the 'normal household' a married couple with three children, Bowley examines whether the official index number is 'an adequate measurement of the change in the cost of living' [1921h, p. 72]. His investigations show that the official figures are not seriously wrong: the official reckoning of the value of £1 spent on food in July 1914 fell to 7*s.* 9*d.* after six years, this last figure being 8*s.* 4*d.* under the modified reckoning given by Bowley.

In the last chapter in Part I Bowley considers the comparison of retail and wholesale food prices. The problem is however one of great complexity, and only 'tentative inquiries' could be made. The only foods that could be investigated were flour, bread, meat, bacon, butter and potatoes.

> We ought to distinguish as clearly as possible between two methods by which middlemen's and retailers' charges are made, namely, the method of adding a definite sum ... and the method of adding a fixed percentage.
> [Bowley, 1921h, p. 77]

The first two chapters of Part II are devoted to general comments on wages, while Chapters VIII to XIV are concerned with wages in separate trades, with Chapter XV dealing with women's wages.

Bowley begins Chapter VI with a quick look at the position in 1914 and follows with a study of war-time conditions. The Board of Trade index figures show that by 1913 the wage index number had risen to 109 (with 1904 taken as 100). It follows from Part I of this study that in that period retail food prices had risen by 12% while wholesale prices had increased by somewhat more than that, so that, in purchasing power, wages were either stationary or had fallen slightly. During the first half of 1914 both wholesale and retail prices fell while wages showed a general increase.

The demand for labour increased as the War progressed, until 1920, though there was some difficulty in finding work for demobilised men who were not skilled in any particular trade. A difficulty in carrying out wage comparisons was that during the War not only was men's work done by lads, old men and women, but also by promoted apprentices and the unskilled. Overtime and night work also increased. Increased demand led to patriotic extra effort, and machinery was improved.

'It is partly for these reasons that women and others not previously trained were able rapidly to learn to carry out processes formerly regarded as skilled,' wrote Bowley [1921h, p. 89]. The increased demand for and diminished supply of labour had the general effect of raising wages to give an appearance of prosperity, though the inflated earnings ceased after the Armistice. Some relevant figures are given in Table 4F.

In Chapter VII estimates of wage movements and the increase in the cost of living are brought together.

> So far as a general statement can be made, we may say that *rates* of wages for the same work increased less rapidly than the cost of living in the first three years of the War, in the fourth year wages gained and their increase over four years was nearly that of the modified index. In 1918-19 wages gained rapidly and reached the official cost of living measurement, and they kept pace with it in the year 1919-20. [Bowley, 1921h, p. 106]

Chapters VIII-XIV are devoted to the following trades: building; engineering, shipbuilding, iron and steel manufacture; coal mines; printing; railways, docks; agriculture; cotton and wool. We shall look only at Chapters VIII and IX.

Since wage rates in the building trades in 1914 varied considerably from town to town eight towns were selected for particular study, viz. London, Southampton, Bristol, Birmingham, Leeds, Manchester, Glasgow and Belfast. Labourers' wages often moved at different dates from those of artisans, but the increase was generally the same number of pence per hour. For example, in London bricklayers' weekly summer rates went from 47*s.* 11*d.* in July 1914 to 102*s.* 8*d.* in July 1920, and the similar wages for bricklayers' labourers at the same dates went from 33*s.* 4*d.* to 91*s.* 8*d.* The corresponding figures for the same groups and the same periods for Belfast were (for

Table 4F. Aggregate changes in rates of wages 1914-1919.

Groups of trades	Number of workpeople whose rates of wages were reported as changed				Total net increase in the weekly wages of those affected			
	1914 000s.	1915 000s.	1917 000s.	1919 000s.	1914 £000	1915 £000	1917 £000	1919 £000
Building	134	116	250	290	13	16	100	197
Coal mining	364	887	1,000	1,110	−26	278	473	597
Other mining & quarrying	33	37	51	59	−1	10	24	31
Pig-iron, iron & steel manufacture	67	126	160	184	−5	37	95	118
Engineering, shipbuilding, etc.	213	882	1,489	1,715	15	158	946	500
Textile	23	621	866	521	1	55	272	153
Clothing	6	104	260	390	1	15	59	104
Road transport & docks	26	178	200	267	5	41	82	59
Paper, printing, etc.	20	55	92	157	2	6	31	61
Glass, brick, pottery & chemical	26	119	201	208	2	14	71	59
Other trades	29	213	310	561	4	31	101	182
Local authorities	19	132	150	185	2	17	54	50
Total	960	3,470	5,029	5,647	13	678	2,307	2,111

bricklayers) 40*s.* 6*d.* and 106*s.* 4*d.* with no figures being available for labourers. For Glasgow the figures were (bricklayers) 43*s.* 9*d.* and 102*s.* 8*d.* and (labourers) 27*s.* 1*d.* and 85*s.* 3*d.*

Bowley concludes Chapter IX on the engineering trades with the following summary:

> for men engaged in the manufacture of iron and steel the change from twelve to eight hours per shift resulted, generally, in the highest-paid men suffering a reduction in earnings up to $33\frac{1}{3}$ per cent. In the case of medium-paid men a reduction in earnings, according to the scale, was made, but this was accompanied by all-round promotions in order to provide the necessary skilled labour for the third shift. The reduction in earnings was, therefore, largely counterbalanced by the increase in wages consequent upon promotions. In the case of the lower-paid men little or no reduction was made, and in many cases earnings were actually increased owing to the promotions necessary to provide semi-skilled labour for the third shift. [Bowley, 1921h, p. 147]

Although women's wages had been considered in other chapters[12], Bowley devotes a fair part of Chapter XV to this topic, saying that

> they need also separate treatment, both because these trades [i.e. cotton and wool] do not include the majority of women workers, and because women's wages were subject to special regulation in connexion with munitions work. [Bowley, 1921h, p. 184]

In July 1914 and July 1920 the numbers (in 000s) of women working in industry, commerce, domestic service, agriculture etc. (including home workers) were 5,966 and 7,311. The numbers of working women not thus covered or who were not in paid occupations were 12,946 and 12,496, giving totals, and including girls over 10 years of age(!)

of 18,912 and 19,807 (the numbers are rough estimates). Bowley, as an example, examines the case of industry in more detail, and notes that of the increase from 1914 to 1918 of nearly 800,000 (2,179 to 2,971 in thousands) some 704,000 was the estimated number directly replacing men or boys.

Between February 1916 and September 1918 the Ministry of Munitions issued 46 different official Orders covering 'factories coming under control in respect of production, costs, profits, etc.' [Bowley, 1921h, p. 185] and regulating women's wages. One effect of this bureaucratic barrage was that the minimum wages for women in the munitions trades rose from 20*s.* (for about 53 hours work) in February 1916 to 40*s.* (for 47 hours) in January 1919. Average wages were considerably higher than the minimum, though for women on 'women's work' (e.g. dressmakers and cigarette makers) the average would probably have been somewhat lower than the overall average.

Bowley considers the wages for a number of trades (e.g. cotton, worsted, linen, jute, printing, pottery), and concludes

> These examples all support the general view that women's weekly rates were at the end of 1918 rather more than double the pre-war level, while their earnings had increased (owing to overtime, to undertaking men's work, or to temporary bonuses) more than this, while by the summer of 1920 the excess of earnings above rates had tended to disappear, and weekly rates were $2\frac{1}{2}$ times those before the War. Average hourly rates of course rose more, from about 3*d.* to some amount between 8*d.* and 9*d.* [Bowley, 1921h, p. 193]

This chapter is concluded with a discussion of minimum wages as established under the Trade Boards Act of 1909 covering ready-made tailoring, paper box, and lace-finishing industries, with other industries being included in 1915 and 1916. Some 320,000 women and 80,000 men were estimated to have been affected in 1914. The general idea of this and similar Acts was to fix minimum hourly rates for adults, ‘without any legal definite number of hours per week’ [Bowley, 1921h, p. 194], though piece-work meant that many workers got more than the minimum.

Augustus Webb gave *Prices and Wages in the United Kingdom, 1914-1920* a favourable review. He writes ‘Dr. Bowley, as we should expect, presents [the data] with admirable skill. The result is a highly valuable epitome’ [Webb, 1921, p. 498].

In 1920 Bowley published an article in the Monthly Bulletin of the League of Nations, criticising an official report on retail prices in a number of countries. Again he found that data had been compiled by different methods and were of unequal value. A number of countries, including Greece, Austria and Hungary, had been excluded from the study because of their unstable currencies and the sporadicity of information. Basing his remarks on the cost of food and assuming that consumption had not changed, Bowley was able to conclude only that expenditure had increased less than officially stated.

4.5 *Wages and Income since 1860*

Wages and Income in the United Kingdom since 1860 was an attempt to consolidate investigations carried out over the preceding forty years. Bowley notes in the Introduction that

> In a summary book of this kind it has not been possible, or even desirable, to exhibit much detail or analysis of the accuracy of the sources, which are to be found in the originals, [1937f, p. ix]

and it is in some respects regrettable that he was able to include neither the full material used nor details showing how figures for different industries were combined.

Wages and Income starts roughly at 1880, by which stage figures were sufficient and sufficiently precise for computation. Three periods were examined: 1880 to 1914, 1914 to 1924 and 1924 to 1937.

Chapter I is devoted to the estimation of 'the changes in average wages of the working class of the United Kingdom during the period 1880-1936, with some reference to earlier dates' [Bowley, 1937f, p. 1]. The data dictated that wages for a normal working week be examined before the effects of unemployment, illness, holidays and short- or over-time are considered. *Wages* and *earnings*, Bowley notes, are in general different things:

> the wage is taken to be the contractual time-rate, while earnings are either the receipts from piece-work or the actual amount received in the week allowing for over- or short-time. [Bowley, 1937f, p. 1]

In considering the period 1880-1914 Bowley remarks that the data for the earlier part of this period are mainly time-rates, while later Wage Censuses consisted of records of earnings. These two methods had to be reconciled.

Factors that had to be considered in moving from time- or piece-rates to earnings led to different conceptions of wage-changes[13]:

> One is the movement due to changes in wage-rates, the other the shifting of the relative numbers in occupations, normally towards higher or rising wages, which makes the increase of the general average greater than that of the average of the occupational or industrial series.
> [Bowley, 1937f, p. 4]

Bowley's examination of the data and the adjustment of the weights finally used resulted in Table 4G (the data in columns headed 'Wood' were provided by George Wood). The general movement of the average 'is the resultant of very unequal changes in different occupations and industries' [1937f, p. 7], as shown in Table 4H.

The period 1860-1880 is more difficult to work with, and only a general account of the movement of the average can be, and is, given.

For 1914-1924 Bowley relies extensively on his [1921h], and he notes that

Table 4G. General wage & earnings index-numbers, 1880-1914.

Year	Allowing for change in numbers in occupations		Not allowing for change in numbers in occupations	
	Bowley 1	Wood 2	Bowley 1	Wood 2
1880	100	100	100	100
1884	103	102	102	101
1889	110	106	105	103
1890	114	111	109	108
1894	115	110	108	106
1899	123	117	115	111
1900	130	122	120	116
1904	123	120	116	113
1909	129	125	121	115
1910	130	127	121	115
1914	138	—	130	—

> The factors whose effect was lasting were the increase in piece-rate and bonus systems, the different rates of change for skilled and unskilled labour, a specially rapid increase in the wages of some women, and a considerable change in the relative importance of industries.
> [Bowley, 1937f, p. 11]

Some of the changes are presented in Table 4I. (It appears that more than half the women listed under 'Professions' were teachers.)

For the period 1924-1937 Bowley concluded that

> average earnings of the employed working class have approximately doubled between 1914 and 1937, while working hours have decreased more than 10 per cent. In the same period retail prices have risen about 50 per cent.
> [1937f, p. 19]

Table 4H. Index numbers for separate industries. Wages or earnings, 1880-1914.

Year	Agriculture	Coal	Building	Engineering & Shipbuilding	Cotton	Wool	Printers
	1	2	3	4	5	6	7
1880	100	100	100	100	100	100	100
1884	94	107	100	94	105	—	—
1889	97	123	103	97	109	—	—
1890	100	140	104	100	110	—	—
1894	99	124	107	99	116	—	101
1899	103	136	113	103	120	—	102
1900	109	163	115	109	123	—	—
1904	110	134	115	110	124	—	—
1909	110	145	115	110	—	—	—
1910	110	146	115	110	—	—	—
1914	122	160	123	122	—	—	—

Table 4I. Relative numbers occupied in industrial groups in England and Wales, 1911 & 1921. (per 1,000 occupied)

	Males		Females	
	1911	1921	1911	1921
Agriculture	99	86	20	17
Coal	85	93	0	0
Bricks, pottery, glass, cement, &c.	25	23	8	11
Chemicals	9	12	5	10
Metals, engineering, vehicles, &c.	124	157	21	44
Textiles	45	40	136	129
Clothing	30	26	145	99
Food, drink, tobacco	28	28	31	39
Paper, printing	17	18	20	23
Wood, furniture	19	17	5	5
Building, public works	75	62	0	2
Other manufactures	18	18	16	23
Gas, water, electricity	10	13	0	1
Transport	97	96	4	8
Finance, commerce	144	127	96	147
National & Local Govt., Defence	55	76	16	38
Professions, entertainments	33	34	77	90
Personal service	52	43	386	298
Miscellaneous	35	31	14	16
Total	1,000	1,000	1,000	1,000
Numbers occupied	11,454	12,113	4,832	5,065

Chapter II is concerned with the matter of Real Wages, that is, wages as seen in relation to changes in the purchasing power of money. If over some period wages rose at the same rate as prices, one might be tempted to say that real wages were unchanged, but Bowley notes that intermediate movements may not have been parallel.

A difficulty is the absence of detailed budgets before 1904, and Bowley therefore decides to take as basis for comparison the cost of a budget of goods that seemed to be of a reasonable standard in that year. A cost of living index (dating from 1914) is given as 'the weighted average of five series relating respectively to food, rent, clothing, fuel and miscellaneous items' [1937f, p. 28] (see Table 4J). One problem in drawing up such an index arises in connexion with rent, with which rates are usually combined. The problem Bowley finds similar to that in the case of a workman in a small town whose combined rent and rates are 8*s*., and who moves to smaller accommodation in London at 12*s*. Whether the increase is worthwhile then depends on whether the amenities of the capital are more valued than those of the provincial town.

There had been criticism of the cost of living index computed for the period 1914-1936. This criticism Bowley finds to be based to a large extent on 'confusion between change of cost and change of standard of life' [1937f, p. 37]. There was insufficient information available about expenditure on items other than food, and Bowley thus gives special attention to the food budget. He presents a hypothetical family budget (food, rent, fuel, clothing and sundries) and, relying on Ministry of Labour figures, he concludes that the average town working class family enjoyed a free margin of £1 per week in 1936 as against 4*s*. 6*d*. in 1914.

'Average earnings and their distribution' is the matter of Chapter III. Bowley begins by noting that a reason for separating change from actual amounts is that the former is based on series reflecting the wages of all operatives, whereas it would be silly to average actual wages for men, women, boys and girls. Further, the age rates of wages and the kinds of occupations differ between these groups,

Table 4J. Index numbers of money wages and the cost of living, 1880 to 1936.

Year	Index numbers			Year	Index numbers		
	Wages	Cost of living	Quotient		Wages	Cost of living	Quotient
1880	72	105	69	1910	94	96	98
1884	75	97	77	1914	100	100	100
1889	80	89	90	1924	194	175	111
1890	83	89	93	1929	193	164	118
1894	83	85	98	1930	191	157	122
1899	89	86	104	1934	183	141	130
1900	94	91	103		(186)		(132)
1904	89	92	97	1936	190	147	129
1909	94	94	100		(197)		(134)

Column 4 is 100×[Column 2 ÷ Column 3], while the parenthesised figures are adjusted for increase of earnings relative to rates.

Table 4K. Industries, excluding mining & agriculture.

	1886	1906	% increase
	s. d.	*s. d.*	
Lowest decile	16 7	19 6	15
Lower quartile	20 0	23 4	16
Median	24 2	29 4	21
Upper quartile	29 5	37 2	26
Highest decile	34 7	46 0	33
Mode	22 8	23 10	
Average	24 11	30 6	22
Per 1,000 earners	%	%	
Under 20*s.*	24	10	
20*s.*–25*s.*	33½	21½	
25*s.*–30*s.*	24¼	19½	
30*s.*–35*s.*	11½	16½	
35*s.*–40*s.*	4¼	16½	
40*s.* and over	2½	16	
	100	100	

and the age distribution of males is very different to that of females. Bowley concludes that when one is concerned with changes in distribution of wages, the only reliable figures are those for men. Here the only satisfactory survey was the Wage Census of 1906 (which excluded mining and agriculture).

The figures that were available allowed the comparison for men's earnings in a full normal week shown in Table 4K (Bowley suggests that the figures 'are not reliable within a few pence nor within say 2 per cent' [1937f, p. 42]).

London is given particular consideration, with reference to Booth's *Life and Labour of the People of London* and the *New Survey of London Life and Labour.* The Survey in the latter was taken in 1929, and shows remarkable continuity in the distribution of men's wages.

Table 4L. Type of man representing families.

	1860	1880	1914
Lowest decile	Average agricultural labourer	Top of agricultural labourers	Bottom of unskilled
Lower quartile	Bottom of unskilled	Average unskilled	Top of unskilled
Median	Top of unskilled	Average unskilled	Top of semi-skilled
Upper quartile	Ordinary semi-skilled	Top of semi-skilled	Skilled

Estimates are cautiously offered for other years. Earnings were no longer divided into two groups—skilled and unskilled—as they had been a century before.

Tables are given for 'Estimated adult men's wages for a full normal week' and 'Estimated income of heads of households' in the whole United Kingdom, though it is noted that reliable estimates for post-war distribution could not be made by generalising figures for London. Table 4L shows 'the type of man who represented the median and quartile families of the United Kingdom' [1937f, p. 46].

Using Wage Censuses and other reports Bowley concludes that

> the average wage or earnings of men and boys in a full normal week, including agriculture and all other wage-earning occupations, may be put, for the United Kingdom, at about 20*s.* in 1886 and about 26*s.* in 1906.
> [Bowley, 1937f, p. 49]

In Chapter IV, 'Earnings and needs', Bowley finds it necessary for the first time to try to define the working class (neither Booth nor Rowntree, he notes, gave an explicit definition). The best he can do is to try to delimit, rather than define, this class, 'for there is no logical line to draw between the working and the middle class' [1937f, p. 54]. It is easier to define the manual working class than the middle class, and rough estimates can then be made of the proportion the working class bears to the whole population.

The constitution of the working-class had changed considerably over the period studied, and Bowley noted in his Introduction that

> When we are considering the progress of the working class we should have regard to the fact that, especially since the introduction of compulsory education, there has been a transfer of the more intelligent, at least in book knowledge, from manual labour to clerical work, teaching and other professional occupations. The existing middle class must be very largely recruited from the children of working-class parents or grandparents. [1937f, p. xvi]

Since the early 1800s wages had increased more than prices, hours of labour had greatly diminished, there had been an increase in the use of ‘luxuries’ (sugar, tea, tobacco), there had been an increased variety of amusements, the mortality rate had fallen and pauperism in Giffen’s sense was no longer a suitable measure of poverty (see Giffen [1886]).

In Bowley’s surveys published in *Livelihood and Poverty* and *Has Poverty Diminished?* (and other surveys) samples were used and the economic conditions of the households chosen were determined as accurately as possible. The aims of the recent surveys were

> to classify the incomes of families in relation to their needs over the whole scale of working-class families; to find what proportion and what numbers were in poverty; and to make comparisons from place to place and from one year to another. [1937f, p. 55]

It is thus clearly necessary to describe the use of the word ‘poverty’, making it exact and intelligible and keeping to the same description throughout. Factors of importance in such a description are food (not the question of an optimum diet, but rather the determination of a definite scale below which a family would definitely be said to be in want), clothing (necessaries would include boots, hats and socks if we regard protection against cold and rain as essential), fuel (for cooking, heating and washing) and rent (most surveys seemed to have taken the rent actually paid as the minimum).

Bowley's investigation leads him to conclude that

> The minimum as defined or described by Booth or Rowntree, and followed to ensure comparability by later investigators, is more inadequate than was formerly believed for the families where there are young children.
> [1937f, p. 58]

The problem is partly this: there are certain requirements by age and sex based on the number of calories needed. From this the cost for an adult is calculated, and this cost is then applied to the number of calories supposed needed by a child to deduce the cost for a child. Bowley notes, however, that this assumes 'that the *cost* of 1000 calories is the same for the diet of a child as for that of an adult' [1937f, p. 58]. This need not necessarily be true: for instance, children need milk, which is expensive—but other (cheaper) foods could perhaps be substituted for it.

Bowley gives an idea of the standard of living reached on his description of the poverty line for a London family consisting of a workman (earning 39*s.* weekly), his wife and two young school-going children: rent, 9*s.* 4*d.*; travel and unemployment and health insurance, 2*s.* 4*d.*; clothes, 4*s.* 2*d.*; fuel, 3*s.*; cleansing materials, 1*s.* 2*d.*; food: 19*s.*, with nothing remaining for beer, tobacco, amusements, trade-union subscriptions, etc.

A more realistic meaning can perhaps be attached to the poverty line by considering the minimum as 'the total of fixed charges on income', the surplus sometimes being used by the poor for optional purchases (such as funeral insurance, amusements, sweets, tobacco, beer and newspapers) as opposed to necessaries. These items may then be followed by the spending of money on an improved diet or better clothes.

Generalisation from London to Urban England as a whole is not permitted, and in this respect Bowley finds Rowntree to be unwise in likening the percentage in his study of York to that given earlier in Booth's survey of London. Comparisons in *time*, of course, may well be possible.

Chapter V is devoted to the national wage-bill, that is 'the aggregate paid in wages in a year in a defined country' [1937f, p. 71]. The problem here, when it comes to 'country', is that during the period of the study Southern Ireland left the United Kingdom, and this required some adjustment to published figures. From the occupation tables of the censuses of production the delimitation of manual workers and others is worked out (shop assistants prove difficult, since they appeared in different surveys sometimes as manual workers and sometimes not). The method used was the following:

> The index-numbers of average wages estimated above are the starting-point. These are multiplied by a series, also in index-form, proportionate to the number of persons employed in working-class occupations. The product is discounted by the percentages unemployed. The result is a series of index-numbers representing the change in the wage-bill. The actual amount of the wage-bill is then estimated from the Census of Wages or otherwise at any one date, and thence its amount can be computed at other dates. [1937f, pp. 72-73]

Here only those between the ages 15 and 65 are considered, and allowance is made for holidays, unemployment and sickness. Long calculation shows, for example, that the national wage-bill (in £million) was 439 in 1880, 726 in 1900, 863 in 1914, 1600 in 1924, and 1720 in 1936.

The last chapter is concerned with the national income, and once again Bowley provides no definition of this term. For the years 1911 and 1924 he merely recapitulates estimates given in his earlier, and even by this date out-of-print, books *The Division of the Product of*

Industry and *The National Income, 1924* (works we have discussed elsewhere).

The period 1924 to 1936 was difficult to handle: changes in salaries were not known, there had been two variations on the income-tax exemption limit and there had been a depression and a recovery. Only a rough estimate of the movement of income was possible, and things were similarly difficult for the period 1880-1913. The question of the evasion of income tax is a problem in this latter period, and Bowley somewhat humourously likens it to 'that of estimating the deficit in import statistics due to smuggling' [1937f, p. 91].

The general conclusion is the following:

> there was no important change in the proportion of earned to total income between 1880 and 1913 or between 1911, 1913 and 1924 ... There is a stability between the relations of the various classes of income considered. There is some evidence of slight variations within the first period, and it is futile to try to make any estimates during the war period and in the years immediately succeeding it. [1937f, p. 97]

Attention here is also given to the period 1860 to 1913, in which Bowley's [1904b] is used, but Bowley reluctantly concludes that

> I do not think that the statistics are sufficient for any fine measurements of income, earnings or wages prior to 1880; there is indeed sufficient uncertainty after that date. [1937f, p. 99]

Six appendixes, each a series of notes containing 'more technical matter and the collation with former estimates' [1937f, p. ix], follow. The first deals with the Wage Censuses of the United Kingdom, the second with the separation of the factors making for changes in average wages, the third with the table of average earnings, the fourth with retail prices, the fifth with the increase in middle-class occupations and the sixth with earlier estimates of national income.

Reviews of *Wages and Income* were detailed and exceedingly positive[14]. Henry Macrosty remarked that ‘It is superfluous to say how useful to the student it is to have the results of all this work brought together and summarised in brief compass’ [1938, p. 461], while Witt Bowden wrote ‘Recommendation of the work to specialists would be superfluous’ [1939, p. 620]—though he expressed some surprise at Bowley’s ambiguous use of terms in some of his tables.

A telling review of Bowley [1937f] and Jürgen Kuczynski [1937] was published by Frederick Brown, who concluded his observations on these two very different books as follows:

> Where Professor Bowley sees ground for optimism, though none for complacency, Mr. Kuczynski sees the horrible results of the inevitable oppression of capitalist exploitation. It is generally possible, by selecting appropriate statistics, to convey any impression consciously or unconsciously desired. If Professor Bowley is biased, it is impossible to convict him of it; for he explains his methods, reproduces the stages in his computations, and defines exactly the meaning of his results, with a minimum of interpretation or comment, leaving the reader to draw his own conclusions. ... Mr. Kuczynski’s figures are not so easily checked, and it is difficult to believe that any rational system of adjusting published data would give such results as the sharp fall in the cost of living in 1884-5 and again in 1900-1. [1939, pp. 217]

One cannot do better in summarising *Wages and Income in the United Kingdom since 1860* than by recalling the final paragraph of the review by Simon Kuznets:

> In conclusion one should note the admirable soberness and caution of Professor Bowley’s treatment, his awareness of the pitfalls of the data and of the manifold aspects of the questions upon which the data shed light, and the dry humour and pithiness of some of his comments. [Kuznets, 1938, p. 458]

4.6 RSS presidential address

In 1938 Bowley delivered his presidential address to the Royal Statistical Society. He began by surveying the change in number of Fellows since his election in 1894, noting that

> unless more candidates are found from the large number of young and eager economists and statisticians that now exists, we have reached a stationary population and a stationary income [1939b, p. 1.]

and, as Lord Stamp, whose connexion with the London School of Economics and Bowley we have noted in Chapter 1, wrote, 'What do I want to do in a stationary population?' [1937, p. 104]. The situation, said Bowley, was such as to require that economies be made, and one of his suggestions was that Fellows 'prune' their manuscripts of the irrelevant before submitting them for publication. This brought him naturally to the subject of his address, viz. 'Production and efficiency'.

Almost immediately he sets out an apology:

> I have nothing new or old to offer, and I propose instead to set problems to the Fellows... Such statistics as I may give are to be regarded as illustrative and subject to all kinds of criticism and amplification, rather than as finished products. For I have never worked at all intensively on the subject of productivity; I do not know all the sources of information nor the definitions or limitations of the material. [1939b, p. 2]

'Improvement,' writes Bowley, 'depends on greater efficiency in producing goods and in rendering services... the less effort is wasted, the greater the efficiency' [1939b, p. 3].

As an illustrative example Bowley supposes that, under the organisation of industry, and at the prices, then extant, goods and services in the United Kingdom had an aggregate value of £4,000 million—that is, about £85 per head or £300 per family per annum. Per occupied person this amounted to about 1*s.* 6*d.* per hour for 50 weeks work at 46 hours' work a week[15].

It was useful, Bowley suggested, to compare these results with those holding for 1860—i.e. in the lifetime of the oldest Fellows. In the 78 years since that time the average value of an hour's work had increased from 4*d.* to 18*d.*, consistent with the estimate that the wages for a week's work had increased in the ratio 100:350, while the actual hours worked had decreased from 60 to 46. In 1860 4*d.* would buy 3 lb. of bread, or 10 ozs. of middling meat, or 3 pints of milk or 4 miles of slow rail transport. There was no electricity or motor transport, but one could get energy generated from about 40 lb. of coal—and perhaps, Bowley noted, 'one-twentieth part of a lawyer's letter' [1939b, p. 3]. In 1938, however, the 18*d.* would buy 2 quartern loaves of bread, or 1 lb. of best meat, or 3 quarts of milk, or $1\frac{1}{2}$ miles in a taxi, or 2 minutes of a consultant's time, or 9 units of electricity.

Using the fall in wholesale prices as a yardstick Bowley concludes that 'the productivity of an hour's work had increased five-fold' [1936b, p. 4], but he hastens to add that this did not refer to services but only to commodities. If the increase in productivity, and reduction in the hours worked weekly, since 1860 could be repeated,

> we should have gone a long way towards the four hours' work per diem for the present output, and no doubt optimists would hold that with an improvement in allocation of the product we should approach Bellamy's millenium[16]. [1939b, p. 4]

Bowley considers the general question in more detail. From a statistical point of view a six-fold division of the problem seemed to be appropriate: (1) primary products, (2) energy, (3) manufacture, (4) agriculture, (5) distribution and (6) personal service. However, he notes, 'Though this division may be practical, it is hardly logical' [1939b, p. 4]. These six groups are then examined in some detail, their history during the preceding 150 years being considered and their possible future course being predicted. It is of interest to the present generation to read that, even in 1939, Bowley found need to say in his discussion of Primary Products,

> New discoveries of sources are less probable now that so much of the earth's surface has been surveyed, and for timber and other vegetable products the problem tends to be that of replacement of the violated forests and plantations. [1939b, p. 5]

Bowley finished off his address as follows:

> In concluding what has, I hope, been a common-sense review of the possibilities of greater efficiency, I make a statement which I should have thought was commonplace, if the contrary were not so often assumed. However great the increase may be in the future, we cannot enjoy its more lavish fruits in the present. [1939b, p. 16]

Presidential Addresses to the Royal Statistical Society were traditionally not a matter for discussion. However, Lord Kennet, Bowley's predecessor as President, proposed a vote of thanks (seconded by ex-presidents[17] Major Greenwood and Lord Stamp and carried unanimously) expressing the pleasure of the Society.

Kennet spoke of the appreciation Bowley's older colleagues felt for his contributions to statistics and the pleasure his younger colleagues must have taken in hearing one speak to whom they owed so much in their education. He injected a note of humour by saying

> He had been wondering, while hearing the President's analysis and suggestions regarding the growth of production and the increase of efficiency, how soon the time would come when the greater part of the human species might desist from all kinds of labour, and leave all active toil to a minority. When that time came he himself would, as usual, be on the side of the majority.
> [Bowley, 1939b, p. 17]

Further, in order to prepare himself for the future, Kennet wondered whether

> the idle part of the species would be in the position of masters or in the position of servants of the race. Were the capitalists and the aristocrats to become parasites in the sense of living at the expense of the community, or were they to be the victims of the few who controlled the productivity of the world? [loc. cit.]

Greenwood said that, having heard Bowley's address, he had begun to wonder 'first of all, whether leisure was a desirable thing, and, secondly, whether leisure *was* going to increase' [1939b, p. 17]. Put perhaps more prosaically, 'Given that material goods could be produced with progressively less effort, how far was the whole of human exertion lightened?' [1939b, p. 18]. He further praised Bowley, saying

> There had been very few statisticians who had so consistently made a purely intellectual examination of the problems which they had had to consider. Taking as his field to a large extent economic statistics, and particularly what one might call the human factor in economics, being the first authority on the study of statistics of wages, the President had never allowed himself to be diverted from the intellectual side of the problem. [1939b, p. 18]

Lord Stamp noted that Bowley's influence had been felt throughout the world. He likened it to that of Alfred Marshall, and said that that evening Bowley had 'allow[ed] himself a sort of mental explosion of the valuable fragments now lying around them' [Bowley, 1939b, p. 19]. Remarking on the new features that had appeared in Bowley's work, Stamp praised Bowley's conduct at the International Statistical Meetings—so suddenly cut short in Prague in 1938—as follows:

> Any good man could do good work in good conditions, but it took a great man to do good work under ad-

> verse conditions, and he had seen Professor Bowley recently applying his mind with full scientific devotion to abstract problems in the city of Prague with the world going to pieces all around him. Having been extruded from Prague, he continued econometrics unperturbed in Poland amid similar distractions. [1939b, p. 19]

Finally, Stamp uttered a comment that could well be taken to heart by those who still today make use of statistical computer packages without thinking about what they are doing: Bowley, he said, had expressed in all his teaching

> the condemnation of those who got busy with their statistical technique without first studying in every respect their material, going right back to the person who made it up, going into the statutory basis on which so many official figures were computed. [1939b, p. 20]

This page is intentionally left blank

Chapter 5

Miscellaneous Books

5.1 Introduction

In addition to his many papers and surveys the seemingly indefatigable Bowley published—and contributed to—a number of books. Some of these, valuable though they might have been at the time of publication, seem to have lost their relevance today, while others, like the *New Survey of London Life and Labour*, are still seen as milestones in social welfare. In this chapter we shall look at some that were specifically concerned with trade, wages and unemployment[1].

5.2 *England's Foreign Trade*

The essay for which Bowley won the Cobden Prize at Cambridge in 1892 was published, with additions and changes, in 1893 as *A Short Account of England's Foreign Trade in the Nineteenth Century, its Economic and Social Results*, and re-issued in a second edition in 1905. This latter edition is essentially the same as the first[2], with errors and misprints corrected and tables and diagrams continued to 1903. Notes in an Appendix brought certain points up to date. There are six chapters: 'Introduction', 'The French Wars, 1793-1815', 'The battle over free trade, 1815-1850', 'Success of free trade, 1850-1870', 'Trade in the period, 1870-1892, 'England's present position'. The style of this book is very different to that of other works by Bowley, and one can have no doubt of its being written as an Essay.

Even at this early stage of his career Bowley tried to avoid tables of figures, believing that more could be shown in, and deduced from, diagrams[3].

> By [diagrams] it is possible to present at a glance all the facts which could be obtained from figures as to the increase, fluctuations, and relative importance of prices, quantities, and values of different classes of goods and trade with various countries; while the sharp irregularities of the curves give emphasis to the disturbing causes which produce any striking change. [Bowley, 1905d, p. iv]

Before the first chapter there is a two-page list of the principal dates that are connected with England's foreign trade[4] in the nineteenth century, ranging from the declaration of war with France in 1793 through various commercial crises, the Irish potato famine, the opening of the Suez Canal, the American Civil War to the Cape Colony's preferential tariff in 1903.

The most important part of foreign trade, up to after the Napoleonic War, was with Britain's colonies, who, for financial reasons, were obliged to export—and import—only through the motherland[5] (Ireland was similarly treated). Trade with the East had to pass round the Cape of Good Hope; London was thus a fitting distribution centre, and Britain became an emporium for goods from the East and tropical produce. Of great importance was the trade between the United States and England. The popular view—and that of the politician—was, Bowley suggests, that trade benefitted only the vendor and not the purchaser, though traders were beginning to realise that this view was fallacious. 'All exchange is a sign of division of labour, and is an advantage to both parties concerned, for otherwise the exchange would not be made' [Bowley, 1905d, p. 11].

The indefinite division of labour, Bowley suggests, is limited 'in the social, intellectual, and moral objections to specialisation' [1905d, p. 16] (Bowley draws a parallel with the division of *intellectual* labour). The method advocated is that of the use of index numbers. The construction of such, in the standardisation of prices, is described, and Bowley states that he will generally in this essay take 1871 as the base year. In this year the value of £100 of goods is given in terms of the value of gold, and the price in any year can then be valued in gold by simple proportion.

Chapter II, 'The French Wars, 1793-1815', is begun with a brief history of the war. Particular mention is made of the Berlin and Milan decrees issued by Napoleon, that effectively put the British Isles under a blockade. Nevertheless some trade got through, mainly by devious means—ships sailing under false colours, goods landed under cover of darkness, etc.

The permanent effects of the war on commerce are also discussed. On the cessation of hostilities England was left in control of the seas. Further, her credit was better in Europe than that of other countries (like France, Germany and Russia) that had experienced fighting within their borders. A problem, however, was the general poverty that prevailed in Europe, and this resulted in England's turning her attention to her colonies and the United States as trading partners.

Free trade, the subject of Chapter III, is based on the following principles: (1) mutual advantage to each party; (2) mutual advantage for traders of different countries under all circumstances (though such trade may be unstable being liable to disturbance by war or politics); (3) if goods are cheaper in one market than another, this probably indicates the use of more efficient methods or less labour, and buying in such a market leads to a further division of labour; (4) the disadvantages or suffering that may attend the development of trade come to more than one country exactly as they do to one, and 'the temporary misfortune of the few is the price of the permanent advantage of the many' [1905d, p. 42]; (5) imports are paid for by exports, and a country's internal trade is thus only altered, and not decreased, by an increase in foreign trade; (6) governments cannot increase trade; all they can achieve is the erection of barriers.

In the 1820s it was the practical need for Free Trade, rather than its theory, that led to merchants in London and Edinburgh petitioning against ‘every restrictive regulation of trade not essential to the revenue’ [1905d, p. 47]. A commission appointed to investigate the matter concluded, among other things, that any of the restrictive conditions then holding were to continue only as a matter of necessity rather than option, a liberal ruling coming at the end of the French Wars that Bowley finds in marked contrast to the behaviour of the French in 1873 and the Americans in 1866 on the conclusions of their wars, when tariffs were raised.

It had been said that in Britain the number of Acts of Parliament concerned with import, export and custody of goods was no less than fifteen hundred. One that perhaps was most controversial was the Corn Law[6] of 1815, prohibiting the importation of wheat when the price was under 80*s.* per quarter-hundredweight and making it free when the price was above 80*s.* The Corn Laws were finally repealed in 1846, largely through the work from 1823 to 1827 of the President of the Board of Trade, William Huskisson (who achieved somewhat dubious fame as the first person to be killed in a train accident).

What were the immediate effects of the introduction of Free Trade? Wages or incomes had changed little from 1815 to 1850, but prices, and hence the purchasing power of money, had varied considerably. After the first reforms by Robert Peel the index number began to fall, changing from 103 in 1840 to 75 in 1851, while incomes and wages increased slightly. The success of Free Trade in the period 1850 to 1870 was the result of ‘multitudinous discoveries’ rather than any one cause. Bowley [1905d, p. 55] instances the following important events: (1) the opening of the first English railway (1830), (2) Wheatstone’s telegraph (1837), (3) first ocean steamer (1838), (4) settlement in New Zealand (1840), (5) reduction of duties on raw materials (1842), (6) repeal of corn laws (1846) and (7) the commercial treaty with France (1860). Despite the fact that one could neither estimate nor obtain any criterion of the enormous effects of the adoption of Free Trade, three things were patent: firstly, trade could not have expanded without the abolition of the 'suffocating restrictions'; secondly, Free Trade was actually the method, and not the source, of commerce; and thirdly, the marked beginning of expansion coincided exactly with the reductions and abolitions of duties.

The progress England made in the construction of railways led to her building the railway line from Paris to Rouen in 1842. Four thousand navvies were sent to carry out the work and teach the French labourers, and later on, while the French were found to be more skilful in certain parts of the work, 'the dangerous and hard work continued to require English hardihood' [Bowley, 1905d, p. 58]. Workers were also later sent to construct the Grand Trunk Railway in Canada and a line in New South Wales.

Perhaps in tribute to Cobden, perhaps not, Bowley now describes the reformer's 'almost royal progress' through Europe and the enthusiasm with which, in each country he visited, 'he was received by the men of most original or advanced opinions' [1905d, p. 61]. Cobden's influence led to the signing of a commercial treaty between England and France. Other treaties followed, and still other countries reduced their tariffs or became free-traders. In many cases there was a 'most favoured nation' clause, which Bowley quotes as follows:

> Each of the two high contracting powers engages to confer on the other any favour, privilege, or reduction in the tariff of duties of importation on the articles mentioned in the present treaty which the said power may concede to any third power. [Bowley, 1905d, p. 63]

These treaties naturally led to increased trade with the pertinent countries. For instance, imports and exports from and to France increased from £18 million and £5 million in 1860 to £38 million and £12 million in 1870.

The next section is concerned with the American War and its effects and with trade with India and the United States. An examination of English trade figures showed ‘some great disorganisation’ between 1860 and 1868. The fluctuations were in the cotton industry and related directly to India and the United States. The war between the Southern and the Northern States resulted in a marked decrease in the supply of cotton and an immense increase in price. During this time, however, the cotton fields in India and Egypt grew considerably. The whole affair showed the effect that dependence on people of another continent may have serious effects—paralysis of trade or thousands thrown out of work—in the case of ‘an unexpected rebellion, a new political party or a change in fiscal laws’ [Bowley, 1905d, p. 67]. In the case of cotton, however, things looked somewhat brighter, for it seemed highly unlikely that there would be disturbances in the same year in both hemispheres.

The war in the United States affected her finances so seriously that a severe blow was dealt to Free Trade. Between 1861 and 1863 tariffs were doubled to meet war expenses, and increased to a maximum of 46% in 1868. British exports thus experienced violent fluctuations, though by 1893 (when Bowley published his essay) things seemed to have improved. ‘The Americans have the English qualifications for work. ... it is not the American’s declared intention to separate himself from the rest of the world; his hope is to tax the foreigner and sustain no loss himself’ [Bowley, 1905d, p. 70].

While exports to India from the United Kingdom had progressed regularly throughout the nineteenth century, Bowley says that ‘The East Indian Company did more harm by monopolising the trade outside their immediate territory ... than by neglecting to develop the resources of India’ [1905d, p. 71]. The opening of the Suez Canal altered the direction of Indian trade.

In considering England’s foreign investments and her balance of imports and exports Bowley divides her trading partners into three:

> those who receive from us in imports more than they return in exports; those who receive and return the same quantity; and, the largest class, those who return more than they receive. [Bowley, 1905d, p. 73]

No countries, he asserts, remain for long in the first class, though Australia was there when gold was discovered. Of the other British Colonies South Africa was in the first class, Canada in the third and Australia and India 'fluctuated' near the second. The United States, Germany, Russia, Holland, France and Belgium were in the third.

Chapter V deals with trade from 1870 to 1892, beginning with the consideration of the crises of 1873 and 1883. Bowley's diagrams of imports and exports from 1870 to 1892 indicate a fall in exports that started in 1873, reached a low point in 1878, and was followed by an increase until 1882, the year after which there was a fall in both imports and exports, with recovery only beginning in 1886.

Bowley finds that commercial depressions in Britain tended to have a ten-year cycle throughout the nineteenth century. The general cause of the depression of 1873 he ascribes to the inflation of 1871 and 1872 that followed the Franco-German (or Franco-Prussian) war of 1870-71.

The second section in this chapter begins with a study of the history of railways. In 1840 England had 800 miles of track, the Continent 800 and the United States 2,800. By 1850 these figures were 6,600, 7,800 and 9,000, and by 1890 19,800, 110,200 and 156,000. Development of railways on the Continent (and in India, Australia and elsewhere) led to British exports of iron and steel rising from a value of £8,000,000 in 1850 to £24,000,000 in 1860. Drawbacks to railway construction include (1) weekly wages are enormous during construction, (2) materials are bought at inflated prices and (3) no return is seen until the whole project is finished. Thus the British railway boom of 1869-1872 'was the most effective cause of the long depression which followed' [Bowley, 1905d, p. 89]. The boom of course also had a great effect on British iron and steel production and export and her ship-building trade.

British railways had to compete with canals and navigable rivers in the matter of transport. Of the time Bowley was writing the choice was between rapidity and convenience on the one hand and economy on the other. Progress on the British canal system was slowed down by ‘the sudden mania for railways’ [Bowley, 1905d, p. 91]. However transport had become so good (and competition so keen) that wheat was delivered at the same price at London, no matter whether it came from Australia, Canada, Germany or Essex, no matter what the cost to produce the grain might have been, and no matter whether it was delivered by railway, canal or sea.

There is then a short digression on silver and its use, as opposed to that of gold, as legal tender in various countries, and the effect of changes in the value of silver on British trade. While Britain and many of her dependencies used gold, China, India and other Eastern nations used only silver.

Bowley does not discuss bimetallism (‘the theory that by universal agreement the ratio between gold and silver can be fixed and kept constant’ [Bowley, 1905d, p. 101]) here, though he writes

> The objections to its adoption are chiefly based on the apparent impossibility of coming to an agreement, and the alleged impossibility of preserving the fixed ratio without immense loss; it is also contended that bi-metallism would not have the remedial effects that its advocates suppose. [Bowley, 1905d, p. 101]

The final section looks at the steady increase of imports and exports and its effect on the population. Although the depression that began in 1873 was severe, the general fall in prices that took place in conjunction with the decreased value of imports ensured that the amount (as measured in goods and not in value) of foreign trade had actually increased. The total imports and exports are as given in Table 5A. Shipping values showed a corresponding increase.

Estimation of the proportion each person’s income contributes to the purchase of foreign goods requires certain data: (1) the total value of imports, (2) the proportion of raw materials to goods available for consumption and (3) the population at every date. Using these data and computing index numbers relative to some base year Bowley finds that the quantity of goods imported for each person

Table 5A. Total imports and exports.

Year	Millions of £
1870	547
1873	682
1876	631
1879	611
1882	719
1885	642
1888	685
1890	748
1895	703

(or for each earner of an income?) has continually increased from 1840 to 1905. Though this shows that depressions do not affect the consumer disadvantageously one should not conclude 'that depressions are good for commerce or for the production of wealth' [Bowley, 1905d, p. 105].

It is also shown that real incomes, i.e. 'wages or salaries reckoned by their purchasing power' [Bowley 1905d, p. 105], have continuously improved (apart from short periods) since 1775. Bowley does warn, however, that the figures presented

> merely state that with the average income there could be obtained in 1901 three times as much of many of the commodities commonly consumed, directly or indirectly, as could be obtained with the average income in 1820; and similarly for other years. [Bowley, 1905d, p. 106]

Missing from the calculation were things like 'rent, the value of land, the difference between wholesale and retail prices, the price of personal services, rendered directly or by officials' [Bowley, 1905d, p. 106]. Further calculation shows that while the quantity of imports has increased for each person, the fraction of real income that went on imports had increased very little since 1871.

The last chapter is a survey of England's present position, with special emphasis on the relative importance of her greatest industries and the circumstances affecting her hold on them. Three divisions are considered: minerals and hardware, textiles, and transport.

At the time Bowley was writing the coal and iron trades were in a transitional state. The supply of coal, a most important trade component, was not expected to last more than a generation or two unless new fields were discovered. It was thus probable (though Bowley noted that stress should not be laid on such predictions) that more coal would be required for home use and less would be available for export.

When it came to the textile trades Britain's command of the cotton trade was perhaps less strong than it had been. Other countries had started their own manufacture for their own needs and for export. In 1861 the percentage of cotton yarn and manufactures among total British and Irish exports was 37%, and in 1890 it was 28%. Manufacture of such goods had increased in India, where factories, only established after the American Civil War, made the export of cotton and the import of yarn no longer necessary. This abolition of double transport meant that in 1888 the Indian spinner had an advantage of $1\frac{1}{2}\,d.$ per pound over the Lancashire spinner.

Bowley's final conclusion in this section is that with the inevitable rise in the price of coal, export of iron and steel would decrease. The case of cotton was more doubtful: much depended on India and the cost of coal.

Even if British exports were to cease altogether the services of British ships (our "invisible exports") would pay for imports to the amount of £90,000,000, and Britain would still rank in trade as second only to France, Germany, the United States, Holland, Belgium and Austria. (Britannia, apparently, no longer ruled the waves!) In 1902 the United Kingdom had (measurements in thousands of tons) 10,055 of shipping in her merchant navy, while the United States had only 883 for foreign trade but 4,915 for lake, river and home trade.

A table of British exports shows that India (together with

Ceylon) was the best customer (£31 Mn. and £34 Mn. in 1891 and 1903 respectively), followed closely by the United States (£27 Mn. and £22 Mn.), Germany and Australia. Roughly equal quantities were exported to Europe and to British possessions. The decrease in exports to the United States Bowley attributes in some measure to their tariff.

In his discussion here Bowley once again praises English workers and their working conditions, saying 'the English artisan, with his skill and common sense, very soon understands new machinery' [1905d, p. 129] and mentioning 'the energy, common sense and inherited skill of the English labourer' [op. cit., p. 132]. Although claiming that 'wages ... are higher in England than in any European or Asiatic country' [op. cit., p. 129], he notes that the important thing is not the wage but the amount of work done for a given quantity of wages. Examples are instanced in support of the statement that competition of badly paid labour is not necessarily injurious—for instance, in 1893 wages in Mexico were 10 to 15 cents per day, while in New England they were about $1 per day. Yet New England competed successfully with Mexico.

While it was true that 'badly-paid labour is not necessarily the most profitable' [Bowley, 1905d, p. 130], certain matters needed to be considered. Firstly, Country A is handicapped if wages increase faster than efficiency while wages and efficiency increase at the same rate in Country B. Secondly, comparison of wages in two countries must take account of the cost of food and other necessaries. Thirdly, if the standard of living is higher in A than in B it may mean that better living results naturally in better work, or it may mean that in B less is spent on useless luxuries.

In concluding this section Bowley writes that if, 'in the distant future', British foreign trade should decrease (as a result of new discoveries, increased skill or natural advantage elsewhere)

> it will mean that Englishmen will gradually forsake the home country, that English enterprise will be successful in some other quarter of the globe, and that England will

> still be the home and mother of successful and multitudinous offspring. [1905d, pp. 132-133]

A touch of chauvinism?

Imports and agriculture are dealt with in the third section. 'Our foreign trade,' writes Bowley, 'is to a great extent an elaborate machinery for supplying us with food' [1905d, p. 133]. Of the £470 millions'-worth of goods reaching English shores some £200 millions'-worth is food. Part of this food bill is paid by the cotton trade, exports of textiles realising an excess of £45,000,000 over imports of raw materials. Similarly the iron and steel trades contribute £50,000,000 to pay for imports.

Mention is made here of the law of diminishing returns, which Bowley illustrates as follows. Suppose that improved methods would lead to more arable land being used for the production of corn and hence to a reduction of imported corn. However it was evident that sufficient corn for home needs could not be obtained, and either more and more less suitable land would have to be used or productive soil would be overburdened to its detriment. Thus

> as more grain was demanded, increasing labour for each new increment would be necessary, and the price would rise with each increase of supply. [Bowley, 1905d, p. 137]

As things actually were, however, corn-growing land became pasture, the price of grain fell and corn could be more cheaply obtained from foreign countries.

Bowley summarises the historical aspect of the agricultural question from the repeal of the Corn Laws. From 1870 onwards the competition of foreign countries where land and labour were cheap or the soil readily yielded crops affected English agriculture disadvantageously. Labourers therefore moved to industry to help pay for imports, and while artisans' wages rose, so did those of agricultural workers, though not as rapidly.

Bowley did not however altogether dismiss the idea that English agriculture could again improve. Nevertheless he viewed with some measure of alarm the exchange of the relatively peaceful, healthy and easy-going country life for the bustle, crowding, unsanitary conditions and strain of the city.

Section 4 contains the conclusion of the work. The first effect is on individual incomes: money laid out on foreign goods will often go further than that on home produce. It was mainly the working classes and those with small incomes who benefitted from the increase of purchasing power, the wealthy benefitting less because they were able to buy food at almost any cost. The variety and cheapness of home goods could also be partly attributable to foreign trade, foreign countries being able to use modern inventions to produce goods more cheaply.

There is also some discussion of the political effects of the growth of foreign trade. A war, for instance, might so affect merchant shipping that peace would be swiftly concluded.

There were, however, unpleasant aspects to foreign trade as well as good: an increase of pace led to an increase of risk, a great development of competition—its evils and its benefits—and 'an endless complication of interests, methods, and chances' [Bowley, 1905d, p. 145]. Periodic commercial depressions were now more wide-ranging and of longer duration, and the effects of such depressions were harder to foresee and calculate. Man seemed to be set on a phrenetic course—like a dog chasing it's own tail. Yet gloom was not to be Bowley's final message for his reader.

> The history of this century may be repeated; nations learn to be at peace with one another, if the burden of vast armies and the sense of the silent majority induce disarmament; the advantages of Free Trade may be rediscovered; as steam and mechanical inventions in this century, so electricity and physics in the next may further increase man's power over nature; and perhaps we shall see the beginning of an expansion of foreign trade, which will surpass even the unprecedented growth of this century. [Bowley, 1905d, p. 147]

5.3 British Trade in Wartime

Based on four lectures given at the London School of Economics in January and February 1915, Bowley's analysis of the monthly statistics from 1906 to 1914 resulted in the publication in the same year of *The Effect of the War on the External Trade of the United Kingdom*. The declaration of war on Germany sent an immediate shock through the United Kingdom's external trade, and the aim of these lectures was to examine the effect of this shock and the adjustments made to cope with it. Things were however moving so fast that Bowley suggested that the historian had to become a journalist, the journalist a reporter

> and the statistician must forget his customary caution and hesitation, and offer, with a confidence that is more apparent than real, crude results and undigested opinions, if his work is to be of immediate practical service. [Bowley, 1915a, p. 1]

There are four chapters: I. Values of imports and exports in the aggregate, II. Trade in the principal commodities, III. Aggregate quantities and prices, and IV. Trade with special countries. There is also an Addendum on trade in January 1915.

After the Introduction to Chapter I Bowley gives figures for the trade of the United Kingdom (excluding bullion and specie) from 1901 to 1914. His conclusions were summarised as follows:

> trade as a whole was very prosperous till the end of March 1914, and was in July still on a level that would have reckoned extremely high in any year prior to 1913. [Bowley, 1915a, p. 11]

In the matter of the balance of trade of imports over exports (with bullion and specie now included) things seemed to be healthy, the excess in 1914 being the greatest recorded at any time (excluded, of course, was any money due to, or owed by, the enemy). Bowley decided, in Chapter II, to limit the question of trade in commodities to those that appeared to be most important in aggregate value, rather than those that were perhaps of particular interest at that time. Quantities of commodities, their prices and countries exported to or imported from are considered separately. The consideration of countries presented unique problems in that countries to which goods are consigned may not in fact be those of final destination, and similarly goods imported from Country A may well have been produced in Country B. It is perhaps interesting today to read that in 1914 the United Kingdom imported three-fifths of her supply of petroleum from the U.S.A., and one-fifth from Russia or Roumania.

After 'the catastrophe' of August 1914 the value of exported manufactures declined sharply (similarly for imports), though this was perhaps not as widely felt as it might have been since many men were absorbed into the armed forces and many others into the provision of the necessaries for war. Surprisingly, the supply of food seemed to have been unaffected. In the matter of exports only herrings and coal were important commodities in the 'food, tobacco and materials' section, while metal products, machinery and textiles feature under 'manufactures'.

To compare recent with previous trade Bowley proposes, in Chapter III, to remove the effect of prices. To this end average prices are calculated by dividing the value of goods (both imported and exported) by their weight or quantity. This is done for each of the commodities listed on three pages for the first half of 1914. The goods for the first half of 1913 and each month in the second halves of 1913 and 1914 were then re-valued at these prices. These lists essentially covered those commodities that accounted for some 70% to 80% of the value of all commodities, the residue being calculated by proportion.

One possible source of error lies in this proportional calculation for the residue: exceptional price changes may not be catered for. Another source of error may arise in the wideness of the categories used (e.g. while woollens are all included together, the export of the cheaper ones might have decreased more than that of the more expensive). The change in the quantity of trade was marked most significantly by the difference in imported food and materials. The greatest increase had been in the price of sugar: with the average for January to June 1914 taken as 100, the average for 1913 was 103, with that for December 1914 being 188. Beef, wheat and mutton followed.

The final chapter is devoted to the matter of trade between the United Kingdom and foreign countries. Difficulties arise here in view of the time taken for official statistics to be published.

As expected, trade with Germany and Austria-Hungary declined considerably during the second half of 1914, and trade with Russia, France and Belgium also dropped. These five belligerent countries took some one-sixth of the United Kingdom's exported manufactures. Export trade with other foreign countries also declined, it being almost normal only with Spain and the United States. Aggregate imports from the Empire in the third quarter of 1914 were almost normal, with an excess from Canada balancing defects from New Zealand and Australia, and a small increase from India balancing a corresponding fall from South Africa. The export position, though not as favourable, was still good. The final conclusion is:

> it appears that our dependence on foreign and colonial supplies and our possible vulnerability at sea have had as yet hardly any visible effect on our production or consumption; for prices must rise, credit could be temporarily disorganised, capital cease to accumulate, production be checked and industry diverted, in any country engaged in a serious war, whether it be insular or continental, trading or self-sufficient. [Bowley, 1915a, p. 54]

5.4 *The Third Winter of Unemployment*

In August and September 1922 a committee including Bowley undertook a survey of unemployment in the United Kingdom at that time.

(The names of the committee members, given in our bibliography, were listed in alphabetical order in the preface to the report, with John J. Astor's first.)

The aim of the investigation, it is distinctly stated in the preface, was not to formulate a policy or to analyse the deeper causes of unemployment in general or the depression then being experienced, but rather 'exclusively to provide objective information of which account must be taken in any policy' [Astor et al., 1923, p. vi].

The report is in two parts: the first part is a general survey by the committee, while the second consists of local reports by investigative collaborators. There are also appendixes. While we shall look in some detail only at Part I, the reports in Part II make fascinating reading, and they can be thoroughly recommended to all who, in the light of the present world economic climate, want to see the results of a severe depression and the methods implemented to cope with it.

Part I has seven chapters, the first beginning as follows:

> The years 1921 and 1922 are the worst in the records of unemployment in this country. In only one month in the present century before 1921 did the percentage of unemployed trade union members exceed ten; in no month since March, 1921, has it fallen below fourteen. [Astor et al., 1923, p. 3]

The autumn of 1922 thus heralded the start of the third winter of unemployment. While a lower proportion of workers may by that time have been unemployed, nevertheless the strain had not decreased, savings and other reserves that had been available having become exhausted. While the class that had been worst off before the war was now receiving relief, the workers who were better-paid, more skilled and responsible were now 'feeling the pinch'.

The extent of the problem is considered, difficulties presenting themselves in that not all the unemployed were recorded in the Employment Department Records and that the general average hid the intensity of the depression in the trades that were worst affected.

When it came to the matter of the distribution of unemployment major elements were found to be the problem of unemployment due to trade depression and the effect that the war had had on industrial development. The survey had also examined the prospects of re-absorption, and to this end districts that showed an abnormal increase in male population between the censuses of 1911 and 1921 were investigated, with similar attention being paid to industries that showed an increase in male workers (unemployment was a less serious problem among women).

The measures of relief that had been introduced were next examined. These included the Unemployment Insurance Scheme (now becoming more relief than insurance), the Poor Law relief [7], Governmental assistance with attempts by local authorities to provide work and by stimulating private firms to expand. The effects of the various relief measures emerged from the studies presented in Part II, but here the conclusions were summarised as follows:

> (1) That the worst effects of unemployment in the way of privation [8] and physical deterioration have been prevented; (2) that the chief incidence of distress is on a different section of the wage-earning classes from that on which it fell in pre-war depressions; (3) that the demoralisation that, according to pre-war theories, would have been expected to result from the provision of maintenance without work has not yet shown itself.
> [Astor et al., 1923, p. 10]

Interestingly, the investigation seemed to show that *health* had not suffered. It transpired too that the unemployed preferred 'honest earnings' [Astor et al., 1923, p. 11] to receipt of the dole. On the matter of demoralisation the Committee was more cautious, saying that 'Maintenance without employment may be demoralising, but unemployment without maintenance is much more certain in its demoralising effect' (loc. cit.) [9]. An exception to this, though, was the case of the young men who had spent years in the army while older men were being trained for and taking up careers; they, it seemed, were liable to severe demoralisation.

The Government's encouragement of relief work, it turned out, had not been entirely successful—for example, conditions attached by local authorities to such work had 'tended to defeat the object in view' [p. 12], since relief work often increased the cost of works and thus neutralised the financial assistance provided.

Chapter II is devoted to a discussion of the extent of the unemployment problem. While the measurement of unemployment was usually the number of persons unemployed in insured trades as a percentage of the total number of insured persons, this figure was subject to certain qualifications (for instance, the figures relating to women were unreliable, as were those for boys under 18 years of age). Figures giving the numbers of unemployed and the percentages of the insured were presented for the principal industrial groups in each of eight divisions of Great Britain and North Ireland. Unemployment was found to be most severe in Scotland and North Ireland, the main stress being in the engineering, shipbuilding, iron and steel trades and in the districts where these were of most importance.

The abnormal conditions of the post-war trade situation made the re-absorption of the unemployed after the depression 'a highly speculative economic problem' [p. 18]. To this end two specific matters were examined: (1) the abnormal increase of the male population in particular towns, and (2) the abnormal increase in particular industries. The question of short time, or under-employment, had its own problems, since short time varied from trade to trade and even from factory to factory—for example, worsted combers and spinners were very busy, weavers were slack, and some woollen mills were short of orders while others were working overtime. Rough estimates suggested that in industry in September 1922 under-employment was about half as considerable as unemployment.

'The public provision for unemployment relief' is the topic of Chapter III. The matter is best handled, it is suggested, by considering the Provision of Relief without Work and the Provision of Work.

The provisions of the Unemployment Insurance Scheme of 1911 and its various changes over the years are described. All employed in specific industries were to be insured, with contributions being made by the workman himself, his employer and the Government. By 1920 the scheme covered all who enjoyed industrial employment, some 12,000,000 in all. Unfortunately it was only when the depression had started that the law whereby a general and compulsory contributory insurance scheme came into operation was enacted, and the scheme could therefore not build up sufficient funds to meet later demands on it. Further, it was not a complete relief scheme.

On the 8th of November 1921 the Unemployed Workers' Dependants (Temporary Provisions) Act became law, thus ensuring payment of temporary grants to unemployed workmen, who were receiving benefits, towards the support of their dependent children, wife, housekeeper or dependent husband.

Large numbers of the unemployed relied on the Poor Law for relief, though there was a 'gap' between the relief provided by this law and the Insurance Scheme[10]. Variation in scales of relief were caused by the different administrative policies adopted by the different Poor Law authorities.

One further form of relief was the provision of school meals by local education authorities. In the year ending 31st March 1922 just under a seventh of school-going children were fed, about 100 meals each in the year; more explicitly, 592,000 were fed from a school population of 4,110,000 receiving 60,676,000 meals in all.

As a final factor the report mentions war pensions received by ex-service men. Widows and dependants also received pensions, and these pensions, though not intended to relieve unemployment, were certainly of help in so doing.

The provision of work for the unemployed is the subject of Chapter IV. Local and State-Aided relief works are first discussed, the latter being funded, or at least assisted, by grants made by the appropriate Government Department and the Treasury, acting on the advice of a special Unemployment Grants Committee, the money being either a direct grant or a loan. These measures were mainly directed to districts where the problem was particularly urgent, but the Trade Facilities Act and the Export Credits Scheme were introduced to stimulate a revival in industry in general. Essentially these schemes allowed the facilitation of trade with less important European countries, the granting of credits to British exporters and, in cases where the risks seemed abnormal, the undertaking of insurance.

Chapter V is concerned with the cost of unemployment. The obtaining of exact figures was impossible, so only the approximate expenditure on relief could be presented. The main sources of income for the unemployed were the Unemployment Insurance Act, the Poor Law and Relief Works. For the year ending the 31st of March 1922 the first source had provided an estimated £67$\frac{1}{2}$ million, and the second about £7$\frac{1}{2}$ to £8 million. The contribution of Relief Works was harder to estimate. Such works take time to organise, and in some cases no payment may have been made on Government guarantees. Nevertheless the Committee estimated such relief at £6 million. A further £1$\frac{1}{2}$ to £1$\frac{3}{4}$ million had been spent on miscellaneous items such as school meals and valuation of risk in guarantees.

When it came to the question of the distribution of the cost the Committee found two points to be of importance: '(1) how far is the cost being met out of current taxation and contributions? and (2) in what proportions do different authorities and agencies contribute what is raised currently?' [Astor et al., 1923, p. 63]. The general conclusion was that

> the Unemployment Insurance Scheme is by far the most important element in the public provision for unemployment, and that the cost of it, and therefore of relieving unemployment, is borne in the main by industry itself, by the contributions present and future of employers and workpeople in work. [Astor et al., 1923, p. 65]

An analysis was also done of the cost of unemployment to the nation, it being found that in 1921-2 maintenance at the Poverty Line or above would cost about £200 million, or from $5\frac{1}{2}\%$ to $7\frac{1}{2}\%$ of the National Budget. Of this the unemployment insurance payments amount to somewhat less than half the amount required.

Chapter VI deals with the effects of the depression—on both the unemployed and their dependants, and caused by the depression itself and the methods adopted to alleviate the situation. Physical distress seemed to be of little significance, it being noted that insurance allowances, Poor Law relief and school feeding schemes had done much to improve matters. It is particularly noted here that the Medical Officer of Birmingham had stated that 'the health of Birmingham is better after two years of trade depression than it has ever been before' [Astor et al., 1923, p. 70]. This was supported by evidence gathered in other towns, and even where the strain was being shown by adults, the children seemed to stay healthy. The reason was patent: 'Health is better than in pre-war depressions, because the pre-war starvation is prevented' [Astor et al., 1923, p. 70].

More of a problem was the mental strain experienced. Different classes reacted differently. Thus those who in good times were low-paid or irregularly employed now had their incomes maintained (and in the case of the worst-off the family situation might actually have improved). Those who in the 'fat years' were well-paid, skilled and regularly employed artisans and tradesmen (and even the lower rungs of the salaried class) felt the strain of being unemployed more.

> The insurance allowance represents a much greater fall from their accustomed standard of living; they are much more reluctant to seek Poor Relief to supplement insurance benefit in "gap" weeks and in case of exceptional family need; the relief works instituted are usually unsuitable for them; and the worry of enforced idleness is more oppressive. [Astor et al., 1923, p. 71]

The effects of the depression were cumulative. The skilled artisan first suffered a reduction in wages, then short-time, then frequent periods without work, and then complete unemployment.

Discussion of public relief raises the spectre of demoralisation, and a report issued by the Ministry of Health stated that some of those who had received Poor Law assistance 'are now suffering the progressive deterioration which inevitably attaches to the condition of being maintained without work' [Astor et al., 1923, p. 72]. This was specifically examined in *The Third Winter*, it being noted that 'The true demoralising influence—of which the wage-earners are only too conscious—is the loss of regular useful occupation to exercise a man's powers and sustain his self-respect' (loc. cit.).

Discarding the 'vague and question-begging' word 'demoralisation' the Committee investigated whether the mental and spiritual state of the respondents could be measured. Aspects examined were the following: (1) did there appear to be an increasing inclination to depend on public relief and a similarly increasing reluctance to work? (the answer seemed to be 'no'), and (2) what was the effect on the spirit of the former workers, now unemployed? (clear evidence of worry and mental health that in some cases was affecting physical health, an increase in gambling and a decrease in thrift and foresight). Young men who had been in the army during the war seemed to be exceptional. Not having had the opportunity to become used to industrial work and possessing no special skills some (many?) of them seemed fairly happy to exist on public relief. 'Their case presents a special problem of progressive demoralisation, and places a special responsibility on the society that let them come into their present condition' [Astor et al., 1923, p. 74].

The local investigations showed little evidence of political unrest arising from the unemployment that had accompanied pre-war depressions.

Alleged general causes of the attitude to relief measures as stated by influential business men in districts studied included the following: (1) 'the dislocation of European markets and the uncertainty of the economic future of Germany' [Astor et al., 1923, p. 75] and (2) 'the uneven movement of wages and prices' [op. cit., p. 76].

The last chapter is concerned with some general conclusions. In the matter of the provision of relief it seemed that the absolute physical suffering experienced in less severe pre-war depressions had been prevented. 'The chief defect of the present provision is that the relieving authorities overlap, and divide the work between them on no consistent principle' [Astor et al., 1923, p. 77].

> We believe that the principle of contributory insurance is the proper principle on which to base provision for ordinary trade fluctuations; the extension of the Insurance Act, therefore, to meeting other needs seems unfortunate and inexpedient. [Astor et al., 1923, p. 79]

Other needs are (1) the exceptional distress caused by the abnormal post-war world situation and (2) exceptional distress due to special circumstances that arise in individual cases (e.g. a large family of dependants). The Committee found that national finances should be used in the first case while the second need should be met by local organisations.

Grounds for grave concern in the extensive growth and continuance of relief include the following: (1) 'the expenditure on allowances to unemployed persons is a charge, that tends to increase, upon a fund that diminishes in proportion to the increase of the charge upon it' [Astor et al., 1923, p. 81], (2) 'the danger of sapping independence is not the only danger that a system of allowances involves. There are also to be considered the possible indirect effects of allowances in retarding wage-adjustments which may be needed to make trade recovery possible' (loc. cit.) and (3) there is still a danger of demoralisation.

When it came to the provision of work the Committee left no doubt on its view of ordinary relief works as a remedy for distress.

> They are uneconomical, suitable only for general unskilled labour, and calculated to impair rather than maintain the industrial quality of more skilled workers. The only use of relief works would seem to be as a test of willingness to

> work in the case of youths demoralised by army service, or other unemployed persons whose *bona fides* there is reason to suspect. [Astor et al., 1923, p. 82]

The Trade Facilities Act (then one year old) was held to be 'the most novel in principle' of measures introduced to relieve depression despite the fact that it had done nothing decisive in the way of relieving unemployment in the present depression. It was however encouraging that the Committee charged with the implementation of the Act had been able to carry out the experiment. Some positive and non-negligible results had indeed been achieved, for instance: (1) some very useful schemes had been brought to fruition and (2) it would appear to be possible to provide encouragement and stimulus so that this kind of development is accelerated during a trade depression. Negative results were also evident: for example, (1) unless expanded, the scheme would probably not help to control the course of a general trade depression, (2) the scheme would do little to provide direct relief of unemployment and (3) present experience had shown that there was a danger in the Government's attempting by this kind of scheme 'to supplement the existing financial machinery of banks, issuing houses, etc. for financing ordinary commercial enterprise' [Astor et al., 1923, p. 87].

The main practical conclusion to be drawn from the various attempts to provided employment was the following:

> in general the Government's attempts to stimulate employment will be most effective, if they are not restricted to particularly depressed localities and to the kinds of work that can be done by the local unemployed in these localities. [Astor et al., 1923, p. 89]

Reviews of *The Third Winter* were generally favourable, even if the reviewers perhaps tended to dwell on the gloomy picture painted by the survey in general[11]. Thus Eveline M. Burns wrote

> It must have been a sad day for the *Daily Mail* when the *Third Winter of Unemployment* was published! ...

> The book as whole is disheartening to read. It is a story of industrial paralysis and waste ... of dull misery and suffering; of repeated and fruitless efforts to find work; of dwindling savings and more constant visits to the pawn-shops; of inadequate though well meant Government attempts to enable people to "carry on". [1923, p. 247]

In his review Otto Mallery singled out by name 'the ever illuminating employer-statesman B. Seebohm Rowntree' and 'the distinguished economist A.L. Bowley'. Perhaps more than other reviewers Mallery focused on the positive rather than the negative aspects of the report. He noted that

> To all believers in the progress of social science the condition of the workers during this long period of unemployment of unprecedented magnitude is most heartening. Investigators find that starvation, privation and physical deterioration have been largely prevented through the measures enumerated. Public health has not declined. [1923, p. 224]

Mallery found the main contribution of the study to be the examination of the relative effectiveness of the different methods that had been introduced to fight unemployment. He also drew a comparison with the 1921 findings of the President's Conference on Unemployment in the United States. The recommendations of this Conference, however, were more concerned with the policies of private employers rather than governmental assistance, direct or indirect, to industry or the unemployed.

5.5 *Is Unemployment Inevitable?*

The investigations in *The Third Winter of Unemployment* were continued and published as *Is Unemployment Inevitable?* in 1924. The book is in four parts: Part I, Survey and Forecast; Part II, Economic

Memoranda; Part III, Condition of British Industries and Part IV, Statistical Inquiries (we shall consider only the first and fourth parts here). Apart from the collectively written Part I the chapters in the other three parts are individually written memoranda. Bowley on his own was responsible for two of the three of the chapters in the fourth part, the remaining chapter being a collaborative effort with Frank Stuart.

The first chapter of Part I sets out the scope of the problem. The subject of unemployment being so vast, only two aspects of the problem are considered in this study: the Trade Cycle and 'the possibility of finding employment for the steadily increasing industrial population of Great Britain in the changed conditions of the world, without a serious reduction in the average standard of life' [Astor et al., 1924, p. 3].

Natural unemployment may be due to a number of things: seasonal unemployment (the building trade, for example, is at its best in summer); changes in fashion, in industrial methods or in class of goods consumed (e.g. motor cars, or the substitution of steel for wrought iron); the transfer of industry from one place to another (workmen possessing homes cannot move freely, especially when there is a shortage of housing); loss of industrial capacity on the part of the workers (old age, or mining pits full of water—in some cases since the mining dispute of 1921) and finally the number of industrial workers in the country is sufficient to carry on all the old-established industries in good times but unemployment will ensue when times are hard.

In connexion with the future occupied population it is mentioned that Bowley has shown that if emigration continued as in the decade preceding the 1911 census, then

> 1. The problem of the increasing working population is one of the next few years only. 2. Normal emigration would reduce this problem to comparatively small dimensions. 3. If emigration became normal, the immediate

> problem could be entirely removed by raising the school age to 16, which would withdraw from the labour market some 700,000 lads. [Astor et al., 1924, p. 9]

Chapters II-IV are concerned with the Trade Cycle. In Chapter II the well-known phenomena of the ebb and flow of trade in (then) recent experience is examined. Countries, in particular industrial countries, that lived by exchange were more sensitive to world-wide movements than those mainly dependent on their own agriculture. Here some measure of insurance was provided by having a large and wide-ranging foreign trade.

In Chapter III analysis and causes of the trade cycle are identified. Noting that there are found to be fluctuations in production and trade, the authors wonder why these ripples grow into tsunamis. 'There is no simple answer to this conundrum ... a very large variety of explanations have been given' [Astor et al., 1924, p. 26].

A detailed discussion of a normal trade cycle is provided together with explanations therefore (starting with Jevons's connexion of the trade cycle with sun-spots—not so surprising when one recalls the intimate relationship between agriculture and manufacture—and passing on to an explanation in terms of flaws in the economic system—e.g. 'over-saving' or under-consumption).

On the matter of solutions to trade cycle problems the authors note at the beginning of Chapter IV that

> there is unanimity of opinion ... that control of either credit or currency, or both, might be used to limit upward movements of prices and production, when a forward swing of the cycle is in progress. [1924, p. 41]

Whatever the cause of trade cycles may be it is common cause that 'production ... tends to run ahead of consumption, and stocks accumulate' [Astor et al., 1924, p. 47]. Information is needed under a number of headings: for example; '(1) The course of prices, (2) The volume of production of important commodities and (3) Stocks of leading commodities' [Astor et al., 1924, p. 49]. A suggestion is made as to a way to gain control over the trade cycle (the numbers here refer to the above three points) e.g. (1) find 'an index number which will measure the booms in those activities which are most sensitive to movements in the trade cycle' [Astor et al., 1924, p. 49], (2) arrange for the trades that produce sufficiently standardised goods to provide output figures and (3) the state and stocks of finished goods in major trades could probably be determined as in (2). 'These suggestions do not provide a panacea, for statistics only record what has happened, not what is going to happen' [Astor et al., 1924, p. 51].

Bowley and Stuart had investigated whether public bodies could adjust their activities so that production was increased in bad times and decreased during booms. The chief difficulty seemed to be administrative.

> The scheme can only be worked with the co-operation of local authorities; but experience shows that the latter are not very willing to take long views, and that continuity of policy is likely to be interrupted by the issues raised at local elections. It is said that in practice local expenditure is always put off, in order to economise the rates, until it is absolutely imperative, and then it has to go forward in several directions at once. [Astor et al., 1924, p. 53]

The second main subject of the study concerns things that will affect Britain's economic future. First, though, the authors consider 'what permanent effects, if any, on our competitive position have been produced by currency depreciation abroad' [Astor et al., 1924, p. 55]. On this latter point it is important to distinguish between depreciated and depreciating currencies, and, after some reference to Germany, it is concluded that since depreciation cannot go on for ever, when it ceases 'there seems to be little evidence that when that stage is reached British competitive power will be permanently affected' [Astor et al., 1924, p. 56].

Bowley's first monograph in Part IV is entitled 'The future population of Great Britain'. Figures for the age distribution in Scotland having become available in February 1924, his analysis—prediction of the population in 1931 and 1941—covered the whole of Britain (there was no census of Ireland in 1921).

In the case of men born between 1876 and 1926 (thus looking slightly into the future) Bowley considers three groups: those born before 1901, those between 1901 and 1914 (too young to fight in the first world war) and those born after 1914. War casualties had reduced the employable population in the age group 30 to 55 in 1931 and in the group 40 to 65 in 1941.

The population is estimated in two ways: (1) the application of the death-rates of 1910 to 1912 to the 1921 population, year by year, and (2) the assumption of the same percentage decrease in each quinquennial during ten years as was shown in the decade 1901 to 1911. Bowley's findings were summarised as follows

> at most there will be 180,000 additional applicants for work (male and female) annually from 1921 to 1931, unless the age of retirement is raised, or the relative number of women occupied is increased, and this is at present being reduced to about 120,000 by emigration. From 1931 to 1941 the most to be expected is 47,000, which will also be reduced by emigration. So far from there being an excessive working population, the annual rate of growth after 1931 will be only 0.2 per cent. The growth after 1941 depends on the birth-rate after 1926, as to which no judgment can be formed[12]. [Astor et al., 1924, p. 363]

The second monograph in this Part is Bowley and Stuart's 'Regularisation of the demand for labour by advancement or retardation of public works'. Believing that 'it is possible *a priori* that public bodies not working for measurable profit, and employing large numbers of workmen, should so regulate their demands as to counteract partly or completely the variations of private demand' [Astor et al., 1924, p. 366],

> We have to consider (A) whether such a policy would defeat itself by causing as much unemployment as it cured, (B) whether transference in time could be on such a scale as to make a serious improvement, (C) how funds could be provided, and (D) what practical difficulties there may be in carrying out the proposal. (loc. cit.)

The authors' analysis leads them to conclude that (A) may be ignored if the other three difficulties are not insurmountable.

In consideration of (B) the authors propose to take the general average of unemployment before the War as 5.0% (Trade Union figures for 1894-1913 gave an average of 4.0% for the first ten years and 4.86% for the second ten). Computing the wage bill in 1911 as about £800 million Bowley and Stuart construct a typical trade cycle over ten years (the first being one of maximum employment) and deduce that the wave of unemployment would be levelled at 5% if a total of £36 million in wages were held over for three years and paid out in the next three, and if £16 million in wages were advanced in the seventh and eighth years.

> If these sums were in fact transferred, it is probable that unemployment would not then reach 5 per cent in the best years, the labour set free from public work being absorbed in private employment, so that the total of unemployment in the cycle would be reduced. [Astor et al., 1924, p. 368]

It is concluded that the number of those employable was at that time about 8% more than in 1911, and the expenditure to be postponed in the first three years was thus £81 million, that to be advanced in the seventh and eighth years being £37 million.

A look is then taken at the expenditure by Central and Local Governments in 1890, 1895, 1900 and each year from 1903 to 1915. Much of the expenditure was not necessarily allocated to any particular year, and it was concluded that 'there has been a great variation in the amount expended year by year, in fact nearly half that required by the proposal, but at the wrong dates' [Astor et al., 1924, p. 371]. One good consequence of the War was that some postponement of expenditure had been forced, and arrears were now being made good during the early post-war years of unemployment.

The difficult of funding from rates in a year in which the expenditure is needed 'is insuperable'. Local Authorities, it is suggested, could raise it by loan; statutory bodies (e.g. the Port of London) and railway companies could merely time their extensions to take place when capital and labour were abundant. Although the Central Government would have to be the main governor of the policy, 'Any effective scheme ... would require courageous and persistent handling, and would need to be removed from the sphere of party politics' [Astor et al., 1924, p. 374].

Bowley and Stuart had apparently consulted authorities on these matters, and the latter somewhat depressingly decided that, for one reason or another, 'the policy would break down in detail' [Astor et al., 1924, p. 374]. The reasons given included the following: the making of roads, building of schools and extension of municipal services are needed simultaneously and cannot be discontinued when unemployment is low; little work was in fact transferable in time; Borough Councils experienced difficulties in that unpopular policies resulted in the council members losing their positions at the next election and the policies being reversed by the new council; 'the system of working by committees makes a general financial policy very difficult' [Astor et al., 1924, p. 375]. The authors concluded that the difficulties were extremely serious, and even more depressingly,

> that the only possible way of influencing the amount of employment provided by Local Authorities, without whose co-operation the policy of regularisation can be only partly successful, is by exercise by the Central Government of its powers of compulsion, of making or withholding grants, of granting or refusing power to borrow, and above all, of providing capital on easy terms at times when it is desirable on national grounds that public works should be set in hand. [Astor et al., 1924, p. 376]

Yet, Bowley and Stuart conclude, all is not lost.

> We reach the conclusions that it is possible to provide funds for regularisation of the labour market, if a strong policy is framed and carried out, without otherwise disturbing the demand for labour, that the practical difficulties of administering a scheme are serious but not insuperable, and that the transference of expenditure from one year to another could be on such a scale as to make an important reduction in the cyclical oscillation of unemployment; but its effect would be principally on men's unskilled labour, and under the best possible administration would leave a considerable part of the problem unsolved. [Astor et al., 1924, pp. 376 & 377]

The final monograph is Bowley's 'The effect on employment of adjusting rates of wages in accordance with (a) the level of prices (b) the state of trade'. In examining (a) he considers adjustment by an index number of the cost of living. It is first noted that wages tend to lag behind (both in a rise or fall) in a rapid movement of retail prices if changes in wages are reliant on intermittent bargaining. As a remedy it is suggested that

> The assessment of wages automatically in terms of a stable currency by means of a valid index-number would remove one variable factor, and make it possible to limit the considerations under which occasional wage-adjustments are made to those relating to the condition of trade.
> [Astor et al., 1924, p. 378]

The process is however attended by disadvantages, both theoretical and practical, in the construction of a suitable index number and in the requiring of different numbers for different classes with different standards of expenditure. Bowley's investigation of wage regulation leads him to conclude, somewhat depressingly, that

> when money is on a sound basis, regulation of wages by a cost of living index-number is more likely to cause un-

> employment than to prevent it. In any case, it is evident that any permanent standardisation of real wages, so as to preserve the standard of living reached at any particular date, will cause unemployment whenever the conditions of industry are worse than at that date.
> [Astor et al., 1924, p. 380]

In his part (b), 'Adjustment in accordance with the state of trade', Bowley notes that real wages should depend on the output of industry and not on some pre-War standard. However he was strongly against the regulation of wages by an index number that measured the national output (one difficulty is in the obtaining of suitable units of output: in Britain essentially the only available measurements were for pig-iron, steel and coal). Bowley concludes

> It appears to be certain that the flow and ebb of employment cannot be prevented by any adjustment of wages, and probable that no artificial regulation of wages can make any substantial improvement. But a much greater elasticity of wages than at present exists ... would no doubt diminish the oscillations of employment.
> [Astor et al., 1924, p. 382]

There was, however, a caveat:

> It is of course possible that labour as a whole should demand higher real wages than those at which all can be employed; and in such a case a reduction would in due course diminish unemployment. (loc. cit.)

Let us now look at some reviews of this book[13]. Although calling it 'a most valuable piece of work', Norman Dearle [1924, p. 602] found that it perhaps compared unfavourably in comparison with its predecessor in form, being a collection of essays by experts resulting in a lack of a sense of unity. Willford King singled out for special praise the chapters written by Bowley. Yet while he considered the book as a great success as a discussion of conditions in British industries, King regarded it as 'somewhat disappointing' from the viewpoint of answering the question posed in the title. He found that three fallacious assumptions marred the report:

> first, that the value of products depend upon their respective costs of production; second, that costs of production, including wages, tend to be fixed and inevitable; and, third, that the volume of unemployment is dependent primarily upon conditions of foreign trade and has little or nothing to do with the system at present in vogue in England of subsidising idleness by means of unemployment insurance. [King, 1925, p. 127]

A harsh opinion of the dole!

King claimed that on reading the report he constantly gained the impression of the tacit acceptance that there were too many people for the number of jobs, and that emigration or a reduction in population would reduce unemployment. King's opinion, on the contrary, was that a great increase in population would result in poverty so serious 'that everyone would be forced to find work at once without regard to the price he could get for his labor, and unemployment would rapidly diminish' [1925, pp. 127-128]—a situation he found to obtain in India and China.

5.6 Economic Census of India

In the 1930s Bowley and Dennis Holme Robertson visited India at the request of the Government of that country and under the sponsorship of the Tata Foundation, their task being to carry out a survey on the provision of economic and statistical data.

There had been earlier initiatives by the British in connexion with official statistical work in India[14]. For instance, on the 7th of January 1807 the Court of Directors of the East-India Company sent the following despatch to the Supreme Government of Bengal:

> We are of opinion that a statistical survey of the country, under the immediate authority of your Presidency, would be attended with much utility; we therefore recommend proper steps to be taken for carrying the same into execution. [Martin, 1838, p. vii]

In 1905 the Viceroy, Lord Curzon, abolished the post of Director-General of Statistics and established the Directorate General of Commercial Intelligence and Statistics. In 1930 George Schuster, finance member of the Viceroy's executive council, recommended that David Meek, director of commercial intelligence and statistics, be involved in the planning process. Their joint initiative had as consequence the visit by Bowley and Robertson during the viceroyalty of Freeman Freeman-Thomas, Lord Willingdon.

In his survey of the Indian population George Shirras described the 1931 census in India (then including Burma) as 'a triumph of organisation', and he noted further that 'it [had] attained high accuracy in the count of population' [1933, p. 57]. An area of 1,800,000 square miles had been covered and a population of 353,000,000 surveyed, the average density being 195 persons per square mile. The carefully trained enumerators[15] (Land Revenue officials and village schoolmasters in many instances) conducted preliminary surveys about a month before the official census, with only a revision then being necessary on the actual occasion (unfortunately the subordinate staff were somewhat inexperienced).

The data obtained by enumerators were entered on slips of paper by clerks, and the slips were then arranged so as to allow the extraction of the required information. Shirras noted the language difficulty caused by the fact that some 225 vernaculars in addition to dialects (he mentioned 544 of the latter) were involved with twenty scripts, each, barring three, spoken by about 2,000,000 people[16]. Thus, at that time, it was cheaper to copy and sort slips of paper rather than to use machines.

Certain curiosities emerged from a comparison of the results of the 1921 census with those of that of 1931. For instance, there appeared in the latter to be a greater number of women in the 25 to 30 age group than was expected, which was ascribed to the fact that women overstated their ages after marriage and understated them before marriage. Further, there were more women in the 25-30 age group in the 1931 census than in the 15-20 group ten years before.

The 1931 Indian Census, succesful though it seems to have been, was not taken without difficulty: a tiger sprang upon the front of the car of the Administrator of Bastar State; some people refused to have their houses numbered on religious, and others on artistic, grounds; some exterior castes in the Punjab, having been asked by one political party to register as Hindus and by another to register as Moslems, eventually declared themselves as Ad Dharmi[17] or 'adherents of the original religion' [Shirras, 1935a, p. 435].

Suitably fortified for their task by having studied reports of committees and commissions and the general volumes of the 1931 census, Bowley and Robertson arrived in India on the 22nd of December 1933. About half their five-month stay was spent on tours, and either singly or together they visited some seventeen major cities and villages, with prolonged stays in Calcutta and Bombay. Closely associated with the survey were the Indian economists Parakunnel Thomas (Professor of Economics, Madras University), D. Ghosh (Lecturer in Economics, Bombay University) and C.P.K. Fazal (Assistant Secretary to the Punjab Board of Economic Enquiry).

Bowley and Robertson's brief was as follows: (a) to make recommendations in connexion with the establishing of a Statistical Department for the whole of India, (b) to comment on the practicability and scope of a Census of Production, (c) to consider what was needed for the measurement of national income and (d) to investigate series of index numbers of prices, wages and production.

One outcome of the investigation was the recommendation that the title of Director-General of Commercial Intelligence and Statistics be abolished and that a permanent economic staff of four members be established. The senior person would be responsible for the organisation of all economic intelligence, two members would be economic experts and the fourth would be the Director of Statistics. The first three members were to be allowed as much freedom as was compatible with membership of the Civil Service, while the Director of Statistics should be responsible for the conducting of the population census, the census of production and the co-ordination of central and provincial statistics. In addition to these four (national) board members, there should be a full-time and independent statistician in each major province who would conduct the Population Census under the new Director of Statistics.

After having discussed the total national income in general the investigators suggested that rather than an estimate of this income as a whole it was better to take censuses of production focussing on agriculture, mining, industry, etc. Further, the estimation of rural income could be effected by estimating the quantity and value of all produce and services arising from the land, with intensive surveys in selected villages. In the case of urban income it was recommended that sample surveys be taken in the larger towns. To get an accurate account of at least a certain section it was suggested that an intermediate urban census be taken of mines, factories using power, and some other industries.

Using data from the main surveys an attempt should be made to estimate the income of the smaller towns, the investigations being extended to the Indian States where possible. A further difficulty was seen in the measurement of non-factory industrial activities, though successive censuses of production might be useful here.

The time-scale proposed for this whole process was as follows: 1934. Appointment of Director of Statistics; 1935. Organisation of the enquiries and training of investigators; 1935-6. Rural survey; 1936. Census of Production. Urban surveys. Urban population and occupation census; 1937. General report. The estimated costs[18] of the programme are shown in Table 5B.

In 1932 British India contained about 500,000 villages[19], and Bowley and Robertson took pains to describe the sampling scheme

Table 5B. Cost of the investigations.

	Rs lakhs
Census of production	2
Rural Survey	22
Urban Surveys	3
Urban Census	2
Report	1
Total	30

Table 5C. Rural sampling allocation.

Province	No. of Villages in Province	No. in Sample
Bengal	86,000	250
Bihar & Orissa	83,000	300
Bombay	21,000	200
Central Provinces	40,000	200
Madras	51,000	200
Punjab	35,000	200
United Provinces	106,000	300
Total	422,000	1650

they thought necessary: a province was to be divided into a number of areas, each homogeneous within itself, and a typical village in each area was to be examined. As Bowley had stressed elsewhere, once a particular village, no matter how remote, had been chosen, none other should be substituted for it, and each village must *a priori* have the same chance of being sampled. A possible allocation is shown in Table 5C.

Detailed recommendations were made about the Census of Production: communication of the demanded facts would be compulsory, and following the line taken in similar American and English censuses it would be limited to factories employing twenty or more people and using mechanical power, although extension *might* possibly be made to smaller workshops or to larger non-mechanical factories[20].

It was important that, for each factory, the aggregate value of sales and the aggregate cost of materials be ascertained. The aggregate of what was termed 'net output' in Britain and 'value added by manufacture' in the United States would provide an estimate of the contribution made by the (factory) industry to the national income, allowances having been made for plant depreciation and change in value of stocks of materials and the finished products.

Trade representatives should be consulted in connexion with the obtaining of details of the amounts and values of the goods produced, of materials bought and of the power used. Products should be classified in agreement with imports and exports. Employees should be grouped as salaried or wage-earning, male or female, and young or adult, with precise definition of the age division. It was suggested that, in order to get an annual average, these data be obtained for a week each month (this would allow identification of seasonal change).

Finally, details of wage-rates need not be taken in the first year of census (possible overloading): however, the total wage-bill, with possible sub-divisions, should be determined. If workmen were employed and paid by a *sirdar*[21], one should try to ascertain how much the sirdar kept. Annual and cash bonuses should noted, together with any advantages in the way of land or housing.

The problem of foreign trade control in India in the 1930s arose as a problem of balance of payments. The more fundamental question was whether the expansion of both imports and exports should be aimed at or whether the target should be contraction of both (that is, to achieve rapid growth of 'real comfort' or to obtain greater stability of economic life). Or, in other words, should one retain the maximum freedom of action with respect to the profitable foreign trade, or should one enter into regional agreement for the mutual expansion of trade.

In this matter statistics should be used with caution. The general conditions of supply and demand could be affected by many causes, and the effects of tariff policy may be only uncertainly illuminated by figures of consumption and production.

The problem of harmonious development between countryside and towns was both urgent and acute. Bowley and Robertson suggested that at that time in India (and the rest of the world) there had been overproduction of agriculture relative to manufactured goods. The majority of the population depended on agriculture, thus the country used a money economy. On the other hand, the food-producing village to a large extent was a self-contained unit. The pressure of population growth might make make it desirable to have more people on the land: in the final resort it might be better to go short of manufactured products.

Bowley and Robertson also found the yearly statistics of permanently settled areas to be almost worthless. There was uncertainty as to whether the quantity of food produced was keeping pace with the population growth. Further, improvement in quality, as distinct from quantity, does usually not show itself in yield statistics, and Bowley and Robertson thus recommended that alternative estimates that avoided dependence on the standard strain be used.

The report noted that unemployment in India was to a large extent seasonal, agricultural operations involving cultivation extending on the average over only three-quarters of the year. A solution might be the diversification of agriculture. 'Laid-off' agricultural labourers spent much time devising subsidiary occupations to fill their 'spare time', two main classes of such occupations emerging: (a) supplementary employment in local seasonal factories and (b) handwork at home. Unfortunately while spinning lent itself readily to part-time work it was a job that was more easily carried out by machines.

Approaching the end of their report, Bowley and Robertson emphasised that governments can do as much harm by guiding production into unsuitable channels as they can do good by making use of idle resources of land, labour and capital. It is further suggested that a systematised knowledge of the physical output of various branches (and not the value of the output revealed by a Census of Production and related enquiries) should serve as a background for Government activities related to particular branches of production.

What was the effect of this investigation? In *The Economic Development of India* Vera Anstey wrote

> Almost the only results of this report were that the registration of inland trade was resumed in 1935, and a Statistical Research Board of the Department of Commercial Intelligence was established in 1934. [1977, p. 546]

Naik [1963, p. 288] in fact remarks, perhaps more harshly, that the recommendations made in the report were not acted upon until 1942. And Prasanta Mahalanobis, in speaking particularly of the rice crop in Bengal, commented that ‘since October 1942 the official estimates clearly indicated a large deficit; and yet we know that the position was not considered serious by Government’ [1944, p. 71] (and the Governments of both Bengal and India were at fault in ignoring the seriousness of the position). He also bemoaned the fact that ‘In India unfortunately collection of statistics often starts when it is usually too late with tragic consequences’ [1944, p. 69].

Mahalanobis notes further the danger posed by controversial or vested interests in the taking or reporting of statistics. As an example he cites the case of a plot-by-plot survey of acreage under jute in 1939 in Bengal. The results obtained from the very expensive survey were found to be so unreliable that the Bengalese Government decided to scrap them. Mahalanobis managed to see the results of some districts, and found them to be so inflated that ‘in certain cases the acreage under jute had exceeded the total geographical area of the region’ [1944, p. 73].

In 1939 the results of the inquiry were published by Thomas and Sundararama Sastry. In his review of this work Kendall notes that the investigators had suggested the appointment of a permanent economic staff and the conducting of an economic census every five years. The latter proposal was apparently regarded by the Indian authorities as too expensive, and the appointment of an economic general staff, though generally favourably received, was also abandoned because of the cost.

It is perhaps most likely that the failure to implement the proposed programme was to a large extent attributable to the fact that Bowley and Robertson had substantially underestimated the difficulty involved in the taking of the surveys. Many investigators were unqualified, there were transportation and seasonal difficulties and relations between the investigators and the villagers were not always of the best[22]. And as Artemus Ward wrote: 'Takin the senses requires experiunse, like any other bizniss' [1891, p. 86].

Chapter 6

Statistical Papers

6.1 Introduction

In his presidential address 'What is statistics?' to the Royal Statistical Society in October 1994 David Bartholomew said 'The question of my title is simple enough but there is no short answer' [Bartholomew, 1995, p. 2]. He regarded the two elements distinctive of statistics as typified by the (British) *Annual Abstract of Statistics* and the *Journal of the Royal Statistical Society*, Series B. The hallmark of a statistical problem are uncertainty and variability. Further,

> Statistics is concerned with understanding the real world through the information that we derive from classification and measurement. [Bartholomew, 1995, p. 5]

It is often difficult to decide whether any particular paper by Bowley should be classified under 'Statistics' or 'Econometrics'—if, indeed, either. Perhaps some indication might be given by examination of Kendall & Doig's three-volume *Bibliography of Statistical Literature*, but the only paper by Bowley to be listed there is his [1909c][1].

Would one perhaps be more accurate in describing Bowley's work as Applied Statistics? As we have seen, Bowley's work with public statistics is concerned with official reports, the summarising of the data and the drawing of conclusions. But one must not think that Bowley merely takes some data and then tries to extract some information[2]: his exceptional work in sampling shows that his studies were carefully designed to answer specific questions, and his frequent calculation of probable, or standard, errors suggests that he is perhaps more concerned with estimation than with inference[3].

In his opening address to Section F of the British Association for the Advancement of Science (the Section from which the Statistical Society had grown) W. Stanley Jevons said

> In order, however, that any subject can be fitly discussed by a Section of this Association, it should be capable of scientific treatment. We must not only have facts, numerical or otherwise, but these facts must be analysed, arranged and explained by inductive or deductive processes, as nearly as possible identical with those which have led to undoubted success in other branches of science. [1870, p. 309]

In the introduction to his *The Nature and Purpose of the Measurement of Social Phenomena* Bowley offered a description rather than a definition: 'Statistics itself I regard as a method rather than a science' [1915b, p. 4]. A pithier remark was given on the 14th January 1861 by Edmond and Jules Huot de Goncourt in the first volume of their *Journal*: *'La statistique est la première des sciences inexactes.'*

6.2 Averages

In December 1897 Bowley published a paper in the *Journal of the Royal Statistical Society* in which he explored the relations between the accuracy of an average and that of the elements of which it is composed. The results given here, Bowley notes, were in fact found in connexion with his Newmarch Lectures of the same year[4].

Bowley records that it is important to take account of the precision of the measuring instrument[5], and mentions too that it would often be silly to expect great accuracy in a statistical total. For instance, one might well say that estimates of census figures are correct to the 100,000s, although the tens and hundreds may well vary from one estimate to another. Further, using probability one may be able to give not only a *possible* but also a *probable* error in the estimation of a mean (say).

The estimation of an average is enhanced by consideration of weights, and Bowley suggests that experience and theory have both shown that a weighted average is not as much affected by errors in the weights as it is by errors in the quantities whose average is being found. Further, when a large number of individual items are considered, errors in the separate items tend to cancel out. In this paper [1897c] he considers the effect errors in the initial observations have on the finding of ratios and averages. Three cases are considered: in the first no assumption as to the size of the errors is made, in the second approximate results are given when the relative sizes of different classes of errors are given, and in the third the relation between an error in the final result and errors in the initial items is explored, and the probable error of that result given the errors is estimated. The paper is set out in three sections—A: simple instances; B: errors in comparison of two estimates; C: probable values of errors—each being subdivided in turn. (The *error of measurement* e in the measurement of a quantity u whose true, but unknown, value is u' is defined as $e = (u' - u)/u$.) We shall sketch some of the results.

Firstly, consider Case A1, 'error due to omission of part of data'. Suppose that each of w persons earns an average of m shillings, and that rw persons are accidentally (or perhaps purposely) omitted, each of whom earns an average of $m(1+\rho)$ shillings (note that nothing is said about either the sign or the magnitude of ρ). Then the error in the (grand) average caused by the omission is

$$\left(\frac{1}{m}\right)\left[\frac{wm + wrm(1+\rho)}{w + wr} - m\right] \quad = \quad \frac{r\rho}{1+r} \quad = \quad r\rho, \qquad (6.1)$$

it being assumed that r^2 is small in comparison with r (similar approximations are frequently made)[6].

For Case B1, 'general case', suppose that m and m' are estimates of two similar quantities, these estimates being made under similar conditions and by similar methods. Let $m(1+e)$ and $m'(1+e')$ be the true (again, presumably unknown) values of m and m' respectively. Then the error in the ratio is

$$\left(\frac{m}{m'}\right)\left[\frac{m'(1+e')}{m(1+e)} - \frac{m'}{m}\right] \quad = \quad \frac{e'-e}{1+e} \quad = \quad e'-e,$$

where terms of second and higher order (including cross-products) are neglected. In his discussion of C1, 'probable value of error in an arithmetic average', Bowley uses the fact that in A3, 'error in arithmetic average', he showed that when there are n such estimates m_i, the error E_1 in the arithmetic average is approximately $\sum_1^n (m_i\, e_i)/\sum_1^n m_i$. The probable error in the arithmetic average is thus $[(\sqrt{\sum m_i^2})/\sum m_i]\, E_1$, which reduces to approximately $E_1/\sqrt{n}$ when the m_i are almost equal.

Note that in the discussion of A5, 'effect of bias, simplest case', it emerges that biassed errors are of more effect than unbiassed ones, and that the latter may be neglected in the presence of the former (here the bias is presumed to be caused by the investigator).

Towards the end of the paper Bowley observes:

> Thus the effect of dispersion of material, and of all the various errors in estimate, can be traced separately in result; and if the data are carefully examined, the relative values of these errors can be estimated.
> [1897c, pp. 865-866]

Further, although it may usually be impossible to estimate the initial errors, limits superior can often be placed on them.

This paper was followed by one published by Bowley in 1911, 'The measurement of the accuracy of an average', in which the formulae previously given were, as a result of the progress of statistics, improved upon. Brief discussions are first given of the standard deviation of a weighted sum, not only for data coming from a curve whose abscissal origin is the centre of gravity of the curve, but also for data from a Normal curve of error.

In the first case Bowley supposes that $e_1, e_2, \ldots, e_n$ are independent quantities from a curve of the given kind, the standard deviation of e_i being σ_i. Setting $E = \sum a_i e_i$, the a_i being constants, he deduces that the standard deviation σ of E satisfies $\sigma^2 = \sum a_i^2 \sigma_i^2$. It is then supposed that the deviation E arises in the estimation of a quantity H, where $H = \sum_1^n a_i h_i$ and e_i is the error in h_i.

In the section on the weighted sum and the Normal distribution Bowley makes the following statement:

> It will be shown that (under certain conditions to be defined) the first, second and all successive moments of the normal curve ... are the same as the corresponding moments of the curve of frequency of E... It is assumed that this is equivalent to saying that this normal curve *is* the curve of frequency of E. [1911a, p. 78]

That the assumption in the last sentence here is not always true is well known: a distribution is not in general uniquely determined by its moments[7].

Before the real work of the paper is undertaken, Bowley proves two propositions, the conclusions of which we may write as follows: (A) the odd moments about the origin of a random variable $X \sim N(0, \sigma^2)$ are all zero while the even moments satisfy $\mu_{2n} = [(2n)!/(2^n\, n!)]\sigma^{2n}$ and (B) the mean of a product of independent variable quantities tends to the product of their means. Unfortunately Bowley does not define what *independence* means in this context. The absence of any definite idea of random variables makes the concept of 'independence' used here difficult to understand from a perhaps more usual measure-theoretic background[8]. Edgeworth, incidentally, puts Bowley's (B) as follows (emphasis added):

> the *fundamental principle* that the mean of the product of two independent statistical quantities—the pairs being supposed to be repeated with indefinitely great frequency under unaltered conditions—is equal to the product of their respective means. [1905, Part I, p. 41]

Bowley's proof of Proposition (A) is of the usual kind: integration of $x^p f(x)$ over the whole real line, (where $f(\cdot)$ denotes the density function of a random variable $X \sim N(0, \sigma^2)$), results in a recursion formula for the moments, and choice of p odd or even gives the asserted results.

To prove (B) Bowley supposes that u and v are two variables with deviations d and δ, so that $u_i = \overline{u} + d_i$, $v_i = \overline{v} + \delta_i$ and $\sum \delta_i = \sum d_i = 0$. Then

$$\sum\nolimits_1^n u_i v_i = n\overline{u}\,\overline{v} + \sum d_i \delta_i + \overline{u} \sum \delta_i + \overline{v} \sum d_i.$$

Under the assumption of independence of the d_i and the δ_i one finds that $\sum_1^n d_i \delta_i \to 0$ as $n \to \infty$, and hence the mean of a large number of products uv 'tends to equal' (better: 'for large n is approximately') $\overline{u}\,\overline{v}$. The extension of this to a number of independent variables is a proposition, says Bowley [1911a, p. 79], 'often regarded as self-evident'.

Bowley now returns to the weighted sum $E = \sum a_i e_i$ of the independent variables e_i. Writing $\exp(\theta E) = \prod_{i=1}^n \exp(\theta a_i e_i)$ and expanding each exponential in a Taylor series Bowley finds that

$$1 + \sum_{k=1} \frac{\theta^k E^k}{k!} = \prod_{i=1}^n \left(1 + \sum_{k=1} \frac{\theta^k a_i^k e_i^k}{k!}\right).$$

Essentially taking expectations on both sides of this last expression (though Bowley prefers to describe his procedure as taking a large number of such equations, summing them and then finding the mean) and using his Proposition (B), Bowley deduces, on taking logarithms, that

$$\log\left(1+\sum_{k=2}\frac{\theta^k}{k!}\mu_k\right)=\sum_{i=1}^{n}\log\left(1+\sum_{k=2}\frac{a_i^k\theta^k}{k!}\,{}_i\mu_k+\cdots\right) \tag{6.2}$$

where ${}_i\mu_k$ denotes the kth moment of e_i (recall that the first order moments are zero).

Now, to approach a limit: let n be large, let the σ_i (the standard deviations of the e_i) be small and much the same, and of the order of $1/\sqrt{n}$. Further, let the a_i be finite and suppose that each is less than unity (which can be ensured by dividing $H=\sum a_ih_i$ by a constant). Suppose also that $\sigma^2=\sum a_i^2\sigma_i^2$ as before. The important assumption is next made that the ranges of the frequency functions of every e_i are bounded 'so that the extreme values whose frequencies are at all considerable are of the same order as σ_i' [1911a, p. 80]. Assuming too that each a_i^p is finite, it then follows that ${}_i\mu_p$ and $a_i^p\,{}_i\mu_p$ are of order $n^{-p/2}$. 'There is no other limitation as to the shape of the frequency curves' [1911a, p. 80].

Expanding each logarithm, thought of as an expression of the form $\log(1+z)$, on the right-hand side of (6.2), we find that that side reduces to $\theta^2\sigma^2/2$. Here terms of order $1/\sqrt{n}$ or lower may be neglected.

Thus from (6.2) we have

$$\left(1+\sum_{k=2}\frac{\theta^k}{k!}\mu_k\right)=\exp(\theta^2\sigma^2/2).$$

On expanding the right-hand side of this last expression and equating coefficients of θ^k one obtains finally

$$\mu_k=\begin{cases}0\,, & k=2n+1\\[2ex] \dfrac{k!}{(k/2)!}\,\dfrac{\sigma^k}{2^{k/2}}\,, & k=2n\end{cases}$$

It thus follows that H and $H/\sum a_i$ are Normally distributed with standard deviations $\sqrt{\sum a_i^2\sigma^2}$ and $(\sqrt{\sum a_i^2\sigma^2})/\sum a_i$ respectively[9].

As an example Bowley considers the sum of ten digits taken at random, the experiment being repeated 1,000 times. Fitting a Normal distribution with mean 45 and modulus 12.845 he finds a good fit, a result one finds supported today on carrying out a χ^2 goodness-of-fit test with two parameters estimated. (The *modulus*, usually denoted at this time by c, is defined by $c = \sqrt{(2/n)\sum(x_i - \overline{x})^2}$.)

The third section of the paper is entitled 'Application to the precision of an average', consideration being taken in turn of the unweighted average, the weighted average, the ratio of two unweighted averages and the ratio of two weighted averages. We shall look only at the first of these four cases: the other cases are similar but more complicated.

Consider the quantities $M_1, M_2, \ldots, M_n$ with $M_i = \overline{m} + m_i$, where $\sum m_i = 0$. Let σ'_m be the standard deviation of m_i and let $\sigma_m = \sigma'_m/\overline{m}$. Further, let M_i be the (imperfect) observed value of the actual value $M_i(1 + e_i)$ and let $\overline{m}(1 + e')$ be the true mean. Finally, suppose that the errors e_i come from distributions having the same standard deviation σ_1 and let σ denote the standard deviation of e'. Using the results of Sections I and II Bowley deduces that, unless the m_i have extremely unsymmetrical distributions, and on neglecting terms of order $1/n$, the distribution of e' is Normal with $\sigma = (\sigma_1/\sqrt{n})\sqrt{(1 + \sigma_m^2)}$.

In a little-known (or at any rate, seldom referred to) paper[10] in 1911 John Maynard Keynes investigated

> what laws of error correspond to given assumptions respecting the algebraic relation between the measurements and the most probable value of the quantity, and *vice versa.* [1911, p. 325]

Under certain assumptions (e.g. that (a) before any measurements are made, one has no reason to suppose that the quantity to be measured is as likely to have any one of its possible values as any other, and (b) the errors are independent), Keynes shows that when the most probable value of the quantity is the arithmetic mean of the measurements, the law of error has the general form

$$f(x_q, x) = \exp\big(\phi'(x)(x - x_q) - \phi(x) + \psi(x_q)\big).$$

Here $f(x_q, x)$ is the probability $\Pr[X_q|A_s H]$, where X_q denotes the evidence that the measurement made is x_q, H denotes all relevant evidence and A_s represents the conclusion that the real value of the quantity in question is a_s. The $x_q, q \in \{1, 2, \ldots, m\}$, are observations and x is the value that maximises $\prod_{q=1}^{m} f(x_q, x)$. On putting $\phi(x) = -k^2x^2$ and $\psi(x_q) = -k^2x_q^2 + \ln A$ one finds that $f(x_q, x) = A\exp(-k^2(x - x_q)^2)$. Moreover, under the assumption that errors of the same positive or negative absolute amount are equally likely this law is unique.

Similarly, in the case of the median the appropriate law of error is found to be[11]

$$f(x_q, x) = A\exp(-k^2|x - x_q|).$$

6.3 On goodness-of-fit

Pearson introduced the χ^2 test in his [1900b], applying it in due course to the testing of goodness-of-fit of data to frequency curves and to the question of independence in contingency tables[12]. Although the problems raised by the estimation of parameters in such situations were noticed by Greenwood and Yule [1915], it was only in 1922 that Fisher correctly analysed the 2×2 table. (Incidentally, Fisher here commended Bowley for distinguishing the use of χ^2 in contingency tables from its use in goodness-of-fit tests.) This paper was immediately followed by one by Yule expanding on Fisher's work.

Pearson seems never to have fully accepted what we now regard as the correct method of determining degrees of freedom[13].

In 1923 Bowley and Connor published a paper in which they examined whether the grouping of data according to some pre-assigned classification was consistent with some specified hypothesis about that grouping. 'Consistency' was defined [1923, p. 1] in terms of the statistical significance of discrepancies between the experiment and the formula, where

> A complex of errors is significant when the individual errors taken in conjunction with one another are greater than can be accounted for by the fluctuations of sampling.

The controversy between Pearson and Fisher on the application of the χ^2 test is mentioned, and it is stated that

> An attempt is here made to define the hypotheses under which the competing methods of application are respectively valid; for the divergence of opinion does not depend upon correctness of mathematical reasoning, but on the selection of the assumptions proper to each problem. [Bowley & Connor, 1923, p. 1]

Suppose that N objects are observed and placed in n groups, the ith group containing $m_i + e_i$ items while the frequency expected[14] under some assigned law is m_i, and let $\chi^2 = \sum_{i=1}^{n}[m_i - (m_i + e_i)]^2/m_i$. It is shown that the probability P of getting a complex of errors equal to or more improbable than $e_1, e_2, \ldots, e_n$ is

$$P = \int_{\chi}^{\infty} \chi^{n-2}\, e^{-\chi^2/2}\, d\chi \Big/ \int_{0}^{\infty} \chi^{n-2}\, e^{-\chi^2/2}\, d\chi. \tag{6.3}$$

Citing the fourth edition of Bowley's *Elements of Statistics* (to be discussed in our next chapter) the authors note that

> if χ^2 does not exceed $(n-1)$ the divergence is not at all unexpected, and that if χ^2 exceeds $2n$, its improbability is so great as to indicate that the law is not satisfied. [Bowley & Connor, 1923, p. 2]

Thus a value of at most $n-1$ for χ^2 will not lead to rejection of the hypothesised distribution, whereas a value of $2n$ will.

The problem is now to see what effect further restrictions may have on the value of P. To this end two situations (illustrated by numerical examples) are considered: (A) frequency groups and (B) contingency tables. (A) is further subdivided into two cases: in the first the distribution of the universe is completely known and in the second the form of the supposed law is known but not its parameters. In the latter case the population parameters are estimated by the appropriate sample characteristics, and Bowley and Connor note Fisher's argument that, in the case of fitting a Normal distribution, two degrees of freedom should be subtracted for the estimation of the mean and the variance (see [Fisher, 1922]). In the illustrative example used here in which $n = 4$, Bowley and Connor note that the exponent in the integrals would then be zero.

> This is equivalent to supposing either (a) that we are dealing with samples whose average and standard deviation are constant, or (b) that each sample is taken from a universe specially adapted to it. [1923, p. 3]

Suppose, however, that one is concerned with the fitting of a specified distribution whose parameters are unknown. Bowley and Connor note that the estimation of these parameters by the corresponding sample statistics need not necessarily result in a minimal value of χ^2, and the corresponding value of P may not be the most favourable. However, if the value of χ^2 is slightly bigger than n one may conclude that the distribution thus determined is certainly possible, but should it exceed $2n$ it is extremely unlikely that the chosen distribution is correct[15]. As regards the initial choice of distribution it is noted that

> The more constants there are, the better the fit, but generally the aim in specifying the law is to use the simplest possible[16]. [Bowley & Connor, 1923, p. 4]

From this discussion Bowley and Connor draw the somewhat depressing conclusion that not only is Fisher's rule (subtract one degree of freedom for each parameter estimated) 'not applicable to the problem of determining whether observations are consistent with a law of assigned form', but also that the equating of sample to population moments is not sufficient 'to give a final result in the case of apparent inconsistency between the sample and the law' [1923, p. 4].

In Case III, the first in Bowley and Connor's (B), the section on contingency tables, it is supposed that the distribution sampled is known and that marginal totals are fixed. This is quite straightforward. Things are less clear, however, in Case IV, where it is supposed that the universe is unknown and is to be estimated, in the absence of correlation, from a sample. Supposing that the total number n of observations is in fact a random sample from such a universe, one wants to find that universe from which the data could have arisen with the least improbability. This is achieved by estimating the frequency in the $\{i, j\}$th cell of the 2×2 table as $f_{i.}f_{.j}/f_{..}$, the f's being marginal frequencies. Once again Fisher's method seems unnecessary.

The paper is concluded (apart from a mathematical appendix) with an example in which it is unclear whether the data are an instance of Case III or Case IV. In such a situation the value of P may well differ markedly depending on whether the marginal totals are supposed to be known or unknown, and it is concluded that 'Writers differ as to which is the more appropriate method' [1923, p. 6].

In June 1923 Fisher considered Bowley and Connor's paper. While noting that, prior to his own 1922 paper, Bowley was the only person who had correctly used the value of χ^2 for the evaluation of P in the test of independence in a 2×2 table[17], Fisher was not only uneasy about Bowley and Connor's 'Doubtful Case', but also criticised the degrees of freedom used in their Case IV. Defending his argument, Bowley wrote 'The dispute is not about the mathematics' [Fisher, 1923, p. 147]; the problem was whether the variation exhibited in the observed sample was more appropriately treated by the methods he and Connor had used or those suggested by Fisher and Yule.

6.4 The double median

In a series of papers from 1888 to 1923 on the calculus of observations Edgeworth suggested a method analogous to the method of least squares in which medians rather than means are used (see, for instance, his [1888a], [1888b] and [1923]). Essentially the problem (for two variables) was the minimisation of the sum of absolute errors

$$\sum\nolimits_{i=1}^{n} |l_i - x - yb_i|.$$

The method was described by Edgeworth as follows:

> find a locus such that if we substitute any assigned value of y in the original equations, the *Median* of the corresponding n values of x may be given by the locus. ... A second Median Curve is afforded by the Medians of the y components; and the intersection of these Median Curves gives the *Median Point*. [1888a, p. 282]

Criticism of this method by Herbert Turner[18] in 1888 resulted in Edgeworth's proposing (see his [1888b]) a new method to minimise the sum of the absolute errors without using the median loci[19]. In essence he plotted the equations of condition $x + yb_i = l_i$ on a graph and used a method of steepest descent to find a minimum value. In the case of five equations in two unknowns we have a situation like that shown in Figure 6.1. Edgeworth [1888b] noted that in general his method might result in a line segment rather than a unique point, and suggested that in such a case the mid-point of that segment should be used. In the situation shown in the figure this led to the choice of the mid-point of BC as the *best* solution.

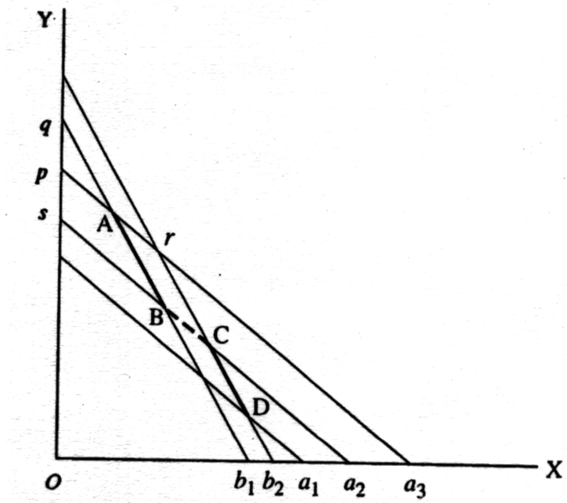


Figure 6.1. Sketch for the double median method.

In 1923 Edgeworth considered the more general problem involving the minimisation of the weighted sums

$$\sum_{i=1}^{n} w_i|l_i - x - yb_i|.$$

This he solved using a graphical variant of his first method.

In 1902, in his only joint paper—one written with Bowley—Edgeworth provided some theoretical considerations on this topic, while Bowley studied applications to wage statistics, the ages of school children and Booth's statistics of London wages. Edgeworth examined two general situations: in the first the groups of observations 'though not perfectly normal, yet seem to have some affinity to the normal curve' [1902, p. 325], while in the second the groups clearly belong to a different category.

In his examples Bowley used Edgeworth's approximations to the law of error. These approximations are defined as follows: let N be the whole number of observations and let Y_τ be the number of observations between the centre of gravity of the law of error (with modulus c) and the deviation $x = c\tau$. Further let i and j denote the kurtosis and skewness respectively. The first, second and third approximations are then

$$Y_\tau^{(1)} = NF(\tau) \equiv \frac{N}{\sqrt{\pi}} \int_0^\tau e^{-\tau^2}\, d\tau$$

$$\begin{aligned}
Y_\tau^{(2)} &= N\Big\{F(\tau) \mp \frac{j}{3\sqrt{\pi}} \pm jf(\tau)\Big\} \\
Y_\tau^{(3)} &= N\Bigg[F(\tau) \mp \Big((j/3)\sqrt{\pi}\Big) \pm jf(\tau) \\
&\qquad + \frac{e^{-\tau^2}}{\sqrt{\pi}}\bigg\{i\Big(\frac{\tau}{2} - \frac{\tau^3}{3}\Big) + j^2\Big(-\frac{5\tau}{3} + \frac{20\tau^3}{9} - \frac{4\tau^5}{3}\Big)\bigg\}\Bigg]
\end{aligned}$$

respectively, where

$$f(\tau) = \frac{1}{3}\Big(1 - 2\tau^2\Big)\frac{e^{-\tau^2}}{\sqrt{\pi}}.$$

As an example of Bowley's investigations let us look at the application made to the wage census figures for 1886 (the other examples mentioned before are similarly treated). Bowley uses two methods (the method of moments and the 'percentile' method) to evaluate the constants c and j (the term in i is not considered here since the last group of wage earners in the frequency table is merely described as those who earn more than 40s., and one has no idea of the centre of this group).

Using the method of moments Bowley obtains estimates for c and j from his grouped frequency table and substitutes them into $Y^{(2)}$. The sum of the absolute values of the differences between the observed and expected values is then used to test the goodness-of-fit to the observed data (Bowley tends to refer to the 'misfit' rather than the goodness-of-fit). This sum, as a percentage of the total number

N, gives a misfit of 6.6%. (For comparison, Bowley notes that if one fits a Normal distribution to the data with the computed values of c and j the misfit is 22%.)

The percentile method, as Bowley defines it,

> consists in observing what fractions of the given observations lie between the median and certain limits, and then choosing the constants so that these fractions and the calculated fractions up to the same limits shall be as nearly equal as possible.
> [Edgeworth & Bowley, 1902, p. 339]

Using the results of the first section of the paper Bowley sets up six equations of the form

$$x' = a_k y' + b_k, \quad k \in \{1, 2, \ldots, 6\},$$

where $x' = c$ and $y' = 2/(3jc)$. The six equations (labelled (a) to (f) to correspond to the six classes in the grouped frequency table) are sketched on the same system of axes, and a figure similar to our Figure 6.1 is found. Bowley notes that Edgeworth's 'double median' method could yield a point, a line segment or a small area. If all six of the equations are used it is found that one vertex of the quadrilateral (corresponding roughly to one vertex of $ABCr$ in our figure) yields the 'best' median value. If the extreme lines (a) and (f) are ignored (that is, if one considers the extreme observations to be unreliable) then any point in the quadrilateral may be chosen. If precisely one of (a) and (f) is ignored then in each case any point on an indicated side bounding the quadrilateral is indicated. The values of c and j are found to differ only slightly from case to case.

In 1928, after Edgeworth's death in February 1926, Bowley published a book entitled *F. Y. Edgeworth's Contributions to Mathematical Statistics*. In the introductory paragraphs to his chapter on the generalised law of error, Bowley notes that when it comes to the fitting of a curve to observations two methods are generally found:

the method of moments and the method of percentiles. Further, it needs to be determined whether differences between observations and curve should be measured by aggregate misfit (which is the method he used in his joint paper with Edgeworth) or by Pearson's method (by which, presumably, the χ^2 goodness-of-fit test is meant).

The use of the method of moments to estimate c, i and j, Bowley comments, is not of necessity the best. His reasons are set out as follows:

> First, the probable errors of j and i, as determined from n observations, involve respectively the sixth and eighth moments of the curve divided by $\sqrt{n}$, and consequently are subject to considerable errors of sampling, which may be of as great order as the terms neglected in the formula, which depend among other things on $m^{-3/2}$, where m is the (hypothetical) number of elements that go to form a single observation. Secondly, though in the normal curve the values of the average and of the modulus determined from the observations are those which lead to minimum improbability ... there is no such sanction for j and i. Thirdly, the observations may be in such wide or irregular grades that even with Dr. Sheppard's corrections the moments cannot be accurately determined, while the method of percentiles is particularly adapted to use the observations accurately however they are graded.
> [Bowley, 1928g, p. 56]

Bowley provides a quick description of the method of percentiles: first write Edgeworth's second approximation to the law of error in the form

$$Y_x^{(2)} = \frac{1}{c\sqrt{\pi}} \exp^{-z^2/c^2} \left[1 - 2j\left(\frac{z}{c} - \frac{2}{3}\frac{z^3}{c^3}\right)\right],$$

where $z = x - \frac{1}{3}jc$ (i.e. take the median as the origin). On expanding the right-hand side of this expression, and ignoring terms in j^2, one gets

$$Y_x^{(2)} = \frac{1}{c\sqrt{\pi}}\, e^{-(x/c)^2}\Big[1 - \frac{4}{3}j\Big(\frac{x}{c} - \frac{x^3}{c^3}\Big)\Big].$$

Integration then yields

$$\int_0^x Y_x^{(2)}\, dx = \frac{1}{\sqrt{\pi}} \int_0^\xi e^{-\xi^2}\, d\xi.$$

Bowley now notes that the left-hand side of this expression gives the proportion of the observations above the median and less than x, and standard Normal tables will hence provide the upper limit ξ of the integral on the right-hand side. Integration, and neglect once again of terms involving j^2, yields $x = c\xi + \frac{2}{3}jc\xi^2$. Although x is unknown (since the median is unknown), the difference between two successive observations x_i and x_{i+1} is known, and hence

$$x_{i+1} - x_i = c(\xi_{i+1} - \xi_i) + \tfrac{2}{3}(\xi_{i+1}^2 - \xi_i^2).$$

This allows the determination of c, j and hence of the median too. (It is also noted that if the median (M, say) and the quartiles Q_1 and Q_2 are accurately known, j and c may be found by setting the differences $Q_2 - M = c\xi + \frac{2}{3}jc\xi^2$ and $M - Q_1 = -c\xi + \frac{2}{3}jc\xi^2$.)

As illustrations of the two methods (moments and percentiles) and Edgeworth's double median method Bowley repeats here his work on English wage statistics[20] from Edgeworth and Bowley [1902].

6.5 Inference from sample to population

In 1923 Bowley published a note on the precision of measurements estimated from samples. Here, motivated by work by Edgeworth and Karl Pearson, he made quite clear his views on inference.

> One of the inverse problems of statistics, that of estimating the value of frequencies, averages, etc. in a universe from similar quantities measured in a sample[21].
> [1923b, p. 494]

Bowley begins by noting as an illustration that it is easy to calculate the probabilities of obtaining a sample correlation coefficient[22] of values $r_1, r_2, \ldots$ for a sample of size 1,000 (say) when one supposes the known values of the population correlation coefficient ρ to be $\rho_1, \rho_2, \ldots$. Nothing however can be be deduced about the chance that ρ lies between two given numbers from these calculated chances unless some prior distribution on ρ is assumed. He categorically dismisses the bland use of a uniform prior:

> the hypothesis that every value from 0 to 1 is equally probable is not only baseless, but also inconsistent with an equally plausible hypothesis that all values of $\arcsin r$ from 0 to 1 are equally probable. [1923b, p. 494]

The thing to note is that one's main concern is with a small range of possible values of r, since values outside of this range would give negligible chances of obtaining the value actually observed. This is easily accomplished.

> All we have to assume is that in a certain small range there is a continuous function representing the *a priori* chance of the occurrence of assigned values of r in the universe; then it is shown that the exact form of the function [i.e. the prior] is indifferent and that it need not even be symmetrical. [1923b, pp. 494-495]

It is also required that derivatives of the second and higher orders carry coefficients $(1/n)$ where n is of course assumed to be large.

Three cases are considered: attributes, variables and the more general case. We shall look in detail only at the first of these, the other two being extensions of the methods used in discussing the first.

Suppose then that n things are taken independently and at random from an 'infinite universe' and that the value of the number of 'successes' S, i.e. the number of items having a certain attribute, in the sample is pn. Let p' be a value of the unknown proportion P of items having this attribute in the universe, and let $F(\cdot)$ be the prior density (Bowley writes simply of 'the function in question') of P (we have changed Bowley's notation slightly). The following assumptions are made: (a) F has a Taylor series expansion and is integrable, (b) the derivatives of F are finite and (c) n is so large that $(1/n)$ may be neglected in comparison with unity. Let $q = 1-p$ and $q' = 1-p'$. Then

$$P_x \;\equiv\; \Pr[P = p' \wedge S = pn] \;=\; F(p')\,\frac{n!}{(pn)!\,(qn)!}\,(p')^{pn}\,(q')^{qn}.$$

On using Stirling's formula and setting $p' = p + x$ we have

$$P_x = \frac{F(p+x)}{\sqrt{2\pi pqn}}\left(1+\frac{x}{p}\right)^{pn}\left(1-\frac{x}{q}\right)^{qn}.$$

Now take logarithms, expand the logarithms of the terms of the form $(1 \pm y)^m$ on the right-hand side in Taylor series and let $z = x/\sqrt{(pq/n)}$. Discarding terms in $(1/n)$ one gets

$$P_x = F(p + z\sqrt{(pq/n)})\,\frac{1}{\sqrt{2\pi pqn}}\,[\exp(-\mathrm{z}^2/2)]\,\mathrm{Q}, \qquad (6.4)$$

where

$$Q = 1 + z^3(q-p)/[3\sqrt{pqn}].$$

Then

$$C_x \equiv \Pr[p - x < P < p + x] = \int_{-x}^{x} P_x\,dx \Big/ \int_{-p}^{q} P_x\,dx.$$

Substitution of k for $(q-p)/\sqrt{pqn}$ in Q and expansion of F in a Taylor series results in

$$C_x = \int_{-z}^{z} \frac{1}{\sqrt{2\pi}}\, e^{-z^2/2}\, dz,$$

that part of the integral in the denominator that lies outside of the interval $[-\sqrt{np/q}, \sqrt{np/q}]$ and terms involving $k/\sqrt{n}$ (i.e. terms in $1/n$) being neglected. Thus, no matter what the form of F, provided that it satisfy the earlier mentioned criteria,

$$\Pr[p-x < P < p+x] = \int_{-x}^{x} \frac{\sqrt{n}}{\sqrt{2\pi pq}}\, e^{-x^2/(2pq/n)}\, dx.$$

In Case II, 'Variables', Bowley supposes that n magnitudes are chosen at random and independently from an infinite universe and that the average is found to be $\overline{x}$. The unknown average magnitude in the population $\overline{x'}$ is taken to have a prior distribution F of the same kind as that assumed in the first case. Using the second approximation (the ordinary Normal distribution being the first) to the density of the sample mean of a large randomly chosen sample of independent quantities[23], Bowley writes the joint probability that the population average is $\overline{x'}$ and the sample average is $\overline{x}$ as

$$Q = F(\overline{x'})\, \frac{1}{s\sqrt{2\pi}}\, \exp^{-y^2/(2s^2)} \left[1 - \frac{\kappa}{2}\left(\frac{y}{s} - \frac{1}{3}\frac{y^3}{s^3}\right)\right], \tag{6.5}$$

where $y = \overline{x} - \overline{x'}$, $\kappa = \mu_3/(\sigma^3\sqrt{n})$, μ_3 is the third population moment about the mean and $s\sqrt{n} = \sigma$ denotes the standard deviation 'of the magnitudes in the universe', in Bowley's words.

Proceeding as before Bowley finds that the chance that the population average lies between the limits $\overline{x} \pm x$ is

$$\int_{-x}^{x} \frac{\sqrt{n}}{\sigma\sqrt{2\pi}}\, e^{-x^2 n/(2\sigma^2)}\, dx.$$

Noting that σ is most often unknown Bowley suggests that it be replaced by the sample standard deviation, and he discusses the ensuing changes in the preceding formula.

A more general statement is investigated in Case III. Here a population of size N contains p_iN magnitudes of type x_i, where $i \in \{1, 2, \ldots, k\}$ (say). A sample of size n yields yields a proportion $p_i + e_i$ of item x_i. Let $V' = \sum a_i p_i$ where $\{a_i\}$ is a sequence of constants and let $V = V' + v = \sum a_i(p_i + e_i)$. On neglecting terms involving $1/\sqrt{n}$ we find that v has a Normal distribution with variance[24]

$$\sigma_v^2 = \left[\sum a_i^2 p_i - V'^2\right]/n.$$

So σ_v is of order $1/\sqrt{n}$.

Bowley clearly specifies the conditions under which normality is obtained: the a_i are finite and, for each integral k, the ratio m_k/s^k of the kth sample moment to the kth power of the sample standard deviation should differ only finitely from the ratio of the corresponding statistics of the Normal curve[25].

Taking F as the prior distribution for V', where F satisfies the same conditions as before, Bowley has

$$P_v = F(V') \, \frac{1}{\sigma_v\sqrt{2\pi}} \, e^{-v^2/(2\sigma_v^2)}.$$

The method used in Case I leads this time to

$$C_v = \int_{-v}^{v} \frac{1}{\sigma_v\sqrt{2\pi}} \, e^{-v^2/(2\sigma_v^2)} \, dv.$$

In the calculation of σ_v terms of order $1/\sqrt{n}$ have been neglected, and Bowley now investigates what the effect will be on the solution if this is not permissible while yet neglecting terms of order $1/n$. This investigation is not carried out here: Bowley merely reiterates that 'σ_v can be computed from the observations if $1/\sqrt{n}$ is neglected' [Bowley, 1923b, p. 450].

Bowley concludes the Note with the observation that appropriate choice of the a_i leads not only to the results of the earlier section but also, in the two-dimensional case, to a result for the correlation coefficient. Further, the substitution of sample moments for unknown population moments where necessary is justifiable provided that terms of order $1/\sqrt{n}$ can be neglected, 'and the principle of inverse probability here used applies' [Bowley, 1923b, p. 450].

6.6 Statistics in economics

In 1927 Bowley delivered the Newmarch Lectures at University College, London. An abridged version [1928c] was published the following year, and this paper is in a sense a bridge between Bowley's work on statistics and that on economics.

Bowley begins by pointing out that the difficulties faced by statisticians whose field of application is economics are greater than those experienced by those working in, say, the biological sciences: 'they can seldom collect the material at first hand or obtain it in a pure state or control experiments' [1928c, p. 253].

In working with public statistics—for instance, census data—it is important not only to examine the meaning of the data but also to test their accuracy. Such tests may be either internal, the data being verified by statistical methods applied to the data themselves, or external, two independent measurements being made of the same objective. The latter type of test is the main topic of this paper.

External tests may be carried out by using different parts of the same report to check each other, or different reports by the same government department, or data compiled by independent officials.

Bowley begins with a study of population and movement thereof. He first considers the definition of population, pointing out that a census excludes British inhabitants such as soldiers, sailors and travellers abroad, while visitors from other countries are included. Any tests must therefore be conducted on what is actually an 'accidental' population. Bowley's investigations lead to the conclusion that

> the total population (as defined in the Census) of the United Kingdom as a whole can be estimated at any time correct to from 1 to 5 per 1,000 for any date since 1881. [1928c, p. 255]

The error, he concludes, is most likely due to passenger movement[26].

After examination of population totals Bowley considers the distribution of age. Information about ages is usually obtained from schedules completed by the householders themselves, and Bowley suggests that checks for accuracy are needed; for wrong figures might be put down through a warped sense of humour, desire for pensions, misunderstanding of the question, etc. Here the census figures for age and sex distributions may be misleading through the absence of members of the armed forces and the merchant navy.

Comparison of census figures with birth records suggests that age-statistics should not be used as being correct within 1%. Bowley notes that census figures should be compared with birth, death and migration figures. Continuity, with possible exceptions, may be expected over several centuries, and life-tables in which the irregularities are smoothed out will prove useful. Comparison of the figures in the 1921 census with annual births from 1911 to 1920 shows (a) the accuracy obtained from an appropriate life-table and (b) the danger in assuming a high degree of continuity in an actual age distribution. Marriage records, Bowley suggests, may be similarly unreliable, it being suspected that some brides take an 'optimistic view' of their age.

The British Census is also changeable from one census to another when it comes to a classification by occupation or by industry. For instance, classification by occupation would result in clerks being classified as clerks, carpenters as carpenters, etc., while classification by industry would result in employees being classed with reference to their employers (bankers, manufacturers, etc.). This change makes comparison from one census to another very difficult.

Attention is next turned to external tests of consistency as provided by the Census of Production and the Unemployment Insurance

Acts. The former of these is compiled from statements made by employers of wages and salaries paid and the latter from descriptions on insurance cards, as checked in part by the Labour Exchanges. Bowley details various compilations of unemployment statistics (e.g. those compiled by labour exchanges or trade unions) and then passes to the consideration of some tests by sampling. He notes that one may either repeat part of an investigation in a special investigation using a sample, or, conversely, one may test the adequacy of a sample by comparing it with the results for the whole population.

As an example of the first approach Bowley considers an investigation of working-class homes in Warrington, a sample of one in thirteen being taken[27]. It was found that the frequency curve of houses according to number of rooms provided a very bad match to the curve from the figures in the 1921 census. The houses examined in the sample were re-visited and more care taken about the classification of rooms (e.g. should kitchen and scullery be counted as one or two rooms?), but the fit was still poor. ‘This experience results in uncomfortable doubt on the complete validity of the Census statistics of housing’ [1928c, p. 266].

As an example of the converse method, the testing of the accuracy of a sample by comparing it with the whole population, Bowley considers the industrial and insurance history of insured persons, using the results of an enquiry published by the Ministry of Labour in 1927. Some 17,500,000 separate ledger-accounts, one for each of the insured, were recorded in ledgers each holding about 220 names. The sample taken consisted of the last name in each of these ledgers, about 58,000 accounts being examined. Using the formula $pn \pm \sqrt{pqn}$ Bowley finds that the expected number of women in the sample is $15,924 \pm 108$, which agrees well with the observed number of 15,800. The tabulation by industries is less satisfactory, it being shown that in twelve industries the number of men in the population differs from that in the sample by more than three standard deviations. In a footnote Bowley explains that since this paper was written it had been found that there had been a misclassification of industries.

Similar analyses are undertaken of (a) wages, (b) income, production and capital and (c) foreign trade.

In summarising his investigations of situations where both internal and external tests may be applied Bowley notes that

> On the whole the results shown by the sample are disquieting. The two reckonings have not tallied, and, instead of obtaining close confirmation, we have had to seek for explanations of differences. If two measurements have agreed within five per cent. it has been an agreeable surprise. Public statistics appear to have advanced no further than astronomy before the invention of telescopes. [1928c, p. 275]

He concludes however that while population census data should be precise, data obtained on passenger movement (say) may well be rough.

> It is reasonable to hold that we have ascertained the inaccuracy of the rough measurement, rather than thrown doubt on the better way. [1928c, p. 275]

The chief use of his analysis, he suggests, is to emphasise that public statistics have varying degrees of accuracy, and in this respect he notes 'the changes in totals, ratios and averages are often more accurate than the totals themselves' [1928c, p. 276].

Addendum. Two papers by Bowley that should have been included in this chapter came to my (A.I.D's) attention when the MS. had been completed. Published in 1937 in the *Comptes Rendus du Congrès International des Mathématicians, II, Oslo 1936* they are 'Standard deviation of Gini's mean difference' (pp. 182-185) and 'Slightly unsymmetrical frequency curves' (pp. 190-192).

Chapter 7

Statistical Books

7.1 Introduction

So far our discussion of Bowley's books has been of those concerned with official statistics or matters of social importance (poverty, conditions in various cities, employment, trade, etc.). Here we shall examine three of Bowley's books that are predominantly and distinctly on statistics. (His 1928 work *F.Y. Edgeworth's Contributions to Mathematical Statistics* is more Edgeworth than Bowley[1].)

7.2 *Elements of Statistics*

In 1901 Bowley's first book on statistics appeared. In all it went through six editions: the fifth edition of 1926 will be used here, unless otherwise stated. The first edition was based on lectures Bowley gave at the London School of Economics from 1895 to 1901, the eighth chapter, 'Accuracy', being based on Bowley's Newmarch Lectures of 1897. Changes in subsequent editions were few, except for the fourth edition, which saw the expansion of the first part of the book and a rearrangement of some of the material, while the second part was completely rewritten and extended.

The aim of the book, wrote Bowley in the fourth edition, was

> to form a general introduction to the theory and practice of statistics for all persons whose business it is to handle them or to whom a general understanding both of the utility of statistical results and the limitations of statistical investigation is important. [1926, pp. v-vi]

The book is divided into two parts: the first part deals with general statistical methods and techniques, such as require only elementary mathematical knowledge, while the second, which is concerned with applications of mathematics to statistics, requires some calculus. The treatment is general, and the illustrative examples are more from sociological and economic research than biological and actuarial work.

While noting that there are controversial areas when it comes to fundamental conceptions[2] concerning the application of probability to observations, Bowley states that he has tried to avoid such matters as far as possible[3]. However, he has adopted a certain course 'which will not meet with universal approval':

> in my opinion the standard deviation has only limited utility unless it is connected with a table of probability by which the chances of exceeding given multiples of this deviation can be calculated. [1926, p. vii]

With this in mind he stresses that arithmetic means (and other functions) that are needed when working with data derived from samples, have Normal distributions. It is also necessary to go from direct to inverse probability, and here, Bowley believes, he goes further than many writers[4].

Part I, 'General Elementary Methods", has ten chapters. The first is entitled 'Scope and meaning of statistics'. Noting that many definitions of statistics have been given[5], Bowley declares that he will merely *explain* what is meant. Despite this declaration, he provides what are *almost* definitions. Thus, for instance, he says that it is better to define statistics *a posteriori*, as providing methods of analysing data that have already been collected.

> It is clear that, under our tentative definition, statistics is not merely a branch of political economy[6], nor is it confined to any one science. [1926, p. 4]

After saying what statistics is *not*, he gives a positive definition[7]: '*statistics is the science of the measurement of the social organism, regarded as a whole, in all its manifestations*' [1926, p. 7].

Even today there is discussion of the exact stage at which the statistician's involvement in an experiment or investigation stops. Bowley was in no doubt about it:

> it may be held to be the business of the statistician to collect, arrange, and describe, like a careful experimentalist, but to draw no deductions; even in an investigation relating to cause and effect, to present evidence but not conclusions. [1926, pp. 8-9]

In the second chapter, 'The general method of statistical investigation', it is suggested that a useful general scheme for all statistical investigations is the following[8]: '(1) The collection of Material, (2) its Tabulation, (3) the Summary, and (4) a Critical Examination of its results' [1926, p. 14].

Bowley notes at the end of this chapter that statistics has an important rôle to play in the providing of evidence showing how two groups of phenomena are related to each other, and in investigating how changes in one measurable quantity are connected to those in another. It is here that probabilistic methods become important.

Chapter III is concerned with the definition of a unit and the collection of data. In connexion with the former, Bowley suggests that an investigation is essentially framed by two questions: 'What is to be counted?' and 'What has been counted?' [1926, p. 18]. The answer to the first of these questions provides at least a preliminary definition of the unit, while the answer to the second shows the modification that the practical investigation necessitated. The importance of *definition* is stressed, as is the use of common terms like *population* and *income* in technical work. The simplicity or complexity of the form will depend on whether it is to be completed by the householder (say) or the census-taker. Further, the form should make it quite clear for what purpose the information is required.

The wage census, taken in England in 1886 and 1906, is then compared to the 1911 population census. The former census differs from the latter in being an exercise voluntarily undertaken and in requiring 'a higher degree of intelligence and education' [1926, p. 30] for the completion of the form. Bowley suggests, for reasons of accuracy, simplicity and cheapness, that the information should be supplied by the employer rather than the employé.

The question of tabulation is the subject matter of the fourth chapter. Once the data have been collected, it is important that they be tabulated in such a way that answers to the questions of interest can easily be determined. If the data that have been taken are such that they do not precisely provide the information one wants, one should simply do the best one can to get results as close to what one requires as possible.

A consideration of averages of various kinds is undertaken in the next chapter. The first sections are devoted to the arithmetic (for grouped and ungrouped data) and the weighted average, and in connexion with the latter Bowley stresses the need for careful examination of the data before concluding that weights may be neglected. Once again he notes that the exact determination of the appropriate weights may not be necessary.

There is then a section that at first seems somewhat uncommon: one on *statistical coefficients*. Bowley defines this term as follows:

> A statistical coefficient is a number, whole or fractional, by which a total (*e.g.*, population) must be multiplied to give an allied number (*e.g.*, number of births).
> [1926, p. 94]

As an example Bowley supposes that the birth-rate is 28 per thousand. The *coefficient* is then 0.028. Thus these coefficients are essentially a special kind of average.

Bemoaning the fact that the mode and the median are not as well known to the common folk as they should be, Bowley next proceeds to an examination of these statistics. The mode, which, like the median, is unaffected by changes in extreme values, he finds most nearly exemplified by Quetelet's *l'homme moyen*[9] [Quetelet 1835]. He mentions the ease with which the median can be found, and points out in particular that it has the pleasing property of being applicable to quantities that are not susceptible of measurement. (It might be noted that in his [1909a] Bowley suggested that, when examining wages, it might well be sufficiently accurate for many purposes—correct within 10%, say—to use the median wage rather than the arithmetic average[10].)

After a short discussion of the geometric mean, a statistic that Bowley finds useful 'when emphasis is on the ratio between two quantities rather than on their absolute difference' [1926, p. 108], the chapter is concluded with a description of a good mean[11]:

> If there is a type it shows it; it gives due influence to extreme cases; it is not easily affected by errors or much displaced by slight alterations in systems of calculation; and it is easily calculated. [1926, pp. 107-8]

Bowley follows this discussion of means with an examination of measures of dispersion and skewness. He motivates this by noting that if one wishes to summarise grouped data, or to compare two groups, one needs (i) some measure of central tendency (a mean), (ii) a measure of dispersion and (iii) a measure of presence or absence of symmetry.

To this end he defines the mean deviation (essentially $\sum|x_i - \overline{x}|$), the standard deviation (with divisor n rather than $n-1$), the quartile deviation ($(Q_3 - Q_1)/2$) (where Q_1 and Q_3 are the first and third quartiles) and the probable error when the group is symmetric.

Perhaps less common today (though it still remains popular in Italian literature) is his next measure, Gini's mean difference[12] g (in Bowley's notation). Suppose one has n observations $\{a_i\}$ arranged in increasing order $a_{(1)} \leq a_{(2)} \leq \ldots \leq a_{(n)}$. Then

$$g = \frac{2}{n(n-1)} \sum_{i=1}^{n} \sum_{j=1}^{n} (a_{(i)} - a_{(j)}), \quad j < i.$$

Note that g may also be written in terms of the unordered observations as

$$\begin{aligned} g &= \frac{1}{n(n-1)} \sum_i \sum_j |a_i - a_j|, \quad j \neq i \\ &= \frac{1}{n^2} \sum_i \sum_j |a_i - a_j|, \quad \text{for all } \{i, j\} \end{aligned}$$

However the standard deviation may also be written in terms of differences between observations. To see this, let $S_{aa} = \sum(a_i - \bar{a})^2$. Then

$$E^2 \equiv \textstyle\sum_i \sum_j (a_i - a_j)^2 / n^2 = 2S_{aa}/n.$$

Letting $s^2 = S_{aa}/n$, we thus have $s = E/\sqrt{2}$, which shows that the usual expression for the standard deviation (if we ignore any argument about a divisor of n or $n-1$) may also be written in terms of the actual observations.

A similar result is given by Bowley, who proceeds as follows: let $\mathbf{m}$ denote the median of the n observations and let $d_i = |a_i - \mathbf{m}|$. Then the median difference may be written as

$$\frac{1}{n}\Big\{d_1 + d_n + d_2 + d_{n-1} + d_3 + d_{n-2} + \cdots\Big\}$$

while

$$g = \frac{1}{n}\Big\{2(d_1 + d_n) + 2\frac{n-3}{n-1}(d_2 + d_{n-1}) + 2\frac{n-5}{n-1}(d_3 + d_{n-2}) + \cdots\Big\}.$$

The concept of *skewness* is then introduced[13]. Bowley notes that skewness of a distribution is found when the quartiles (or similar pairs of quantiles) are unequally spaced about the median. Since skewness is concerned with the shape rather than the size of a distribution, it should be measured by a ratio rather than an absolute measure. He therefore suggested, as the simplest measure of this characteristic of a curve (called the *dispersion* in earlier editions of this book), the ratio $s = (q_2 - q_1)/(q_2 + q_1)$, where q_2 and q_1 represent respectively the excess of the upper quartile over the median and the excess of the median over the lower quartile. (Note that $|s| \leq 1$.) In terms of the quartiles Q_1 and Q_2 and the median M this ratio may be written

$$(Q_2 + Q_1 - 2M)/(Q_2 - Q_1).$$

As an example here Bowley considers the average time of the four quickest trains from each of three stations in London to and from Leatherhead in 1898, a time at which he was commuting between these places[14]. He comments, with perhaps a touch of humour, that while the matter of punctuality can be treated statistically, comfort and picturesqueness of the route cannot.

Continuing with grouped data, Bowley finds that the median, quartiles and skewness can be meaningfully used in considering adjectival[15] responses (crudely what we might today call categorical data).

The seventh chapter is devoted to graphical representation, it being claimed that this method and that of averages are best suited to handling large quantities of data. Several caveats in the construction and interpretation of graphs are mentioned: for example, the pointlessness of trying for too great accuracy in such representation, and the impossibility of ascertaining causes (though general trends may be detected). Diagrams may *suggest* correlation, but cannot *prove* much.

Periodic data are then discussed, the elimination of non-seasonal or seasonal causes being examined.

Finally in this chapter it is suggested that logarithmic scales be used to allow the more expeditious representation of data of certain kinds (for instance, diagrams of *ratios* rather than of *quantities*, as in the consideration of index numbers of prices).

Chapter VIII deals with accuracy. Though absolute accuracy is unobtainable, one may well be able to find estimates that are sufficiently accurate for appropriate purposes: for instance, public transport time-tables do not show seconds (and very often even the minutes might be doubtful), while the arrival or departure of ships may be given to hours. Bowley defines the *relative error*[16] as the ratio of the difference between the estimate and the true value, to the estimate, and 'the error is to be reckoned positive when the true value exceeds the estimate' [1926, p. 180].

Rules are given for the finding of the error in an estimate compounded of several elementary errors. For example, let $u_1, u_2, \ldots, u_n$ be estimates of n quantities with sum u, and let the respective errors be $e_1, e_2, \ldots, e_n$. Then e, the error in the sum u, is given by $e = \sum e_i u_i / u$. Similarly, the error in the product $\prod u_i$ is $\sum e_i$, and that in the ratio u_1/u_2 (say) is

$$\left[\frac{u_1(1+e_1)}{u_2(1+e_2)} - \frac{u_1}{u_2}\right] \Big/ \frac{u_1}{u_2} = \left[\frac{1+e_1}{1+e_2} - 1\right] = e_1 - e_2.$$

(Here the Taylor series expansion of $(1+e_2)^{-1}$ is approximated simply by the first term, namely 1.) Note that in these two last cases terms of second and higher orders in the e_i are neglected.

The difference between biassed and unbiassed errors is illustrated by a number of examples—one involving the distance of a bicycle ride (recall Bowley's passion for cycling)—and the effects of these two kinds of error are discussed. In summary, Bowley finds that unbiassed errors are diminished by averaging, while comparison decreases biassed error. Further, the errors in the choice of weights tend to be less important than other errors in the estimates. (Keynes [1983, p. 94] in fact notes that the errors in even a roughly chosen system of weights, if that system be rationally chosen, will be of little consequence.)

Index numbers form the matter of Chapter IX, a topic that Bowley finds to be almost co-extensive with statistics, 'for we have limited the term statistics to the measurement of complex groups and their changes' [1926, p. 196]. As usual a precise definition is given[17]:

> Index-numbers are used to measure the change in some quantity which we cannot observe directly, which we know to have a definite influence on many other quantities which we can so observe, tending to increase all, or diminish all, while this influence is concealed by the action of many causes affecting the separate quantities in various ways. [1926, p. 196]

A series of index numbers is used in both a restricted and an unrestricted sense: in the former it refers to a periodically calculated series of weighted averages, while in the latter it is merely a series whose trend and fluctuations reflect the movement of some related quantity. Three general points need consideration: (1) the nature and extent of the group and the nature of the quantity being investigated, (2) the choice of samples and (3) the effect of the weights.

A caveat is then issued against the earlier assertion that the choice of weights is usually (but not always) of little effect. Bowley notes that this is not the case when an abnormal year is chosen as base year, and suggests that in such a case the geometric mean rather than the arithmetic should be used in working with unweighted averages[18].

Assuming that $\{x_i\}_1^n$ and $\{y_i\}_1^n$ represent respectively the quantities whose movement is being investigated and their observed values, and that, at least approximately, $y_i - 100 = b_i(x_i - 100)$ (where the b_i are constants), Bowley shows that

$$I - 100 \approx k(J - 100),$$

where $I = \sum w_i x_i / \sum w_i$, $J = \sum w_i y_i / \sum w_i$, the w_i are weights and k is an average of the b's.

As a way of avoiding the difficulties occasioned by the use of a mean (for instance, the choice of weights) Bowley suggests that thought be given to using the median. He notes that American statisticians[19] have used comparison of totals rather than weighted or unweighted price ratios, a method that he finds to involve no new principle.

Difficulties not encountered when wholesale prices are considered arise when the purchasing power of different classes is considered. An example of working-class budgets in 1914 and 1918 shows that, at least in this case, the arithmetic, geometric and harmonic means yield very nearly the same index, and Bowley suggests that, for convenience, one might well use

$$\frac{\sum(Q+q)p}{\sum(Q+q)P} \times 100,$$

where $\{Q, P\}$ and $\{q, p\}$ denote the quantity and price in 1914 and 1918 respectively.

In concluding Bowley notes that index numbers may also be used 'to measure the action of a cause, which affects quantities which have no common measure' [1926, p. 212].

The final chapter in Part I is concerned with interpolation, a point that becomes of importance in, say, the estimation of population distribution in intercensal years for sociological or actuarial purposes.

As illustration of the various methods that may be used, Bowley considers the graphical method (noting that a carefully drawn freehand sketch may well be as good as one drawn on mathematical principles), periodical figures (interpolation of figures for a month, say, when annual data and the existence of periodicity are known) and the use of subsidiary curves (using the behaviour of one series to interpolate missing data for a similar series).

Passing on next to a treatment of algebraic methods, Bowley shows the usefulness of finite difference methods (including a discussion of the relationship between finite differences and differential coefficients). A problem that arises in interpolation in applied statistics is that the form of the continuous function $y = f(x)$ to be fitted is generally unknown. Under reasonable assumptions any functional relation between x and y may be written (perhaps only approximately) as $y = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$, the convergence being assumed sufficiently fast to allow the neglect of terms beyond the fifth (say). Once again the merits of a freehand curve are noted.

Several pages are devoted to a discussion of mathematical methods of interpolation, covering Newton's Method, Horner's Method, central differences and Lagrange's Formula[20].

A further ramification occurs when the initial data are themselves needful of correction—for instance, the finding of an age distribution from census data, the returns being thought to be either inaccurate (respondents may tend, for one or other reason, to give an age that is slightly 'out') or insufficient in number. In this case one requires a smooth curve near a number of points but not necessarily passing through any of them.

One way of handling this case is to apply any of the methods previously discussed to averages over 'fairly large' groups. Another is to use moving averages, while a third is the using of finite differences until the differences of some pre-determined order (say, the fourth or fifth) vanish. Finally, in a brief discussion (more details follow in Part II), Bowley advanced least squares as yet another method.

This brings us to Part II, 'Applications of mathematics to statistics'. It was this part of the book that saw most changes from edition to edition, but we shall again consider only the fifth edition.

In the first chapter frequency groups and curves are discussed, along with means, measures of variation, skewness κ (the square root of Pearson's $\beta_1 = m_3^2/m_2^3$) and kurtosis $\kappa_2 = m_4/m_2^2$ (here m_i denotes the ith moment about the origin). These characteristics are found for several actual situations. In one of the examples (concerned with the sampling of digits from tables of logarithms) Bowley applies methods given by Elderton[21] [1953] that are particularly useful for the calculation of moments using an adding and multiplying machine. How much easier complex calculations are today!

In Chapter II, 'Algebraic probability and the normal curve of error', Bowley summarises all the methods and fundamental theorems of algebraic probability in one paragraph[22]: suppose that N events are equally likely to occur, and that exactly M of these have a certain characteristic. Then the chance that an event that has occurred has this characteristic is M/N. This leads to a discussion of the well-known multiplication and addition laws for chances and the binomial distribution. The mean, variance, skewness and kurtosis of a random variable $X \sim \mathrm{Bin}(n,p)$ (Bowley does not use this notation) are found and the Normal density function is then derived, first as a limit to the binomial with $p = \frac{1}{2}$ and then for other values of p. (The Stirling-de Moivre theorem for the approximation of factorials gives

$$n! \sim \sqrt{2\pi}\, n^{n+(1/2)}\, e^{-n}.$$

Here "$\sim$" indicates that the ratio of the two sides tends to unity as $n \to \infty$.)[23]

Note that in the case of $p \neq \frac{1}{2}$ the usual expression

$$y = [1/(\sigma\sqrt{2\pi})]\, e^{-x^2/2\sigma^2}$$

is obtained on neglecting terms of order $1/\sqrt{n}$, while the retention of such terms and neglect of terms of order $1/n$ yields

$$y = [1/(\sigma\sqrt{2\pi})]\, e^{-(x^2/2\sigma^2)}Q,$$

where $Q = 1 - \frac{\kappa}{2}(\frac{x}{\sigma} - \frac{x^3}{3\sigma^3})$. (Here κ and σ denote, as usual, the skewness and the standard deviation respectively.) The mean, variance, kurtosis and skewness for the Normal distribution are then found. It is also noted that the probable error is that value of z for which

$$\int_0^z \frac{1}{\sqrt{2\pi}}\, e^{-z^2/2}\, dz = \frac{1}{4},$$

yielding, more generally, $x \equiv z\sigma = 0.67449\sigma$. This measure Bowley finds to be often preferable to σ.

Bernoulli's limit theorem is stated (but not proved) to show the correspondence between an experimental and a theoretical frequency, and some examples are presented. An important application of this result is to sampling, and Bowley discusses this with reference to a number of examples.

Especial attention is then given to the case in which the universe sampled may not be regarded as 'infinite' for all intents and purposes, or when the samples are not independent. Suppose that, of N items, pN have a specific property and qN do not, where $p + q = 1$. Let n items be drawn, and let P_x denote the probability that $pn + x$ of these have the property in question. Then, under a suitable limiting procedure, P_x has a Normal distribution with $\sigma^2 = pqn(1 - (n/N))$.

A discussion of the 'law of small numbers' follows[24]. This yields, of course, the usual approximation of the binomial by the Poisson distribution. Bowley notes that the derivation shows what he terms the *permanence of small numbers*: experience shows that, if a certain characteristic is found among a small number of a vast number of items, then this small number is either seldom greatly exceeded or seldom vanishes.

From the small we pass to the great: Chapter III is concerned with 'The law of great numbers (the generalised law of errors)[25]'. Here Bowley shows that the Normal density can in fact be deduced from a wider set of hypotheses than the expansion of $(p + q)^n$ and a suitable limiting procedure. He first finds expressions for the standard deviation and the mean cube of error for a sum and an arithmetic mean, viz. $\sigma_a = \sigma/\sqrt{n}$ and $\kappa_a = \kappa/\sqrt{n}$, the subscript a denoting average. He then shows essentially that, if $\{X_i\}_1^n$ is a sequence of (independent) Normal random variables then $\sum X_i$ and $\overline{X}$ are also Normal. It is then observed that the same result, at least as a first approximation, obtains even when the X_i are not Normally distributed, Bowley viewing this as so important that he gives two proofs.

In the first of these, based on the expansion of a multinomial distribution, Bowley shows that the moments of the latter are the same as those of the Normal, when terms smaller than $1/n$ are ignored. As we mentioned in the previous chapter, Bowley was well aware that a sequence of moments need not necessarily determine a density uniquely, and here he notes that his result is proved 'if we may take identity of standard deviations and of all moments as implying identity of curves' [1926, p. 295].

The second proof, more mathematically challenging, was given by Edgeworth in 1905. While Edgeworth's result yields a number of approximations[26], Bowley here gives only the first two; the first of these is a Normal density of the usual form, while the second is

$$Q = \frac{1}{s\sqrt{2\pi}} \exp^{-x^2/(2s^2)} \Big[1 - \frac{\kappa}{2}\Big(\frac{x}{s} - \frac{1}{3}\frac{x^3}{s^3}\Big)\Big]. \tag{7.1}$$

(Here s denotes the standard deviation 'of the frequency curve' of the sum, in Bowley's words.)

Hitherto it has been supposed that the universe of possible samples is so large (infinite?) that the chance of any specific item's being chosen does not affect the chance of selection of any other item. This condition is now removed, the universe being assumed finite. Supposing thus that n items are randomly chosen from N Bowley shows that $s^2 = \sigma^2(1 - (n/N))$, where s^2 and σ^2 denote respectively the variance of a sum and the variance of the universe sampled.

Mention is also made of Isserlis[27] [1918], in which paper it is shown that, for large values of the population size N, the arithmetic mean of a random sample of size n is approximately Normally distributed, no matter what the distribution sampled may be or whether n/N be negligible. What Bowley modestly does not mention, however, is that it was his suggestion to Isserlis of the importance of this problem in connexion with random sampling and the results that might be expected that led to the paper.

Eight examples are then given concerned with the fitting of a Normal distribution, in its second approximation form, to data. These are carefully chosen to illustrate a number of different situations.

The first example has three parts. In the first, (A), the 'number of letters in each of the first completed words in 10,000 consecutive lines' [1926, p. 304] of a long book are counted. In (B) the first 10 entries are then added, and 1,000 batches are made in this way. Finally, (C), 100 sums each of 100 entries are similarly made. Integration of (7.1) (with s^2 being replaced by σ^2 and $z = x/\sigma$) shows that, if $X \sim N(\mu, \sigma^2)$, then for $z > 0$,

$$\Pr[(X - \mu)/\sigma < z] = F(z) - \kappa f(z),$$

where

$$F(z) = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-x^2/2}\, dx$$

and

$$f(z) = \frac{1}{6\sqrt{2\pi}} \left[1 - (1 - z^2) \exp^{-z^2/2}\right].$$

Further, $\Pr[|(X - \mu)/\sigma| < z] = 2F(z)$. (Note that we have retained Bowley's notation here: f is not the density function corresponding to F—and F is not a distribution function.) Expected frequencies (using an arbitrary origin at 8 and the sample variance) are then calculated using tables given for $F(z)$ and $f(z)$.

While the original frequency curve, although continuous and unimodal, was not Normal, most of the area under the curve seemed to lie between the limits $\overline{x} \pm 2\sigma$, thus suggesting that the law of great numbers in the usual form should hold for items randomly chosen.

In (B) Bowley finds an improvement in the fit on using the second approximation (getting a close fit), but finds that the very close fit obtained in (C) using the first approximation is not perceptibly improved by using the second approximation. In answering (B) he concludes that

> analysis shows that the author's style changes from the earlier to the later part of the book, so that there is some correlation between 10 words taken consecutively,
> [1926, p. 306]

an opinion that is borne out by a re-arrangement of the 100-word samples used in (C). One may well ask oneself what consequences Bowley's observation of the change of style through the book has for questions of disputed authorship.

The second example is concerned with the measurement of the length of bricks forming the boundaries of paths in a garden. Variations in lengths may be due to a number of independent factors—weathering, differences in the original formation, difficulties of measurement, etc.—each of small effect. The effects being thus expressible as a sum of errors, a Normal distribution is fitted and, when corrected, is seen to be good.

The two biometrical examples are concerned with (1) skull and stature measurements of the Dinka[28] race and (2) the length of plaice in the North Sea. In both cases the Normal fit is found to be satisfactory. Example 5 deals with a report[29] on the numbers of school children of various ages in the sixth grade in the public schools in St Louis, U.S.A., in 1899. Again the second approximation (7.1) gives a close fit.

The last three examples, none of which is worked out in full, deal with (1) speeds of 100 pedestrians as measured by the time taken to walk between two fixed points, (2) the expenditure on food by 970 working class urban families and (3) the cost of flour in 272 places in the U.S.A.

Chapter IV deals with applications of the law of error. After a general examination of the question of the precision of sums and averages, Bowley produces an example on the Normal distribution of averages, using the distribution of the last digits in a table of seven-figure logarithms. The Normal fit, although adequate, is perhaps not as good as might be expected, and Bowley, citing Nixon [1913], draws attention to the fact that the assumption of independence is not completely satisfied[30].

Passing on next to absolute errors in weighted sums and averages, Bowley assumes again that n quantities are selected independently from n frequency groups, the means and variances of these groups being known. The square of the standard deviation (Bowley again avoids the word 'variance') of the average of the sampled data is shown to be given by $s_a^2 = \sum w_i^2 \sigma_i^2 / (\sum w_i)^2$. Further, if $\overline{\sigma}^2 = \sum w_i^2 \sigma_i^2 / \sum w_i^2$, then it follows that

$$s_a = \frac{\overline{\sigma}}{\sqrt{n}} \sqrt{\left(1 + \frac{\sigma_w^2}{\overline{w}^2}\right)},$$

where $\overline{w} = \sum w_i / n$ and $\sigma_w^2 = \sum (w_i - \overline{w})^2 / n$.

Bowley turns next to an examination of relative errors (or deviations). Such an error e is defined in general as $e = (x - x')/x'$, where x is the observed value of the true (or mean) value x'. Starting off with products and quotients, Bowley supposes that two independent factors F_1 and F_2 are erroneously measured as $F_1(1 + e_1)$ and $F_2(1 + e_2)$ respectively. Then the error e in their product P satisfies

$$P(1 + e) = F_1(1 + e_1)\, F_2(1 + e_2),$$

which yields $e = e_1 + e_2$ when $e_1 e_2$ is negligible. Hence, by an earlier result, the variances satisfy $\sigma^2 = \sum \sigma_i^2$. (Here σ^2 is the variance of e and σ_i^2 is the variance of e_i.) This is immediately extended to a product of n factors.

When it comes to a quotient $Q = F_1/F_2$ of independent factors, one has

$$Q(1 + e) = F_1(1 + e_1)/F_2(1 + e_2),$$

which reduces on expansion to $e = e_1 - e_2$ when squares and products of the e's are ignored.

As a general remark here Bowley notes that if e is the error in a function f and e_1 is the error in x, then $f(x) \times (1 + e) = f\big(x(1 + e_1)\big)$ and hence, at least approximately,

$$e = x\, [f'(x)/f(x)]\, e_1.$$

Relative errors in averages are then examined, the expression for the standard error of the mean being of the same form as that for the absolute error of a weighted average given before. In considering relative errors in weighted averages Bowley draws on his [1911a]. Here it is assumed that the weights w_i are also subject to error.

When it comes to the comparison of averages Bowley supposes firstly that $Q = F_1/F_2$ where F_1 and F_2 are independent quantities. If e, e_1 and e_2 are errors in Q, F_1 and F_2 respectively, then, as already shown,

$$Q(1+e) = F_1(1+e_1)/F_2(1+e_2) \approx (e_1 - e_2)(F_1/F_2),$$

and the standard error of Q is given by $\sigma_Q = \sqrt{\sigma_1^2 + \sigma_2^2}$. However errors may very often be both positive or both negative (e.g. deliberate underestimation of wages on two different occasions). In this case let $d = e_1 - e_2$. When e_1 and e_2 are independent (with zero means), the expectation of d^2 is simply the sum of the expectations of e_1^2 and e_2^2. Detailed formulae for the standard errors for the ratio of weighted and unweighted averages are given in Bowley's Appendix. It is further noted that under generally satisfied conditions the ratio of the weighted averages may be determined with considerable accuracy even if the errors in the weights and the original measurements are large.

As a final topic in this section Bowley considers whether an observed difference between two averages could be ascribed to observational error (for example too small a sample size) or to actual differences between the characteristics under examination. A useful definition is provided:

> If the observed difference is greater than is to be expected in chance selection, it is said to be *significant, i.e.* significant of a real difference between the phenomena.
> [1926, p. 329]

The general procedure is as follows: let $\overline{x_1}$ and $\overline{x_2}$ be the observed means of samples of size n_1 and n_2 respectively. Form the ratio $|\overline{x_1} - \overline{x_2}|/\sigma$, where σ is the standard deviation of the distribution of $\overline{x_1} - \overline{x_2}$, and compare this with various percentage points of the standard Normal distribution. It is to be concluded that there is no evidence of any real difference if $|\overline{x_1} - \overline{x_2}|$ is not greater[31] than 0.674σ, while the event may be said to be improbable unless the difference is really present once it exceeds 2σ and significant once it exceeds 3σ.

This is then illustrated by examples involving (1) the examination of a binomial for which N and p are known, (2) two binomial samples drawn from the same universe and (3) two binomial samples drawn from different universes (or from a stratified universe) where the p_i may differ from one universe to another[32].

Passing next to the case of a universe containing N measurable objects, Bowley considers two separate cases. In the first of these it is supposed that the mean $\overline{x}$ and standard deviation s of the population are known. The quantity computed is of the same form as before, with the standard deviation now being given by $s\sqrt{(1/n) - (1/N)}$ where n denotes the sample size. In the second case one's knowledge about the population is supposed to be only that obtained from a sample, n, $\overline{x}$, s. A sub-sample n_1, $\overline{x_1}$, σ_1 is then taken, and it is shown that the 'test statistic' (for want of a better phrase) is

$$|\overline{x_1} - \overline{x_2}| \Big/ \sqrt{\left(\frac{\sigma_1^2}{n_1} + \frac{s^2 - 2\sigma_1^2}{n}\right)}.$$

In the examples Bowley considers it is found that stratification improves (if only slightly) the precision of the average.

Two applications of the principles of this chapter in a time-series context are then handled, the aim being to see if fluctuations are random or the result of either a trend or periodicity. Two examples (the first on recorded times in 'The Oaks[33]' between 1850 and 1899, and the second on marriage rates in England and Wales from 1860 to 1909) on the existence of trend are discussed. In neither of these examples is there any evidence of sudden change over the entire time period when the entire set of data is considered, but in each case division of the data into two groups shows (at least to the eye) a significant difference between the two time periods.

On the matter of periodicity Bowley notes that while the question of the *existence* of a period is properly in the domain of harmonic analysis, the *influence* of a given period may be handled statistically. Three examples are considered. In the first of these monthly measurements are taken over t years, the standard deviation about $\overline{x}$ being $\sigma = \sqrt{\sum(x-\overline{x})^2/(12t)}$. The use of Normal tables, where possible, is then suggested either to compare two averages each of t records or to compare the average of the t January records with that of the t February records (say). This method has certain difficulties however (for instance, the examination of the fact that the averages seem generally to fall from month to month over part of a year and then to rise similarly), and so it is suggested that one might well rather examine the numbers of rises and falls from one month to the next. Once again a definite conclusion seems difficult to reach.

A short, though theoretically deep, chapter on empirical frequency equations then follows. Noting that in general frequency groups cannot be expressed by the law of large numbers, the independent causes needed for the generation of the Normal distribution not necessarily being able to be assumed, Bowley asserts that one needs to consider either a wider class of distributions than the Normal, or curves that are empirically found to fit specific data sets. Naturally least squares is a method that presents itself, but Bowley claims that Pearson's method of the equating of the population and sample moments is in general use in the case of frequency curves for observations. This method and others are discussed here.

Pearson's system of frequency curves results from solving the differential equation[34]

$$\frac{d}{dx}\,y = \frac{(x+a)}{b_0+b_1x+b_2x^2}\,y$$

(we follow Bowley in using y to denote the frequency function).

By appropriate choice of the constants seven principal types of solution curves may be found for the fitting of observations. For example, the Normal density and the second approximation to the general error curve result respectively from the equations

$$\frac{d}{dx}\,y = -\frac{x}{\sigma^2}\,y$$

and

$$\frac{d}{dx}\,y = -[x + (\kappa\sigma/2)]/[\sigma^2 + (\kappa\sigma x/2)]\,y.$$

[To solve these equations write them in the form $y' + g(x)y = 0$ and multiply by the integrating factor $\exp(g(x))$.]

Edgeworth's method, requiring a transformation of the Normal curve, and treated here simply by referral to relevant papers, is of use in cases where the skewness is so large that the second approximation to the generalised law of error (see equation (7.1)) is inapplicable. In its simplest form Pareto's Law[35], $y = Aa/x^{a+1}$, results from solving $\frac{d}{dx}y = -(m/x)y$ (use an appropriate integrating factor as indicated above). Finally, a formula that is particularly useful in actuarial work is Makeham's formula[36], which results from solving $\frac{dy}{dx} = -(a+bc^x)\,y$. The solution is

$$y = ks^x g^{c^x},$$

newly-introduced constants being functions of a, b, c.

The next three chapters are concerned with correlation: the first with the theory, the second with examples and the third with partial and multiple correlation. The concept is described as follows. Suppose we have pairs of observations $\{X_i, Y_i\}$.

> If the average or shape of the frequency curve of the Y's associated with a given X is not the same as that for all values of Y when the sorting by values of X is not made, then there is something common to the two quantities and they are said to be correlated. [1926, pp. 350-351]

The regression curves of X on Y and Y on X are then mentioned, and for n pairs of magnitudes it is shown that

$$E(XY) = \overline{X}\,\overline{Y} + E(X - \overline{X})(Y - \overline{Y})$$

(notation altered).

Noting the presence of $E(X - \overline{X})(Y - \overline{Y})$ in formulae when X and Y are not independent[37], Bowley suggests that this be used as an indicator of correlation, and suggests that it be divided by $n\sigma_x\sigma_y$ to get rid of its dependence on the units of measurement. This leads to the usual expression for the correlation coefficient r.

John Maynard Keynes examined the question of correlation in *A Treatise on Probability* in 1921. Although citing with approval the work of writers like Yule and Bowley ('the best and most systematic writers on the subject' [Keynes, 1973, p. 461]) Keynes finds the general treatment of the transition from the mathematical definition of the correlation coefficient r to its use in inference to be far from clear.

Discussing the nature of r, Bowley supposes that

$$X_i = \sum_{j=1}^{p} {}_jU_i + \sum_{k=1}^{q} {}_kV_i \quad \text{and} \quad Y_i = \sum_{j=1}^{p} {}_jU_i + \sum_{k=1}^{q} {}_kW_i,$$

where the U, V and W are chosen at random from different frequency groups and are independent of each other. It transpires then that $r = p/(p+q)$, and

> This is the simplest conception of the numerical value of r; ... it shows that the correlation coefficient tends to be the ratio of the number of causes common in the genesis of two variables to the whole number of independent causes on which each depends. [1926, p. 356]

Supposing that the frequency curves for the U, V and W are all Normal, Bowley derives the customary expression for the correlation surface, viz.

$$e^{-Q}\frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-r^2}}e^{-Q},$$

where

$$Q = \frac{1}{2(1-r^2)}\left[\frac{x^2}{\sigma_x^2}+\frac{y^2}{\sigma_y^2}-\frac{2rxy}{\sigma_x\sigma_y}\right].$$

Mention is also made of the possible extension to consideration of weighted sums.

The fact that Normal distributions are required for the above work means that the importance of the usual product-sum formula is somewhat attenuated, and other measures are therefore explored. Under the same kind of assumptions made before, an expression, due to Edgeworth, for the correlation surface of the Normal form is derived under the assumption that n is large and $1/\sqrt{n}$ small. Some properties of the Normal correlation surface are given (e.g. moments and the form of cross-sectional distributions—again Normal). Further, if the conditions ensuring that the regression curve be a straight line should not be satisfied, it may nevertheless be found that the line is approximately rectilinear, even in the absence of initial Normality of the Xs and the Ys.

> If, however, there is nothing in the genesis of the measurements, or in their results, to justify the assumption of rectilinearity, r ceases to be an intelligible measurement of the *amount* or degree of commonness of causation, though it may still be a useful function of the quantities in analysis. [1926, p. 365]

Pearson's correlation ratio is then discussed (see Pearson [1905], [1911a] and [1923a]). Consider a bivariate table and let ${}_ps_x$ denote the standard deviation of the pth array of the Xs. Further, let

$${}_a\sigma_x^2 = (1/N)\sum_p n_p\,{}_ps_x^2,$$

where n_p is the number in the pth array. (So ${}_a\sigma_x^2$ is the weighted mean of ${}_1s_x^2$, ${}_2s_x^2$, etc.) The correlation ratio is defined by

$$\eta_{xy} = \sqrt{1 - ({}_a\sigma_x^2/\sigma_x^2)}.$$

(In the Normal theory $\eta^2 = r^2$.)

The question of correlation when data are presented in a 2×2 table is then examined, it being desired to see whether there is a relationship between the two factors and, if so, to measure it. When Normality obtains Bowley notes that a 'troublesome equation for r' [p. 368] has been found by Pearson [1900a]. Under the assumption that the underlying distributions are Normal and that separation of the data at the median is possible, it can be shown that $r = \sin 2\pi q$ where $a = d = (\frac{1}{4} - q)N$, $b = c = (\frac{1}{4} + q)N$ and a, b, c, d are the entries in the 2×2 table in the north-west, north-east, south-west and south-east cells respectively.

Next the matter of association in general is discussed. Referring to the following 2×2 table,

a	b	n_1
c	d	n_2
m_1	m_2	N

Bowley notes that Yule has used $Q = (bc - ad)/(bc + ad)$, the *coefficient of association*, and $\omega = (\sqrt{bc} - \sqrt{ad})/(\sqrt{bc} + \sqrt{ad})$, the *coefficient of colligation* (see, for instance, Kendall & Stuart [1973, p. 559])[38]. While noting that considerable experience is needed for the correct interpretation of statements like '$Q = \frac{1}{4}$', or that '$Q_1 > Q_2$' indicates a greater degree of association in the first case than in the second, Bowley states that a good deal of the difficulty arises from the fact that 'no definite measurable meaning has been given to the term "association" ' [1926, p. 370]. It is then perhaps better to restrict one's attention to the *existence* rather than the *amount* of association, and this leads to a discussion of contingency and Pearson's *coefficient of contingency* defined by $C = 1/\sqrt{1 + (N/\chi^2)}$, where N is the total number in the 2×2 table and χ^2 is the well-known expression in terms of observed and expected values,

$$\chi^2 = \sum_i (o_i - e_i)^2/e_i.$$

In 1963 Anthony Edwards examined the question of association in a 2×2 table of paired attributes with unfixed marginal totals. He showed that any logical measure of association should be a function of the cross-ratio, and this rules out Pearson's coefficient of contingency but allows Yule's two coefficients.

It is perhaps worth pointing out that Pearson [1900b], when discussing the fitting of observed data to a theoretical frequency distribution, derives in terms of m_i' and m_i, the observed and expected frequencies respectively, $\chi^2 = \sum(m_i' - m_i)^2/m_i$. However he writes[39]

> This result is of very great simplicity, and very easily applicable. The quantity
>
> $$\chi = \sqrt{\sum(m_i' - m_i)^2/m_i}$$
>
> is a measure of the goodness of fit.

It is now known that χ^2 has 'nicer' properties than χ (for example, the sum of two independent χ^2's is again a χ^2. See Barnard's introduction to Pearson [1900b] as given in Kotz and Johnson [1992]).

The second last section in this chapter is concerned with correlation between two time series. Generally in such a situation the value of x_i at one time is not independent of that at another, and hence the existence of a relationship between x and y may merely suggest a periodic progress over time. The presence of trends in the same direction may result in a high correlation coefficient even if x and y are 'otherwise independent'. It is thus necessary to get rid of the time factor, and here Bowley recommends smoothing, calculating the correlation coefficient between $x_i - \overline{x_i}$ and $y_i - \overline{y_i}$ over an odd number m of years, where x_i, y_i are a pair of measurements in the central year of the m years, and $\overline{x_i}$ and $\overline{y_i}$ are the arithmetic means of measurements over the whole period[40]. There is also mention of Pearson's 'variate difference correlation' [Cave[41] & Pearson, 1914], but this Bowley finds generally unsatisfactory on account of a lack of precision or the small number of significant figures in the data.

A final note in the chapter is concerned with a verbal description of how to draw a scatter diagram and to sketch the regression line.

Chapter VII is devoted to some examples illustrating the use of the preceding theory. In some only the values of the correlation coefficient and the correlation ratio are found; in others, the equation $y-\overline{y} = r(\sigma_2/\sigma_1)(x-\overline{x})$ is drawn on the scatter plot; and in one there is a detailed comparison of 1,000 data points with the theoretical correlation distribution.

As a final Note here Bowley discusses the change that would be made to the Normal correlation surface if terms of order $1/\sqrt{n}$ were to be retained in the development of the correlation surface. As a reference Bowley cites Edgeworth [1917a], where the appropriate expression is seen to be

$$z = z_0 - \frac{1}{3!}\left(k_{30}\frac{d^3}{dx^3}z_0 + 3k_{21}\frac{d^3}{dx^2\,dy}z_0 + 3k_{12}\frac{d^3}{dx\,dy^2}z_0 + k_{03}\frac{d^3}{dy^3}z_0\right),$$

where

$$z_0 = e^{-Q}\big/[2\pi\sqrt{(1-r^2)}],$$

$Q = (x^2+y^2-2rxy)/2(1-r^2)$, x and y are the differences between the observations and their means, divided by their standard deviations, and the k's are the moments $k_{ij} = E(X^iY^j)$.

Partial and multiple correlation form the topic of Chapter VIII. Attention is restricted in the main to three variables, though the more general theory is also considered. The partial correlation coefficient between x and z, say, y being held constant, is shown to be

$$(r_{xz} - r_{xy}r_{yz})\big/\left(\sqrt{1-r_{xy}^2}\sqrt{1-r_{yz}^2}\right).$$

Although based on Yule's [1897], Bowley's treatment differs in certain respects[42]. The latter avoids the method of least squares

> a method which is not used (except very rarely) in this book, because of the difficulties that underlie the principles involved [1926, p. 400]

but rather uses the law of error and the assumption that the standard deviation of z is independent of x and y. It is supposed here that $z = ax + by + c$, and the constants a, b, c are to be determined

> so that the observed deviations of observed values of z from the values given by this equation have the least improbability. [1926, p. 398]

Suppose that there are n pairs of values (x_i, y_i), the ith pair occurring k_i times, and let $N = \sum_1^n k_i$. Bowley's method then requires the assumption that each $\eta_i \equiv \overline{z_i} - (ax_i + by_i + c)$ should have a Normal distribution, and the further assumption of independence allows the estimation of a, b, c from the minimisation of $\sum_1^n k_i \, \eta_i^2$.

Three examples are provided. In the first, concerned with working-class families, the variables involved are weekly expenditure on food, number of persons under 14 years and number of persons over 14 years. The average weekly expenditure on food was found to be 51s, the data pertaining to the skilled classes. The second example is concerned with the number of rooms in a tenement, the number of children under 10 and the total number of people in the family. The third, in similar vein, considers income, rent and family make-up.

The chapter is concluded with a mathematical development of the multiple correlation surface for $X = U + V_1$, $Y = U + V_2$ and $Z = U + V_3$ where U, V_1, V_2, V_3 are assumed to have Normal distributions. There is also discussion of the case in which X, Y, Z are finite sums of Normally distributed variables, and of the case in which, instead of three variables, we have n.

Chapter IX, 'Precision of measurements of averages, moments and correlation', begins with a discussion of inverse probability. Previous problems were concerned with the representation of a given population by a sample; the more practical problem, however, is the inference from a sample to a population. 'This involves the difficult and elusive theory of inverse probability[43]' [1926, p. 409].

This opinion perhaps puts Bowley squarely in the anti-frequentist (or non-frequentist) school of probabilists—at least if we follow a modern author's opinion. For in 2002 David Howie, in considering the rise of the frequency interpretation of probability in the nineteenth century, stated that in this context probability could safely be used for modelling and estimation, but had place neither in inference nor in matters of judgement.

Some simple examples (of the balls and coins types) are given. As one of the examples of some interest as being perhaps somewhat out of the run of the usual simple examples on this topic, Bowley has the following. Suppose that a bag contains n balls (each either black or white), the number of white balls being determined by spinning a wheel on the circumference of which the numbers $0, 1, \ldots, n$ are equally spaced. The number taken is that nearest a fixed point next the circumference when the wheel is spun. The *a priori* probability of there being k white balls in the urn is then $q_k = 1/(n+1)$. The probability that, a white ball having been drawn, the urn originally contained k white balls, is $p_k = k/n$. Then, using a discrete form of Bayes's Theorem, one has

$$\frac{q_k\, p_k}{\sum_i q_i\, p_i} = \left[\frac{1}{n+1}\frac{k}{n}\right] \Big/ \sum_{j=1}^{n} \frac{1}{n+1}\frac{j}{n} = \frac{2k}{n(n+1)}.$$

Let X denote the number of white balls originally in the urn. Then

$$F(x) \equiv \Pr[X \leq x] = \sum_{k=1}^{x} [2k/(n(n+1)] = x(x+1)/n(n+1).$$

If it is as likely as not that there were originally as many as x white balls in the urn, i.e. if $F(x) = \frac{1}{2}$, then $x(x+1)/n(n+1) = 1/2$. For large n Bowley suggests that x be approximated by $n/\sqrt{2}$; a more accurate approximation is $x = \sqrt{n(n+1)/2} - \frac{1}{2}$, but the difference between the two is slight.

Bowley notes that

> the result depends on the hypothesis made as to the relative *a priori* chances of the unknown events between which we have to choose, and as indicating that we get a more comprehensive result by aggregating the chances than by taking them singly. [1926, pp. 411-412]

The next section of this chapter is concerned with the precision of a proportion P of items having a specific characteristic in a population of size N when a sample of size n yields np' items having that characteristic. Under the assumption that all values of P in $[0,1]$ are *a priori* equally probable Bowley deduces that

$$P_x \equiv \Pr[p' < P < x] = \frac{\binom{n}{p'n} \int_{p'}^{x} u^{p'n}(1-u)^{q'n}\,du}{\binom{n}{p'n} \int_{0}^{1} u^{p'n}(1-u)^{q'n}\,du},$$

where $q' = 1 - p'$. (Bowley merely writes that he is finding 'the chance that the original value of p was between p' and x', but he clearly assumes that $p' < x$.) On taking n to be so large that terms of order $1/\sqrt{n}$ may be neglected, and passing to Normal approximations[44], Bowley deduces that

$$P_x = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-u^2/2}\,du,$$

where $x = p' + z\sigma$ and $\sigma^2 = p'q'/n$. Bowley is careful to point out that his result is the converse to something similar obtained in finding $\Pr[p < P < p + p_1]$ when sampling from a *known* universe, the appropriate variance now being in terms of p rather than p'. Indeed, in his Chapter IV, p. 330, Bowley had shown that when N and P are known the sample proportion p' is approximately Normally distributed with mean P and variance $P(1-P)(n^{-1}-N^{-1})$. For negligible n/N this becomes $P(1-P)/n$. There is also a short discussion of the more general (i.e. not necessarily binomial) situation. It is noted that while inference about an unknown sample parameter in terms of a known population parameter is comparatively easy, the converse is not necessarily so.

> In order then to determine the precision of any measurement based on a sample, we have to take three steps, the first to find the standard deviation of the difference between the true value and the observed value, the second to find the chance that any assigned deviation would arise, the third to apply the principle of inverse probability. [1926, p. 415]

A numerical example illustrates the result, today well known, that the influence of the prior is most felt in the neighbourhood of the mode of the prior density. Over that region it is suggested that a good first approximation is achieved by taking the *a priori* probability to be proportional to the area[45].

Further examples concern the precision of the arithmetic mean and the standard deviation. In the first case an expression in terms of a Normal distribution function is given for the chance that the (unknown) average in the population differs from that in the sample (known) by no more than a certain amount. It is assumed here that the sample size is large and that sampling is effected from a group 'in which no large portion is distant more than, say, twice its standard deviation from its average' [1926, p. 416], this assumption allowing a Normal approximation. In the case of the precision of the sample standard deviation and variance, Bowley derives expressions for these characteristics in terms of population moments and *vice versa*. The appropriate simplification obtained if the population is

Normally distributed is noted, as is the fact that the method was communicated to Bowley by Edgeworth. However, it is also noted that 'The standard deviations and frequency curve of errors in higher moments or in the correlation coefficient cannot be, at any rate readily, calculated by this method' [1926, p. 418]. To this end the rest of the chapter is concerned with the finding of the standard deviation of various moments and the correlation coefficient without using inverse probability.

Bowley [1928d] notes that Sheppard had given the expression

$$\sigma_r^2 = \frac{r^2}{n}\left\{\frac{M_{22}}{r^2\sigma_x^2\sigma_y^2} + \frac{M_{40}}{4\sigma_x^4} + \frac{M_{04}}{4\sigma_y^4} - \frac{M_{31}}{r\sigma_x^3\sigma_y} - \frac{M_{13}}{r\sigma_x\sigma_y^3} + \frac{M_{22}}{2\sigma_x^2\sigma_y^2}\right\}$$

in his [1899], where $M_{rs} \equiv |x^r y^s| = \frac{1}{n}\sum_1^n x_i^r y_i^s$. While this expression holds in general, it can be shown to reduce to that holding in the Normal case when the appropriate moments are used.

The final chapter in this second part is headed 'Tests of correspondence between data and formulæ', and is devoted to goodness-of-fit tests. As an example at the outset Bowley considers the weekly expenditure per 'unit' in $N = 970$ families, and fits a Normal distribution. The first method he mentions to test the goodness of fit is to evaluate what we may write symbolically as $(1/N)\sum|o_i - e_i| \times 100\%$, the sum being over all values of i. He finds this method unsatisfactory on two scores: (i) it has no probabilistic connexions and (2) one has no idea how good the fit is. He then presents Pearson's method[46] (see Pearson [1900b]), which runs as follows. Suppose that N items are chosen at random from an indefinitely large universe. Let m_i be the number expected in the ith class (or 'grade' as Bowley terms it), where $\sum_1^n m_i = N$. The chance that $m_i + e_i$ observed items fall in the ith group, where $\sum e_i = 0$, is then given by

$$e^{-e_i^2/(2\sigma_i^2)}/(\sigma_i\sqrt{2\pi}),$$

where $\sigma_i^2 = p_i(1-p_i)N$ and $p_i = m_i/N$. The joint chance of the errors e_i is then $P = Ke^{-\chi^2/2}$, where $\chi^2 = \sum(e_i^2/m_i)$ with K constant.

When it comes to the evaluation of P Bowley refers the reader to Karl Pearson's work, merely noting here that

$$P = \begin{cases} \sqrt{\dfrac{2}{\pi}}\Big[\displaystyle\int_z^\infty e^{-x^2/2}\,dx + \Big(\frac{z}{1} + \frac{z^3}{1.3} + \cdots + \frac{z^{n-3}}{1.3\ldots(n-3)}\Big)e^{-z^2/2}\Big] \\ e^{-z^2/2}\Big(1 + \dfrac{z^2}{2} + \cdots + \dfrac{z^{n-3}}{2.4\ldots(n-3)}\Big) \end{cases}$$

for n even and n odd respectively. Whether the representation of the observed data by the chosen theoretical distribution is adequate or not is simply given 'by picking out values of χ^2 which for a given n make $P = 1/2$ or slightly more' [1926, p. 431].

It is noted that the value of P is affected by the combining of classes. A further complication arises from the fact that, while the observed $m_i + e_i$ are generally integral, the expected m_i need not be, and this may require some adjustment to the number expected in the class having the smallest number of observations, should that number not be large. Finally, the expected fit cannot be supposed to be good in the tails of the distribution. 'Hence, only a broad, but often sufficiently definite, result can be obtained' [1926, p. 432]. In conclusion Bowley notes that in the case of $m \times n$ tables the procedure used depends on whether the marginal totals are given or not.

The only part of the book remaining to be discussed is the Appendix, 'Mathematical Notes'. Here Bowley explores the following: 'Wallis's Theorem for the Value of π'; 'Sum of Powers of Integers'; 'Stirling's Formula for $m!$'; 'The Euler-MacLaurin Theorem, which connects Summation with Integration'[47]; 'Dr. Sheppard's Corrections for the Moments of Frequency Curves'[48]; 'The Moments and Constants of the Second Approximation to the Generalised Curve of Error'; 'Ratio of Unweighted Averages'; 'Ratio of Weighted Averages'; 'Normality of Standard Deviations of the Errors in Moments, etc.'; 'The Method of Least Squares'; 'Simpler Method of obtaining formula (130)[49], p. 429'.

The appearance of the first edition of the *Elements* in 1901 was glowingly noted[50]. Worthington Ford[51], while generally enthusiastic about the book, expressed some reservations about whether the exposition of the practical application of statistical method was the best possible. He found, however, that Bowley had clearly mastered the introduction of the methods of pure mathematics into statistics, and that 'his book may thus be accepted as the latest and best summaries of its methods' [1901, p. 444].

Charles Sanger's review was similarly enthusiastic[52]. While expressing surprise that this was the first book in English on the elements of statistics, he commented that 'this book is the best book on the Elements of Statistics written in English, French, German, or Italian' [1901, p. 194.]. There were, however, defects, he concluded, including too great a concentration on the work of the Labour Department, a lack of satisfactory definition of various statistical terms, a confusion of a life table with an age distribution table and a penchant for stating dogmatically what may well be only a matter of opinion.

On the 3rd March 1901 Alfred Marshall wrote to Bowley saying that his view of economic statistics was very different to that which he found implied in the *Elements*. The mathematical method was, he wrote, best used in the same way as scales were used by aspiring pianists: for training one in sound instinctive habits. On the other hand, Marshall was more enthusiastic about Bowley's use of statistics. In a letter to Bowley on the 20th December 1901 he wrote

> There is scarcely any question in economics which might not be advanced by bringing to bear on it (i) a knowledge of what statistics have to say: combined with (ii) a knowledge of what statistics can't be made to tell, but which has to be reckoned for in a realistic solution.
> [Marshall, 1961, p. 775]

Yule reviewed the fourth edition in 1921. He noted that the section on the Labour Department, adversely mentioned in Sanger's review, had now been replaced by a far more informative one on the condition of the working-classes. However he found the section on the proper choice of weights unsatisfactory. Yule suggested that the emphasis should have been on a full treatment of standardisation, since differences were liable to arise precisely because of the 'logical' system of weighting. While noting with some reservation the number of examples drawn from fields other than the economic, Yule mentioned with pleasure the abundance of applications of the theory of sampling to social investigation. He also commented favourably on Bowley's introduction and use of the second approximation to the Normal distribution.

The sixth edition of the *Elements*, essentially the same as the fourth, with corrections and some new material being incorporated as Notes or Supplements, appeared in 1937. In his short review of this edition Maurice Kendall said

> One can only hope that before long every student will be required, for the good of his soul and the collective benefit of social research, to get a grasp of the nature and scope of the tools which he will later have to handle. Such a student would naturally turn to Dr. Bowley's "Elements" in the first instance. [1938, p. 459]

7.3 *Groups and Series*

In 1902-1903 Bowley gave a course of six lectures, *The Measurement of Groups and Series*[53], to the Institute of Actuaries at Staple Inn Hall in London. This course was the fifth in a series begun in 1897 to expose aspiring actuaries to topics not in their official syllabus, the earlier courses being on legal matters and stock exchange securities.

The topics presented in these lectures are much the same as those covered in Bowley's *Elements of Statistics*. The intention here, however, was to present *theoretical* rather than *practical* aspects of matters that fell within the larger actuarial ambit, while, it transpires, not overstressing mathematical aspects.

In the first lecture, after a short introduction in which he expresses his diffidence at addressing members of the Institute on matters with which they might well be familiar, Bowley discusses, under the general heading of *Measurement of Groups*, the graphical method, histograms and ogives, and defines several averages. The second lecture is concerned with various deviations and introduces the measurement of skewness, while the third is devoted entirely to the curve of error. In his fourth lecture Bowley considers the method of least squares, the fitting of formulae to observations, uses of the curve of error and how to construct a group from samples. This leads on naturally in the fifth lecture to a treatment of correlation. The final lecture, headed *Measurement of Series*, is devoted to the classification of series, periodic curves, symptomatic series, the correlation between series, and a discussion of the significance of the correlation coefficient[54].

Starting off, then, with groups, Bowley begins by defining a group as 'a number of persons or things each of which possesses a measurable characteristic, the group being arranged according to the magnitude of the characteristic' [1903a, p. 1]. The importance of the choice of the right size of a group is stressed, and its investigation or summary in terms of the graphical method or the method of averages is discussed. In considering the former Bowley introduces histograms, his definition being unusual. If one thinks of a bar graph, the 'histogram' is defined here as the dotted line made up of straight lines joining the mid-points of the tops of the bars[55]. Karl Pearson first gave the word 'histogram' in print in 1895, defining it in the way that is now commonplace, as 'a common form of graphical representation, *i.e.*, by columns marking as areas the frequency corresponding to the range of their base' [1895, p. 399].

Bowley describes the assumptions to be made and the precautions to be taken in drawing diagrams most carefully, and the aspects he notes are exactly what one would wish a student of applied statistics to consider today.

The second method of investigating groups, that of averages, is then discussed. Here we find the mean, mode[56], median, quartiles (Q_1 and Q_2 in Bowley's notation) and percentiles. Bowley was not very enamoured of the arithmetic mean, writing

> The arithmetic average facilitates certain computations, but, in my experience, it is the least valuable of the means or averages which can be calculated ... It is very liable to error. [1903a, p. 14]

In his second lecture Bowley considers the standard deviation and the modulus. (The mean—today, absolute—deviation is also mentioned but it does not receive much attention.) The former is defined as the square root[57] of $\mu_2 = (1/n)\sum(x_i - \overline{x})^2$, while the modulus is $c = \sqrt{2\mu_2}$. This is followed by the *average deviation* and the *probable error* ('one of the most erroneous terms[58] in use in mathematics' [1903a, p. 25]). The former is defined as $\sum f_i|x_i - \overline{x}|$, the latter as $[Q_2 - Q_1]/2$, where Q_1 and Q_2 are the lower and upper quartiles. Once again Bowley takes care to consider the advantages and disadvantages of the different measures.

Noting that the discussion so far has been applicable to groups that do not conform to the law of error (even approximately), Bowley next considers the situation in which such conformity is evinced. The concept of skewness is introduced, and a number of possible measurements of this characteristic are discussed. The first of these is simply the difference between the arithmetic mean and the median. The second is given in terms of the quartiles as $|OQ_2 - OQ_1|$, where O denotes the position of the median. The third measure results from taking the first power of the deviations of the x_i about $\overline{x}$, and comparing the excess on one side of the centre of gravity with the defect on the other side. The final method is to use $|\sum(x_i - \overline{x})^3|$. In his subsequent discussion of these different methods Bowley points out that the fourth method yields a result having the wrong dimensions (being a cube), and that it should be divided by the cube of the modulus.

In his third lecture Bowley turns to the curve of error. He begins by deriving the equation of the distribution of deviations about their centre. His method follows that we may crudely ascribe to de Moivre[59] (a purely algebraic treatment, rather than one using calculus) and Laplace, working with a non-symmetric binomial distribution (though mention is also made of the symmetric case). Bowley arrives at the following form for the probability density function (as we would call it today):

$$y = \exp\{-t^2[1 - 2j(t - 2t^3/3)]\} \,/\, [c\sqrt{\pi}]$$

with c denoting the modulus, $j = \mu_3/c^3$ denoting the skewness and $t = x/c$. Interestingly Bowley's derivation allows the choice of a scale for the ordinates, and it is noted that the choice making the greatest ordinate equal to $1/(c\sqrt{\pi})$ results in unit area under the curve, while the choice $N/(c\sqrt{\pi})$ for the greatest ordinate gives an area equal to N, the number of experiments[60].

The cumulative distribution function (Bowley refers merely to 'the equation in its integral form' [1903a, p. 36]) and the existence of various tables are then examined. This is followed by a consideration of the appropriate (Normal) curve to be fitted to data (that is, essentially the question of the determination of the appropriate mean and variance). From this Bowley notes that 'in a sense there is only one symmetrical curve of error' [1903a, p. 38]. A problem however arises here in the estimation of the skewness $j = \mu_3/c^3$, the presence of outliers having great effect on μ_3. Bowley suggests that

> A good way out of the difficulty is not to calculate j by the above method at all, but to calculate it by an *à posteriori* method, to choose that value of j which makes the misfit least. [1903a, p. 41]

The method, which Bowley ascribes as being due partly to Edgeworth and partly to Karl Pearson, is 'to obtain figures ... in such a form that it can be seen what value of j will make the sum of the absolute differences least' [1903a, p. 41]. As an illustration Bowley (following Pearson) considers first of all the case in which it is supposed that heights in a population have a certain distribution with c and j being estimated from the given heights. What is the probability that a certain number of people drawn at random from this population would have the actually observed heights? The calculation of this probability when the curve is symmetric can be easily carried out using Pearson's table of the Normal distribution; the problem is not so easy when the curve is not symmetric.

Bowley next defines a *frequency function.* The definition is essentially the one we still use today, including the requirement that the 'area under the curve' should be unity. Here too Bowley defines the *precision*, h, as the reciprocal of the modulus, it being noted that the greater the precision the more accurate are the predictions that can be made as to a random magnitude.

The important extension is then made to the finding of the frequency curve for a sum of elements each having possibly different frequency curves (i.e. finding the distribution of $\sum X_i$ given the distribution of each of the summands). It is assumed here that the frequency curves have small practical ranges and finite moduli. The well-known results for the mean and modulus of the sum are given in terms of the corresponding characteristics of the summands: $\sum \overline{X_i}$ and $\sqrt{\sum c_i^2}$ respectively.

It is also pointed out here, that if one sample is chosen at random from each of a number of frequency-curves having similar moduli, the sum will follow the curve of error, no matter whether the summands themselves did or did not follow this curve. Bowley attributes this result partly to Laplace and partly to Edgeworth [1892b].

This brings us to the fourth lecture. Bowley starts off with a discussion of the method of least squares, one which he regards as solving two problems, (i) to find the most probable value from a sequence of partly erroneous values, and (ii) to examine the precision of the result. At the very outset Bowley delivers a caveat: the method relies completely on the assumption that the frequency curve is Normal. A verbal, rather than a symbolic, discussion of the procedure (minimise the sum of squares of the deviations, and estimate the unknown true quantity by the arithmetic average) is followed by the somewhat optimistic statement that 'When we have grasped that initial principle, the rest of the investigation is only a matter of the differential calculus' [Bowley, 1903a, p. 47].

Bowley notes the difficulty that arises in solving the system of equations obtained after differentiation when the polynomial used to express the relation between the quantities being measured is of high degree (even one of third degree results in considerable work).

Consideration is then given to the question of the justifiability of the assumptions needed for the correct use of the method of least squares. The conclusion is that the assumptions are in general not justifiable, the Normal curve not being appropriate in many practical situations. Bowley therefore suggests that some other method be used when the method of least squares is either too complicated or not justified. An empirical approach he suggests is to minimise $\sum|x_i - \overline{x}|$ rather than $\sum(x_i - \overline{x})^2$, though this can also be difficult. As a second alternative method he suggests the careful choice of the coefficients in the equation $y = a_o + a_1x + a_2x^2 + a_3x^3$ so that this equation passes exactly through four prescribed points. Repeat this for four other points, and so on, ending up with a system of curves. Then choose those coefficients that seem to give the best fit; it is, says Bowley, 'really a makeshift method' [1903a, p. 50].

The third method, and one that Bowley finds of particular importance, runs as follows: from the data calculate as many moments about the mean as there are unknowns in the assumed polynomial y (as given above, for instance). In terms of these unknowns calculate the moments for the assumed curve, and equate the two sets of moments[61]. (Were we trying to fit a Normal curve by this method only two moments, the mean and the modulus, would be needed.)

In concluding this section Bowley raises the following philosophical (or methodological?) point, one which he asserts we do not have enough experience yet to decide on:

> How far ought we in such investigations to take empirical formulæ which are only justified by their results, and how far should we base our reasoning on *à priori* assumptions as to the nature of error, and as to its occurrence, assumptions which underlie the theory of probability, and from such assumptions obtain our equations? Should we obtain our equations with the view to fitting the result, or should we obtain our equations from *à priori* reasoning and see how far they fit the results? [1903a, p. 52]

Consideration is then given to various uses of the curve of error. He essentially repeats his earlier derivation of the curve, and then illustrates by considering the frequency of the first ten natural numbers as obtained by averaging each of a thousand samples of three numbers drawn at random from *Chambers' Mathematical Tables*. The fit of these data, as judged by the naked eye, to a Normal curve is seen to be good.

Attention is then paid to the construction of a group from samples. To illustrate the method Bowley took a known group (official wheat prices for 636 months) and assigned numbers chosen from *Chambers' Mathematical Tables* from 001 to 636 to these months (knowledge of the *exact* prices in this case—knowledge that would usually be lacking—making it possible to group them in 10*s*. groups). Next 100 three-digit numbers were drawn at random, and the prices corresponding to these chosen months were grouped in 10*s*. groups (i.e. a frequency table was constructed). Then only the first 25 items in the previous sample were examined, and their prices grouped in 20*s*. groups. The question is to determine how near the sample (either the first or the second) is to the group sampled.

In the example considered here it is known that 21% of the population sampled lay between 30*s*. and 40*s*, the number in the sample of 100 in this class being 16. Using the fact that the binomial distribution approximates to the Normal distribution as the sample size n increases, Bowley deduces that in the case of the sample of size 100 the modulus is approximately

$$\sqrt{2p(1-p)n} = \sqrt{2 \times 0.21 \times 0.79 \times 100} = 5.8.$$

Note that $21 - 16 = 5$, which is less than this modulus. All other similar differences between theoretical and observed values are found to be less than the probable error (=0.47× modulus), and it is then concluded that

> We have thus found a criterion of the divergencies to be expected between the distribution of magnitudes in a group of samples and the distribution in the unknown group from which they arise. [1903a, pp. 59-60]

The fifth lecture was begun with a discussion of correlation. Bowley carefully frames the question to be considered: given n pairs of measurements (x_i, y_i), what is the most probable value of the y corresponding to a given one of the x's? Phrased thus, it seems that the only x's to be used are those that have already been observed. Bowley considers separately the cases in which there is no causal connexion between the x's and the y's, and then those in which such a connexion obtains. He also considers the situation in which one can say neither that the x's are the cause of the y's nor *vice versa*, but only that the two groups are not completely independent.

This leads on naturally to a discussion, in the next section of the lecture, of the correlation coefficient. The case of a straight-line fit is first examined, the method involving as usual the choice of the appropriate constants to minimise $\sum[y_r - (ax_r + b)]^2$. Choosing the axes in the scatter diagram (Bowley does not use this term) so that the x's and y's are both measured about their means, Bowley defines[62] the correlation coefficient r as the average of products of the form $(x_i/\sigma_1)(y_i/\sigma_2)$, where $\sigma_1^2 = \sum x_i^2$ and similarly for σ_2^2. He takes care to emphasise that the division by σ_1 and σ_2 allows the comparison of *absolute* rather than *concrete* quantities. A long discussion of the interpretation of various values of r follows, the importance of obtaining the *significance* of a correlation being stressed (Bowley returns to this point in his sixth lecture). An interesting feature of the example Bowley considers, of 610,100 pairs of observations of the ages of husbands and wives in the County of York in 1901 grouped in five-year periods, is the remark that the *medians* rather than the *means* could be used with as much success. Interesting too, from a practical point of view, is Bowley's suggestion that when fitting a straight line to data one might just as well draw a system of axes through $\overline{x}$ and $\overline{y}$ and then rotate a ruler about this centre until there are the same number of points in the scatter diagram on both sides of the ruler. The very astute observation is made that 'It is often absurd in cases of probability to work out the results with very great accuracy' [Bowley, 1903a, p. 70]. The lecture is concluded with a justification for the formula for r, the conclusion being that r as given by the formula is a reasonable measure of 'correlation' no matter what the frequency distribution of the data may be[63].

The correlation coefficient can then be used to see whether there is any connexion between two series of phenomena, and this in turn leads naturally to consideration of the nature of series, the topic of the sixth and final lecture.

Just as Bowley began his first lecture with a definition of groups, so he starts this lecture on series with the following words:

> By a series I understand a list of numerical events recorded at regular intervals, for example, recorded once every year. [1903a, p. 74]

that is, a series is roughly what we would call a time series today. A general discussion of the diagrammatic representation of a series is followed by a classification of the curves representing series into periodic curves, symptomatic curves and others. The first term requires no discussion today; the second covers curves that have definite tendencies either increasing or decreasing (a 'symptom'), while 'others' means 'curves with random fluctuations' [Bowley, 1903a, p. 75]. These different classes are then discussed.

A problem that arises immediately when one examines periodic curves is the disentanglement of the period from the symptom when that curve is also symptomatic, or the estimation of the period when the curve is not symptomatic. Bowley exhibits a record of monthly observations with an annual period, and it is desired to find the movement separate from the period. A continuous representation is given by considering the twelve averages obtained by taking the average from January 1st to December 31st, from February 1st to January 31st, etc. The resulting curve shows the movement when the periodicity has been eliminated (in the absence of symptoms the resulting curve should be a straight line). If, on the other hand, one wishes to measure the period separate from the symptom, Bowley suggests one should work with a curve passing through the monthly means, medians or modes of the entire set of observations—say of data collected over 50 years. (It is assumed that the causes affecting the period and those influencing the symptom are independent.)

In his discussion of symptomatic series Bowley writes 'All statistics representing sociological phenomena that I have had experience of are symptomatic' [Bowley, 1903a, p. 77]. He notes the difficulty of distinguishing the symptom from small fluctuations, and, in the illustration provided (male and female death rates from 1845 to 1894) he suggests fitting a curve using five-year moving averages.

Attention is then turned to the matter of the correlation between series. While the remarks and methods given before in the discussion of groups are applicable to series in the absence of periodicity and symptomatology, things become more difficult when either a symptom or a period is present. If the curve is periodic Bowley suggests that one replace it by one given in terms of its averages and then effect a comparison with another similar curve. Should the period be irregular he suggests that one act as though it were symptomatic with no period. He notes too that successive annual deviations from year to year may not be completely independent.

It is far more complicated to find correlation when the curve is symptomatic. The presence of correlated fluctuations but opposite symptoms will yield a negative correlation for the symptoms ignoring the fluctuations, while the fluctuations in the absence of the symptoms may be positively correlated.

Bowley also considers whether deviations of imports, say, for a specific year should be compared with deviations for (say) the marriage rate in the same year or in the subsequent year[64]. On this point Bowley cites a paper by Hooker [1901], whose method is the following. In the presence of a curve having a period of p years, the deviation for a specific year should be taken about the mean, the latter being calculated as the average for the p years of which the specific year is the middle. Further, wishing to see whether the marriage-rate in fact responds at once to general prosperity, or whether there is a lag, Hooker calculates the correlation of the marriage-rate with the trade figures for the present year, the past year, the following year, six months before and six months after. Bowley considers this in some detail in a specific instance (involving marriage-rates, death-rates and rate of imports), again emphasising the usefulness of a graph.

The penultimate section of this lecture is devoted to the matter of the significance of the correlation coefficient[65]. Bowley argues that the significance should be a function of $1/\sqrt{n}$, where n is the sample size. He gives the usual 'Normal distribution' type formula for the deviations from the actual correlation coefficient, the modulus being $(1-r^2)\sqrt{2/n}$ and the probable error[66] $0.67(1-r^2)/\sqrt{n}$.

This brings Bowley to the conclusion of his series of lectures. Noting that as soon as one tries to represent (or summarise?) groups and series by quantities obtained by algebraic methods, Bowley points out that one is then led to approximations that need probability theory and the theory of errors. Aspects of these latter 'tools', Bowley suggested, might still be controversial, and the lack of time resulted in his being able to do little more than introduce some of the difficult problems that could arise.

7.4 *An Elementary Manual of Statistics*

In 1910 Bowley published the first edition of his *An Elementary Manual of Statistics*. We discuss the fifth edition of 1934 here.

This book, intended 'for the use of those who desire some knowledge of statistical methods and statistical results without going deeply into technicalities or undertaking mathematical analysis' [Bowley, 1910a, p. v], is set out in two parts. The first of these starts with a discussion of the nature and use of statistics, and then goes on to deal with averages, diagrams, sampling, rules for published statistics and methods of statistical analysis. The second part, devoted in the main to official statistics, considers the population census, vital statistics, trade and transport, prices, production, wages, employment, income and capital, and taxes and rates. Two appendices contain exercises based on the methods of the first part and a list of references. Even though the first part is more 'theoretical' than the second it contains numerous examples.

Bowley starts Part I with a clear definition of 'statistics':

> Statistics are numerical statements of facts in any department of inquiry, placed in relation to each other; statistical methods are devices for abbreviating and classifying the statements and making clear the relations.
> [1934e, p. 1]

With this in mind, Bowley notes that three of the main uses of statistics (all eminently reasonable) are

> (i) to give correct views, based on facts, as to what has happened in the past ... (ii) to afford material for estimates of the present ... (iii) to make possible a forecast for the near future [1934e, pp. 4-5]

The second chapter is concerned with accuracy and approximation. Bowley points out the foolishness of spurious accuracy (for example, giving census figures to the nearest units, tens, hundreds or even thousands). He uses two notations here that are perhaps uncommon: the first is the symbol ‰ for *per mil*, or *per thousand*, and the second is the writing of numbers in the form 123^{000} for 123,000 to show that the figure is correct only as far as the 3.

In the third chapter, dealing with various averages, Bowley reiterates his earlier remark that small errors in weights have little effect on averages, while attention is given to absolute, relative, biassed and unbiassed errors in Chapter IV.

Attention is next turned to the representation of data by diagrams. Bowley is generally not in favour of an excessive use of diagrams (he believes, for instance, that they are sometimes used in advertisements deliberately to mislead), though he admits that there are cases when they may be of use.

Chapter VI is devoted to the question of tabulation. The concern here is with the analysis of tables drawn up after the original data have been ordered, but Bowley stresses the importance of deciding what one is trying to find out from the data before tabulating.

Sampling is the subject of Chapter VII. Pointing out the importance of ensuring that each item in the population being investigated should have the same chance of being included in the sample, Bowley states that this can be achieved in two ways: either through *mixture* or *random selection*. The first of these is illustrated by supposing one is concerned with the detection of gold in a number of barrels containing sweepings in the Mint. Draw equal samples from the top, middle and bottom of each barrel, mix each sample, take the same size sample from each of the original samples from the tops (say), mix a number of these (say four) together and repeat until a sample of size suitable for assaying is reached. The method of random selection may be carried out in various ways: for example, one may (i) spread out the population and choose items simply at random, (ii) divide the items into equal groups and draw one item at random from each of these groups, or (iii) number the items successively, write the numbers on slips of paper, shuffle these slips and draw tickets at random, examining the items bearing the numbers drawn. It is stressed that 'No formal rules can replace judgment and experience in the selection and interpretation of samples' [1934e, p. 62].

The eighth chapter is devoted to a consideration of the importance of being fully seized of the meaning and limitation of published statistics. What is the definition of the terms used? Has grouping been carried out, and if so, how homogeneous are the groups? Do the tabulated results in fact relate to the quantity in which we are actually interested? What statistics *should* have been collected? If certain quantities have been compared, are they actually comparable? Are the samples, and indeed the populations, sufficiently large, and taken over a sufficiently long time period, for meaningful conclusions to be drawn? Should further data be examined?

The final chapter in the first part of the *Manual* is concerned with methods of statistical analysis. In his last paragraph Bowley gives the salutary warning 'All statistics which cannot bear full criticism should be put aside, even if the inquiry has to be given up' [p. 82].

Part II of the *Manual*, as we have already said, is concerned with the presentation of public statistics. Bowley starts off, appropriately, with a brief description of the census in the United Kingdom[67]. He notes various difficulties: for instance, in 1921 the census was taken in Great Britain but not in Ireland, while in 1926 separate censuses were taken in North and South Ireland[68].

Extracts from the census reports for 1921 are given, and mention is also made of the census of the United States of America[69]. For detailed examination Bowley considers statistics for 'The City and County of Bristol'. He notes the necessity to distinguish between urban and rural populations, and illustrates this by comparing certain statistics of Bristol with comparative measures of Boston.

Bowley notes too the need to distinguish distribution of population by locality from distribution by occupation, the latter being entered under one of the headings 'Professional', 'Domestic', 'Commercial', 'Transport', 'Metals', 'Building'. 'Undefined' and 'Without specified occupations or unoccupied'. The chapter is concluded with a discussion of vital statistics. As a serious problem here Bowley notes that[70]

> Age is stated inaccurately for the very young (through misreading of the instructions), for the very old (through ignorance or through the desire to magnify old age), and by women who are unwilling to confess even under the cover of secrecy to advancing age, and generally there is a tendency to return the age at the nearest round number, instead of at the last birthday ... There may be a tendency to overstate age, with the idea that an old age pension may depend on it. [1934e, p. 101]

Chapter II, on vital statistics[71], uses the Registrar-General's Annual Report. It is interesting to note that Bowley mentions that as late as 1925 neither births nor deaths were registered in seven of the 49 Continental States in the U.S.A. and deaths only were registered in eight. Making the astute observation that the relative number of both the old and the very young to the total population has a profound effect on the death-rate, Bowley states that comparisons of the death-rates in two populations can only be validly effected by the elimination of variations caused by age and sex.

The matters of morbidity, case-fatality rates and death-rates may be seriously affected by inadequate or mistaken definition. Bowley hastens to point out that these difficulties may arise (i) from a general difficulty in the classification of a disease or in the lack of uniform agreement on its definition, (ii) on unconscious bias exhibited by the medical personnel concerned, (iii) on the identification of the cause when two diseases are present, and (iv) from a reluctance to identify the cause of the problem in certain diseases. As an example of the latter Bowley cites alcoholism, but one that springs more readily to mind today is AIDS.

In Chapter III, 'Trade and Transport', the term *trade* refers to external trade of the United Kingdom. Again some figures for the United States of America are also presented. One thing Bowley notes here is that there are at least four measurements of a ship's size—

'displacement, dead-weight, gross register tonnage, and net register tonnage' [1934e, p. 136]—and for this reason one can neither rely on shipping statistics for fine measurements nor compare such statistics from one year to another. A short chapter on prices then follows.

Production, the subject of the fifth chapter, also presents problems. It is suggested that often the best one can do is to consider the *quantity* of raw material used and the *value* of the manufactured goods that are exported. North American figures are given for comparison with those for England and Wales.

Chapter VI is devoted to a consideration of wages. Once again this is a topic that poses problems to the investigator: for instance, payment may be by time or piece. In the first case one has to consider whether the wages are per week, per hour, or some other period. What constitutes a day's or a week's work? Further, in certain trades or occupations a week's work in winter may be different to that in summer. In the building trades one job may be quickly followed by another, which might not be the case for instance with a farm-worker at harvest time. How much overtime, if any, is there?

Piece work is work for which payment is made for the completion, or at least the carrying-out, of specific tasks. Here a week's work may depend on the skill or strength of the performer (as an example Bowley cites the case of printers who are paid at higher rates for setting smaller type than for larger).

Employment forms the matter of Chapter VII. The statistics given were all provided by the Ministry of Labour, and this ministry in turn derived its information from (i) trade unions, (ii) communications from employers and (iii) the enactment of various Unemployment Insurance Acts. The records were in part scanty: for example, the building trade figures depended only on carpenters and plumbers, the nature of whose work is such that they enjoy more work in winter than others in the trade (so seasonal fluctuations must be considered). For this and similar reasons Bowley suggests that the figures presented be used to give an *index* rather than a *measure* of unemployment. He states at the end of this chapter that 'There are no comprehensive statistics on unemployment in the United States' [1934e, p. 185], and presents the only data he could find, viz. of unemployment among 250,000 workers in Massachusetts in 1920.

Chapter VIII is concerned with statistics relating to the working classes, in particular with 'tables relating to Trade Disputes, Trade Unions, Friendly Societies, Co-operation, and Cost of Living' [1934e, p. 186]. In the matter of disputes the tables Bowley reprints classify people as *directly* or *indirectly* affected by strikes. This he finds an awkward distinction, on account of clear definition of the terms. By the latter term is officially meant 'other workpeople employed at the establishments where the dispute occurred, and thrown out of work by the dispute' [1934e, p. 186]. But suppose that carpenters are laid-off at a firm whose workers are on strike. If these carpenters were employed by the firm they would count as 'indirectly affected'; if they were employed by a contractor to work at the firm they would not be so described—though they would still be out of work. 'In fact the effect of a strike cannot be measured' [1934e, p. 187].

Considering next the matter of household budgets Bowley notes that when one relies on reports from the workman or his wife one can expect some inaccuracies. For instance, the amount spent on luxuries and alcohol may well be underestimated, and the investigator can perhaps only rely on responses from thrifty householders. The figures given in this portion, however, are not subject to such inaccuracies. Yet while the figures may be satisfactory the interpretation of them may be open to objection.

> As soon as we attempt to compare the well-being of two groups of people we find that statistics of incomes, prices and methods of expenditure only take us part of the way; habits, desires, thriftiness and skill in domestic economy vary greatly from class to class and from nation to nation, and cannot be reduced to statistical measurement.
> [1934e, p. 193]

The figures for the United Kingdom given in this chapter are supplemented not only by some from the U.S.A. but also by those for some French and German towns. Bowley notes that in each case the proportion spent on food decreases by about one-sixth as one moves from the lowest income to the highest, and that the proportion spent on meat stays much the same (although the actual amount spent rises). Differences between countries are however evident in the case of the amount spent on bread, flour and sugar (the British seemed to consume a large amount of the latter).

In Chapter IX Bowley considers income and capital, giving a clear definition right at the outset:

> By *Total National Income* is generally meant the aggregate of the incomes (including earnings) of the persons composing a nation; income is taken as meaning the money, or money value of goods, coming into a person's possession during a year for his own use (subject to rates and taxes), after all expenses connected with obtaining it are subtracted. [1934e, p. 199]

He noted the difficulty of ever giving a *precise* meaning to his term, and, as a curiosity, we mention as an example of the money value of goods 'the value of a week's sojourn at a hotel and the equal value of 180 quartern loaves[72] of bread or 134 oz. of tobacco.' [1934e, p. 199]. Pipe-smokers and hotel visitors today might well be interested in this! He also notes that the utility of £1 to anyone decreases in general with an increase in his income, an observation that is often made today in university courses in utility theory.

He notes too that to say that the inhabitants of the United Kingdom had an average income of £90 in 1924 is almost a meaningless statement, depending as it does on the present method of giving value to services and commodities. Warnings about the drawing of conclusions from official statistics of income are constantly made in this chapter, for example 'The statistical tables in the Annual Reports are full of pitfalls even for the wary' [1934e, p. 202], and Bowley is not sparing even of his own methods: in his presentation of his estimate of the change in average wages, he writes 'The method is open to a great many fairly obvious criticisms' [1934e, p. 208].

This brings us to the last chapter, 'Taxes and rates'. Once again Bowley finds that the officially presented statements 'involve many difficulties of definition and interpretation' [1934e, p. 215]. For example, his compilation of figures from the *Statistical Abstract of the United Kingdom* gives the total Revenue for 1925-1926 as £758 Mn. as opposed to the £812 Mn. actually given in the *Abstract*. The difficulty lies, he asserts, in the definition of *revenue*. For example, all the receipts from the Post Office are officially included as Revenue, whereas Bowley claims that expenses incurred in conducting the postal, telegraph and telephone services should be excluded. This would reduce the official figure of £57.4 Mn. by £54 Mn., and even then it might be argued whether the balance should be seen as a *tax* or a *trading profit*.

Details are given of the receipts from customs and excise[73], with the highest customs receipt in 1925-6 coming from tobacco (£53.5 Mn.), followed by sugar, running a poor second at £18.4 Mn[74]. The greatest excise was obtained from beer at £76.3 Mn.

Mention is also made of the Inhabited House Duty (repealed in 1924), and Bowley notes that it might be interesting, though difficult, to compare income-tax returns and the value of houses. A cursory examination of the figures suggests that the higher the income the larger the value of the house that is occupied but the smaller the proportion of the household income that is spent on rent.

There were a number of reviews in scholarly journals of various editions of the *Manual*. It is interesting to look at some of them to see not only the matters to which exception was taken (or which were approved of), but also the change in opinion of reviewers over the years.

In the first of his two reviews [1910a] of the first edition of the *Manual* Alfred Flux[75] raises a number of criticisms. He finds some of the new notations introduced by Bowley to be unattractive, and also declares that Bowley transgresses his own precept about accuracy in giving certain figures more 'precisely' than is justified. While Flux agrees with Bowley in discouraging the excessive and careless use of diagrams, he suggests that Bowley might have gone too far in saying that certain specific diagrammatic presentations are 'incorrect'.

Flux finds Bowley's illustrations in no wise limited by his frequent use of the statistics of wages, this being a matter in which Bowley was particularly experienced. As a further example of Flux's criticisms we mention that he considers Bowley's discussion of the distinction between *gross* and *net* tonnage in shipping. While Bowley gets the latter from the former by subtracting, according to certain rules, space occupied by the engines and the crew's quarters, he does not take account of the space occupied by the passenger's quarters. This is corrected by the time of the fifth edition.

In his second review of 1910 Flux was perhaps more negative. He noted that it is far more difficult to write an elementary manual than an advanced text, and while he commended Bowley for a valiant attempt, it seemed, in his opinion, that there were certain defects in the *Manual*. He wondered whether these could be a result of the book's being a somewhat hasty compilation of lecture notes—though there is no suggestion of such an origin in the Preface to the first edition. While Bowley emphasised the importance of attention to the correct definition, meaning and limitation of every estimate, Flux once again stated that he did not find this precept to be followed by the author. As an example Flux noted that Bowley wrote [1910a, p. 16] 'The national expenditure of the United Kingdom is about £160,000,000 ... the total national income is estimated at £1,800,000,000'. Flux did not believe that 'national' is used in the same sense in these two places, for national expenditure surely refers to total governmental expenditure while national income refers to the aggregate income of the citizens. Nor did he find the word 'total' of any effect.

As in his earlier review Flux doubted whether the statistical neophyte is in any way helped by being exposed to either the terminology or the results of error theory. Further, he found Bowley's writing (on p. 77) that 'It is shown that they (i.e. the weights) need not be taken with great accuracy' to be misleading, since what had been shown earlier in the *Manual* was at best a heuristic discussion, the reader having been referred for the proof to Bowley's *Elements of Statistics*.

Flux concluded by suggesting that most of the defects could have been corrected if the author had been more careful, and that attention to matters such as he had commented on would make the text far more useful to the reader.

Negative though this review may in general have been perceived, Bowley certainly paid attention to it, for by the fifth edition of 1934e 'the total national income' had been replaced by 'the aggregate of personal incomes'. Similarly, Flux's discussion of the weights probably brought about Bowley's change of the phrase 'It is shown' to 'It is known'. Other precise criticisms made by Flux (trivial though some of them seem to be today) were similarly met.

In his review James Field[76] found Bowley 'not wholly successful' [1910, p. 563] in his attempt to write a small book that is both a fundamental treatise and a register of official statistics. Field found the first part of the *Manual* generally useful, but he bemoaned the fact that in this 'brief and rather casual guide' [1910, p. 563] only *British* official statistics were quoted, and he said further that 'this insularity of scope is a defect which makes that part of the book ... comparatively uninteresting' (loc. cit.). He also criticised the writer for having cited none but works published in England in his list of selected references. Although Field wished that Bowley had doubled the size of Part I (it was certainly increased in later editions), he concludes by saying that the work 'adds another to the respectable list of statistical books which are partly useful' [1910, p. 564].

Allyn Young[77] published a short review of the *Manual* in 1910, describing it as 'a handbook of statistical criticism' [p. 386]. He too finds certain aspects of the book unsatisfactory, saying that

> it is hardly constructive enough to serve as a formal text-book, but it is a good book to put into the hands of students or of others who are entering upon their statistical apprenticeship. [1910, p. 386]

The seventh and last edition of the *Manual* was published in 1952. It is dangerous to generalise from a negative review of the first edition to a glowing one, by a different author, of the last. But the review of this last edition by Buckland perhaps indicates that the *Manual* had proved its usefulness over the years.

Buckland finds that this edition will serve the student 'in the same sturdy way' [1953, p. 89] as the first, and that Bowley's comments in the preface on the danger of the correlation coefficient and least squares in the hands of the beginner should warn him of the advantage to be gained from consulting the expert. He also notes that the last two chapters in Part I

> contain truths which are all too frequently overlooked in these days of statistical tools which are sometimes too sharp for the available data. [1953, p. 89]

This sentiment should today be inscribed on the desk of every indiscriminate user of statistical computer packages!

Part II had again been updated for the 1952 edition, with a new appendix giving pertinent material from 1938 to 1947-50. While commending Bowley for this, Buckland regrets that the then most recent statistics from the British Transport Commission were not given. A warm conclusion:

> Teachers, particularly those in evening institutes or connected with other forms of adult education, will be glad to have this new edition of an old favourite. Its continued influence should do much to spread the idea of the statistical approach to problems of modern society.
> [1953, p. 90]

What can one say about this book in general today? Firstly, whether the prefaced intent was met is debatable. It is difficult from a modern background of a training in mathematical statistics to see whether anyone ignorant of statistics could draw much in the way of knowledge of statistical methods from this text: perhaps Bowley's use of the phrase '*some* knowledge' is correct. The range of topics covered is such that one who completed reading the *Manual* would certainly have some idea of terms like 'average' and 'error' and would be able to look critically at diagrams and decide whether claims based on results deduced from sampling are in fact justified or even justifiable. Perhaps the enquiring and intelligent layman would have benefitted more from Part II and the up-to-date figures that were presented there in the various editions.

Chapter 8

Index Numbers

8.1 Introduction

The theory of index numbers—a topic that may be viewed as part of the general theory of aggregation[1]—has by now an extensive literature. Wynne Maunder, for example, lists some 2,600 entries (up to 1968) in his bibliography of index numbers[2].

Early work on index numbers[3] can be traced back to *Chronicon Preciosum*[4], a 1707 monograph by William Fleetwood, Bishop of Ely, described by Edgeworth [1925a, p. 381] as 'one of the best' among such works. (In brief, Fleetwood was asked by a Fellow of All Souls College, Oxford, whether the possession of an estate of £6 per annum at that time should require his vacating his Fellowship in terms of the college statutes of 1439 that set the limit at £5. Fleetwood's conclusion, based on the comparison of prices of different commodities and on stipends, salaries, etc. at the two times, was that resignation was not necessary. This is an example of the tabular standard, commodity standard or 'fixed basket' approach to index numbers [Diewert, 1993, §2].)

In 1822 Joseph Lowe introduced the basic idea of weighting[5]. Two German authors, Étienne Laspeyres in 1871 and Hermann Paasche in 1874, publicised this approach and their indices, respectively

$$\begin{aligned} P^L_{rs} &= \sum_i p_{is}\, q_{ir} \,/ \sum_i p_{ir}\, q_{ir} \\ P^P_{rs} &= \sum_i p_{is}\, q_{is} \,/ \sum_i p_{ir}\, q_{is}, \end{aligned}$$

(where r and s denote the reference and current situations respectively and p and q stand for price and quantity), are still extensively used[6].

Kendall [1969, p. 10] notes that, as these authors put it, it appeared that the arithmetic and geometric means were both to be rejected. In actuality, though, what was rejected was the idea of *unweighted* averages. Keynes also supported the use of weighted rather than unweighted averages, noting that this view was illustrated by 'the well-known paradox' that whereas wages in each individual trade may be falling, the average wages could well be increasing. This phenomenon would not arise if unweighted averages were used.

When one considers that the construction of an index number (see Edgeworth [1925a] for a discussion of the many such indices) relies heavily on both statistical technique and economic theory, one is little surprised that Bowley should have contributed to the development of this matter (see Frisch [1936]).

8.2 Papers on index numbers

Throughout his career Bowley wrote a number of papers dealing explicitly with index numbers, from an early one of 1897, through his series of papers with George Henry Wood on the statistics of wages in the United Kingdom during the nineteenth century and several reports for the London and Cambridge Economic Service, to one of his last papers in 1952.

In his paper of 1897 Bowley considered index numbers for import and export data that differed from those made earlier

> in choice of 1881 as base year, which affords means of comparing the effect of different base years, and in the curtailing of the numerical work to its smallest dimensions by leaving out all figures which could not influence the result. [Bowley, 1897a, p. 274]

In emphasising the last point in this quotation Bowley writes ‘All index numbers are intrinsically approximate, and are so far from being exact that it is generally useless to evaluate them within 1 per cent’ (loc. cit.).

Difficulties that arose in the calculation came from things like (1) headings used in 1895 had no corresponding headings in 1881, and (2) horses, machines and pictures were so indefinite that one could not find any statistical value in their average price.

The index is calculated in the following way: let $\{a_i\}$ represent units of goods (Bowley supposes four goods for purposes of illustration) of values $\{\alpha_i\}$ imported in the base year with $\{a_i'\}$ the goods and $\{\alpha_i'\}$ the values in another year. The desired index number is then $100 \times \sum \alpha_i' / (\sum \alpha_i \, a_i'/a_i)$.

Tables are given for index numbers for 1885 (1881 = 100) and different systems of weights, using the arithmetic, geometric and harmonic means together with figures from the *Economist.* (Bowley noted that the geometric mean of index numbers may perhaps be the best thing to use when computing an index number for years that are far apart.) The figures with 1881 as base year corresponded closely with those given by Augustus Sauerbeck using 1873, a fact that ‘may give statisticians new confidence in weighted averages’ [1897a, p. 278].

In 1919 Bowley examined the methods used in questions of prices and cost of living. He noted that ‘cost of living’ is a vague phrase, and that it ‘only becomes explicit when it is expanded into “The cost of maintaining a defined standard by a defined family” ’ [Bowley, 1919b, p. 343]. The cost of living could be measured by calculating an index number of retail price changes, estimating then the rise in the cost of living and the change in the purchasing power of money.

A number of methods are suggested for the study of such an index. In examining the first of these, 'Index-number of retail prices', Bowley concludes that no improvement in a retail index number is made by weighting, and he suggests a number of the form

$$(100/n)\sum_{1}^{n}(p_i/P_i),$$

where there are n commodities with prices P_i and p_i in the base year and a subsequent year respectively.

The second method, 'Change in cost of a constant standard', is a process that

> takes the simple form of weighting the changes of prices of commodities by their estimated relative importance in working-class consumption. [Bowley, 1919b, p. 347]

The Board of Trade in the United Kingdom had used

$$100 \times \left[\sum_{1}^{n} Q_i p_i \Big/ \sum_{1}^{n} Q_i P_i\right], \tag{8.1}$$

where the Q_i are the quantities of the commodities in the base year.

Bowley is somewhat critical of the *Labour Gazette* here, suggesting that 'Enormous and very expensive mischief has been caused by this publication' [1919b, p. 348]. He had served on the Sumner[7] Committee that had been tasked with estimating the increased cost of living to the working class caused by the war, and he notes here that the *Labour Gazette* appeared to have paid no attention to this committee's report, which

> showed quite clearly that while the Ministry of Labour's index-number showed a rise of 108 per cent. in food prices from July, 1914, to June, 1918, actual expenditure had only increased 90 per cent., while the fall in standard of living was very small, and necessitated by war conditions. [Bowley, 1919b, p. 348]

The third method was concerned with the change of purchasing power when standards were different. Here Bowley notes that statisticians had used Equation (8.1) above and something similar 'working backwards' from the later to the earlier year, viz.

$$100 \times \left[\sum_1^n q_i p_i / \sum_1^n q_i P_i\right], \tag{8.2}$$

where the q_i are the quantities in the later year. The arithmetic, geometric or harmonic mean of these two indices is then found[8]. This method Bowley finds satisfactory for estimating the purchasing power of money for two countries or at two dates when living habits and commodities bought are not dissimilar, but he finds it unsatisfactory for countries as far apart as England and India (say) or between 1815 and 1914.

Bowley's fourth method, 'Change in expenditure irrespective of standard', results from comparing the actual expenditure in the later of two years with that in the earlier. The index number is now

$$100 \times \left[\sum_1^n p_i q_i / \sum_1^n P_i Q_i\right]. \tag{8.3}$$

Noting that in the period under consideration increase in wages led to an increase in prices and this in turn led to further demands for higher wages, Bowley remarks that measurement by realised consumption is only applicable, as far as wages are concerned, when there is a shortage of supplies: in normal conditions when supplies are sensitive to price it seems unsatisfactory.

Can a change in 'the degradation or improvement of the standard of food, clothing and housing actually attained' [Bowley, 1919b, p. 350] be measured? As a tentative index[9] Bowley suggests

$$100 \times \left[\sum_1^n q_i P_i / \sum_1^n Q_i P_i\right], \tag{8.4}$$

in which quantities weighted by prices are compared rather than prices weighted by quantities. As a rather crude result Bowley finds that the urban workman spent 90% more on food and received $8\frac{1}{2}$% less in 1918 than in 1914. It is assumed that quality has stayed the same, though Bowley suggests that the bacon of the summer of 1918 would not have been valued at $11\frac{1}{2}\,d.$ per pound in 1914. In general he finds it more satisfactory to use the existing standard of expenditure of 1918 for weighting rather than the pre-war standard of 1914.

A method that had gained considerable support at the time Bowley was writing was measurement by calories. However, 'man cannot live by calories alone, even if the elusive vitamines are added and enough protein is present' [Bowley, 1919b, p. 353]. He noted that the urban workman got 3% less in 1918 in nutrition. Had he attempted to keep the nutritional value the same in 1918 as it had been in 1914 he would have spent 96% more on food but obtained $5\frac{1}{2}$% less satisfaction (a loss that would have been valued as $1s.\ 5d.$ per family per week in 1914).

General statistics of expenditure on items other than food not being available, housing budgets of expenditure are needed. It is however pointed out that what is considered in this paper is that the class sampled is that of whom budget-keepers are representative and not the whole working class.

Three important points that require attention are the following: (1) the accuracy of the average yielded by sampling, (2) the variation shown and (3) the relation between the variance shown and the age and sex profile of the family. The first matter had received considerable attention before; when it comes to the second Bowley mentions the use of the standard deviation of the average and the probable error (better termed the 'quartile deviation', he suggests). The variation in consumption, which depends 'partly on income and partly on family size, and partly on local custom' [Bowley, 1919b, p. 357], can be reduced—at least as far as the contribution from family size is concerned—either by using partial correlation or by reducing all budgets to that of a standardised family.

Bowley notes, for instance, that while a man may need only twice the nourishment of a child, his food may well cost considerably more than the child's if he has meat, bacon and bread and the child bread, dripping and sugar. Using the expenditure on food per 'equivalent man' for 971 working-class budgets Bowley calculates the standard deviations of the averages for various classes (skilled, semi-skilled, unskilled and together) in seven districts in the United Kingdom (from Scotland through London to Wales). He concludes that

> The [quartile] deviation thus found when the groups are drawn from fairly homogeneous districts, and when the correction has been made for the size of family, is very remarkable, especially since many purchases are rationed. I doubt if this phenomenon has been systematically observed hitherto, and it appears to me very important. It appears to depend on variability of income rather than on any other factor. It also tends to confirm the suggestion that standardisation is imperfect. [1919b, p. 359]

It is interesting to note that Bowley next considers the merging of all classes to form a continuous group, comparing the observed figures with Edgeworth's second approximation to the Generalised Law of Great Numbers and Pearson's Type III distribution[10]—the comparison is effected by a goodness-of-fit test with the total number of cases divided into ten expenditure groups of two shilling width from under 5.5*s.* to greater than 21.5*s.* In each case the 'fitted' number of cases is given with $\pm$ the standard deviation.

> The fit is remarkably good, by any test, except for the seven families of hearty eaters who confessed to over 21*s.* 6*d.* per "man," and in the case of Type III for the 18 families who starved on less than 5*s.* 6*d.* [1919b, p. 359]

Partial correlation is applied to family expenditure (E), in shillings, on food for 390 families in the skilled class in which one or more persons were over 14 years (x) and two or more children (y) were under 14. The appropriate equation was found to be

$$E = 14.5 + 9.4x + 3.7y.$$

The agreement between the predicted and observed expenditures was found to be 'as close as can be expected' [Bowley, 1919b, p. 360]. A somewhat depressing conclusion is that 'As usual young children in the larger families are found to have the worst chance of adequate nourishment' [Bowley, 1919b, p. 361].

Ending with a plea for better and more careful collection of data, Bowley somewhat apologetically excuses his paper by saying that 'it is but a poor contribution to the solution of practical questions to indicate that the data are insufficient and the methods open to question' [1919b, p. 361].

In Chapter 1 we mentioned the Newmarch lectures given by Bowley at University College, London in the 1920s. The gist of the 1928 series was reproduced in his paper 'Notes on index numbers' in 1928.

Bowley begins with a lucid description of such numbers:

> A series of index-numbers is a series of indicators of the movement of a phenomenon variable in time, whether the phenomenon can be completely or directly measured or not ... An indicator must satisfy the conditions that the times of its maximal and minimal readings synchronise (or definitely precede or follow) the maxima and minima of the phenomenon, and that the amplitudes of its fluctuations are greater or less in sympathy with those of the phenomenon. The indicator becomes more exact when its movements, or the amplitudes of its fluctuations, are directly proportional to those of the phenomenon, and to obtain such a direct relation is the objective of most index-numbers. The words indicator and index-number may be conveniently used respectively for the cases where the movements are only sympathetic and where the relations are direct and measurable. [Bowley, 1928b, p. 216]

In Section I of the paper Bowley considers unweighted means. Here it is supposed that 'each series of observations is *a priori* of the same valency in relation to the objective' [Bowley, 1928b, p. 217], no enquiry being made as to whether the assumption is justified or not. One has then to decide whether the unweighted arithmetic, geometric or harmonic mean, or even the unweighted median or the mode, should be used[11].

Let $\{y_i\}_1^n$ be a sequence of observations with objective (i.e. index number) I. Write $y_i = I + e_i$, where the errors (or residuals) e_i are Normally distributed. Then the value of I that would cause these errors with the least improbability is the arithmetic mean.

In the case of the unweighted geometric mean it is supposed at the outset that $y_i = I \times \eta_i$. When the $\ln \eta_i$ are Normally distributed one finds that the observed variations would arise with least improbability from a value of G given by $\ln G = \sum \ln y_i + (\sum \ln y_i)/n$.

> A reasonable method of deciding whether the arithmetic mean or the geometric is the better is to ascertain whether, in fact, the variation of the quantities or the variation of their logarithms is the nearer to that given by normal distribution. [Bowley, 1928b, p. 217]

To decide whether the arithmetic or the geometric mean is 'better' Bowley considers the deciles $D_1, \ldots, D_9$ with median $M = D_5$. When the distribution about the arithmetic mean is Normal,

$$M = (D_1 + D_9)/2 = (D_2 + D_8)/2 = \cdots ,$$

and as a coefficient of asymmetry he suggests

$$v_1 = (MD_6 - MD_4)/(MD_6 + MD_4),$$

with v_2, v_3 and v_4 being defined similarly in terms of the pairs (D_7, D_3), (D_8, D_2) and (D_9, D_1) respectively.

Applying this to the figures shown in Table 8A (taken from *The Statist* and based on 45 price-relatives, and with 'Nat.' indicating figures based on price-relatives and 'Log.' figures using logarithms) Bowley finds that the figures in the Table are too far from the entries that would be zero if the numbers were Normally distributed or if their logarithms were. He concludes that there is little to choose

Table 8A. *Statist* Index.

	Year 1913 Base 1867-1877		Year 1926 Base 1867-1877		Year 1920 Base 1913		Year 1925 Base 1913	
	Nat.	Log.	Nat.	Log.	Nat.	Log.	Nat.	Log.
v_4	−.25	−.50	.054	−.15	.23	.04	−.15	.27
v_3	−.33	−.42	.15	−.01	−.10	−.24	.05	−.03
v_2	−.36	−.40	.04	−.06	−.3	−.38	−.07	−.2
v_1	−.60	−.55	.07	−.01	−.6	−.6	−.2	−.2

between the two methods, and also expresses doubt as to the advisability of using this approach for as few as 45 data points.

As a further, and more satisfactory, method he suggests one select two points in the distribution and examine how points between these two points are dispersed in comparison with the Normal curve. He supposes, for illustration, that D_1 and D_9 are fixed here, and the other deciles used as the intermediate points.

Examination of the figures in Table 8B shows that in times of rapid changes in prices the geometric mean is safer. When the quantities averaged are widely spread out the geometric and arithmetic means will be far apart, and when the observations are closer together so will be the means[12]. Bowley suggests therefore that one is generally better off with the geometric rather than the arithmetic mean. The harmonic mean is a variant of the arithmetic, obtained by taking the base (=100) at the end, rather then the beginning, of the period under investigation. The unweighted median, advocated by Edgeworth, has the advantage of being unaffected by fluctuations in extreme observations and tends to correct asymmetry in the data.

Bowley concludes this section by stating four arguments for the use of the geometric mean rather than the arithmetic: he finds none of these to be based on relevant premises, and suggests that the former be used, not only for ease of computation in general, but also on the grounds of its being safer when price-changes are widely dispersed and of its independence of arbitrary choice of the base year.

Table 8B. *Statist* Index: test by distribution.

	Year 1926			Year 1920			Year 1925		
	Base 1867-1877			Base 1913			Base 1913		
	Expected	Actual		Expected	Actual		Expected	Actual	
		Nat.	Log.		Nat.	Log.		Nat.	Log.
D_1	77	77	77	176	176	176	117	117	117
D_2	$93\frac{1}{2}$	$93\frac{1}{2}$	88	236	$204\frac{1}{2}$	213	132	138	128
D_3	$104\frac{1}{2}$	98	97	281	220	244	139	128	137
D_4	114	$106\frac{1}{2}$	106	316	$254\frac{1}{2}$	273	149	$153\frac{1}{2}$	144
D_5	$123\frac{1}{2}$	121	114	351	299	304	157	163	152
D_6	133	$137\frac{1}{2}$	124	386	$310\frac{1}{2}$	340	165	169	160
D_7	$142\frac{1}{2}$	146	134	421	343	381	173	176	169
D_8	$77153\frac{1}{2}$	158	148	466	377	426	182	$190\frac{1}{2}$	180
D_9	170	170	170	526	526	526	197	197	197

While noting at the start of his second section—'Index-numbers and sampling'—that the measurement of general price-changes is often complicated by the sparseness of data Bowley remarks on the importance of recalling that price movements cannot be assumed to be independent.

Denoting by ${}_ty_i$ the price-relative of the ith commodity t years after the base year, and by P_t the index number for the tth year (however computed), Bowley notes that one has

$$({}_ty_i/P_t) = a_i + b_it + c_it^2 + \cdots + v_i,$$

where the constants $a_i, b_i, \ldots$ can be determined from the observations (e.g. by least squares) and the v_i are the residuals. Using *The Statist* figures and measuring t from 1867 to 1913, Bowley finds the significant coefficients and correlations. *The Statist*'s index number was 85: Bowley's calculation adjusts this to 85 ± 2.6 in terms of probable error when correlations are ignored and 85 ± 2.2 when they are taken into account. 'Owing to correlation the 45 entries are equivalent to only $39\frac{1}{2}$ independent entries' [Bowley, 1928b, p. 222].

Bowley concludes by noting that experience and theory combine to show that when prices are regarded as samples and the index numbers are unweighted, there is a certain degree of imprecision that increases the further we move from the base year. The same sort of error occurs when the averages are weighted, but it may perhaps be damped down if the weights are chosen in such a way as to bring the sample nearer to the objective.

The third section of the paper, 'Cost-of-living index-numbers', is decidedly theoretical in nature, and concerned with the question 'what change in expenditure is necessary after a change of prices to obtain the same satisfaction as before?' [Bowley, 1928b, p. 223]. To answer this the first thing one needs are records of the expenditure at both dates. Suppose that at one date (1914, say) n commodities were bought, with Q_i units of commodity i being bought at a price P_i, while at the second date q_i units are bought at p_i.

The method used by the Ministry of Labour yields the index

$$I_1 = \sum_1^n Q_ip_i \Big/ \sum_1^n Q_iP_i,$$

(the customary multiplicative factor 100 being omitted), while a committee of Labour associations had suggested

$$I_2 = \sum_1^n q_i p_i \big/ \sum_1^n q_i P_i.$$

While both of these indices had their supporters[13], it had been suggested that an average of the two should be taken: proposed formulae were $(I_1 + I_2)/2$, $\sqrt{I_1 \times I_2}$ and $\sum(Q_i + q_i)p_i / \sum(Q_i + q_i)P_i$.

Bowley now connects his use of the word 'satisfaction' with utility, saying that the satisfaction derived is a function $U(x_1, x_2, \ldots, x_n)$ only of the quantities $x_1, x_2, \ldots, x_n$ bought (the x_i are not assumed to be independent). Over the period of time considered the function U has the same form and constants. Bowley is careful to point out that his analysis is inapplicable to two different countries or to one country for a period of more than 60 years.

A fundamental assumption here is that the given total expenditure E (a constant) is distributed among the commodities in such a way that U is maximised. Thus $x_i = Q_i$ at the first date, giving the maximum of $U(x_1, \ldots, x_n)$ subject to the constraint $\sum x_i P_i = E$. This yields the solution

$$\frac{\partial U}{\partial x_i} = M P_i \ , \quad i \in \{1, 2, \ldots, n\},$$

whence

$$M = \frac{1}{P_1}\frac{\partial U}{\partial x_1} = \frac{1}{P_2}\frac{\partial U}{\partial x_2} = \cdots = \frac{1}{P_n}\frac{\partial U}{\partial x_n}.$$

> Here M is a constant, identifiable with the marginal utility of money for the "average" man, with prices and total expenditure known. [Bowley, 1928b, p. 224]

Similarly at the second date, with m denoting the marginal utility of money at that time, we have

$$m = \frac{1}{p_1}\frac{\partial U}{\partial x_1} = \frac{1}{p_2}\frac{\partial U}{\partial x_2} = \cdots = \frac{1}{p_n}\frac{\partial U}{\partial x_n}.$$

Letting $\mu = M/m$, $U(Q) = U(Q_1, \ldots, Q_n)$ (with $U(q)$ similarly defined) and expanding $U(q) - U(Q)$ Bowley finds that 'the whole expression, which measures the excess of satisfaction given by the second budget of expenditure over the first' [1928b, p. 225] is

$$U(q) - U(Q) = \sum_1^n MP_i \, \partial Q_i + \tfrac{1}{2} \sum_1^n (mp_i - MP_i) \, \partial Q_i$$
$$= \tfrac{1}{2} \sum_1^n (MP_i + mp_i)(q_i - P_i).$$

Assuming further that relatively small changes in the q_i to $q_i + v_i$ would provide the same utility at the second date as Q_i and supposing too that terms like $v_i^2, v_i v_j$ can be neglected, Bowley shows that the expenditure of $\sum(q+v)p$ at the second date provides the same satisfaction as that of QP at the first. If I is the ratio of these expenditures, then[14]

$$I = \frac{\sum(Q_i + q_i)p_i}{\sum(Q_i + q_i)P_i} + (I - \mu) \frac{\sum(q_i - Q_i)P_i}{\sum(q_i + Q_i)P_i}.$$

Since there seems to be no *a priori* way of determining μ, and since it cannot differ much from I Bowley suggests that the second term in the preceding equation be dropped[15]. The resulting index

$$I = \sum(Q_i + q_i)p_i \, / \sum(Q_i + q_i)P_i$$

is also known as the Marshall-Bowley-Edgeworth index I^{MBE}, which one can write as

$$I^{MBE} = \sum p_i Q_i (1 + q_i/Q_i) / \sum P_i Q_i (1 + q_i/Q_i).$$

In this form the ratio of the first terms in the numerator and the denominator is seen to be the summand in the Laspeyres *price* index while the remaining terms are a function of the Paasche *quantity* relative q_i/Q_i. (The Laspeyres and the Paasche quantity index numbers are similarly defined, with the p's and the q's interchanged.)

Calculations made using figures provided by the Ministry of Labour and two Committees show that the index found by using I, I_1, I_2, $(I_1 + I_2)/2$ and $\sqrt{I_1 \times I_2}$ differ very slightly from each other.

Bowley now looks at the special case where $U(Q) = U(q)$. Assuming again that $I - \mu = 0$ and letting $V_1 = \sum q_i P_i / \sum Q_i P_i$ and $V_2 = \sum q_i p_i / \sum Q_i p_i$ he deduces that

$$V_1 \times V_2 = 1 \Longleftrightarrow U(Q) = U(q).$$

Hence consideration of V_1 and V_2 will provide a simple test of the (approximate) equality of the budgets.

Answers become less readily available in Section 4, 'Index-numbers of quantity'. When one is concerned with changes in the standard of living the assumptions of Section 1 are no longer justified, and the equations of Section 3 yield neither an absolute measurement of the total utility nor a measurement of the ratio of two values of U.

> It appears to be only possible either to say that the sought measurement is symmetrically related to V_1, which assigns the relative values of the units of different commodities as at the first date, and to V_2, which assigns those at the second date; or to deduce the quantity index from the price index. [Bowley, 1928b, p. 230]

Again Bowley considers the quantity index numbers $(V_1 + V_2)/2$, $\sqrt{V_1 V_2}$ and $[V_2 (1 + V_1)]/[1 + V_2]$, and in the example presented the results obtained by the different methods are very nearly the same.

When one's concern is rather with the index number of production, Bowley finds that the methods of Section 1 and Section 3 are both inapplicable.

> We cannot assume that there is any utility function of unchanged form for a nation, or that there is any general force increasing production of commodities of all kinds, modified by special variations subject to a law of error. [Bowley, 1928b, p. 231]

Difficulties here are to be found in the nature of the material: in all but the simplest cases there is no unit of the production of manufacture, and annual records are available only for a few raw materials and partly manufactured products. The Board of Trade had apparently used V_1 for quantity and I_2 for prices, and Bowley suggests that $\sqrt{V_1V_2}$ and $\sqrt{I_1I_2}$ would be better.

The fifth section of this paper is concerned with causes of error in index numbers, which Bowley classifies as follows[16]:

> (a) Inaccuracy of prices (or quantity) relatives; (b) Error due to sampling; (c) Errors in weights; (d) Omission of relevant classes; (e) Non-coincidence of the field of selection with the objective; (f) Inappropriateness of the relative to the weight. [Bowley, 1928b, pp. 232-233]

In his discussion of these errors (illustrated where necessary with Board of Trade figures), Bowley notes the importance of taking correlation into account where it is present. He also remarks that the errors in (e) and (f) cannot be measured, though rough estimates of (a), (b), (c) and (d) (classes of errors that are not independent) can be obtained. The section is concluded with the following important observation:

> A general impression from the study of many details is that an index-number, when properly defined and related to an objective, is liable to a probable error of at least 2 per cent. This error is more important than any arising from the wrong choice of the form of a symmetric mean. [Bowley, 1928b, p. 235]

The final section is devoted to wage index numbers and requires a new notation. Let $N_1, N_2, \ldots$ and $W_1, W_2, \ldots$ be respectively the numbers in the different industries (or occupations) and their average wages in the basic year, with the similar figures in another year being $n_1, n_2, \ldots$ and $w_1, w_2, \ldots$. Further, let

$$\overline{W} = \sum N_i W_i / \sum N_i \ , \quad \overline{w} = \sum n_i w_i / \sum n_i$$

and

$$R_1 = \sum N_i w_i / \sum N_i W_i \ , \quad R_2 = \sum n_i w_i / \sum n_i W_i .$$

What is required is $\overline{w} \div \overline{W}$ and possibly also R_1 and R_2, the latter two indices representing movements due to changes in wages on their own, any changes in the relative numbers in the industries being ignored.

In general 'it is useful to separate the effects of the change due to shifting of numbers from that due to increase of wage-rates' [Bowley, 1928b, p. 236]. To this end let

$$n_i = N_i(1 + p_i) \ , \quad \sum n_i = (1 + \overline{p}) \sum N_i \ , \quad w_i = W_i(1 + r_i).$$

(The p_i are provided by the Population Census and the r_i by the Ministry of Labour, but there is no check on the weighted average of the r_i.) Finally let

$$E_i = N_i W_i / \sum N_i W_i \ \text{ and } \ P = \sum E_i(1 + p_i)/(1 + \overline{p}).$$

Then it can be shown that

$$R_1 = \frac{\sum E_i(1 + r_i)}{\sum E_i} \ , \quad R_2 = \frac{\sum E_i(1 + r_i)(1 + p_i)}{\sum E_i(1 + p_i)},$$

and

$$\overline{w} / \overline{W} = P \times R_2 .$$

As an illustration Bowley shows that the value of $\overline{w} / \overline{W}$ in 1924 with 1914 taken as base year was 1.95 for men, 2.10 for women and 1.98 in all. It is noted that the calculation of the movement for women was complicated by the fact that there had been a transfer from domestic service to industry and from industry to salaried work.

Continuing an investigation started in his paper [1913d], Bowley published a further analysis of the relation between wholesale and retail prices in 1922 (the *Labour Gazette* had published monthly retail food prices since July 1914). At the beginning of 1919 the retail and wholesale food prices were both approximately 230 (100 was taken as the figure before the war), with a brief fall being followed by an increase in 1920 with another fall in 1921.

Noting thus that changes in retail prices tend to follow changes in wholesale prices after an interval, Bowley considers here three methods to relate index numbers for retail prices to those for wholesale prices at the same or a preceding date. These methods are (i) an empirical formula due to Moritz Elsas, (ii) the correlation method and (iii) the method of differences. To give an idea of these methods, let p be the retail or cost-of-living index number in some month, let p_{-n} be the corresponding number in the nth preceding month, and let P_{-n} be the corresponding wholesale index number. Elsas's formula is

$$p = \sqrt[3]{P_{-2} \times p_{-2}^{2}}.$$

Bowley preferred an expression using a weighted arithmetic rather than a geometric mean, the weights being chosen by the method of least squares. He notes that his equations are 'quite empirical, and the coefficients may depend on the period taken in their calculation' [1922e, p. 196]. Both formulae give results very near to the figures actually observed.

Formulae based on the correlation coefficients, notes Bowley, only answer the question

> What linear equation between the retail price-index at one date and wholesale or retail price-indexes at earlier date gives the closest approximation to the facts, under the test that the sum of the squares of the residuals shall be least? [1922e, p. 196]

The formulae are suitable for short-term prediction, and are of importance in allowing analysis of the time-lag between wholesale and retail prices and in their relative oscillations.

The third method involves 'the comparison of the change in the wholesale index-number with a subsequent change in the retail index-number' [1922e, p. 197].

Bowley considers three different settings, based on: (A) *Statist* wholesale food index number and *Labour Gazette* retail food index number, (B) *Statist* wholesale food index number and *Labour Gazette* retail food index number (fish, milk, margarine and eggs excluded) and (C) *Statist* general wholesale food index number and *Labour Gazette* cost of living index. In each case standard deviations, averages, correlation coefficients and mean differences between recorded and calculated results are found. We shall consider only the first case (the others are similar).

In this case the correlation coefficients are

$$\begin{array}{llll} r_{p\,P_{-2}} = 0.955; & r_{p\,P_{-3}} = 0.968; & r_{p\,P_{-4}} = 0.967; & r_{p\,P_{-5}} = 0.914; \\ r_{p\,p_{-1}} = 0.622 & r_{p\,p_{-2}} = 0.907 & r_{p\,p_{-3}} = 0.529 & \end{array}$$

Elsas's method gives

$$\begin{aligned} p &= 0.87p_{-1} + 0.13P_{-1};\ \text{ mean difference 5.3} \\ p &= 0.74p_{-2} + 0.26P_{-2};\ \text{ mean difference 9.2} \\ p &= 0.63p_{-3} + 0.37P_{-3};\ \text{ mean difference 11.7} \\ p &= 0.56p_{-4} + 0.44P_{-4};\ \text{ mean difference 14.6} \end{aligned}$$

The coefficients k and l in the general equation $p = kp_{-1} + lP_{-1}$ are chosen so as to minimise $(p - kp_{-1} - lP_{-1})^2$.

The regression equations are

$$\begin{aligned} p &= 81.0 + 0.65P_{-2};\ \text{ mean difference 7.4} \\ p &= 72.2 + 0.636P_{-3};\ \text{ mean difference 6.2} \\ p &= 50.5 + 0.524P_{-3} + 0.213p_{-2};\ \text{ mean difference 6.1} \\ p &= 63.8 + 0.665P_{-4};\ \text{ mean difference 7.5} \end{aligned}$$

Here the coefficients are chosen for appropriate minimisation—for example, in the third equation, $(p - k - lP_{-3} - mp_{-2})^2$ is to be minimised.

Bowley's third method requires the comparison of changes in the wholesale index numbers with subsequent changes in the corresponding retail index numbers. Here a typical equation is of the form

$$p = p_{-1} + n(P_{-1} - P_{-2}),$$

Table 8C. Average intake of calories & proteins per day.

	Calories		Protein (oz.)	
	Per person	Per "man"	Per person	Per "man"
1904	2500	3100	2.7	3.3
1937-8	3000	3700	3.1	3.8

where the numerical coefficient n is the product of the correlation coefficient between the two variables on the right-hand side of the equation and the ratio of the standard deviations.

In 1941 Bowley published a paper comparing earnings and expenditure in 1904, 1914 and 1937-1938. There were certain problems in trying to effect such comparisons: for instance, in 1904 the results obtained were for a normal family consisting of a man, his wife and one or more children, while in 1937-8 the unit was simply 'insured persons', which would have included men alone, women alone, and not necessarily any dependants. The figures showed a reduction from 1904 to 1937-8 of two children per family living at home.

When it comes to proportional expenditure it was concluded that '60 per cent of visible income was devoted to food in 1904, and only 40 per cent of visible expenditure in 1937-8' [Bowley, 1941b, p. 131].

Turning his attention next to the matter of diet, Bowley starts with the 1904 dietary as modified by the exclusion of Ireland, and compares the quantity and prices of various commodities foodstuffs in that year with the corresponding figures for 1937-8.

The calorie and protein content for the food budgets for 1904 and 1937-8 are given in Table 8C, the families having been expressed in equivalent male adult units. It is clear from these figures that there had been an improvement in nutrition.

Bowley next considers cost of living indices, first examining the change from 1914 to 1937-8. Using his new budget he finds that

> Within the margin of uncertainty there is no significant difference between the increase of 57 per cent shown by the existing Cost of Living Index, and the recomputation

Table 8D. Index numbers for November 30th, 1940.

1937-8 =100

	On new budgets		Official
	Urban	Rural	
Food	126	123	122
All items	$125\frac{1}{2}$	125	124

Table 8E. Index numbers for 1937-8 (1914=100).

Rural Budgets	On new budgets	Official
Food	141.7	142.0
All items	158.4	157.3

> which gives 59 per cent. The calculation suggests, however, that a revision would raise the index very slightly over this long period of twenty-three years. ... But it is to be remembered that the preference ratios in expenditure for an average household may have changed simply because the constitution of the average family by number, age and sex has changed. [1941b, p. 134]

Taking 1937-8 as base (=100) Bowley next computes index numbers for Industrial (or urban) households and Agricultural (or rural) households using the new budgets. Comparison with the official figures from the Ministry of Labour Gazette are shown in Table 8D.

The agricultural budgets for 1937-8 are also re-valued with 1914 being taken as the base year, resulting in the figures in Table 8E.

Bowley derives a formula connecting Laspeyre's (I_1) and Paasche's (I_2) indices

$$I_1 = \sum_i p_i\, Q_i \,/ \sum i\, P_i\, Q_i\ , \quad I_2 = \sum_i\, p_i\, q_i \,/ \sum_i P_i\, q_i,$$

where, as before, P, Q refer to the base year and p, q refer to the present year. This relationship is

$$(I_2 - I_1) \sum_i P_i\, q_i = \sum_i [Q_i\, P_i(q_i/Q_i - J_1)\,(p_i/P_i - I_1)],$$

where $J_1 = \sum_i q_i P_i / \sum_i Q_i P_i$ is an index of volume. Noting that $I_2 < I_1$ in the presence of negative correlation, Bowley expresses this in layman's terms:

> part of a general rise in prices can be evaded, by substitution of a commodity the price of which has risen less than the general average for one that has risen more.
> [1941b, pp. 135-136]

Some five pages are devoted to an algebraic analysis under the following assumptions:

> that a quadratic function of the quantities of commodities bought is a sufficient approximation for the total utility to the purchaser, and that the purchaser maximises this function in a given price and income situation.
> [1941b, p. 137]

This function, f say, is found to depend among other things on M, the marginal utility of money, where M is the first derivative (or 'differential', in Bowley's term) of f.

> The major conclusion, so far as it can be expressed in words, is that, when prices are rising, that of the luxury more rapidly than of the necessity, and income rises more than either, then it may easily happen that the ordinary relationship ($I_1 > I_2$) between the index-numbers is reversed. [1941b, p. 138]

In 1952, in his last paper on this matter, Bowley presented the series of index numbers of wage rates and cost of living that had been published over a number of years in the *Bulletin of the London and Cambridge Economic Service*. While there had been difficulties since 1940 in preserving the exact definitions originally adopted, these seemed to have little effect on the general average.

Twenty industries or occupations were chosen for which regular data were available. With December 1924 taken as the base year for the wage-rates index, monthly figures were given from January 1925 to June 1952. There is also a table showing yearly averages from 1924 to June 1952 with 1938 as base year. The workers whose occupations were covered include builders, fitters, railwaymen, lorry drivers, and women in the boot-making, confectionery, shirtmaking and tobacco industries. Cost of living indices are also given for the same periods, and less extensive indices of retail prices.

Bowley mentions two methods of comparing wages and prices. The more common of these, which he finds open to objection, consists of dividing wages by prices and terming the results ‘real wages’. The second, a method by subtraction rather than division, requires the regarding of the 1914 budget as a minimum providing ‘prime necessaries and adequate calories, but little margin for anything that can be termed luxuries’ [1952a, p. 505]. In 1938 it turned out that about 78% of expenditure was needed to meet the 1914 budget, the remainder being free for the purchase of new commodities, ‘semi-luxuries’ or insurance. In 1924 the ’free’ portion had been about 8%, while in 1947 it was 41% and 37% (perhaps more) in 1952.

8.3 Other indices

In commenting on the resemblance between ‘the determination of variations in the value of the monetary standard’ and ‘the determination of the standard of moral action’ Edgeworth remarked that

> With respect to both problems there are wise men who despair of determinateness; there are enthusiasts of whom each is confident that he has obtained *the* solution. With respect to both problems the discrepancy of principles is greater than the difference in practice; within certain limits almost any formula, accompanied with common sense, will lead to good results. [1894, pp. 158-159]

Similar remarks hold for price and quantity index numbers, an enormous number of which, of varying degrees of mathematical complexity, have been proposed over the years (see, for example, Edgeworth [1925b, §III, Parts H and I] and Hill [2004]). In this section we shall mention some that were advocated in Bowley's time.

Augustus Sauerbeck carried out sterling work in publishing price indices year after year[17]. In June 1895, in writing of the effect of fluctuations in small articles on such numbers, he said 'The great fluctuation will seriously affect the system of index numbers—arithmetical mean of all the proportionate figures' [1895, p. 171]. In considering this paper, Nicolaas Pierson wrote 'the system of index-numbers is not to be reconstructed, but to be abandoned altogether, because it is faulty in principle' [1896, p. 127]. Harsh words indeed, but it seems that his dissatisfaction was with the use of *averages* in the computation of index numbers (using either arithmetic or geometric means), for he concluded his paper by saying

> the only possible conclusion seems to be that all attempts to calculate and represent average movements of prices, either by index-numbers or otherwise, ought to be abandoned. [Pierson, 1896, p. 131]

In his response to Pierson's paper Edgeworth exposed what he saw as two serious omissions: the first was connected with the character of probability[18], and the second with the direction to a practical purpose. While admitting the wisdom of the view that questions of utility were concerned, Edgeworth concluded that

> If practical exigencies require that some one measure of utility should be framed by combining the index-numbers pertaining to different strata of society, then presumably more importance should be assigned to that one which pertains to the masses. [Edgeworth, 1896, p. 140]

The measurement of utility, Edgeworth stated[19], 'may nevertheless be a postulate of practical economics' [1896, p. 140], and he recommended here the use of the weighted median in certain cases[20].

In the first of the three parts of his memorandum 'Measurement of change in (the) value of money' read before the British Association for the Advancement of Science in 1887 Edgeworth gave the index, 'suggested independently by Marshall' [see Edgeworth, 1925b, p. 213]

$$\tfrac{1}{2}\sum(q_{i0}+q_{i1})p_{i1} \,/\, \tfrac{1}{2}\sum(q_{i0}+q_{i1})p_{i0}$$

(notation changed). This index, denoted by I^{MBE} earlier in this chapter[21], has the analogous *quantity* index

$$Q_{01}^{MBE} = \frac{\sum p_{in}q_{in}}{\sum p_{i0}q_{i0}} \times \frac{\sum p_{i0}(q_{i0}+q_{in})}{\sum p_{in}(q_{i0}+q_{in})}.$$

Here p_0, p_1, q_0 and q_1 denote the prices of a commodity in the base and the current years and the weights (or quantities) in the same years respectively, and the sum is taken over all the commodities included.

Note that prices are observed at points of time, while quantities are usually referred to periods of time. (It is therefore often assumed that the periods are very short.) Specifically Marshall and Edgeworth suggest that instead of using quantities referring to one of the two points of time (compared by the index) an *average* of the corresponding quantities should be used.

Modifications of the Marshall-Bowley-Edgeworth index are given by Frisch [1930], [1936]. More recent developments provide various *exact* cost-of-living indices derived from classes of quadratic utility functions (see Balk [1981]).

The indices that combine q_{i0} and q_{i1} in a simple manner are particularly suitable in applications. For example, when the distribution of commodities changes drastically between the two periods, both q_{i0} and q_{i1} should enter the weighting system.

In the Introduction to his Second Memoir, 'Tests of accurate measurement', Edgeworth [1925b, p. 298] wrote

> Attention may be called to the advocacy of the Median (for the computation of certain index-numbers) on the

> score not only of its peculiar facility, but also (in certain cases) its comparative accuracy. The Weighted Median, Laplace's Method of Situation, is not so familiar an operation but that its exemplification may be useful.

In this same memoir Edgeworth cites a report of the British Association Committee that had been drawn up by Robert Giffen. The Committee recommended the use of some kind of weighted index number, though it was noted that, when a large number of articles were involved, 'the scientific evidence is in favour of the kind of index-number used by Professor Jevons' [Edgeworth, 1925b, p. 299]. Jevons's price index, in our notation, is

$$I^{J} = \prod_{i=1}^{n} (p_i \,/\, P_i)^{1/n}.$$

Edgeworth, however, presented some alternatives to this index:

> The index-numbers which challenge comparison with those proposed by the Committee may be arranged under four categories, namely: 1. Those which are formed by taking the *Simple Arithmetical Mean* of the given relative prices ... II. What may be called the *Weighted Arithmetical Mean*, each relative price being affected with a factor proportioned to the quantity of the corresponding commodity, the principle adopted by the Committee. III. The *Geometric Mean*, as employed by Jevons. IV. The *Median*, proposed by the present writer as appropriate to certain purposes. [Edgeworth, 1925b, p. 321]

An advantage on the side of the median 'is its insensibility to accidental alterations of "weight" ' [Edgeworth, 1925b, p. 331].

> It appears, therefore, that our index-number, though not likely to be wide of any mark which has been proposed, is not the one which is most accurately directed to a particular, or rather, indeed, the most general object. It is no matter of surprise or complaint that we should not hit

> full in the centre an object which has not been our aim; our index-number being mainly a *Standard of Desiderata*, measuring the variation in value of the national consumption. Our primary aim, indeed, is more comprehensive, not this special, but a collective, or "compromise," scope; not so much to hit a particular bird, but so to shoot among the closely clustered covey as to bring down the most game. [Edgeworth, 1925b, p. 331]

In his contribution 'Wages, nominal and real' to Palgrave's *Dictionary of Political Economy*, Bowley gave the index

$$\tfrac{1}{2}\left[\left(\sum q_{i0}\, p_{i0} / \sum q_{i0}\, p_{i1}\right) + \left(\sum q_{i1}\, p_{i0} / \sum q_{i1}\, p_{i1}\right)\right]$$

(notation altered), where the subscripts '0' and '1' merely refer to two *different* dates, i.e. period '0' does not necessarily precede period '1'. This index Bowley described as a measure of *aisance relative* (relative affluence).

As a final example of an index, one in lighter vein, we remark that in 1914 Cave and Pearson, in their study of variate difference correlations, considered a *tobacco index*, one which was 'of considerable interest as marking the association of indices of trade prosperity with the consumption of a luxury' [p. 352]. The conclusion reached was the following:

> Thus we see that the consumption of tobacco can hardly be considered as a measure of general prosperity; it appears to be greatest when trade conditions are unfavourable, and in particular when savings are least and manufacturing conditions as measured by the importation of coal are slack. The result suggests the pipe of the unemployed at the street corner, rather than the increased expenditure of the fully occupied artisan. [1914, p. 352]

Chapter 9

Sampling

9.1 Introduction

In some form or other sampling can be traced back to Assyrian times and the Old Testament Jews, and an early Roman use of sampling was related by Plutarch in his Life of Lucullus: by questioning three captives, and knowing roughly how large the enemy troop was, the Roman general was able to estimate how long it would be before the enemy's food supply ran out, and on attacking when he believed this period had expired, he won a convincing victory[1].

It might well be argued that the demographic work of William Petty and John Graunt in the 1660s ushered in the serious statistical study in England of social problems[2]. In 1765 Arthur Young published a proposal of the way in which such a survey should be done: the investigation should be conducted by the investigator himself, the survey should cover outlying areas, and data should be obtained for a range of factors. All of these are of course eminently reasonable requirements, and they may be seen, albeit with minor modifications, as embodied in Bowley's methodology[3].

All scientific investigation is to some degree or other based on sampling. We have already noted that the dictates of time and money often prohibit the taking of a full census, and Bowley has indicated that a survey sample is then not only the correct procedure to be followed but also the only procedure to allow the obtaining of deeper, inexpensive and timely data on matters of social relevance. In this chapter we shall discuss Bowley's introduction of certain sampling methods, paying particular attention to what was in a sense his only paper on mathematical statistics.

9.2 Representative Sampling

It was only in 1895 that a term like 'representative sampling[4]' was used in what Kruskal and Mosteller [1980, p. 172] call an 'analytical' way: Anders Kiaer read a paper entitled 'Observations et expériences concernant des dénombrements représentatifs' at the Berne meeting of the International Statistical Institute in which the term was introduced. Crudely put, Kiaer regarded his 'representative sample' as the population 'in small'—that is, as a sort of approximate miniature of the population. Using census information, sample results could be compared with population characteristics, but more detail could of course be obtained in a sample than in a census. But while a carefully designed and executed sampling scheme could certainly provide information about the true proportion that a census would provide, it was necessary not to carry the idea to extremes, as Edgeworth [1913, p. 178] noted.

Kiaer gave more details of his proposed sampling method at subsequent meetings of the International Statistical Institute in St Petersburg (1897), Budapest (1901) and Berlin (1903). Bowley supported Kiaer's work, and in 1903 the Institute accepted a resolution recommending the use of the representative method provided that the conditions under which the observational units are chosen are fully specified. At the 1911 meetings of the ISI in The Hague it was reported that, in co-operation with the International Geographical Congress, an elaborate scheme for a survey had been suggested that would represent the ideal demographic survey.

Kiaer's proposals for the use of sampling did not go uncriticised, and Smith [1997] lists several reasons for this opposition. One of the main arguments was the seen (or perhaps imagined) supposition that homogeneous groups did not exist—or if they did, then they were so small that adequate estimation could only be conducted by a complete enumeration of the population. A further problem was caused by the rise in the late nineteenth century of *monography*, understood as an investigation of a population by case studies. Thus Kiaer was opposed by those who would count all transactions in a comparatively small group of families, and also by those who made fewer observations but examined a much larger number of families.

Kiaer's views[5] were initially received with reluctance, if not disfavour, and it was only after World War I that they became more acceptable. Smith (op. cit.) suggests that this was partly due to the fact that after the war various governments saw the need for more information. By 1925, however, the difficulties that Smith noted had been overcome—to a large extent as a result of Bowley's work[6].

In 1924 a commission was appointed by the ISI 'for the purpose of studying the application of the Representative Method in Statistics', the members being A.L. Bowley, Corrado Gini, Adolph Jensen, Lucien March, Verrijn Stuart and Franz Žižek.

Bowley of course had 'tested the waters' long before his report to the International Statistical Institute. But adequate as his method was it did not generalise readily to more complex sampling schemes, and it was only in 1934 that an appropriate generalisation, particularly useful in the social sciences, was published by Jerzy Neyman[7].

As a result of Bowley and Jensen's work the International Statistical Institute accepted a recommendation[8] that contained, among other things, the following definitions:

> A. Random Selection: A number of units are selected in such a way that exact equality of chance of inclusion is the dominant rule... B. Purposive Selection. A number of groups of units are selected which together yield nearly the same characteristics as the totality. In order to have

> any knowledge of the precision of the estimates, it is necessary that sufficient groups should be included to allow the variations between the characteristics of the groups to be measured. [Seng, 1951, p. 223]

9.3 Bowley's memorandum on sampling

In 1925 Bowley presented a paper—justly described by Smith [1997, p. 30] as a 'tour de force'—at the XVIth Session of the International Statistical Institute at Rome [Bowley, 1926a] in which he proposed a scheme for representative sampling and a method for the suitable analysis of the data[9]. His work, once again notable for advocating what is still regarded as sound practice in statistics, showed that a mathematical measure of precision could be given not only when random sampling was used, but also (and this was an important feature of the study) when the sampling was *purposive*. This second method was later described as follows:

> Here the unit of selection is a district or group, every member of which is included in the sample. The selection is so made that the aggregate of the districts gives the same results as the universe in respect of certain quantities (called "controls") which are known in the districts and universe and which are correlated with the unknown proportions or quantities which are the subject of investigation. [Jensen, 1928, p. 541]

The first section of the monograph begins with an introduction in which Bowley defines *attributes* (characteristics that may be present or absent) and *variables* (quantities such as age). The first thing is to define the population or 'universe' that is to be examined, and an appropriate sample is then taken in such a way that *à priori* each item has the same chance of being chosen.

> Minute precautions are necessary to ensure that the method of selection is completely uncorrelated with the presence of the attribute or the size of the variable.
> [Bowley, 1926a, p. 7]

While the results are proved in the Mathematical Notes (pp. 22-45) to this first section they are summarised on pp. 10-21 in three parts: I. Sampling for the prevalence of one attribute, II. Distribution of alternative attributes by sample and III. Sampling for the determination of the magnitude of an average. In each case both direct and inverse problems are considered. We shall discuss these parts in turn.

In Part I(A), the direct problem, it is supposed that a universe of size N has a proportion P of items having a certain attribute. From this universe n items are chosen at random. Required to find the probability that the number in the sample having the stated attribute is pn. Let $pn = Pn + x$, $Q = 1 - P$ and $q = 1 - p$. Then E_x, the desired probability, is given by

$$E_x = \binom{PN}{pn}\binom{QN}{qn}\Big/\binom{N}{n}.$$

Four separate cases are now considered, depending on the magnitudes of various quantities.

In Case 1, 'Random sample', it is assumed that Pn is so large that $1/(Pn)$ may be neglected. Then (the method is essentially the use of Stirling's Formula and the expansion of logarithms to approximate the hypergeometric distribution by the second approximation to the Law of Error)

$$E_x = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-x^2/(2\sigma^2)}\, R,$$

where $\sigma^2 = PQn[1 - (n/N)]$ and

$$R = 1 - \frac{Q-P}{2\sigma}\Big(1 - \frac{2n}{N}\Big)\Big(\frac{x}{\sigma} - \frac{x^3}{3\sigma^3}\Big).$$

Further

$$\Pr[\,|p-P|<z] = \int_{-z}^{z} \frac{1}{\sqrt{2S\pi}}\, e^{-z^2/(2S)}\, dz,$$

where $S = PQ[(1/n) - (1/N)]$ (the terms involving $Q - P$ disappear on integration). That is, the hypergeometric distribution is approximated by a Normal distribution with mean $n(P/N)$ and variance $n(P/N)(1 - P/N)(1 - n/N)$.

In the second case P is taken to be small and n large, so that $1/n$ is negligible but $1/Pn$ is not negligible. Putting $Pn + x = r$, $n = kN$ and $Pn = w$ Bowley proves that

$$E_x = e^{-w} w^r / r!$$

when k is negligible and

$$E_x = \frac{e^{-w} w^r}{r!}\,(1-k)^{-1/2}\, e^{-x^2 k/[2w(1-k)]}$$

when k is kept and $1/(PN)$ is ignored. It is also stated that the results in these first two cases are almost the same when w is as large as 20 and n is as large as 1,000.

In the 'small sample' situation of Case 3 it is supposed that n is small and n/N negligible. Then

$$E_x = \binom{n}{Pn+x} P^{Pn+x}\, Q^{Qn-x}.$$

The final case is that of proportionate stratified sampling. Suppose that there are d districts of populations $N_1, N_2, \ldots, N_d$ (with $N = \sum N_i$) and that $P_i N_i$ items in the ith district have a certain attribute. Let kN_i items be sampled from the ith group. The frequency group of E_x then has variance

$$\sigma_d^2 = (PQ - \sigma_v^2)\, n\, (1 - n/N)$$

where $\sigma_v^2 = \sum N_i\,(P_i - P)^2/N$ and $NP = \sum P_i N_i$. Then

$$E_x = \frac{1}{\sigma_d\sqrt{2\pi}}\, e^{-x^2/(2\sigma_d^2)}\, T,$$

where

$$T = 1 - \frac{\kappa_d}{2}\left(\frac{x}{\sigma_d} - \frac{x^3}{3\sigma_d^3}\right)$$

and

$$\begin{aligned}\kappa_d\sigma_d^3 &= n\Big(1 - \frac{n}{N}\Big)\left(1 - \frac{2n}{N}\right) PQ(Q-P)\\ &\quad\times\left(1 - \frac{3\sigma_v^2}{PQ} + 2\sum \frac{N_i(P-P_i)^3}{NPQ(Q-P)}\right).\end{aligned}$$

Bowley notes that if P is very small, there will be a perceptible chance e^{-Pn} that the attribute of interest is missed altogether. Should the sample be stratified this chance will be decreased slightly, and there will be a further decrease if the attribute is entirely in one district. On the whole stratification increases accuracy, except when the attribute is evenly distributed over the entire population.

Let us look next at Part I(B), the inverse problem when sampling for the prevalence of one attribute, and consider the following situation:

> Given that in a sample of n persons or things, drawn at random from a universe containing N, pn possess a certain attribute, what can we infer about the prevalence of the attribute in the universe? [Bowley, 1926a, p. 12]

There are two parts to the solution:

> in one the chances that the sample would be drawn from various hypothetical universes are compared; in the other it is considered under what circumstances we can make any inference about the relative chances that in fact the universes contained given proportions. [1926a, p. 12]

The first part, which does *not* involve inverse probability, is treated by simply changing the formulae found before so that they depend on p rather than on P. This yields, for unstratified sampling, the expectation (*sic.*) that $pn + x$ items having the distinguished attribute are found in a sample from a population in which the proportion of that attribute is P, as

$$E_x = \frac{1}{\sigma'\sqrt{2\pi}}\, e^{-x^2/(2\sigma')}\, T',$$

where

$$T' = 1 - \frac{q-p}{6\sigma'}\left(2 - \frac{n}{N}\right)\frac{x^3}{\sigma'^3}$$

and $\sigma'^2 = pqn(1 - (p/n))$.

This holds when $1/(pn)$ is negligible: similar results are given for use when other terms can be ignored. Bowley also notes that one can definitely proceed in the case of stratified sampling only if $1/\sqrt{n}$ is negligible, and gives some numerical examples.

Suppose next that one wishes to make a more definite inference from the sample to the universe.

> This necessitates some assumption about the *à priori* chance that in the universe from which selection was made the proportion should be P. [1926a, p. 15]

The method used here is very much the same as that advanced in Bowley [1923b], a paper we discussed in Chapter 6. The changes are those occasioned by the introduction of sampling. Thus let $F(P)$ be the chance that the *à priori* proportion π (say) in the universe is P and let S be the proportion in the sample. Then

$$\Pr[\pi = P \wedge S = p] = F(P)\, E_x,$$

and thus

$$\Pr[\pi = P | S = p] = F(P)\, E_x \Big/ \sum_P F(P)\, E_x.$$

Assuming that the form of F, although unknown, is determinate, that F is integrable and continuous and that 'its change in the neighbourhood of $P = p$ is finite' [Bowley, 1926a, p. 15], one can show that $\Pr[\,|P-p| \le (x/n)]$ is independent of F and does not involve the unsymmetrical term encountered in the previous sections. Thus

$$\Pr[p - z < P < p + z] = \int_{-z}^{z} \frac{1}{\sqrt{2\pi D}}\, e^{-z^2/(2D)}\, dz,$$

where $z = x/n$ and $D = pq(1/n - 1/N)$.

An essential factor in the derivation of this formula is the fact that E_x falls rapidly as $P-p$ increases (Bowley provides tables indicating this). Compensation for this would be achieved by requiring that extreme values of P should have great *à priori* probability—if it is regarded as important that such values be included.

A similar result obtains in the case of stratified sampling. Bowley emphasises that the assumptions made in the establishing of the inferences made here are difficult to verify and may not be applicable in all cases.

In considering the Direct Problem in Part II, 'Distribution of alternative attributes by sample', Bowley examines the situation in which the results of a sample fall naturally into a number c of classes (e.g. married, single, divorced, widowed), and while the preceding results may be applied to each class separately his concern here is with the distribution as a whole. He finds that

$$E_x = ce^{-\chi^2/2}\left[1 - \frac{(2-k)n}{6(1-k)^2} \sum \frac{(P_i - p_i)^3}{p_i^2}\right]$$

where $k = n/N$ and

$$\chi^2 = \frac{n}{1-k} \sum \frac{(P_i - p_i)^2}{p_i}.$$

If $1/\sqrt{n}$ can be neglected the above formula becomes

$$E_x = c\,e^{-\chi^2/2}.$$

> As a rough generalisation it may be said that the chance is about $\frac{1}{2}$ that χ^2 will not exceed $(c-2)$, where c is the number of separate classes, and more than 20 to 1 against χ^2 exceeding $2c$. [Bowley, 1926a, p. 16]

When it comes to the inverse problem here Bowley begins by saying that the proposition can be inverted under assumptions similar to those made before 'and applied with discretion to the chance that an unknown universe will not differ from an observed sample by errors which make χ^2 exceed given values' [1926a, p. 16].

Two numerical examples follow. In the first of these, involving the age distribution of men claiming unemployment payments, $c = 9$ and $\chi^2 \approx 124$, a disparity that Bowley finds to suggest that 'Either the samples were not properly collected, or they relate to different populations, or there is some mis-statement in the table' [Bowley, 1926a, p. 17].

In the second example, concerned with cases in which men with dependent children received benefits, things seem more satisfactory, with $c = 7$ and $\chi^2 \approx 5.1$.

In Part III(A), the direct problem when sampling for the determination of the magnitude of an average, it is supposed that a population has N items of magnitudes $X_1, \ldots, X_N$. Let $\overline{u}$ be the average, σ the standard deviation, μ_2 the second moment and μ_3 the third moment about the mean of the magnitudes of the population. In the case of random sampling Bowley supposes that a sample of n items is taken with mean $\overline{u} + x$. The standard deviation of x is then

$$\sigma_a = \frac{\sigma}{\sqrt{n}} \sqrt{(1-k)},$$

where k is the sampling fraction n/N. The chance that a value $\overline{u} + x$ is observed is then

$$E_x = \frac{1}{\sigma_a \sqrt{(2\pi)}} \, e^{-x^2/(2\sigma_a^2)} \, V, \tag{9.1}$$

where

$$V = 1 - \frac{\kappa_1}{2}\left(\frac{x}{\sigma_a} - \frac{x^3}{3\sigma_a^s}\right), \quad \kappa_1 = \frac{1}{\sqrt{n}} \frac{1-2k}{\sqrt{(1-k)}} \frac{m_3}{m_2^{3/2}},$$

and $1/n$ is negligible (the m's are sample moments). It can then be shown that the chance that the average in the sample and the average in the population differ by no more than x is

$$E_x = \int_{-x}^{x} \frac{1}{\sigma_a\sqrt{(2\pi)}}\, e^{-x^2/(2\sigma_a^2)}\, dx. \tag{9.2}$$

Various approximations, depending on the size of k or $1/\sqrt{n}$, are also given.

The next case is concerned with stratified sampling. Here it is supposed that the population is divided into c districts of $N_1, \ldots, N_c$ persons respectively, with different averages in the different districts. Suppose that the same proportion k is chosen from each district. The standard deviation ${}_a s_d^2$ is then

$${}_a s_d^2 = (1-k)(\sigma^2 - \sigma_v^2),$$

where $n\sigma_v^2 = \sum n_i(\overline{x_i} - \overline{u})^2/n$ and the x_i are the district averages. An expression similar to that given in (9.1) above is obtained, the term in κ_1 being considerably more complicated.

In concluding this part Bowley notes that

> If the averages of the districts differ considerably from the general average, or if the standard deviations in the districts are considerably smaller than in the population as a whole, the gain in accuracy by stratification may be considerable. [1926a, p. 20]

Part III(B) is again an inverse problem. Here Bowley considers a sample of size n drawn from a population of N magnitudes. Denote the sample average by $\overline{\mu_1}$ and its second moment about that average by μ_2'. The chance E_x of obtaining such an average from a population having mean $\overline{u_1} - x$ is

$$E_x = \frac{1}{\sigma' \sqrt{(2\pi)}} \, e^{-x^2/(2\sigma'^2)}, \tag{9.3}$$

where

$$\sigma'^2 = (\mu_2'/n)\,[1 - (n/N)]$$

in the case of a restricted but unstratified sample and

$$\sigma'^2 = \mu_2'/n$$

when the sample is unrestricted and unstratified. When the sample is both restricted and stratified

$$\sigma_2' = (1/n)\,[1 - (n/N)]\,(\mu_2' - \sigma_v'^2).$$

Here $n\sigma_v'^2 = \sum n_i(\overline{x_i'} - \overline{u_1})^2$, where $\overline{x_i'}$ is the observed mean in the ith sample. Bowley notes that (9.3) cannot be extended to include terms in $1/\sqrt{n}$ unless the population standard deviation is known.

Finally, supposing that the prior density of the population average is a function F having the same properties as before, one can show that the chance that the population average does not differ from the sample average by more that x is

$$E_x = \int_{-x}^{x} \frac{1}{\sigma' \sqrt{(2\pi)}} \, e^{-x^2/(2\sigma'^2)}\, dx, \tag{9.4}$$

the value of σ'^2 relevant to the appropriate sampling situation being chosen.

As an illustration Bowley considers the investigation carried out in Northampton and described in his *Livelihood and Poverty* (he takes care to note that the universe here is ‘working-class houses as defined for the investigation’). The number of houses examined was

693, with a sampling factor of 1/22.7 (a restricted but unstratified sample). A table is provided of the number of houses and the number of persons in each (ranging from 16 houses with 1 person through 146 houses with 3 and 2 houses with 12). Using (9.4) he then finds the chance that in the aggregate of working-class houses the average was outside the interval $[\overline{u_1} - 3\sigma', \overline{u_1} + 3\sigma']$ or within certain other intervals (e.g. $[\overline{u_1} - 2\sigma', \overline{u_1} - 3\sigma']$, $[\overline{u_1} - \sigma', \overline{u_1} + \sigma']$).

This first section of the monograph is concluded with a Mathematical Note (twenty-four pages long) in which the preceding formulae are carefully derived.

The second section is devoted to a consideration of purposive selection. Bowley notes that the problems discussed here differ in *emphasis* rather than in *kind* from those in the preceding section.

> The essential difference is that in purposive selection the unit is an aggregate, such as a whole district, and the sample is an aggregate of these aggregates, while in random selection the unit is a person or thing, which may or may not possess an attribute, or with which some measurable quantity is associated. [Bowley, 1926a, p. 46]

There are two consequences of this approach, viz.: (1) one is now dealing with weighted rather than unweighted averages, and (2) purposive selection very often involves intentional dependence on correlation between the quantity sought and known quantities. It is important then to examine how far correlation increases the precision of the measurements and how an investigation should be carried out to maximise this precision. Two parts are considered: 'Averages and single proportions' and 'Distribution in grades'.

In the first part it is assumed that the country or population under consideration is divided into N districts, the ith such district containing a_i units of interest (say, population or area) and the total number of such units in the country (say) being A. We wish to find P, the proportion of the A units having a certain attribute, or X, the average of some variable that is associated in some way or other with every unit. Thus

$$AP = \sum_1^N a_i p_i \ , \quad AX = \sum_1^N a_i x_i.$$

[The procedure for P is the same as that for X, and we shall accordingly discuss only the latter here.]

Let x_i denote the average in the ith district, and let the N values of the x's be thought of as frequency groups with unweighted mean $\overline{x}$ and standard deviation σ_x. Suppose next (and this is an important point in purposive selection) that there are a number of allied measurements or 'controls' *whose magnitudes are already known in every district.* Let the magnitudes of these controls in the ith district be $u_i, v_i, w_i, \ldots$ and $U, V, W, \ldots$ in the whole population. Then

$$AU = \sum_1^N a_i u_i \ , \quad AV = \sum_1^N a_i v_i \ , \ldots .$$

Further let $\overline{u}, \overline{v}, \overline{w}, \ldots$ denote the means of the N values of the u's, v's, w's, ...

Let $r_{xu}, r_{xv}, r_{xw}, \ldots$ denote the correlation coefficients between x and $u, v, w, \ldots$ and similarly let $\rho_{uv}, \rho_{uw}, \rho_{vw}, \ldots$ denote the correlation coefficients between $u, v, w, \ldots$. Thus, for example,

$$r_{xu} = \frac{\text{Mean}(x_i - \overline{x})(u_i - \overline{u})}{\sigma_x \sigma_u}.$$

Notice that this definition makes Bowley's correlation coefficient $1/N$ times our more usual $\sum(x_i - \overline{x})(u_i - \overline{u})/\sqrt{\sum(x_i - \overline{x})^2 \sum(u_i - \overline{u})^2}$.

Now assume that the regression equation connecting x and the control magnitudes $u, v, w, \ldots$ is, to a sufficient degree of accuracy, rectilinear, and write it in the form

$$x - \overline{x} = \sum_k G_k \left(k - \overline{k}\right) \ , \qquad k \in K \equiv \{u, v, w, \ldots\}.$$

The error e_i arising in the estimation of x_i from the regression equation is then

$$e_i = (x_i - \overline{x}) - \sum_{k \in K} G_k \left(k_i - \overline{k}\right) \ , \qquad i \in \{1, 2, \ldots, N\}, \tag{9.5}$$

where $k_i \in \{u_i, v_i, w_i, \ldots\}$. Standard regression results then give the form of the coefficients. For instance, if u is the only control then

$$G_u = r_{xu}\,\sigma_x/\sigma_u, \tag{9.6}$$

and indeed in general the G_i coefficients are given in terms of standard deviations and regression coefficients.

It is perhaps worth taking a quick look at Bowley's derivation of the above expression for G_u. For ease he lets $z, y_1, y_2, \ldots$ stand for $x - \overline{x}, u - \overline{u}, v - \overline{v}, \ldots$, and writes the regression equation as

$$z = b_1 y_1 + b_2 y_2 + \cdots .$$

To obtain the values of the b's it is necessary to differentiate

$$f = \sum(-z + b_1 y_1 + b_2 y_2 + \cdots)^2 \tag{9.7}$$

with respect to each of the b's in turn and to set the results equal to zero. This gives expressions of the form

$$0 = \frac{1}{2}\frac{\partial f}{\partial x} = N(-r_{1z} + b_1\sigma_1^2 + b_2 r_{12}\sigma_1\sigma_2 + \cdots).$$

Now expand the summand in Equation (9.7), and write the result in the form

$$f = N(\sigma_z^2 - b_1\sigma_1\sigma_z r_{1z} - \cdots) + \frac{b_1}{2}\frac{\partial f}{\partial b_1} + \cdots .$$

Since each partial derivative has been set equal to zero it follows that

$$\sigma_z - b_1\sigma_1 r_{1z} - b_2\sigma_2 r_{2z} - \cdots = \frac{f}{N\sigma_z},$$

a system of equations satisfied by

$$\frac{-\sigma_z}{R_{11}} = \frac{b_1\sigma_1}{R_{12}} = \frac{b_2\sigma_2}{R_{13}} = \ldots \quad \text{and} \quad f = N\,\sigma_v^2\,\frac{R}{R_{11}}, \tag{9.8}$$

where R is the determinant

$$\begin{vmatrix} 1 & r_{1z} & r_{2z} & r_{3z} & \cdots \\ r_{1z} & 1 & r_{12} & r_{13} & \cdots \\ r_{2z} & r_{12} & 1 & r_{23} & \cdots \\ r_{3z} & r_{13} & r_{23} & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \cdots \end{vmatrix}$$

and R_{1k} is the co-factor found by removing from R the first row and the kth column. For one control, e.g. for $K = \{u\}$, the values of G_u from (9.6) and b_1 from (9.8) are seen to be the same.

The 'standard regression results' thus having been derived, let us now suppose with Bowley that

> A number, n, of districts is selected in such a way that the average for each control is the same in the aggregate of them as in the universe. [Bowley, 1926a, p. 48]

Then

$$U\sum_1^n a_i = \sum_1^n a_i u_i \;\;,\quad V\sum_1^n a_i = \sum_1^n a_i v_i \;\;,\quad W\sum_1^n a_i = \sum_1^n a_i w_i \;\;,\quad \ldots$$

The value of the unknown X computed from the sample is then

$$X_n = \sum_1^n a_i x_i \Big/ \sum_1^n a_i, \tag{9.9}$$

and our desire is to find the precision of this estimate. From Equations (9.5) and (9.9) we get

$$\begin{aligned} X_n &= \sum_1^n a_i \left[e_i + \overline{x} + \sum_{k\in K} G_k\,(k_i - \overline{k})\right] \Big/ \sum_1^n a_i \\ &= \left[\sum_1^n a_i e_i \Big/ \sum_1^n a_i\right] + \overline{x} + \sum_{k\in K} G_k\,(L_i - \overline{k}), \end{aligned}$$

where $L_i \in \{U, V, W, \ldots\}$. On writing $X_n \equiv (X_n - X) + X$ in the preceding expression and re-arranging terms we get

$$X = X_n - H - \sum_1^n a_i e_i \Big/ \sum_1^n a_i$$

where

$$H = -(X - \overline{x}) + \sum_{k \in K} G_k\,(L_i - \overline{k}).$$

Now H, which depends on differences between weighted and unweighted averages, may be computed from the data. Further, according to Bowley, H will be small unless the correlation between the sizes of the districts and the variables is considerable.

Let us next examine the error $\sum_1^n a_i e_i \,/\, \sum_1^n a_i$. Let $n\overline{a} = \sum_1^n a_i$ and $n\sigma^2 = \sum_1^n a_i^2 - n\overline{a}^2$. Then

$$\textstyle\sum_1^n a_i e_i = n\overline{a}\,(X_n - H - X).$$

Let σ_e denote the common variance of the e_i, all assumed uncorrelated. If σ_n is the standard deviation of $X_n - H$ then

$$\sigma_n^2 = \sigma_e^2\,\frac{1}{n}\Big(1 + \frac{\sigma_a^2}{\overline{a}^2}\Big). \tag{9.10}$$

The terms on the right-hand side of this last expression can all be calculated from the sample and assumed knowledge of the population. Thus $\sigma_e^2 = (R/R')\,\sigma_x^2$, where R is the correlation determinant defined above and R' is a similar determinant with the r_{ij} replaced by ρ_{ij}. Similarly σ_x can be found from the sample. Equation (9.10) thus gives the standard deviation of the error incurred in estimating X by $X_n - H$ as

$$\frac{\sigma_x}{\sqrt{n}}\sqrt{\Big(1 + \frac{\sigma_a^2}{\overline{a}^2}\Big)}\sqrt{\frac{R}{R'}},$$

and Bowley remarks 'The advantage obtained by the use of the controls depends solely on the value of $\sqrt{(R/R')}$, being greatest when this is least' [1926a, p. 49].

Several special cases are considered, and it seems in general to be the case that any gain achieved by increasing the number of controls is quite small. Indeed Bowley concludes that

> the standard deviation of the error of the result is in ordinary cases dominated by the value of σ_x (or σ_p) and by n the number of observations, rather than by the controls exercised in purposive selection. [1926a, p. 50]

The effect of stratification is next considered. Bowley notes at the outset that

> It is not easy to distinguish the advantages of the method of stratification, in which the universe is regarded as consisting of divisions, in the districts within each of which the variable in question is confined within a narrow grade and where one district is selected from each division, from the general method of purposive selection. [1926a, p. 53]

As an illustration Bowley supposes that there is one control U and one quantity X whose mean is to be investigated. Let there be $N = k \times \nu$ districts in the country, the districts having equal populations. Further, arrange these districts in ascending order of U in k equal divisions. Let the averages for the k divisions be $\overline{u}+d_1, \ldots, \overline{u}+d_k$ and $\overline{x}+\delta_1, \ldots, \overline{x}+\delta_k$ for U and X respectively, and let the district value of U in the jth district in the ith division be $\overline{u}+d_i+{}_iu_j$ (similarly for X). Bowley then shows that

$$r_{ux}\,\sigma_u\,\sigma_x = r_{d\delta}\,\sigma_d\,\sigma_\delta + r'\sigma'_u\sigma'_x. \tag{9.11}$$

Here $\sigma'^2_u = \sum_{i=1}^k {}_i\sigma^2_u/k$, where ${}_i\sigma_u$ is the standard deviation of U in the ith division, $\sigma^2_d = \sum_1^k d^2_i/k$ (and similarly for X and δ) and r'

is the common value of the divisional correlation coefficients. This equation shows that the smaller σ'_u is, the greater is r'; or, in Bowley's words, 'that is, when the divisions are nearly homogeneous within themselves in respect to U, the greater is the correlation between U and X within the divisions' [1926a, pp. 53-54].

It is noted however that the relationship between the general correlation of U and X in the country and the correlations within the divisions is complex. As a further simplification Bowley now supposes that one district alone is selected from each division, a district whose U is as near as possible to that of the division average, and that X is measured in each of these divisions. We shall not go through the appropriate calculations: suffice to say that under the assumptions that led to Equation (9.10) it can be shown that the variance of the error is

$$\sigma_e^2 = \frac{\sigma_x^2}{k}\,(1 - r'^2)\left(1 - \frac{\sigma_\delta^2}{\sigma_x^2}\right).$$

Comparing this result to that obtained in the case of control without stratification, viz.

$$\sigma_e^2 = \frac{\sigma_x^2}{k}\,(1 - r_{ux}^2),$$

Bowley concludes this section by noting that

> The advantage obtained by stratification therefore, though it exists, may be expected to be slight. It depends on the non-rectilinearity of the regression between the control and the quantity sought in the divisions. [1926a, p. 55]

Bowley begins the second part of this section, 'Distribution in grades', with a clear statement of the matter being investigated:

> The problem being to assign the proportions in various age-groups, in grades of income or in some other classification, districts are selected which each satisfy certain controlling conditions, and the proportions found in their aggregate form the required estimate. [1926a, p. 56]

While the case of one grade is really covered by preceding work, the new thing here is the consideration of several grades. Since we are concerned with proportions (expressed as percentages), the investigation is subject to the constraint that the sum of these proportions should be equal to 100. 'Unless the number of grades is small' [Bowley, 1926a, p. 56] such conditions contribute little to the accuracy of the investigation.

> A more important consideration may be that there is an approximation to a law of distribution, such as Pareto's income formula, or other correlations between the proportions. [Bowley, 1926a, p. 56]

Suppose then that there are m grades with proportions p_i in the ith grade in the whole country and $p_i + {}_jx_i$ in the jth of the N districts. Then

$$\sum_1^m p_i = \sum_{j=1}^N {}_jx_i = 0\ , \quad i \in \{1, 2, \ldots, m\}$$

$$\sum_{i=1}^m {}_jx_i = 0\ , \quad j \in \{1, 2, \ldots, N\}.$$

The important assumption is now made that the $\{{}_jx_i\}_j$ are Normally distributed for each i. For any $j \in \{1, 2, \ldots, N\}$ let

$$\chi^2 = \frac{{}_jx_1^2}{\sigma_1^2} + \frac{{}_jx_2^2}{\sigma_2^2} + \cdots + \frac{{}_jx_m^2}{\sigma_m^2}.$$

If the $\{{}_jx_i\}_i$ were independent, 'the chance that they would be found in a single district' [1926a, p. 56] would be $C\exp(-\chi^2/2)$, where C is constant, and the chance that a value as large as χ_1 will be found would be

$$P_1 = \int_{\chi_1}^{\infty} e^{-\chi^2/2}\, \chi^{m-1}\, d\chi \Big/ \int_0^{\infty} e^{-\chi^2/2}\, \chi^{m-1}\, d\chi.$$

Since $\sum_{i=1}^{m} {}_j x_i = 0$ for each j, the expression for χ^2 can be reduced by one variable, thus reducing the $(m-1)$ in P_1 to $(m-2)$. If one knows in addition that the average in the district j is equal to the average in the country as a whole, an expression of the form $\sum_{i=1}^{m} a_i \cdot {}_j x_i = 0$ (the a_i being scale readings of the averages in the districts) holds, and the index is reduced even further to $(m-3)$.

As contrasted with this case in which only one district has been considered Bowley suggests that if n districts are merged one will have, at least approximately,

$$\chi^2 = \sum_{i=1}^{m} \left[\left(\sum_{j=1}^{n} {}_j x_i \right)^2 / \, (n\sigma_i^2) \right].$$

While noting that this has not been mathematically verified, Bowley concludes that

> Whether this is the form or not, we have the combined effect of the increased precision that arises from averaging, and of the virtual reduction of the number of grades that comes from the controls. [1926a, p. 57]

As a further method of investigating the matter Bowley notes that one might consider the reduction of the standard deviation of one grade (say the first) only. We shall not pursue the matter here: it will suffice to note that in this case the standard deviation σ_1 of ${}_1x_1$ is reduced to $\sigma_1(1-(2/m))$.

The final section in this part is concerned with correlation between the proportions in the grades, and while noting that such correlations are probably not present in the sort of cases in which his method is applicable, Bowley admits that they may sometimes arise. Briefly put,

> The maximum effect of correlation between one pair of grades is to reduce virtually the number of grades by 1, for with perfect correlation between the first pair if we were given ${}_t x_1$ we should know ${}_t x_2$. [1926a, p. 59]

In this case the previously given $C\exp(-\chi^2/2)$ becomes

$$C\exp^{-D/(2R)}$$

with

$$D = \frac{{}_1x_1^2}{\sigma_1^2} R_{11} + \cdots + 2\frac{{}_1x_1 \cdot {}_1x_2}{\sigma_1\sigma_2} R_{12} + \cdots ,$$

where R and the R_{ij} are as given in Equation (9.8).

Bowley notes that it is not necessarily the case that simplification or a reduction in the aggregate of errors will follow if there is correlation. If the correlation is positive an excess in one grade will accompany an excess in another—for instance all lower grades may show an excess and all higher grades a defect. Similarly, if there is no correlation the distribution of defects and excesses would be 'sporadic', and widening the grades would increase the precision.

As his final remark here Bowley notes that when the observations in a district may be expected to 'approximate closely' some specific distribution, one may accurately determine the constants in that distribution for each district. If, for instance, there are two such constants then two grades from each district will suffice to determine their values. Examination of their variation over the chosen group of districts will allow an estimation of their value and precision for the whole country. 'Since, however, we cannot in general expect the existence of any such law, these are mainly theoretic considerations' [Bowley, 1926a, p. 61].

The last section of the paper, 'Tests by sub-samples, and general controls (random and purposive selection)', is despatched in under a page. If the population is large or there are a large number of districts it may, Bowley suggests, be possible to divide it into m parts, each of which satisfies the appropriate conditions. He notes, however, that any bias present throughout the selection may well not be detected by any such sub-division.

If $P_1, P_2, \ldots, P_m$ are the proportions of the attribute under consideration in the m sub-samples and if P is the proportion of the attribute under consideration in their aggregate, then $\sigma\sqrt{1+(1/m)}$ is a measure of the precision of the difference expected between P and any one of the P_i (here σ is the calculated standard deviation of the error in P). If the $|P - P_i|$ are all less than this value then all P_i's may err in the same direction or σ might have been overestimated. Conversely, if the P_i are more spread out then σ may have been underestimated.

Bowley noted that general controls might perhaps be constructed

> by calculating from our sample some quantity whose magnitude in the whole population is known (and one that has not already been used as a control if the selection is purposive). [Bowley, 1926a, p. 62]

Now this estimate may suggest that some rule has been broken—perhaps by a biased sample, perhaps by erroneous information—if it differs from the known magnitude by more than twice the standard deviation (say). On the other hand, even if there is agreement between the estimated and the actual values,

> it is still quite possible that there should be errors in information or in method of collection in respect of the quantities which cannot be verified. (loc. cit.)

9.4 The application of sampling

In 1936 Bowley published a compact expository [Bowley, 1936a] in which he covered many topics related to the theory of sampling, exhibiting his eclectic and egregious experience in the field. The paper should be obligatory reading for anyone dealing with survey samples.

The fundamental object of sampling is to give maximum information about the parent population with minimum effort. The problem, Bowley notes, may be to infer properties of a sample from known proportions in a universe (or population) or, conversely, to infer properties of an unknown population from the examination of a sample or samples. Attention in this paper is restricted to the latter, the population sampled being both actual and finite. Mathematically the problem is analogous to estimating the proportions of the various colours of balls in a 'limited' urn of balls after several trial draws[10].

The first application of his sampling method in the United Kingdom, according to Bowley, was in 1912 when certain persons in Reading asked him how best to use a limited amount of money to investigate the economic condition of the working-class in that town. The investigation led to the books *Livelihood and Poverty*, *Has Poverty Diminished?*, parts of the *New Survey of London Life and Labour* and the work by D. Caradog Jones on the Merseyside and P. Ford on Southampton. (Somewhat earlier Kiaer had used this approach in a survey of Norwegian workers in 1895, with special tabulations from the census in Norway in 1900 and in Denmark in 1901, and in his study of housing in Oslo in 1913.)

The method (stratified random sampling) is to take a list of the houses in the area (possibly given by alphabetical order of streets) and mark for investigation one in n of the houses in the given order[11]. Alternatively, the population may consist of a file of cards on which data relating to persons, households, etc. are recorded.

In the United States of America the method was applied to study unemployment during the depression of 1921 and 1922. In Japan the earthquake in Tokyo in 1923 destroyed a great portion of the tabulated Census data. A sample of 1 in 1,000 (numbers 500, 1500, ...) was then taken from eleven million household schedules, the results for age and sex groups, household size, etc. being published in 1924. The discrepancies between these sampled data and the full results that were obtained at a later date turned out to be within the bounds theoretically obtained. A Swedish extraordinary census of 1935 provided a convincing example of the use of sampling related to total registrations.

Bowley details two other investigations 'by regulated sample' in which he had been involved. The first of these was concerned with participation in a committee to determine immediate effects on employment on the outbreak of World War I. Using a sample of (a) all London firms on the current Census of production, (b) one in twenty from the Home Office list of factories and workshops, and (c) City Offices, a list was obtained of (a) the number of people employed on the 21st July 1914, (b) the number employed on the 21st of August, (c) the number enlisted and (d) the number of part-time workers.

The second investigation involved the 1917-18 returns obtained monthly by the Ministry of Food from a systematic sample of millers and bakers (throughout the country) dealing with stocks and the use of flour to assess the effects of voluntary rationing. The critical time was the unrestricted submarine warfare in the summer of 1917, the concern being with the diminishing supply of food in the country and the fact that the harvest was still some time away.

Bowley notes that it is important to know the population of which the sample is supposed to be 'fair' and to have some method of judging the precision. In the above examples the universe (population) varies from the households from which the census returns were obtained, to the insured persons making claims for benefits (in the unemployed investigation), and in the case of towns' inquiries the main definition was the inhabited houses in an accessible list (occupied by persons designated as 'working-class'). The universe should thus always imply the existence of a list of units, and Bowley discusses in some detail the importance of the defined units having an equal chance of being included (or at least it being known that the chances differ in known ways from section to section).

He also dwells on the thorny problem: how great should n, the number of units in the sample, be? This is clearly related to the standard deviation $\sqrt{pq/n}$, where p is the proportion in the universe having the attribute in question and $q = 1-p$. One can usually form a preliminary idea about the size of p and decide what precision we wish to obtain. As an example Bowley supposes that $p \approx 0.3$ and that the standard error of the estimate is in the vicinity of 0.01. Then $0.01 = \sqrt{pq/n}$, so that $n \approx 2,100$. Bowley suggests that 1,000 is often a reasonable number for n. For $p = 0.3$ this gives 0.014 for the standard error of p.

Bowley claims that usually in his investigations the standard error of a *variable* is small. The size of working-class families (in the *New London Survey* investigations) was on the average 3.69, with a standard deviation of approximately 1.7. On taking $n = 1,000$ one finds that the standard error of the estimate is $1.7 \div \sqrt{1,000} \approx 0.06$, which is 'sufficiently small for many purposes' [1936a, p. 478].

He points out that it is easy to transgress the stipulation of equal chance of inclusion. He also notes the importance of not starting to sample from the unit numbered '1', for that unit might occupy that position on account of its having a specific attribute (e.g. it may be a corner house). Departure from this rule, he remarks, may introduce serious unknown bias.

A survey should contain questions that can be checked from other sources. For instance, in a town sample the results ought to agree with those deduced from the Census, the Education Authority's statistics and other relevant sources. In his study of Reading Bowley in fact found that his results had to be used to correct the official information, incorrect details having been given to him by the Education Authority.

Usually as one handles an increasing sample one finds that the estimates converge 'within progressively narrowing limits' [1936a, p. 479]. Bowley then makes the interesting observation

> that as they [the estimates] become stabilised the natural man has more and more confidence in their adequacy, though his degree of confidence may not be proportional to any mathematical chance. [1936a, p. 479]

What is meant by the last clause here? Is there no connexion between between (mathematical) chance and degree of confidence, or are some probabilities to be seen as non-measurable (as in Keynes's *A Treatise on Probability*)? One is reminded of Bowley's writing in his book on Edgeworth's contributions to mathematical statistics that ‘very slight knowledge is often sufficient to establish a greater or less (though not a measured) probability’ [1928c, pp. 8-9].

In the case of retail prices one can neither catalogue the universe nor have access to any particular unit one may wish, and even the definition of the universe may pose difficulties. In such a case it may well be impossible to find the standard error of the results since the chance of selection is unknown—though one might be able to do something when dealing with index numbers of prices if absence of correlation (say between price changes and ‘the definableness of the article’ [1936a, p. 479]) may justifiably be assumed. When the population is large one may be able to assume that sampling is indeed random, and even that the items chosen are not significantly different from those not sampled. Further, the absence of information is usually not correlated with the size of the quantity we wish to measure. For example, in examining the cards of the *New London Survey* Bowley found that in a considerable proportion of the cards the age of the mother was either not stated or clearly seemed to be guessed. Indeed, Bowley suspected that the absence of information

> had more to do with the zeal or tact of the investigator than with anything connected with the number of children or the occupation of the father. [1936a, pp. 479-80]

This hampered Bowley’s attempt to relate the numbers and ages of the children to the age of the mother and the occupational status of the father.

In specific cases tabulation of the returns may throw some light on the adequacy of the data. In Bowley’s investigation the ages of the children seemed to congregate at particular (not necessarily integral) numbers. This resulted in some measure of roughness and inaccuracy of the data, but not in any bias. Diagrams representing the results elucidated the degrees of resemblance and difference that may be expected.

In the pre-computer days tabulation was ‘usually a dull and tedious job’ [1936a, p. 480] (for instance Bowley notes that the difficulty of checking results increases faster than the sample size n) but, as he points out,

> there is a certain interest in watching the entries accumulating in a cross table and seeing the gradual growth of continuity out of randomness. [1936a, p. 480]

An ideal situation is when we have a Normally distributed population and a random sample of independent observations. However, if the pure conditions of sampling are not satisfied, there will still be doubt as to the true definition of the quantity one has actually succeeded in measuring.

Bearing in mind Bowley's path-breaking work on sampling it is interesting to note his comments on a paper published by Wishart. Bowley mentioned the Royal Statistical Society's motto, *Aliis Exterendum* (see Chapter 1 for Barnett's comment), and emphasised that while 'statistics' was certainly concerned with the 'methods concerned with the collection, tabulation, classification and analysis of numerical data obtained from observation', inference from a sample to a population was 'dangerously near the expression of opinion which the Founders of the Society forbade' [Wishart, 1939, p. 560].

Chapter 10

Economics

10.1 Introduction

In Chapter 1 we noted that the course of Bowley's career was to some degree foreshadowed by his prize-winning Cambridge essays. Recall that he had been Professor of Economics at Reading, and his interest in the subject in general was evinced by the number of papers he contributed to learned journals throughout his life.

In a letter written to Bowley on the 3rd March 1901 Alfred Marshall wrote 'Others have given more time to economics than you: but no one has done so much relative to his opportunities', and Ralf Dahrendorff [1995, p. 213] noted that the interest shown by Bowley and Allen in social, and particularly economic, matters was instrumental in the emergence of econometrics[1]. Claude Ménard in fact goes even further, saying that Bowley 'should be regarded as a founder of modern econometrics' [1987, p. 144].

10.2 Economic statistics

Warren Persons defined *economic statistics*[2] as a topic that 'include[s] all the numerical data of mass-phenomena which have an economic application' [1925b, p. 179]. Clearly much of Bowley's work fits this definition; we shall look at two pertinent papers here.

Early in the twentieth century Bowley published a paper [1905c] on the statistics of the woollen industries. He began by noting the importance of relating developments of industries at that time to the state of things ten, twenty or thirty years before, if a true understanding of the present state was to be arrived at. This was particularly true at that time of industries in the West Riding of Yorkshire, especially in the woollen and worsted trades.

Board of Trade figures for the total exports of these manufactures from the whole United Kingdom were given, attention being drawn to periods of boom and periods of depression. There was little change in the general wage level from 1883 to 1902. Bowley's rapid view persuaded him that it was better to deal with separate years than quinquennial averages, and with this in mind he presented annual summaries from 1890—starting here because this year saw the introduction of a new classification of exports—to 1904.

Regrettably, though, serious qualifications of these figures were needed before the progress of the industry could be ascertained. Firstly, the value of the wool used in exported goods was unclear, since there was no estimate of what proportion of the wool retained for use was to be credited to exported manufactured items. Secondly, values could not be ignored in favour of consideration of quantities—for instance the Board of Trade classified tissues and cloths into fourteen categories. A further difficulty arose from the fact that one could not take a yard as an equally common measure for all fourteen categories, while prices varied from 9*d.* for mixed stuffs to 14*s.* 6*d.* for broad, heavy, pure woollen cloth.

As a way out here Bowley proposed that an index number be determined as follows:

> Take a yard of mixed stuffs as unit, and express all the other cloths and tissues in terms of this, taking the prices of any period (say 1896-99) as basis; then one yard of woollen stuffs is to be reckoned as two yards of mixed stuffs, and so on. [Bowley, 1905c, pp. 588-9]

From 1895 to 1904 the export of quantity of yarns had been almost stationary (with a slight downward trend), whereas the export of tops and noils in that time had increased rapidly, suggesting that foreign nations were becoming less dependent on completed British woollen goods. There seemed, however, to be a rapid increase in the export of Apparel and Slops (the wool being in its final form) from 1890 to 1904, and Bowley was led to conclude that 'till we have statistics of production for the home market ... we can go but a very little way in the statistical history of the West Riding' [1905c, p. 590].

At the start of his presidential address to Section F of the British Association for the Advancement of Science in 1906 Bowley noted that the Section was devoted to 'Statistics' from 1835 to 1855 and that it got its 'curious name' of 'Economic Science and Statistics' in 1856. By the start of the twentieth century statistics had undergone a great deal of improvement, but its real potential had yet to be realised by other scientists. Being the first 'armchair' (to use his own word) statistician, with little in the way of economic credentials, Bowley directed his address mainly to the claims of statistics to be an exact science, worthy to rank as high as the sciences forming the subject-matter of Sections A to L. His intention was to show that the work of the statistician

> resembles the natural sciences[3] in the respect that the most delicate researches in theory lead directly to visible and important practical results. [1906b, p. 541]

Bowley first clarified distinctions between arithmetical statistics and mathematical statistics:

> the distinction to be made is not between the various methods of accumulating and tabulating data, but between the truth and falsity of the reasoning based on the tabulation. [1906b, p. 541]

The mathematical treatment, he noted, not only provided a microscope to observe differences that were 'blurred to the naked eye of arithmetic' [1906b, p. 541], but also furnished a method of measurement to aid in the realisation of the significance of a fundamental fact hidden by its various manifestations. Purely arithmetical work, limited only to the tabulation of exact records, results in 'statistics' that is nothing but accountancy, but the use of mathematical principles like interpolation and approximation results in a vital tool to make firm observations from a shifting base and to 'measure the inaccessible' (loc. cit.).

Because statistical definitions were 'the delimitation of boundaries', there were many difficulties to the clear understanding of economic categories between different countries and to the keeping of consistency over long periods of time. Even though a lot of effort had gone into the unification of conventions, it was still impossible, for example, to compare wages or prices in England and Germany. In the United Kingdom the Board of Trade, and especially the Labour Department, had paid considerable attention to the provision of explicit statements as to the exact meaning of their tables. However, the changing nature of economics presented a permanent dilemma: if an old classification was retained it was found to be out of date in the light of later conditions; if a new classification was adopted, comparison might become impossible.

Bowley advocated strongly the introduction of a 'central thinking department in statistics', noting that essential information that is often missing may very well cause a whole piece of research produced by a government department to become non-viable, even if a great deal of related information had been collected by other government agencies.

In the past official records had been largely arithmetical, and the government, quite correctly, had tried to limit recorded data to ascertained facts. However 'the things counted are not coextensive with the quantity that the scientific inquirer needs to measure' [1906b, p. 543]. What was required was an application of mathematical methods to the records and a sufficient number of people capable of handling such data by scientific methods.

Bowley was particularly unhappy with what he saw as the inappropriate organisation of the study of statistics (be it arithmetical or mathematical) in the major universities in the United Kingdom (e.g. Manchester, Birmingham, Edinburgh, Dublin, Cambridge and Oxford)[4]. He bemoaned the fact that the Public Service had a considerable supply of expert arithmeticians at lower grades, but that no attempt was made to train them up into expert statisticians—and this at a time when statistical reasoning was gaining so much importance in trade disputes and in proposals for social reform.

According to Bowley the use of mathematical reasoning in statistics was very imperfectly understood at the end of the nineteenth century and the beginning of the twentieth. One reason for this was the fact that the statistician's results were often inherently imprecise: unlike the physicist, who may be able to give a result carefully correct to so many significant figures, the statistician (particularly in economic research) may have to give a result whose second or perhaps even first significant figure is doubtful.

> We must candidly accept the fact that our raw material is imperfect, and our business is to remove the imperfections as far as we can, and, above all, to measure those we cannot remove. [1906b, p. 546]

Later in his address Bowley briefly reviewed the history of and significant achievements in statistics, from Gauss and Laplace to Quetelet, Edgeworth and Karl Pearson (he did not discuss mathematical tools like interpolation, least squares and life tables that are of general use to the statistician). Bowley noted that Edgeworth's work on the use of mathematical methods in practical situations had been surprisingly fruitless:

> The attention of mathematical statisticians has been mainly directed to theory, and to actual measurement of anthropometrical and biological correlations; it is time that it was brought to bear on the criticism and analysis of existing industrial statistics. [1906b, p. 549]

His own particular interest, Bowley noted, lay in the area where probability theory was used, this being a tool that not only was of the greatest importance but also was the least understood in general. 'All depends on a complete grasp of the nature of the measurement', he said [1906b, p. 549], and in an example in which data had led to the making of the statement 'the most probable estimate of average wages is 24s.' he discussed the meaning of the phrase 'probable error', standard deviation and finding the odds that the average exceeded a certain amount. All of this could be summarised by giving the result as 24s. ± 6d., the accuracy being measured in terms of the standard deviation. (The fact that measurements of precision were often not given was a serious defect in the presentation of official statistics.)

Following Edgeworth's writings on the Central Limit Theorem, Bowley noted that if a sample of size n is taken from almost any frequency distribution, and if n is sufficiently large, then the mean will have almost a Normal distribution with a standard deviation that diminishes with increasing n. Further, by using this method,

> we are able to give... a reasoned estimate for the real physical quantity of which the average is a local or temporary instance' [1906b, p. 550]

Bowley later used another case to illustrate the application of sampling techniques, which he thought were largely ignored or neglected. He expressed reservations about the use of such techniques by laymen: (1) the theory was still in the course of development, with general rules not yet being laid down and tests of precision being often ignored, (2) unanimity had not yet been reached on the best way to ground the theory and (3) there was still difference of opinion on the definition of technical terms. Further, suitable examples were lacking for educative and laboratory work.

When sampling was applied, one had to be careful to test its independence and precision by internal evidence. However, one had to face the difficulties of classification, which could be overcome without mathematical analysis. This is a reason for Bowley's agreeing that more advanced mathematical methods should not be included in an undergraduate course, 'but I could wish that the line were not drawn quite so low' [1906b, p. 554].

Towards the end of his address Bowley made a most profound social observation:

> If we want to check the growth of ignorant and unadaptable labour, we must save the boys of 13 and 14 from entering occupations that offer no future, and provide them with that knowledge and technique which industry will need five years later. The reason why a not unwilling worker cannot find an employer is not the want of sufficient capital, but the uselessness of the workman to society. [1906b, p. 556]

Sound advice indeed, but it is not always easy to see what the requirements of industry will be in five years time. Do we need more actuaries or more artisans?

In conclusion Bowley stated

> It is because of the immediate and pressing need of information before we commit ourselves to dangerous remedies on an erroneous diagnosis that I have spent my allotted time in pressing the importance of scientific method in statistical research. [1906b, p. 558]

He was of course quite correct, and this address, written a century ago, can still be read profitably by the modern statistician or economist—or indeed any scientist.

10.3 *Mathematical Groundwork*

In 1924 Bowley united his interest in mathematics and economics in his book *The Mathematical Groundwork of Economics. An Introductory Treatise*[5]. Having mentioned the need for such a work, Bowley goes on to set down its (ambitious) design:

> I have attempted to reduce to a uniform notation, and to present as a properly related whole, the main part of the mathematical methods used by Cournot, Jevons, Pareto, Edgeworth, Marshall, Pigou, and Johnson, so far as these are applied to the fundamental equations of exchange and to the elementary study of taxation. [1924d, p. v]

In the Introduction Bowley sets out his basic framework. There are two entities, one 'the satisfaction derived from economic goods or in some cases the desire to obtain them' and 'the physical quantity of goods' [Bowley, 1924d, p. 1]. More precisely, let $U(x, y, \ldots)$ be an algebraic function of measurable quantities $x, y, \ldots$ and let U be related to an entity $S(x, y, \ldots)$ 'where S is not a calculable function but the non-measurable satisfaction' obtained from $x, y, \ldots$ in such a way that the following postulates are satisfied: (1) if $x, y, \ldots$ change without affecting the value of U, with an increase in x balanced by a decrease in y, etc., then S is unaltered; (2) if changes in $x, y, \ldots$ cause an increase, or decrease, in U, then S increases or decreases, and (3) if successive changes in $x, y, \ldots$ cause successive changes in U to U_1 and U_2, and from U_2 to U_3, with $U_3 - U_2 > U_2 - U_1$, then $S_3 - S_2 > S_2 - S_1$ (and similarly for decreases).

Bowley illustrates the difference between U and S by comparing the former to the height of a thermometer and the latter to the sensation of heat. Further, he calls U the utility function, and it is clear that there is some connexion between this work and Edgeworth's *Mathematical Psychics* of 1881.

Chapter 1, 'Simple exchange of two commodities', is begun with a discussion of marginal utility, indifference curves and offer curves[6]. Suppose we have two commodities X and Y and that two persons A and B start with a_1 and b_1 of X and a_2 and b_2 of Y. B gives A an amount x of X and receives y of Y in turn. After this exchange A has

$$ {}_1\xi_1 = a_1 + x \quad \text{and} \quad {}_1\xi_2 = a_2 - y $$

and B has

$$ {}_2\xi_1 = b_1 - x \quad \text{and} \quad {}_2\xi_2 = b_2 + y $$

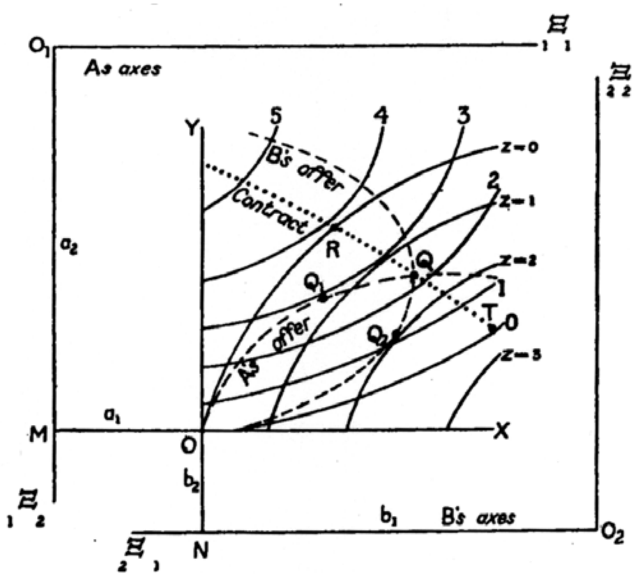


Figure 10.1. Exchange of two commodities.

(see Figure 10.1)[7] (here the subscripted prefixes 1 and 2 refer to A and B respectively). Let ${}_1U(\xi_1, \xi_2)$ and ${}_2U(\xi_1, \xi_2)$ be the utilities to A and B respectively of ξ_1 and ξ_2 units of X and Y. Then

$$
\begin{aligned}
{}_1U(\xi_1, \xi_2) &= {}_1U(a_1 + x, a_2 - y) &= {}_1V(x, y) \\
{}_2U(\xi_1, \xi_2) &= {}_2U(b_1 - x, b_2 + y) &= {}_2V(x, y),
\end{aligned}
$$

these expressions defining the utilities ${}_1V$ and ${}_2V$ of A and B after the exchange. A's indifference curve is found by plotting ${}_1V(x, y) = z$, for $z \in \{1, 2, \ldots\}$. Note that the amount of utility is unchanged by moving from one point to another on the same indifference curve. Using the expression for the tangent to $V(x, y) = c$ at a point (x_1, y_1) on that curve[8], Bowley shows that as the ratio of exchange $p = y/x$ varies, all points of contact of the tangents satisfy $x\, {}_1V_x + y\, {}_1V_y = 0$, where V_x and V_y are partial derivatives. This gives the locus of points

OQ_1Q in Figure 10.1, which is A's *offer curve.*

Suppose next that the bargaining is made as a whole, i.e. without a sequence of exchanges. A and B will each try to pick the most advantageous point on the other's indifference curve, and the *bargaining curve* Q_1QQ_2 will be obtained. The *marginal utility* to A of an increment in X, when he already has x and y, is then ${}_1V_x = D_x\,V(x,y)$, where D_x means partial differentiation with respect to x. (Wicksell [1925] noted that it was clear from the context that x and y here should be a_1+x and a_2-y respectively—see Darnell [1982, p. 165].)

If A and B do not know each other's position but make successive trial bargains, temporary equilibrium may be reached. The *contract curve* (given by RQT in the figure) is the locus of such points, and is given by

$${}_1V_x\,{}_2V_y - {}_2V_x\,{}_1V_y = 0 \quad \text{or} \quad {}_1U_{\xi_1}\,{}_2U_{\xi_2} - {}_2U_{\xi_1}\,{}_1U_{\xi_2} = 0.$$

Elimination of y from the equation

$$p = y/x = {}_1V_x/-{}_1V_y$$

yields an expression of the form $p = f(x)$. If, for instance, Y is money, then this last expression is A's *demand curve*, where p is the price of a unit of the commodity X. Similarly, if y is eliminated from

$$p = y/x = -{}_2V_x/{}_2V_y,$$

then $p = \phi(x)$ is B's *supply curve.* Proceeding from $p = f(x)$, one obtains the *elasticity of demand*

$$\eta = -p/(x\,D_xp).$$

In Figure 10.2 η is given by $\eta = NL/ON$.

Bowley then turns to the matter of money prices and marginal utility. If Y is the amount paid by A to B then

$$\kappa_1 = -{}_1V_y = {}_1U_{\xi_2} \quad \text{and} \quad \kappa_2 = {}_2V_y = {}_2U_{\xi_2}$$

are the marginal utilities of A and B respectively.

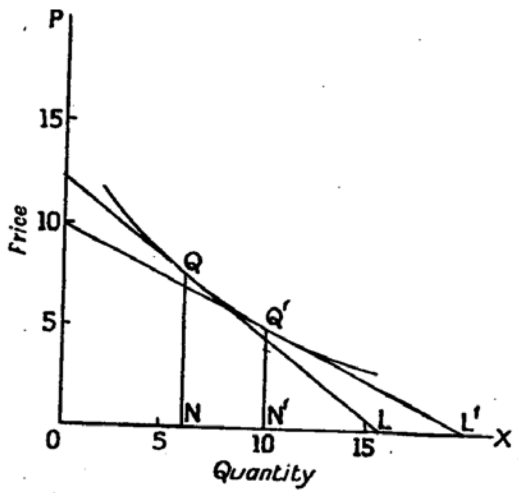


Figure 10.2. Demand Curve.

In the final section in this chapter Bowley considers the utility surface. Suppose the X and Y are both commodities. Then the following expressions are all negative:

$$_1U_{\xi_2\xi_2} = \ _1V_{yy}, \ \ _1U_{\xi_1\xi_1} = \ _1V_{xx}, \ \ _2U_{\xi_1\xi_1} = \ _2V_{xx}, \ \ _2U_{\xi_2\xi_2} = \ _2V_{yy}.$$

Referring to Figure 10.1, consider the contour lines $z = 0, z = 1, \ldots$. These contours define the surface $z = \ _1V(x, y)$.

In an Addendum to this section Bowley considers theoretical aspects of the utility surface that arise when one considers whether the uses of the commodities are independent or correlated (practical aspects are investigated in Chapter VI). For instance, consider A's utility surface and write V for $_1V$. It is shown that the second partial derivative $V_{xy} = 0$ if X and Y have completely independent uses, so that the marginal utility V_x of x is unaffected by changes in y.

There next follows a short chapter on multiple exchange. It is assumed throughout that the cost of production is not in question and that all parties have enough commodities and are prepared to exchange with each other.

Suppose then that there are m commodities $\{X_i\}$ and n persons $A, B, \ldots$. Suppose further that the tth person starts off with ${}_ta_r$ units of X_r and has ${}_t\xi_r = {}_ta_r + {}_tx_r$ after exchange. Let $p_1, p_2, \ldots$ be the price-ratios. The problem is then to determine the $m \times n$ quantities ${}_tx_r$ and the $m-1$ price ratios (it is assumed that X_m is money and therefore that $p_m = 1$). As one final bit of notation let ${}_tU_r$ denote the partial derivative of ${}_tU \equiv {}_tU({}_t\xi_1, \ldots, {}_t\xi_r, \ldots, {}_t\xi_m)$ with respect to ξ_r. The increment in utility due to exchanges is then

$$\delta({}_tU) = \sum_{i=1}^{m} {}_tU_i \ \delta({}_t\xi_i).$$

In the second section of this chapter, 'Equations of equilibrium for perfect competition', Bowley supposes that two of the n people, say A and B, exchange quantities of two commodities[9] X_1 and X_2 (say), quantities that are so small that their exchange does not significantly affect the price-ratios (assumed the same for all persons). Maximisation of the utilities results in $\delta({}_tU) = 0$ for all t, and as in the first chapter it follows that

$${}_tU_1/p_1 = \cdots = {}_tU_m/p_m\,, \quad \forall t \in \{1, 2, \ldots\}.$$

If X_m is money, then, as we have already noted, $p_m = 1$, and in this case each ${}_tU_r/p_r$ is equal to ${}_tU_m$, the marginal utility of money.

> In simple words, in spending money the greatest satisfaction is obtained when the transference of a trifling sum from one purchase to another would have an insignificant effect on satisfaction. [Bowley, 1924d, p. 21]

Bowley shows here that

$$\sum_{r=1}^{m} p_r \sum_{t=1}^{n} {}_tx_r = 0 = \sum_{t=1}^{n} \sum_{r=1}^{m} p_r \, {}_tx_r,$$

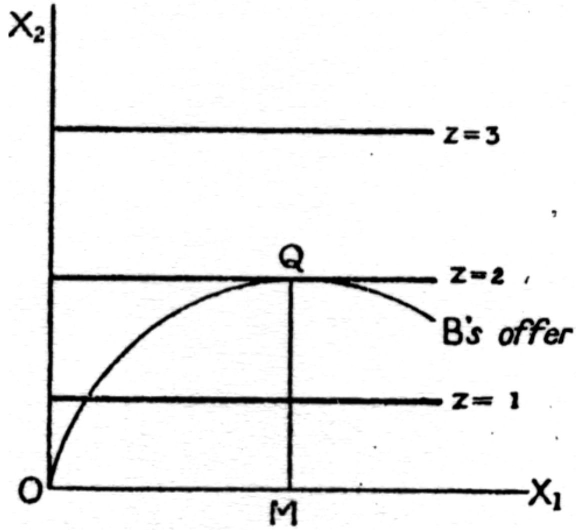


Figure 10.3.

and hence there are only $(m + n - 1)$ independent equations. Combined with the $(m-1)n$ maximising equations this yields $mn+m-1$ equations, which will in general allow the determination of the mn quantities ${}_tx_r$ and the $m - 1$ price-ratios. A corollary of this is that each person is able to maximise his satisfaction at the same time.

Section 3 is concerned with the equations of equilibrium for monopoly. Let us suppose that one person, say A, produces so much of one commodity, say X_1, that he can affect the price, and suppose too that person B is unable to affect prices when exchanging X_2 for X_1. Equilibrium is now at Q_2 rather than Q in Figure 10.1, the difference being occasioned by the fact that we now have more than two commodities.

If A reaches a stage where he has an elegant sufficiency of X_1, or has no use for it, then ${}_1U_1 = 0$ and we have $D_{x_1}(x_2) = 0$. Now x_2 is a maximum in B's offer curve, as shown in Figure 10.3 (the horizontal lines are A's indifference curves). If there are only two commodities, with A having the monopoly of X_1 and B the monopoly of X_2, then the situation is indeterminate without further information.

In his discussion of this section of the *Groundwork* Wicksell [Darnell, 1982] pointed out that Bowley had not used x_1 and $-x_2$ correctly throughout, the error resulting in the reader's conclusion that the monopolist would be forced to lower the monopoly price instead of increasing it if he gained a correct profit from his own goods. Darnell [1982] endorsed this opinion.

Bowley completes this section with a consideration of the case in which A retains his monopoly of X_1 when there are more than two commodities. We shall not go into details here.

The last section of this chapter is concerned with aggregate demand and supply. Let x_r be the sum of the positive quantities among ${}_1x_r, {}_2x_r, \ldots, {}_nx_r$, and let there be $k < n$ purchasers. Then

$$x_r = {}_1x_r + \cdots + {}_kx_r$$

and

$$p_r/p_m = {}_1U_r/{}_1U_m = \cdots = {}_kU_r/{}_kU_m$$

(if $p_m = 1$ then the U's are the marginal utilities).

From these $k + 1$ equations the terms ${}_tx_r$ may be eliminated, leaving an expression of the form $p_r = f(x_r)$ which allows one to look at a change in p_r caused by a change in x_r. This is the aggregate demand equation for X_r.

There then follows a Note on universal monopoly, in which three people each monopolise one commodity, while a fourth person has commodity X_4 but does not monopolise it. Nineteen equations are set up in the price ratios and quantities, and the problem is solved by the method of elimination. Wicksell [Darnell, 1982, p. 166] found considerable fault with this section, and this opinion was endorsed by Darnell, who noted that 'Bowley's attempt to derive a general equilibrium solution by the route of partial analysis is inapplicable' [1982, p. 172]. Darnell also noted that 'the difficulties inherent in the problem Bowley posed for himself are immense' (loc. cit.).

In Chapter III, 'Production', the question of the cost of production comes into play. Once again we have commodities $X_1, \ldots, X_m$, whose production depends on *factors of production* such as capital, labour and materials, and which will be indicated by $Y_1, \ldots, Y_\nu$. Bowley's aim is

> to discover the mathematical formulae which measure the amounts of the different factors used in the production of one commodity, and the relative amounts of one factor used in the production of different commodities. We have further to determine the distribution of each factor among different manufacturers of one commodity. [1924d, p. 28]

To this end Bowley first considers the *law of substitution*, which arises when there is one commodity and one producer[10]. Suppose that there is a quantity x of a commodity that depends in a known way on quantities $y_1, \ldots, y_\nu$ of the factors—say $x = F(y_1, \ldots, y_\nu)$. Let $\pi_1, \ldots, \pi_\nu$ be the known prices per unit of the factors, and let $p'x$ be the cost of producing the x units. The manufacturer's aim is to minimise p' by appropriate choice of the y_s (we shall regard x as constant throughout).

Now $p'x = \pi_1 y_1 + \cdots \pi_\nu y_\nu$, and hence, since x is constant,

$$\delta(p'x) = x\delta(p') = \pi_1 \delta y_1 + \cdots + \pi_\nu \delta y_\nu$$

and

$$0 = \delta x = F_{y_1} \delta y_1 + \cdots + F_{y_\nu} \delta y_\nu.$$

Elimination of δy_1 yields

$$x\, \delta p' = \frac{1}{F_{y_1}} \sum_{k=2}^{\nu} (\pi_k F_{y_1} - \pi_1 F_{y_k})\, \delta y_k.$$

For a minimum of p' we have $\delta p' = 0$ for small changes in the y_s and hence, bearing in mind the independence of the y_k, we have

$$(1/\pi_1) F_{y_1} = \cdots = (1/\pi_\nu) F_{y_\nu}.$$

> This is the law of substitution, which determines the amount of the factors used in the production of a commodity... A consequence is that at the minimum the transfer of a small sum from expenditure on one factor to expenditure on any other leaves the price of production unchanged. [1924d, pp. 29-30]

The supply curve for X is found as follows: from the $\nu + 1$ equations

$$x = F(y_1, \ldots, y_\nu)$$
$$p' x = \pi_1 y_1 + \cdots + \pi_\nu y_\nu$$
$$(1/\pi_1) F_{y_1} = \cdots = (1/\pi_\nu) F_{y_\nu}$$

the terms in the y_s may be eliminated and an expression of the form $p' = \phi(x)$ will be found.

However, if $x = F(y_1, \ldots, y_\nu) = \sum_{k=1}^{\nu} a_k y_k$ then $F_{y_k} = a_k$ and it follows that

$$\min\{\pi_1/a_1, \ldots, \pi_\nu/a_\nu\} < p' < \max\{\pi_1/a_1, \ldots, \pi_\nu/a_\nu\}.$$

Thus if $\min\{\pi_1/a_1, \ldots, \pi_\nu/a_\nu\} = \pi_1/a_1$ (say) then the minimum is reached when only Y_1 is used, the case of *alternative factors*.

If one denotes the cost of a unit of X by $\mu = p'x$, then the producer's offer curve, or the *integral supply curve* (see Figure 10.4) is $\mu = x\phi(x) \equiv \chi(x)$. The curve $p' = \phi(x)$ is the *supply curve*.

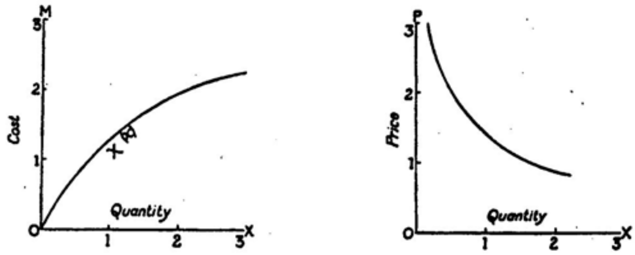
Also of importance is the relationship between changes in quantity produced and the cost of producing these changes. To analyse this relationship either p' or the function μ introduced above may be used.

The *elasticity of supply* e is defined by

$$e = -(p'/\delta p')/(x/\delta x).$$

Proceeding to the limit we get

$$e = \frac{p'}{x\, D_x p'} = \frac{\phi(x)}{-x\, \phi'(x)}.$$



(a) (b)

Figure 10.4. (a) Integral supply curve. (b) Supply curve.

Further, let $\epsilon = \mu/(x\, D_x \mu)$,

> so that ϵ measures the ratio of the relative increase of cost to the relative increase of output, while e measures the ratio of the relative decrease of price to the relative increase of output. [1924d, p. 32]

(Wicksell [Darnell, 1982, p. 166] says that the words 'cost' and 'output' should be switched in the second line of this quotation, and similarly for 'price' and 'output' later on.) Noting that $D_x\mu = p' + x\, D_x p'$ Bowley proves that $e = \epsilon/(\epsilon - 1)$. This is then followed with a discussion of increasing ($\epsilon > 1$), constant ($\epsilon = 1$) and diminishing ($\epsilon < 1$) return. Another view of the supply price p' is given in Section 7.

The graphs of $\phi(x)$ and $\Phi(x)$ (the curve of marginal supply prices), for increasing and decreasing returns, are shown in Figure 10.5. Wicksell notes an error here [Darnell, 1982, p. 166]: these graphs should not be parallel but should pass through the same starting point.

The eighth section is concerned with several manufacturers and one commodity.

The question of whether any one individual can affect the selling price is then examined. Supposing that the price were at the value p

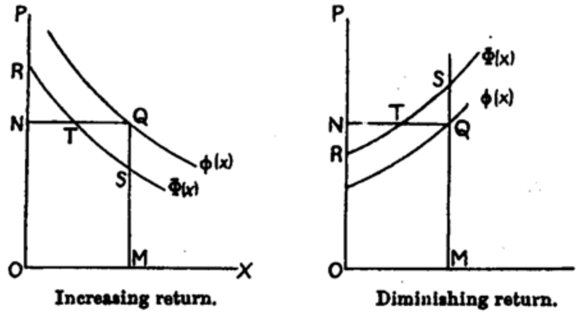


Figure 10.5. Marginal supply prices.

determined above, one finds that any producer (say, the first) could increase his profit by reducing his output to $f(x) + {}_1xf'(x) = {}_1p'_m$, provided that other producers were unaffected. Thus

> the selling price would increase, and then the other producers would push up their production till the marginal supply price of each equalled the new price. This would cause over-production at the new price, which would therefore fall. The above equations therefore give stable equilibrium, if no producer is predominant. [1924d, p. 37]

The chapter is concluded with a short discussion of how the proportions of the factors that are available are distributed when there are several manufacturers who compete for their use.

This brings us to Chapter IV, 'Supply and demand for the factors of production'. Here the aim is to find supply curves of the form $\pi'_s = \phi(y_s)$ and demand curves $\pi_s = f(y_s)$ and to examine the equilibrium between supply and demand.

'The ultimate factors,' writes Bowley, 'are labour, capital, and land' [1924d, p. 40]. Each of these is considered on its own, and Bowley eventually deduces that for each factor one knows either the amount or else equations of the form

$$(1/\pi'_s)\,{}_tW_s = -(1/p_r)\,{}_tU_r = -{}_t\kappa,$$

where ${}_tW_s$ is the marginal disutility[11] for person t of producing the factor Y_s, ${}_tU_r$ is the marginal utility of the commodity t receives in exchange and ${}_t\kappa$ is t's marginal utility of money. The *supply equation* in this case is then

$$\pi'_s = -(1/{}_t\kappa)\,{}_tW_s = \phi(y_s).$$

When it comes to the matter of the equations of demand Bowley supposes that the demand is occasioned by the use of one factor for the production of a single commodity X which may be regarded as typical. Then the price function $p = f(x)$ satisfies

$$\mu = p'x = xf(x) = \textstyle\sum_{i=1}^{\nu} \pi_i y_i$$

(where it is assumed that $p' = p$, that is, that there is no profit). Once again the equations of the minimum cost of production are

$$(1/\pi_1)F_{y_1} = \cdots = (1/\pi_\nu)F_{y_\nu}.$$

From these equations omit μ and eliminate all the y's except for some one—say y_s. This results in an equation in $y_s, \pi_1, \ldots, \pi_\nu$ and p. When only π_s and y_s are supposed to vary, write the demand equation as $\pi_s = f(y_s)$, where f involves the prices of all the commodities. In the case of competition $\pi'_s = \pi_s$ and hence $f(y_s) = \phi(y_s)$ gives the equilibrium point.

If labour or suppliers of factors are combined then $y_s(\pi'_s - \pi_s)$ can be maximised, and the value of y_s can be determined from

$$f(y_s) + y_s\,f'(y_s) = \phi(y_s) + y_s\,\phi'(y_s).$$

Wicksell notes that $y_s(\pi'_s - \pi_s)$ should be $y_s(\pi_s - \pi'_s)$, and Darnell [1982, p. 177] points out that Bowley's formula would in fact apply to monopsony.

In the final section, 'The share of the factors', Bowley considers the elasticity of demand for Y_s, viz. $\eta_s = -\pi_s/(y_s f'(y_s))$. The cases $\eta_s > 1$, $\eta_s = 1$ and $\eta_s < 1$ are considered separately, and it is shown that in the last case

> a trade union could increase the aggregate income and aggregate advantage of its members by raising their rate of wages and causing some to be out of work or to work short time. Every one, including those at play, could get more. [Bowley, 1924d, p. 44]

Chapter V, 'General equations of supply and demand in a stationary population', starts off with a summary of the limitations of the preceding chapters, and Bowley notes that

> In fact the actual determination for any price or quantity involved depends on every other; we can only obtain a complete solution if we restrict our universe to two persons and two commodities ... or extend it and include all conditions in any interdependent series of equations. [1924d, p. 47]

It is the last part of this quotation that is investigated in this chapter.

Suppose then that the tth person produces ${}_tx'_r$ of the rth commodity X_r, that he provides ${}_ty_s$ of the sth factor Y_s and that he uses or saves ${}_tx_r$ of X_r. Further, let the prices of factors of units be denoted by $\pi_1, \ldots, \pi_\nu$.

In deriving the supply equations Bowley denotes by x_r the total amount of X_r that is produced, by y_s the total amount of Y_s that is used (equivalently, the total amount of Y_s that is supplied), by y_{rs} the total used in the manufacture of X_r and by ${}_ty_{rs}$ the amount of Y_s used by person t in making X_r. The average cost per unit of X_r to the t person in the manufacture of X_r will be denoted by ${}_tp'_r$.

A number of equations are set up—for example, the mn equations for the production functions,

$$ {}_tx'_r = {}_tF_r({}_ty_{r1}, \ldots, {}_ty_{r\nu}), \quad \forall t \in \{1, \ldots, n\}; \ \forall r \in \{1, \ldots, m\}, $$

the mn equations for the costs of production,

$$_tp'_r\ _tx'_r = \sum_{s=1}^{\nu} \pi_s\ _ty_{rs}, \quad \forall t \in \{1, \ldots, n\};\ \ \forall r \in \{1, \ldots, m\},$$

and the $n\nu$ equations for the disutility of supply of factors

$$(1/\pi_1)\,_tW_1 = \cdots = (1/\pi_\nu)\,_tW_\nu = -_t\kappa, \quad \forall t \in \{1, \ldots, n\}.$$

In all there are $mn\nu + mn + m\nu + n\nu + m + 2\nu$ equations, and the elimination of certain quantities (e.g. $_ty_{rs}$ and $_tx'_r$) allows the determination of m supply equations involving quantities like x_r, $_tp'_r$ and $_t\kappa$.

The demand equations are considered in Section 3, and in Section 4 the supply and demand equations of the previous sections are jointly examined. Two sets of relations are now introduced.

> The first takes into account the whole income of each person from the supply of factors or the net value of production, which must equal his expenditure together with saving. The second set connects p_1 with p'_1, p_2 with p'_2, &c. [Bowley, 1924d, p. 51]

The equations

$$\sum_{s=1}^{\nu} \pi_s\,_ty'_s + \sum_{r=1}^{m} (p_r - {_tp'_r})\,_tx'_r = \sum_{r=1}^{m} p_r\,_tx_r,\ \forall t \in \{1, \ldots, n\}$$

when combined with the equations of the preceding sections give $2m - 1$ equations in $3m$ quantities x_r, p_r and p'_r.

Finally, to connect p_r with p'_r one needs to consider two cases. In the first, that of exchange under competition, one has $p_r = p'_r$, and in the second, under producers' monopoly, $\delta(p_r - p'_r)\,x_r = 0$, $\forall r \in \{1, \ldots, m\}$ (in the latter case recall that p_r and p'_r both involve x_r). Elimination of p'_r results in $2m - 1$ equations, allowing the determination of $x_1, \ldots, x_m$ and of the price-ratios $p_1 : p_2 : \ldots : p_m$.

There is an interesting discussion at the end of this section. Bowley suggests that the equations considered here that express merely identity should be distinguished from those that depend on volition. Particular attention needs to be paid to the hypotheses relating to the latter, and these hypotheses may be classified as *industrial, commercial* or *hedonistic*. As respective examples Bowley gives the law of substitution involving

$$(1/\pi_s)\, D_s({}_tF_r),$$

the maximising of

$$(p'_r - {}_tp'_r)\, {}_tx'_r, \;\; (p'_r - p_r)x_r, \;\; (\pi_1 - \pi'_1)y_1$$

when it is permissible, and

$$(1/\pi_s)\, {}_tW_s = -{}_t\kappa = (1/p_r)\, {}_tU_r.$$

It is the hedonistic equations that Bowley suggests might be most subject to criticism.

> There remains the general assumption that persons in economic matters act under economic motives with adequate knowledge. There are many transfers of wealth on other grounds, and the equations are not always pressed to the maximum. Also ignorance and miscalculation are common, and the mere clinging to custom may prevent advantageous changes. [Bowley, 1924d, p. 53]

In the final section Bowley considers whether there could be more than one set of solutions and whether the equilibrium is stable. As regards the first question he suggests that, while the nature of the case does not prohibit the existence of more than one solution, one's experience will enable one to decided on the correct solution in practice. The stability of the solution can be judged from the behaviour of the supply and demand curves and their points of intersection, and this is still to be examined. The solution is essentially statical, but one may be able to see how the system will behave if there should be a change in any of the constants (e.g. an increase in the amount of land available).

We now come to Chapter VI, 'Applications of the general equations'. In the first section Bowley considers the direction of a demand curve. The equations for one consumer are

$$\mu = p_1 x_1 + \cdots + p_m x_m$$
$$(1/p_1)\, U_1 = \cdots = (1/p_m)\, U_m = \kappa,$$

where quantities $x_1, \ldots, x_m$ are bought in unit time during which the consumer's total expenditure is μ and his marginal utility of money is κ.

If independent usage is made of the $X_1, \ldots, X_m$ then U_1 depends on no other x_i but x_1. Then $U_{12} \equiv D_{x_2}(U_1) = 0$, and the same is true for any $U_{r,s} \equiv D_{x_s}(U_r)$. The demand curve is $p_1 = (1/\kappa)U_1$ and if κ is not sensibly affected by changes in X_1 then $D_{x_1} p_1 = (1/\kappa)U_{11}$. If the utility grows by diminishing increments as x_1 grows by equal increments, p_1 is negative.

If the uses of the X_i are not independent then

$$\kappa D_{x_1} p_1 = U_{11} + U_{12} D_{x_1}(x_2) + \cdots + U_{1m} D_{x_1}(x_m).$$

Further information is required before one can determine whether the sign is positive or negative.

As an illustration Bowley considers the case of two commodities with μ and p_2 constant and with p_1, x_1 and x_2 changing. The demand equations are now

$$p_1 x_1 + p_2 x_2 = \mu$$
$$p_2 U_1 - p_1 U_2 = 0.$$

Taking the utility surface to be given by

$$U = -a x_1^2 - 2h x_1 x_2 - b x_2^2 + 2g x_1 + 2f x_2,$$

Bowley shows that $D_{x_1}p_1$ is negative if $h \leq 0$ and the uses of X_1 and X_2 are independent or complementary.

Passing next to the case of alternative demand, Bowley notes that when $h > 0$, that is X_1 and X_2 are alternative, one finds that $D_x p_1$ may be positive or negative. As an example consider a case in which the utility surface is $z = -x_1x_2 + 40x_1 + 100x_2$ and the income equation is $p_1x_1 + p_2x_2 = 840$. Let p_2 be fixed at 40. Bowley proves that

$$D_{x_1}(p_1) = p_1/(50 - x_1)$$

which is positive when $x_1 < 50$. Similarly, if the price of X_1 is constant one gets

$$D_{x_2}(p_2) = p_2/(20 - x_2),$$

so that an increase in p_2 leads to an increase in x_2 provided that initially $x_2 < 20$. Thus we are led to the curious situation in which an increase in the price of X_1 may cause an increase in the consumption of X_1 and a decrease in the consumption of X_2.

Section 3 treats the demand for and supply of one commodity in situations of competition and monopoly. Bowley's investigations show that 'the increase in price made by the monopolist is influenced by certain considerations' [Bowley, 1924d, p. 60]. For instance,

> If the price is high there is an inducement to use substitutes, and the public may tend to give up the use of the commodity. If profits are great, there is an inducement for rivals to try to break the monopoly. If in deference to public opinion the monopolist lowers the price he may make a small sacrifice in his profits and increase the output perceptibly. [Bowley, 1924d, p. 60]

The next section—one regarded by Darnell as 'a very difficult and novel passage' [1982, p. 173]—is devoted to some general matters involving monopoly and combination: for instance, monopoly may occur in the *production of all commodities* even if the factors of production are not monopolised. An examination of the question of bilateral monopoly[12], in which the producer is the monopolist and also makes a profit, leads Bowley to the general conclusion that 'universal monopoly of all factors and all production leads to indeterminate results' [Bowley, 1924d, p. 62].

Attention is then turned to the question of whether purchasers of goods can gain any advantage by combining rather than competing—and, in the same vein, whether the only buyer of a certain product has any special power.

The general position when either buying or selling may be competitive or non-competitive is summarised as follows: if A buys and B sells commodity X, and if A pays and B receives Y (money), then A's offer is ${}_1U_x - p\ {}_1\kappa = 0$ and B's offer is ${}_2U_x - p'\ {}_2\kappa = 0$. A and B are both satisfied only when $p = p'$ and ${}_2\kappa\ {}_1U_x = {}_1\kappa\ {}_2U_x$. B can fix the price if he is the only seller and there are several buyers not in collusion. Similarly, if A is the only buyer and there are several sellers not in collusion, then A can choose p' to maximise his net gain in utility.

In Section 5, 'Joint and composite demand and supply', Bowley considers the quantities $y_{r1}, \ldots, y_{r\nu}$ as the *joint demand* for the production of x_r (e.g. labour, coal, ore, transport). The quantities $y_{1s}, \ldots, y_{ms}$ are taken to be under a *composite* or *alternative demand* for use in various manufactures (e.g. Y_s may be the labour required by various manufacturers). The pertinent equations in these two cases have already been given.

Composite or *alternative supply* 'occurs when a want can be supplied by X_r or X_{r+1}' [Bowley, 1924d, p. 66] (e.g. when a need (or desire?) for meat may be met by beef or mutton). *Joint supply* 'occurs when X_r and X_{r+1} are produced by the same process in a determinate proportion (e.g. gas and coke)' [Bowley, 1924d, p. 66]. The demand equations are unaltered and the problem is determinate. There is *joint demand* for commodities X_r and X_{r+1} when each is only useful with the other (e.g. 'love and marriage', as the song has it, though Bowley's more down-to-earth example is pen and ink). Suppose that one unit of X_r is needed with one unit of X_{r+1}. We then have n new equations

$$_tx_r = {_tx_{r+1}}, \quad \forall t \in \{1, \dots, n\}$$

while in the utility function the passage

$$= (1/p_r)\, {_tU_r} = (1/p_{r+1})\, {_tU_{r+1}} =$$

is replaced by

$$= [1/(p_r + p_{r+1})]\, {_tU_{r'}},$$

where $_tU_{r'}$ is the marginal utility of a unit of the combination of X_r and X_{r+1}. The solution is determined uniquely.

The chapter is completed with a discussion of the *derived* or *indirect demand* for factors of production. As a simple illustration suppose that there is one commodity X with demand equation $p = f(x)$ and suppose that the production of X depends on the factors $Y_1, \dots .Y_\nu$. Assuming that the prices of factors other than Y_1 (say) stay constant, Bowley shows that the demand for Y_1 may be given by an equation in π_1 and y_1, this equation involving the constant prices $\pi_2, \dots, \pi_\nu$.

The last chapter is concerned with surplus value, rent and taxation. The first section deals with producers' surplus[13].

> A surplus is obtained when a producer sells for more than his cost price or a consumer buys for less than he is willing to give. [Bowley, 1924d, p. 69]

Although Bowley begins by considering the case of many producers, we shall look only at his discussion of one producer[14]. Let $p = f(x)$ be the demand curve, where p is now unknown, and let

$$p_m = D_x(px) = D_x(xf(x))\,.$$

This is equal to NQ in Figure 10.6, in which DQ is the locus of Q. The producer now changes p' (and therefore x) in order to maximise

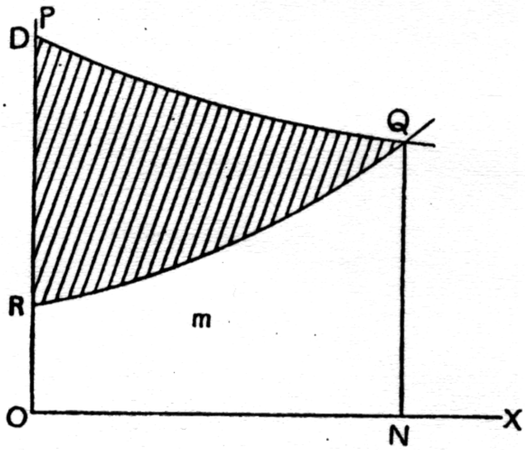


Figure 10.6. Producer's surplus.

$(p - p')x \equiv pm$. At the point of equilibrium

$$D_x(m) = D_x(px),$$

and so $p'_m = p_m = NQ$. Then

$$px = \int_0^x p_m \, dx = \text{area } ODQN,$$

and $m = \text{area } ORQN$. The profit is then

$$\text{area } ODQN - \text{area } ORQN = \text{area } DRQ.$$

The second section here is devoted to economic rent, Bowley being specifically concerned with land. Suppose a producer of commodity X can hire labour and capital and can buy materials for this production at fixed rates. Suppose too that only one plot of land is cultivated and that the production x_1 is varied by varying y_1, the amount of labour, and y_2, the quantity of material. Then $x_1 = F(y_1, y_2)$. Further, if p is the selling price, the producer wishes to maximise $px_1 - p'x_1$, where p' is the cost of production per unit.

Then we have the equations

$$p'x = \pi_1 y_1 + \pi_2 y_2$$
$$(1/\pi_1)F_{y_1} = (1/\pi_2)F_{y_2}$$
$$x = F(y_1, y_2),$$

from which $p' = \phi_1(x)$ can be found.

In the case of decreasing return $\phi_1'(x) > 0$, and x_1 then satisfies

$$p = D_{x_1}(p'x_1) = \phi_1(x_1) + x_1\phi_1'(x_1) = p_m'.$$

The maximum profit is

$$(p_m' - p')x_1 = x_1^2\,\phi_1'(x_1).$$

All plots are cultivated for which $p = \phi_1(x) + x_1\phi_1'(x)$ has a positive root, and the *local margin of cultivation* is obtained when the root is zero. The *intensive margin* on any plot is obtained when $p = p_m'$.

When labour and interest on capital are included in y_1 and y_2 the *profit* $x_1^2\,\phi_1'(x_1)$ is the rent that can be obtained for the first plot.

Next we have a section on 'Taxation in the case of competition'. Let τ be a tax, per unit of X_r, that is paid by the producer. Further, consider the demand $f(x)$ and the supply $\phi(x)$ of X_r only. Let $\psi(x) = f(x) - \phi(x)$, and let the equilibrium before and after tax be $\psi(x_1) = 0$ and $\psi(x_1 - \xi) = \tau$ respectively. A Taylor series expansion then gives

$$\tau = -\xi\psi'(x_1) + \tfrac{1}{2}\,\xi^2\,\psi''(x_1) - \cdots .$$

The receipt from tax is $R = \tau(x_1 - \xi)$, and use of the above expansion of τ then allows the expression of R as

$$R = -x_1\,\xi\,\psi'(x_1) + \xi^2\,\psi'(x_1) + \tfrac{1}{2}\xi^2\,\psi''(x_1) + \cdots .$$

The loss of utility experienced by the consumers (in money) is then

$$C = \int_0^{x_1} f(x)\,dx - x_1 f(x_1) - \int_0^{x_1-\xi} f(x)\,dx + (x_1-\xi)\,f(x_1-\xi)$$
$$= -x_1\,\xi\,f'(x_1) + \tfrac{1}{2}\,\xi^2\,[f'(x_1) + x_1\,f''(x_1)] + \cdots\,.$$

The situation is shown in Figure 10.7. Let Q and L respectively be the positions before and after taxation. Then

$$ON = x_1, \quad NQ = f(x_1) = \phi(x_1), \quad MN = \xi.$$

Further

$$\tau = KJ \approx JQ\,[-f'(x_1) + \phi'(x_1)] = -\xi\,\psi'(x_1)$$
$$C = \text{area } QHSL \approx \tfrac{1}{2}\,\xi\,(-f'(x_1))\,(2x_1 - \xi)$$
$$R = \text{area } KTSL = \tau(x_1 - \xi).$$

If

$$P = \text{area } QHTK \approx \tfrac{1}{2}\,\xi\,\phi'(x_1)\,(2x_1 - \xi),$$

then

$$C + P - R = \text{area } KLQ \approx \tfrac{1}{2}\tau\xi.$$

(Bowley considers here only the case of constant return, but states that the other cases can be similarly treated.)

When there is competition, when the return is increasing and when the producer gains no profit, then

$$C - R = x_1\,\xi\,(-\phi'(x_1)) + \xi^2\,[-\tfrac{1}{2}f'(x_1) + \phi'(x_1)] + \tfrac{1}{2}\,\xi^2\,x_1\,\phi''(x_1),$$

while in constant return

$$C - R = \tfrac{1}{2}\,\xi^2\,[-f'(x_1)].$$

When the return is decreasing Bowley shows that P, the producer's aggregate loss of profit, is

$$P = x_1\,\xi\,\phi'(x_1) - \tfrac{1}{2}\,\xi^2\,[\phi'(x_1) + x_1\,\phi''(x_1)] + \cdots\,,$$

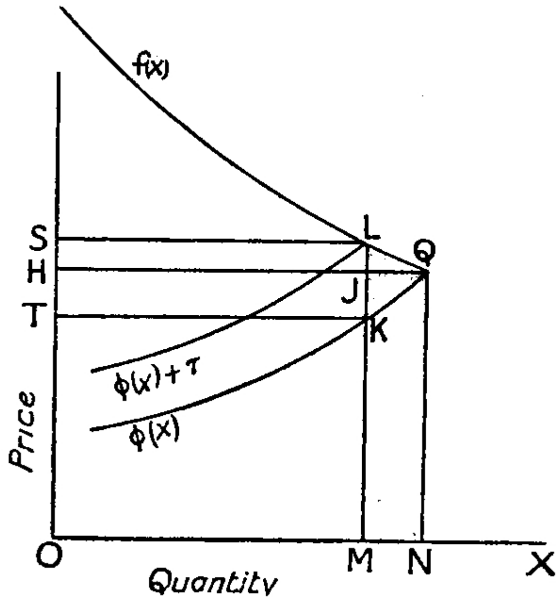


Figure 10.7. Competition: decreasing return.

and hence

$$C + P - R = \tfrac{1}{2}\,\xi^2\,[\phi'(x_1) - f'(x_1)] + \cdots$$

> Hence in all cases the public, producer and consumer together, lose more than the revenue gains.
> [Bowley, 1924d, p. 74]

Suppose next that $f''(x)$ and $\phi''(x)$ may be neglected. Then

$$\frac{C}{P} = \frac{-f'(x_1)}{\phi'(x_1)} = \frac{-e}{\eta},$$

where E and η are the elasticities of supply and demand at x_1. The increase in price is

$$\begin{aligned} f(x_1 - \xi) - f(x_1) &= -\xi\, f'(x_1) \\ &= MN\,(-f'(x_1)) \\ &= JQ\,(-f'(x_1)) \\ &= \frac{\tau}{-f'(x_1) + \phi'(x_1)}\,(-f'(x_1)). \end{aligned}$$

Thus when the return is constant the increase in price is τ, when the return is decreasing it is less than τ, and when the return is increasing it is greater than τ.

The maximum amount of tax is obtained when τ is chosen so that $x_\tau \psi(x_\tau)$ is maximised when x_τ is the amount exchanged.

The final section in this chapter is concerned with taxation in the case of producer's monopoly. For a tax τ the monopolist maximises $(\psi(x) - \tau)x$ at x_τ, where

$$\psi(x_\tau) + x_\tau\, \psi'(x_\tau) = \tau, \quad R = \tau x_\tau.$$

Had there been no tax an amount x_1 would have been produced, where

$$\psi(x_1) + x_1\, \psi'(x_1) = 0.$$

Let P denote the loss of profit and tax. Then

$$\begin{aligned} P &= x_1\psi(x_1) - x_\tau[\psi(x_\tau) - \tau] \\ C &= \int_{x_\tau}^{x_1} f(x)\,dx - x_1\, f(x_1) + x_\tau\, f(x_\tau) \\ C + P - R &= -x_1\, \phi(x_1) + x_\tau\, \phi(x_\tau) + \int_{x_\tau}^{x_1} f(x)\,dx. \end{aligned}$$

On taking $x_1 = x_\tau + \xi$ and supposing that the supply and demand curves are in fact straight lines, so that

$$f''(x_1) = \phi''(x_1) = \psi''(x_1) = 0,$$

one finds on using a Taylor series that

$$C + P - R = \xi^2 \, \phi'(x_1) - (\tfrac{1}{2}\xi^2 + \xi \, x_1) \, f'(x_1),$$

and hence

$$C/P = f'(x_1)/[2 \, \psi'(x_1)] = e/[2(e - \eta)].$$

One then has $C = P/2$, $C > P/2$ or $C < P/2$ according as the return is constant, increasing or decreasing.

Various further cases of the behaviour of C, P and R under monopoly when the return is constant, increasing or decreasing are also examined.

> Under monopoly, if the tax is not per unit but a lump sum, the price is unaffected and the amount sold unaffected; the whole is paid by monopolist and can theoretically be increased till it nullifies his profit, and $R = x_1 \, \psi(x_1)$, viz. twice the maximum under a tax per unit. [Bowley, 1924d, p. 77]

The *Groundwork* is concluded with an Appendix detailing the mathematical ideas and formulae used in the text, and perhaps bearing in mind Hardy's comments on his earlier *A General Course of Pure Mathematics* (see Chapter 11) Bowley hastens to point out that

> The definitions and proofs are not rigid in the mathematical sense, and any careful reader will detect numerous lacunae. [1924d, p. 78]

The first section is devoted to some introductory remarks on functions, including a 'popular' definition of continuity. Aspects of the differential calculus are discussed, including derived functions (or differential coefficients)—using $D_x y$ rather than the more common $\frac{dy}{dx}$ since the latter 'suggests a fraction and not the result of a process' [1924d, p. 80]—Taylor's Series and some standard results in differentiation. Subsequent sections consider functions of two or more variables and partial differentiation, an introduction to integration, beginning with the idea of an indefinite integral as the inverse to the operation of differentiation, and passing on quickly to a view of the definite integral that is perhaps more in the sense of the Cauchy integral than anything else, for Bowley writes 'The most important use of integration in the present connexion is in its relationship to areas' [1924d, p. 93]. The Appendix is concluded with a Note on elimination, extensive use of which is made, as we have seen, in the main body of the text.

Among contemporary reviewers of the *Groundwork* Edgeworth found it to be a 'clear, concise and correct statement of the leading propositions and methods which mathematics contributes to Political Economy... The maturer student will be edified by it' [1924, p. 430]. While he disagreed with Bowley's approach to certain topics as not being that 'which we recommend as best' [1924, p. 431], Edgeworth clearly said that no disparagement was meant by this observation. Bowley's originality was seen to arise not only in his use of new arguments but also in his introduction of new phrases—though Edgeworth queried the value of the innovations. For instance, he objected to the bold substitution of the 'new-fangled symbolism[15]' $D_x y$ for $\frac{dy}{dx}$.

Less flattering, once again, was G.H. Hardy. Lord Robbins noted in his autobiography that when he showed Hardy the *Groundwork* 'he was distinctly uncomplimentary, holding the exposition to be deficient in elegance and the results lacking in depth' [Robbins, 1971, p. 118]. Hardy in fact had refereed Bowley's book for the Oxford University Press, and while expressing his inability to comment on the economic aspects, he found the mathematics to be elementary.

Young noted that this book 'reveals him [i.e. Bowley] as an adept in the difficult field of mathematical economics' [1925, p. 133] and

> His book is the one best guide available to the student who seeks to acquaint himself with the methods and results of modern mathematical economics. [1925, p. 133]

Further 'Professor Bowley is the first to weld the scattered parts of mathematical economics into a consistent whole' [1925, p. 133]. Young singled out in particular Bowley's successful synthesis of the analysis combining utility functions, indifference curves, etc. with the analysis concerned with supply and demand of commodities with reference to money prices. In some respects Bowley's treatment was superior to Pareto's, particularly when it came to production functions.

A much longer review of and commentary on the *Groundwork* was published by the Swedish economist Knut Wicksell[16] in 1925: we have already made use of many of his comments. As one of his general comments we mention the following:

> Bowley belongs to those who love conciseness more than is desirable for the average reader; I am not at all sure that I, for my part, succeeded in gathering his meaning at every point, despite all my efforts to do so. In any case, it seems that the book would be better for those who are already engaged with these problems than for beginners. [Lindahl, 1958, p. 209]

While the *Groundwork* is often seen as an early contribution to mathematical economics, this view was not universally taken. For in his review of R.G.D. Allen's *Mathematical Economics* [1956], Theil wrote

> This is the first textbook in mathematical economics: it will prove, I think, a highly successful one. Strictly speaking, its subject matter is mathematical economics mixed with pure mathematics; which is perhaps the only way to write a book of this kind. [1958, p. 336]

Darnell, on the other hand, said of the *Groundwork* that 'its major role is to be seen as the first readily available source of the spirit of mathematical economics' [1982, p. 175][17].

Chapter 11

Mathematics

11.1 Introduction

In our first chapter we noted that Arthur Bowley's mathematical ability was apparent even in his time as a pupil at Christ's Hospital. We also noted his appointment as a teacher of mathematics at St John's School in Leatherhead, and his later position of lecturer in, and then professor of, mathematics at Reading University College. Further, in his early years at the LSE he lectured in both statistics and mathematics.

Bowley was joint tenth Wrangler in his year in the Mathematical Tripos at Cambridge. Had he been Senior Wrangler the course of his intellectual and academic life might well have been different, and a career in mathematics—no doubt a respectable one, though, as Allen [1968, p. 135] noted, 'As a mathematician Bowley was competent without being very original, and he became increasingly old-fashioned in his mathematical formulations'—might have been pursued to the detriment of economics and statistics. His views on mathematics were recorded in an unpublished note *In Praise of Mathematics*, quoted by his daughter Agatha:

> There is indeed a satisfaction in the development of a mathematical proof; with its rigid hypotheses, clear-cut,

> terse and logical argument, and far-reaching result. It is analogous to the artisan's pleasure in a fine piece of work. Again it is almost an aesthetic joy to discover some general theorem, which is found to contain very many earlier theorems as special cases.
> [A.H. Bowley, 1972, p. 30]

But his personal predilection for the practical was expressed as follows:

> It is not, however, that it is by the pleasure of the practitioner that the importance of mathematics is to be measured, but by its use to the engineer, the physicist, the chemist and others, whether to construct buildings, to examine invalids by X-rays, or to burst the atom.
> [A.H. Bowley, 1972, p. 30]

We have also mentioned at various places comments Bowley made from time to time on mathematics, and in this chapter we shall look at three of his works that are particularly concerned with that subject.

11.2 Kinetic energy

In 1908, while he was lecturing at both Reading College and the LSE, Bowley published a Note that is unique in the corpus of his works: one dealing with the change in kinetic energy caused by the mutual action of two particles.

The changes of momentum between two particles of masses m and m', when an action and a reaction occur, is

$$I = m(u_1 - u_2) = m'(v_2 - v_1),$$

where the u's and v's represent the velocities before and after the action. If $V_1 = u_1 - v_1$ and $V_2 = u_2 - v_2$ are the relative velocities before and after, then $I(m + m') = mm'(V_1 - V_2)$, and the loss of kinetic energy is

$$\frac{1}{2}\frac{mm'}{m + m'}\,(V_1^2 - V_2^2).$$

> Hence, in general, there is a loss of K.E. in all cases where the relative velocity is diminished, and a gain in cases (e.g. attraction) where the relative velocity is increased. [1908i, p. 327]

In the case of an impact, $V_2 = -eV_1$, where e is Newton's coefficient of restitution. The loss of kinetic energy then becomes

$$\frac{1}{2}\frac{mm'}{m + m'}\,V_1^2(1 - e^2),$$

which is positive since $0 < e < 1$.

It is uncertain how original this work was to Bowley. In a footnote Charles Jackson[1] writes that 'This method was certainly familiar at Cambridge in 1889, and was given by Mr. R.R. Webb to his pupils. It was, I believe, due to Professor Greenhill[2]' [1908i, p. 327].

11.3 Mathematics in statistics

In many of his statistical works Bowley commented, almost in passing as it were, on the mathematics needed in statistics[3]. In 1924, however, at the International Congress of Mathematicians in Toronto he read a paper [Bowley, 1928e] in which he gave considerable attention to the use of mathematics in economic, social and public statistics. He assumed further that general methods like classification, tabulation and graphic representation were known, and considered what further was needed for the interpretation of collected material.

To this end he noted that information may either be complete and cover the entire field, or partial, arising from sampling or the representative method. In the first of these cases, where methods not requiring probabilistic considerations are concerned, he considered time series (trends, fluctuations, the method of moving averages), the measurement of seasonal variations in monthly or weekly records over a number of years (e.g. statistics of unemployment), the examination of frequency groups (e.g. distribution by age in an occupation)[4], double tabulation and correlation, index numbers and standardisation (e.g. of the death-rate). In the second case the matter of error or precision of course arises.

In considering methods involving probability theory, Bowley restricted himself to sampling theory, saying that

> two conditions are necessary for pure sampling; first, that the universe to be examined should be exactly defined; second, that every person or thing in the universe shall have an equal chance of being included in the sample. [1928e, p. 913]

He noted something that is perhaps often glossed over in sampling surveys today, viz. 'that no alteration from the objects chosen should be allowed owing to difficulties of observation' [1928e, p. 914].

Distinguishing, like Yule[5], between 'variables' (e.g. measuring the heights of men—a variable characteristic) and 'attributes' (e.g. classifying people according to sex) Bowley considered (i) averages by samples (e.g. the difference between the average number of children per boiler-maker family in Scotland and the average number of children per marriage in the whole of the country, in order to investigate whether this occupation is unconnected with the number of children), (ii) the frequency of attributes by sampling (examination of proportions using the binomial and the Poisson approximation)[6], (iii) correlation by sampling (Bowley stressed the importance of the correct interpretation of r) and (iv) the representative method. Here goodness-of-fit tests were included.

Bowley concluded this Congress paper with the observation that this representative (or purposive) sampling scheme was at that time under investigation by the International Statistical Institute, though by the time the proceedings of the Congress were published (in 1928) Bowley's important report on sampling had already been presented [Bowley, 1926a].

11.4 *Pure Mathematics*

Bowley's book, *A General Course of Pure Mathematics from Indices to Solid Analytical Geometry*, was published in 1913. Described as 'the result of an attempt to bring within two covers a wide region of pure mathematics' [p. iii], the book covered topics in algebra, co-ordinate geometry, trigonometry and the differential and integral calculus. Each Section (the book is in Sections rather than Chapters) contains a number of examples for the reader. While the treatment was rigorous, the aim was to cover only those theorems and formulae of practical application or leading to others of direct practical application. Continuity and irrationals were accorded merely a heuristic treatment, while Conic Sections were summarily dismissed with the assertion that 'There seems no reason why the best years of a scholar's life should be devoted to the Conic Section' [1913a, p. v].

Section I is devoted to algebra. At the start, considering the extension of the meaning of expressions like $a \times b$ from positive integers to a wider field, Bowley makes the useful remark that 'A definition arising in this way from a generalisation of a law suggested by simple cases is known as a *mathematical convention*' [1913a, p. 1]. This section deals with indices, logarithms, inequalities (including the relation between the arithmetic (m), geometric (g) and harmonic (h) means, with $g^2 = hm$), progressions[7], ratio and proportion, permutations and combinations (including Pascal's Triangle) and the Binomial Theorem for positive integral index. In concluding this section Bowley notes the importance of permutations and combinations in both probability and chance, 'subjects of great importance in Statistics, but beyond the scope of this book' [1913a, p. 23].

Section II is concerned with geometry, beginning with a standard treatment of similar plane figures. Bowley starts off with an *axiom* that is perhaps unusual, viz. '*Numerical quantities are equal to one another, if it can be shown that they differ by less than the smallest quantity we can assign*' [1913a, p. 25]. The idea of *homothetic* (i.e. similar and similarly placed) figures is introduced.

In the section on trigonometry Bowley introduces the common trigonometrical ratios (with a short discussion of their origins in toxophily), angles (in degrees) and some common trigonometric identities. Inverse functions of the form $\sin^{-1} y$ are introduced (Bowley preferring this to the 'continental usage' $\arcsin y$). Radian measure is discussed by first defining π as the limit as $n \to \infty$ of $n \sin(180^\circ/n)$.

Section IV is devoted to various aspects of explicit functions $y = f(x)$. Here we find linear, quadratic and inverse ($x = g(y)$) functions and compound variation (with Boyle's Law[8] as an example). When it comes to the solution of equations Bowley examines the remainder theorem, the number of roots of $f(x) = 0$ (including conjugate roots), the relation between the roots and the coefficients of an equation of nth degree and Horner's method.

Section V introduces the concepts of limits and (infinite) series. Considering the series $\sum_1^n 1/2^k$ Bowley notes that the difference between this sum and 1 can be made less than ε by taking n sufficiently large[9], and in a footnote he points out that 'ε means *as small a quantity as can be definitely assigned*, by any mental or numerical process' [1913a, p. 98].

The phrase 'tends to infinity' is stated to be simply a convenient and conventional way of saying 'becomes greater than any finite quantity, however large, we like to choose' [1913a, p. 101]. This allows the usual sort of definition[10] (essentially given verbally rather than in symbols) of $\lim_{x \to x_1} f(x)$. Some specific examples are given; in the first of these Bowley shows that $\lim_{n \to \infty} r^n = 0$ where $r = 1 - d$ and $0 < d < 1$ and d is independent of n, and he stresses that it is essential that the usual simple requirement that $0 < r < 1$ be replaced by the statement that d does not tend to zero, citing $\lim_{n\to\infty}(1\mp(1/n))^n$ as an example.

Infinite series are then considered, the series $S_n = \sum_1^n u_i$ being defined as convergent if $\lim_{n\to\infty} S_n$ is zero or finite 'and unambiguous' [1913a, p. 106]. Tests for convergence are limited to the ratio test and various extensions thereof (there is also some examination of absolute convergence). The series chosen for illustrative purposes is $\sum x^k/k!$, firstly for $0 < x < 1$ and $1 - x$ finite, and then for any finite positive x.

The subsection on the multiplication of power series requires some consideration. Bowley supposes that $S_1 = \sum u_i$ and $S_2 = \sum v_i$ are both convergent 'infinite' series of positive terms. On defining $S = \sum w_i$, where $w_n = u_n v_1 + u_{n-1} v_2 + \cdots + u_1 v_n$ he shows that $S = S_1 \times S_2$, and states that this is still true even when some of the terms in the original series are negative. Notice that Bowley does not claim that S is convergent, though since he shows that $S = S_1 \times S_2$ this is perhaps tacit[11].

Vandermonde's Theorem in the form

$$\binom{m+n}{k} = \sum_{i=0}^{k} \binom{m}{i}\binom{n}{k-i}$$

is next proved. Passing from this form to one in which $\binom{p}{r}$ is replaced by $[p]_r/r!$, where $[p]_r = p(p-1)\dots(p-r+1)/r!$, Bowley shows that the result holds for all values of m and n, with $k > 0$.

This leads on naturally to the Binomial Theorem for arbitrary real index. Bowley does not care for the expression

$$(1+x)^m = \sum_{k=0}^{\infty} \big([m]_k/k!\big)\, x^k,$$

preferring to show that

$$(1+x)^m = \lim_{n\to\infty} \big(1 + mx + \cdots + ([m]_n/n!)\, x^n\big).$$

The discussion of the exponential series again shows Bowley's original approach. He defines e and $E(x)$ by

$$e = \lim_{n\to\infty} \bigl(1 + 1 + 1/2! + \cdots + 1/n!\bigr)$$
$$E(x) = \lim_{n\to\infty} \bigl(1 + x + x^2/2! + \cdots + x^n/n!\bigr),$$

the latter being already known to be convergent for all real x. Clearly $E(1) = e$. Using these two functions Bowley proves that[12]

$$E(x_1)\,E(x_2) = E(x_1 + x_2)$$

and that

$$\lim_{n\to\infty} \bigl(1 + x + x^2/2! + \cdots + x^n/n!\bigr) = e^x,$$

where x is commensurable[13].

The next topic is that of natural logarithms. Bowley notes that e^x has only been defined for x commensurable[14], and he therefore carefully discusses here the calculation of $\ln x$ when x is incommensurable (or, perhaps better, irrational, as he mentions in the Appendix). It is further noted that

$$\lim_{n\to\infty} \bigl(1 + x/n\bigr)^n = e^x$$

for x finite and positive or negative. As an illustration Bowley considers the use of this result in continuous growth (e.g. compound interest). Series expansions of various logarithmic functions (e.g. $\ln[(1 + x)/(1 - x)]$) are derived, and a foreshadowing of the work to be undertaken in the chapter on the differential and integral calculus is found in a section devoted to limits of the form

$$\lim_{h\to 0} \bigl[f(x + h) - f(x)\bigr]/h.$$

Section VI is devoted to plane geometry, dealing with geometric properties of lines that are the loci of some point P. The straight line is considered in considerable detail, and the equation of second degree, given initially in the form

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0,$$

is painstakingly examined (first with $h = 0$). Specific choice of the constants results in circles, ellipses, hyperbolas and parabolas, and the possibility of viewing them as conic sections is examined. The focus, directrix and eccentricity are defined and computed where appropriate. Things that were once common in first courses in this subject, like pole and polar, the semi-latus rectum and the director circle, are discussed, matters that are perhaps at best of historic interest to students of 'pure' mathematics today. The section is concluded with a fairly brief discussion of polar co-ordinates.

Section VII contains work more recognisable to the modern student, viz. differential and integral calculus. Bowley begins by noting that in both pure and applied mathematics it is very often necessary to consider 'quantities so small as to be less than any assigned finite quantity[15]' [1913a, p. 191].

Let $f(x)$ be a function 'which can be represented by a graph drawn by a pencil that does not break contact with the paper' [1913a, p. 191], a passage that may be interpreted as an easy way of saying that f is continuous. For such a function Bowley considers

$$\lim_{\delta x \to 0} \frac{f(x + \delta x) - f(x)}{(x + \delta x) - x},$$

which he names 'the derived function' or 'the differential coefficient' of $f(x)$ with respect to x (here δx is an increment in x). The derivatives of various standard forms are stated and some rules for differentiation (e.g. $D_x y = D_x u \, D_u y$, where $y = u(x)$) are proved. There is also some consideration of implicit functions[16].

Attention is then turned to integration, beginning with some standard results. Bowley notes that the simple forms he gives are obtained by recognising that differentiation of the result yields the integrand. He then writes

> If we start at the other end of the problem and try to find the integral function of the simplest derived functions, we have to devise methods *whose justification is their success* [1913a, p. 208]

(emphasis added). Integration by parts and definite integrals are treated, the latter being used for a procedure as complicated as finding areas and volumes. There is also some consideration of differential equations (e.g. simple harmonic motion) and partial differentiation, the latter being applied to finding maxima and minima of a function of two independent variables.

The most original part of the *General Course* is reached with the treatment of imaginary and complex quantities in Section VIII. Symbols like d in $d \times d = -6$, for example, are described as *imaginary* quantities, 'that is, quantities that are the subject of imagination or thought only, as opposed to *real* quantities whose application to physical measurements is direct' [1913a, p. 223].

An imaginary quantity is defined using an operator $\mathfrak{O}(\cdot)$ that, when acting on a yields a quantity $a_\imath$, called an *imaginary* quantity, such that a similar (sic) operator $\mathfrak{O}$, now operating on $a_\imath$, yields $-a$, that is,

$$\mathfrak{O}(\mathfrak{a}_\imath) = \mathfrak{O}\{\mathfrak{O}(\mathfrak{a})\} = -a,$$

the choice $a = 1$ yielding the answer -1. 'The operation $\mathfrak{O}$,' writes Bowley, 'is an hypothetical or imaginary operation' [1913a, p. 224].

The operations of addition, subtraction, multiplication and division on imaginary quantities are then defined (note that Bowley indicates that these operations, being generally defined in the context of real numbers, cannot be cavalierly applied to imaginary quantities). He gives rules such as $a_\imath \pm b_\imath = c_\imath$ where $c = a \pm b$ and $a, b, c \in \Re$, and definitions like that of the imaginary zero as $0_\imath = a_\imath - a_\imath$. An examination of the rules and definitions shows that

> $a_\imath$ can be taken in all the processes of addition, multiplication, subtraction, and division as if it was $a \times \imath$, where $\imath$ is used just as if it was a real quantity. [1913a, p. 226]

(Here the symbol $\imath$ is written for $1_\imath \equiv \mathfrak{O}(1)$.)

Attention is then turned to the connexion between the operator $\mathfrak{O}$ and geometry, it being shown that $\mathfrak{O}$ is similar to a rotation through a right angle. Bowley supposes that x and y denote the co-ordinates of a point P in two-space, measurement being made from the origin O. The vector OP then represents the *complex quantity* (x, y), or (r, θ) in polar co-ordinates (r being known as the modulus and θ the argument or amplitude), where x is real and $y_\imath$ is imaginary.

The algebra of complex quantities is then considered, the operators $+, -, \times, \div$ being defined in such a way that they coincide with these operators on reals when y is zero and on imaginary quantities when x is zero. The rules for the use of these operators are given in terms of both (x, y) and (r, θ); for instance the work on indices shows that, for $n, m \in \mathbb{N}$,

$$(r, \theta)^n \times (r, \theta)^m = (r^{n+m}, (n+m)\theta).$$

This result is then generalised to get de Moivre's Theorem[17], given by Bowley in the form

$$(r, \theta)^n = (r^n, n\theta + (2k\pi/q)),$$

where n is rational, q is the denominator of n if n is fractional and k is a non-negative integer. Specification then leads to consideration of the nth roots of unity.

Bowley's method of introducing $+, -, \times, \div$ for complex quantities now allows him to conclude that

> we may apply all the rules and processes of algebra up to the solution of equations, ratio, progressions, and indices, whatever the letters stand for, but not as yet to logarithms, limits, or series, since these words have not been defined in connexion with imaginary and complex quantities. [1913a, p. 233]

Conjugate complex quantities are introduced and this leads again to consideration of the solution of equations, it being shown for instance that every equation of nth degree has exactly n roots, different or coincident, real or imaginary.

Using the Binomial Theorem, which also holds for complex quantities, Bowley derives, among other formulae, series expansions for $\sin\theta$ and $\cos\theta$ in the '$\lim_{n\to\infty}$' rather than the '$\sum_1^\infty$' form. Attention is then turned to series. Here Bowley proves that $\sum a_k z^k$, a power series in z with constant coefficients, converges if $\sum a_k r^k$ converges, where r is the modulus of z. An examination of the product of two convergent (complex) series (the coefficients in each being real) leads to results like

$$E(z_1) \times E(z_2) = E(z_1 + z_2)$$

(where E has its previous meaning) for *all* values of the arguments, and Euler's expressions for the sine and cosine,

$$\sin\theta = (1/2\imath)(e^{\imath\theta} - e^{-\imath\theta}), \quad \cos\theta = (1/2)(e^{\imath\theta} + e^{-\imath\theta}),$$

are derived. The trigonometrical ratios of complex angles—including their periodicity, which follows from the observed periodicity of $E(z)$—are further explored, and the section is concluded with a short discussion of hyperbolic functions.

The final section is concerned with co-ordinate geometry in three dimensions. Bowley also discusses *conicoids*, that is, curves represented by the general equation of second degree

$$\begin{aligned} f(x,y,z) = ax^2 + by^2 + cz^2 + 2fyz + 2gzx \\ + 2hxy + 2ux + 2vy + 2wz + d = 0, \end{aligned}$$

and explores in detail the surface generated by such an equation.

This completes our discussion of the *General Course*[18]. Let us look at its reception by the mathematical community at large.

In 1913 an anonymous reviewer stated that 'the only adverse criticism that can be made against this excellent volume is that one is not clear as to the section of the public for which it is intended'

[Anon., 1913, p. 214]. The reviewer found that the book was neither for the mathematical specialist nor the engineering student 'and, apparently, in England these two classes alone are worth consideration!' [loc. cit.]. Bowley's proofs were found to be 'beautifully clear and rigorous'. While the treatment of limits was said to be careful—'the essential point that $|f(x)-l|$ becomes and *remains* less than ε being excellently brought out' [1913, p. 214]—the reviewer found the definition unusual, and declared that it was less satisfactory than that given by Hobson[19]. Bowley's discussion of infinite series was also seen to be rather suspicious, his rearrangement of a double series being carried out without any preliminary investigation of whether this was permissible.

A General Course of Pure Mathematics was somewhat ambivalently received by professional mathematicians. Edmund Whittaker gave it warm approval[20], but Godfrey Hardy[21] was less enthusiastic. While describing Bowley's purpose in writing the book as 'admirable' and finding his choice of topics 'excellent', Hardy has to say

> Unfortunately his knowledge of mathematics seems to be hardly sufficient for the very difficult task that he has undertaken, [1914, p. 395]

and suggests that 'a better acquaintance with good mathematical literature' was needed.

Hardy finds that the book has 'many and serious' defects, among which he mentions the intentional glozing over of the natures of continuity and irrationals. The more analytical parts of the book are also found to be flawed. The definition of a limit was 'both confusing and inaccurate', the discussion of the multiplication of series was wrong (which of course affected things like the exponential series), that of maxima and minima was 'very loose and inaccurate', the theory of complex numbers was 'neither illuminating nor correct', and the proof of the Fundamental Theorem of Algebra was fallacious.

In his twelve page response to a reply from Bowley on this review Hardy said that what Bowley had tried to do was best left to the professional mathematician. In a subsequent letter, however, he did admit that Bowley's proof of the multiplication theorem for infinite series was 'substantially correct' and that his criticism of this point would be retracted in a comment to the *Cambridge Review* (see Maunder [1972, pp. 21, 25] and Darnell [1981, p. 156]).

In 1915 Hardy published a note on the definition of a complex number in which, without mentioning Bowley by name, he refers to 'a recent book (a book too of some merit)' [Hardy, 1915, p. 48], by which *A General Course* is doubtlessly meant. Hardy notes that other authors had also introduced complex numbers via an operator (as Bowley had done) in the following way: consider an operator $O(\cdot)$ operating on real numbers, such that

$$O\{O(a)\} = -a, \tag{1}$$

and $\imath a$ is identified with $O(a)$, a procedure that is 'open to a multitude of objections'. Firstly, $O(\cdot)$ is supposed to operate on reals, but if $O(a)$ is not real, what meaning can then be attached to $O\{O(a)\}$? Recall however that Bowley did not say this, but merely wrote of 'an operator $\mathfrak{O}(\cdot)$ ... acting on a' and 'a similar operator', not mentioning reals at all. Hardy's more fatal second objection is that Equation (1) does not define any unique operation. He shows that if O operates on the complex number $(a, b) \equiv a + \imath b$ in such a way that

$$O(a, b) = (\lambda a + \mu b, \lambda' a + \mu' b),$$

(where λ, λ', μ and μ' are real), then one suitable answer is

$$O(a, b) = (a \sinh\theta - b\rho\cosh\theta, (a\cosh\theta)/\rho - b\sinh\theta).$$

If O operates on the (pure) real number a then, with $\theta = 0$ and $\rho = 1$, we have $O(a) = \imath a$.

In 1916 G.W. Palmer stated his support of the operator method, finding Hardy's analysis to be at fault since Bowley had applied O to real or *purely imaginary* numbers and not to a *complex* number.

Hardy of course was a mathematician *par excellence*, and his opinions on 'real' mathematics (a class that includes 'applied' as well as 'pure' mathematics) are expressed, both forthrightly and forcefully, in his 1940 book *A Mathematician's Apology*[22]. From today's perspective Hardy's assessment of Bowley's book may be seen as harsh and unjustified: we have found the book to be quite adequate in general.

Chapter 12

Miscellaneous

12.1 Introduction

The quantity of material published in any science today is such that there are few researchers, if any, who can claim to keep up with new results in more than a narrow field. Abstracting journals are of great use, and many scholars willingly and freely write short reviews for the benefit of the wider community. Of great importance are longer reviews of books and critical discussions of papers by other scholars, and Bowley contributed a number of such reviews (many were simply signed 'A.L.B.'). Useful too, and of general interest, are obituaries of those who have made noteworthy contributions to a discipline, and Bowley wrote a number of such memorials.

12.2 Obituaries and Reviews

Bowley's professional life spanned some sixty years from the last decade of the nineteenth century to the middle of the twentieth. In the course of that time a number of those who had been his colleagues and, in some cases, more intimate friends, died, and Bowley undertook the melancholy task of writing their obituaries. His printed comments not only shed light on his professional relationship with other economists and statisticians but also broaden our knowledge of the deceased.

On the 8th of April 1935 Edwin Cannan (*b.* 1861) died[1]. He and Bowley had been intimate friends for many years, and the latter contributed a very touching warm obituary to the pages of *The Economic Journal*. Cannan began lecturing at the London School of Economics when it started in 1895, giving public lectures on the History of Rating and sharing classes in Economics with William Hewins, the first Director of the School. He became Professor in 1907, and Emeritus in 1926.

Bowley's introduction to Cannan was in 1895, when the latter read a paper entitled 'Probability of a cessation of the growth of population of England and Wales during the next century' at an Ipswich meeting of the British Association, a paper Bowley described as 'very remarkable for its statistical foresight'. Citing *Who's Who* for 1905 Bowley records that Cannan's recreations, all taken seriously, were given as 'Cycling since 1888, history of English roads, municipal government, demographic statistics, and (formerly) correspondence with government departments' [Bowley, 1935e, pp. 388-9].

When it came to Cannan's academic work, Bowley wrote

> From the first to the last the object of his teaching and writing was to apply the criticism of common sense, couched in non-technical language, to economic theory, without respect of persons and without acceptance of any formulæ. [1935e, p. 389]

George Henry Wood (1874-1945) set a long and profitable collaboration in motion by introducing himself to Bowley at a meeting of the British Association in Bristol in 1898. Like Bowley, Wood had been working on wages and their representation by index-numbers, and the collaboration between the two investigators led to the publication of the joint detailed and important research on wages in the United Kingdom from 1899 to 1906. Subsequent papers on the cotton industry were by Wood on his own, and after completion of this study he moved to Yorkshire as Statistician to the Woollen industry[2]. Papers on the wool textile industry and cricket were subsequently published in the *Journal of the Royal Statistical Society.*

In that same year Bowley wrote a memoir for another collaborator and friend: Sir Hubert Llewellyn Smith. Born in 1864 and educated at Bristol Grammar School, Llewellyn Smith went on a mathematical scholarship to Corpus Christi College, Oxford, where he obtained a double first in mathematics. His interest, though, seemed to lie more in political economy than in mathematics, and like Bowley he won, in 1886, the Cobden Essay Prize. In the Civil Service examinations he came second and was offered a place in the War Office, but his connexion with the Society of Friends led to his declining this offer [Collet, 1945, p. 483]. The Government established a Labour Department in 1893, and Llewellyn Smith was appointed Labour Commissioner, charged not only with the gathering and publication of data but also with negotiations in labour disputes. Having retired from his official pursuits, in 1928 he undertook the direction of the *New Survey of London Life and Labour*, his earlier work with Charles Booth making him eminently qualified for this task.

Bowley's last publication was an obituary for Henri Methorst (1868-1955), and it is the only one for a foreigner [Bowley, 1956]. Methorst was born in Amersfoot in Holland, and on completing his doctorate at the University of Utrecht he joined the Dutch official statistical service, being Director-General of the Central Bureau of Statistics at the time of his retirement in 1939. He played a prominent rôle in the International Institute of Statistics, and it was largely due to his influence that the Permanent Office of that society was established in The Hague. Methorst filled the position of Secretary General of the International Institute and Director of the Permanent Office until he resigned in 1947 (then almost 80). His research interests were mainly in the fields of population and health.

From 1894 to 1955 Bowley contributed some 120 reviews to various journals; most appeared in *The Economic Journal*, followed by

Economica, *Journal of the Royal Statistical Society* and *The American Economic Review*. There were a few years when no reviews appeared, but in most years there were two or three, and in 1907 there were nine! While most reviews were of works published in English, Bowley showed his linguistic skills by reviewing a number of French, German and Italian books.

The reviews are listed in our Bibliography: here we shall merely mention some points from a few of the reviews to give an idea of Bowley's opinions. All the reviews exhibit the usual thoroughness and care with which Bowley approached his work.

Bowley's first published review was of a book by Henry de Beltgens Gibbins, *British Colonies and Commerce*. While commending the appearance of an addition to 'the wretched insufficiency of ordinary historical text-books' [Bowley, 1894, p. 75], he found that the author had tried to cover too wide a range of material and that while the text was 'crammed with hard facts', the underlying principles were 'very scantily dealt with' [1894, p. 76].

Bowley was critical of various official reports published in the United States. For example, in his review of a book of essays on the American XIth Census—'that extraordinary medley of figures' [1899c, p. 428]—Bowley bemoaned the fact that in the United States, as in England, 'there is no permanent Census office, and there is an almost complete lack of continuity... in the Census organisation' (loc. cit.). He did however say that the book was 'of the very greatest importance', and that it would be of great interest to English readers[3].

In 1923, in a review of Irving Fisher's book *The Making of Index Numbers* Bowley wrote 'Professor Fisher's treatment would perhaps be less arbitrary if he had spent more thought on the definition and purpose of an index-number and on the principles of weighting' [1923c, p. 90]. Bowley clearly did not care for Fisher's interpretation of index-numbers, finding it to be too vague to be of use in scientific analysis. However he did conclude that 'There is much that is interesting and useful, and something that is novel and useful, in the book' [1923c, p. 94][4].

Harshest of all Bowley's reviews though was that of William Hurrell Mallock's *Social Reform, as related to Realities and Delusions*[5]. Bowley writes of 'the extremely confusing arrangement of the book, which contains no table, no cross references, an erroneous index, much repetition, and interminable verbosity' and further 'The way in which Mr. Mallock garbles his quotations is nearly as serious as his abuse of statistics' [1914b, p. 438].

Some reviews were glowing. Thus Harald Westergaard's *Die Lehre von der Mortalität und Morbilität* was described as a 'very valuable and exhaustive treatise' containing 'many wise remarks as to the general methods of statistics' [Bowley, 1902c, p. 92]. Similarly, in his review of a paper by Edgeworth on the law of error Bowley not only praised the work for its theoretical importance, but also noted that, in his opinion, 'its importance for practical statistical work will in the end prove yet greater' [Bowley, 1905c, p. 750].

George Udny Yule's *An Introduction to the Theory of Statistics*[6], while doubtlessly being of inestimable values to statisticians, is seen to be less appealing to economists. Noting that the book is meant to be elementary, Bowley writes 'We regret, however, that the author has, in spite of his elaborate mechanism of analysis, left the test of significance of variations so vague' [Bowley, 1911b, p. 264].

Bowley's last review, published in 1955, was of two volumes on Consumers' Expenditure, one for 1910-1919 and the other for 1920-1938. Bowley noted the 'colossal' amount of research that had gone into these and similar reports. It was, he suggested, 'perhaps safe to say that there is nothing more to be known about the *aggregate* private expenditure in the United Kingdom on most consumers' *goods*' [1955b, p. 114].

12.3 Demography

In the Supplement to the *35th annual Report of the Registrar-General* of 1875 William Farr wrote[7]

> How the people of England live is one of the most important questions that can be considered; and how—of what causes, and at what ages—they die is scarcely of less account; for it is the complement of the primary question teaching men how to live a longer, healthier and happier life.

It was with such questions in mind that Bowley delivered his two Chadwick Lectures on ‘The growth of the suburban population in England’ in 1921 (see Chapter 1 for further details).

At the start of the first of his first lecture, given on the 24th June, Bowley noted that[8]

> The increase of suburban population in the last 70 years has raised many questions of health and sanitation, and is thus within the province of the Chadwick Trust.

He then discussed the definition of a Suburb—‘a district in which the density of population is between 1 per acre and 29-30 per acre and from and to which a considerable number travel daily to earn their living’—and studied the rates of growth in the environment of London from 1881 to 1911.

Three tables of statistics and four cartograms were handed out at this lecture, the area examined being composed of five regions: (1) the then present county of London, (2) eight contiguous boroughs, (3) the remainder up to a ten mile radius from Charing Cross, (4) a 10 to 12 mile ring and (5) a 12 to 15 mile ring. Using his definition of a suburb Bowley noted that the suburban population of London was 27% of the total population in 1851, 27% in 1881, 28% in 1891, 32% in 1901 and $26\frac{1}{2}\%$ in 1911. The decrease in 1911 was attributed to the fact that many of the former suburbs in the 5 to 10 mile ring had essentially become urban in character.

In his second lecture (1st July) Bowley considered tests of suburbanity other than population density (his main concern in the first lecture). He now looked at things like the relationship between gardens and semi-rural amenities in proportion to administrative area.

He noted further that health depends on things like effective number of people in a house, ventilation, the availability in cities of parks etc. He concluded that although health cannot be measured, death rates can, and noted further that attention should be paid to the relationship between number of persons per room and death rates—though one should not conclude that overcrowding causes deaths. Bowley's formula for the logarithm of the death rate per 1,000 of persons per square mile, was 3.05× the logarithm of the population. This differed slightly from the formulae given in 1915, 1919 and 1920 by John Brownlee and based on earlier research by William Farr, to whose work we have already referred in Chapter I. Farr's Law states that the death-rate R and the population density (say the number of persons per square mile) D are related by an expression of the form $R = cD^m$, where c and m are constants to be determined. The most recent data that Brownlee could cite in 1920 gave $R = 9.90 \times D^{0.1023}$.

Once again the conclusion was somewhat negative: the development of suburban populations was too complicated and required such a specialised local knowledge that general conclusions were impossible.

In 1923 Bowley read a paper, based on these two lectures and abundantly illustrated with tables and graphs[9], before the Royal Statistical Society on formulae connecting 'death-rates with measurements of the local aggregation of population' [1923a, p. 516]. His field of investigation was the urban districts of England and Wales, with some merging of districts in certain cases.

Three measurements were taken for testing the effect of population density (in a crude sense and as measured by number of persons per square mile) on health: (1) Persons per room (i.e. 'the population enumerated in private houses in an urban district divided by an aggregate number of rooms in those houses' [1923a, p. 517]), (2) "Crowding" (a house is 'crowded' if there is more than one person per room. One's concern is with the proportion of persons in this condition to all persons) and (3) "Overcrowding" ('the proportion of persons in houses where there are more than two people per room).

Bowley notes however that 'These terms are only used in default of better' [1923a, p. 518]: it was not to be assumed that an adequate test of overcrowding was provided simply by counting rooms.

Because density and population both vary over such large ranges Bowley decided, for graphic and tabular work, to use logarithms rather than actual numbers.

> The object of the analysis was as much to find empirical formulae connecting the death-rates with the five measurements chosen, as to calculate correlation coefficients, and for this purpose the choice of functions only needs justification by results. [Bowley, 1923a, p. 519]

Denoting the variables (the logarithms of population, crowding, etc.) by $x_1, x_2, \ldots$, the quantity z of interest (not measured from its average) is given by

$$z = a_1x_1 + a_2x_2 + \cdots + b.$$

Letting d denote the difference between the observed value of z and the calculated value, and by Δ the mean square divergence for n observations, one has

$$n\Delta^2 = \sum_1^n d^2 = \sum(a_1x_1 + a_2x_2 + \cdots + b - z)^2.$$

The constants $a_1, a_2, \ldots, b$ are then determined by minimising Δ.

Considering only the cases $n = 1$ and $n = 2$ Bowley shows that the expression for z may be written

$$z = a_1x_1 + \overline{z}\,(\pm\Delta) \;\text{ and }\; z = a_1x_1 + a_2x_2 + \overline{z}\,(\pm\Delta),$$

where a_1 is given respectively by

$$a_1 = r_{zx_1}\frac{\sigma_z}{\sigma_{x_1}} \;\text{ and }\; a_1 = (r_{zx_1} - r_{zx_2}r_{x_1x_2})/(1 - r^2_{x_1x_2})\,\frac{\sigma_z}{\sigma_{x_1}},$$

(and a_2 is similar). As usual the r's denote correlation coefficients and the σ's standard deviations.

Bowley adopts the sound statistical procedure of next examining the data carefully rather than rushing ahead to calculate correlation coefficients and regression equations, and certain regional characteristics are observed. Consideration of the regions as units clearly shows that 'density is not the main determinant of death-rates' [Bowley, 1923a, p. 521]: in fact both death-rates and infant mortality rates show high correlation coefficients with the three measurements of crowding (persons per room (R), per cent. crowded (C) and per cent. overcrowded(V)). Indeed in almost every region the correlation between both these rates with $\log D$ (density), $\log P$ (population), R or V is less than with C.

The regression equation is given as usual by an expression of the form $z = \overline{z} + r(x - \overline{x})\,\sigma_z/\sigma_x$, and in most regions this is found to be satisfactory. However in the London region the regression seems to be parabolic rather than rectilinear. Bowley notes that in 1919 John Brownlee had given the specific equation (found to give good results for the period 1891-1900)

$$\text{Death-rate} = 10.82\,(\text{density per square mile})^{0.10179}. \tag{1}$$

In Bowley's notation this becomes

$$\log S = a \log D + b,$$

where S denotes the death-rate. For the 1,095 urban districts in Bowley's study taken as one group it is found that

$$S = 9.4(640D)^{0.36},$$

which in fact gives no improvement on the rectilinear equation

$$S = 1.14 \log D + 12.3.$$

The suggested parabolic graph turns out to be

$$S = 12.39 + 0.28 \log D + 0.65(\log D)^2,$$

an equation that Bowley finds to give a better fit than Farr's (he finds this latter expression to be 'incapable of representing the higher death-rates observed where the density is specially low' [Bowley, 1923a, p. 528]).

Observing that density and crowding are both correlated with death-rates Bowley also finds the pertinent partial correlations (e.g. between R and S for given $\log D$).

> In the case of London the new formulae show no improvement on the old, but the interesting result is found that for districts with the same number of persons per room there is no significant correlation with density at all. [Bowley, 1923a, p. 531]

The following conclusions are reached:

> This paper is confined to a purely statistical study of the relative closeness of relationship between death-rates and various measurements of crowding, and the resulting lessons are left for the student of these subjects to deduce. Housing conditions seem to be more closely related to death-rates than is density by its ordinary measurement; but before the war (the period to which the figures relate) bad housing and high death-rates may have been both manifestations of poverty, so that the improvement of housing without the raising of incomes would not necessarily be effective. A further analysis based on the prevalence of poverty would be necessary to carry the investigation more deeply into ultimate causes. [p. 535]

In 1924 Bowley published a short paper on the connexion between the birth-rate and the population growth in Great Britain, investigating, when neither emigration nor immigration was assumed,

> what birth-rate is necessary to prevent a decrease of the population, and what would be the ultimate age-distribution

> in a population in which the number of births was constant and the death-rates stationary.
> [Bowley, 1924a, p. 188]

The age-distribution assumed was that shown in the Census of 1921, the death-rates those in England and Wales in 1911-12 and the number of births the average over Great Britain in 1921-23.

A number of variable factors affected the age-distribution in 1921: for example, those born before 1876 had survived from a period when neither birth-nor death-rate had diminished, emigration had affected those over 18, and many men had died during the War. 'The composition of the population in 1921 was therefore abnormal ... we have the phenomenon of a considerable natural increase combined with a diminishing number of births' [Bowley, 1924a, pp. 188, 189]. Figures are given estimating the population in 1931, 1941, 1951, 1971, 1991 and 2011. Under the hypothesis of a stationary number of births Bowley shows that

> The proportion of children under 15 falls rapidly to 1931 and then slowly to 1971 ... The excess of women over men, due to the war losses and emigration, is gradually reduced. ... The total of men and women aged 15 to 65 is is from 66 to 68 per cent. of the population throughout. The evident movement is the replacement of the young by the old. The active members of the population will be supporting the survivors of a former generation in place of a rising generation. [Bowley, 1924a, p. 190]

An examination of France showed a similar situation.

Bowley concludes by noting that if the then current rates of births, deaths and emigration were to continue, 'the population of Great Britain would increase to 45 or 46 millions about 1941 and then diminish' [1924a, p. 192].

In his discussion of methods used in the 1920s for forecasting population size de Gans contrasts the logistic population growth model with the demographic forecasting methodology (later called the cohort-component projection model). He records that Bowley was at the forefront of this latter approach after World War I, and notes that, in his [1924a], Bowley

> wanted to work out the future age distribution and size of the population under the clearly defined hypotheses of a constant number of births and of unchanged death rates. [de Gans, 2002, p. 93]

Bowley, of course, nowhere stated that such constancy was expected; he merely found it of interest to see what the birth-rate should be to obviate a population decrease.

In the discussion to Thomas Stevenson's [1925] Bowley began by saying that he 'regret[ted] that so much prominence has been given to the logistic equation' [Stevenson, 1925, p. 76][10]. He took population data for England and Wales, the United States and France and showed that equations of widely varying forms could be used to fit the data: for example,

$$y = 19.09 + 24.56t + 0.112t^2$$

$$y = 107.85\left[1 - 2\int_0^t \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\,dt\right]$$

$$y = 9.730 + (1 + e^{-t})$$

The ordinary quadratic parabola (fitted by least squares) was found, in the case of England and France, to give results almost exactly the same as the logistic equation (one form of which is $1/y = \alpha + \beta\gamma^x$). The cubic parabola was found to behave similarly in the case of the United States. The second of the above equations was found to fit English data admirably. In summary, Bowley said

> I agree that the logistic equation is well adapted to represent rather roughly the recorded changes of population in selected countries; I do not admit that any further use for it has been justified. [Stevenson, 1925, p. 80]

12.4 The Economic Service

On the 5th July 1921 a meeting was held at the London School of Economics to discuss the formation of a Service on the lines of the Harvard Economic Service. Those present were William Beveridge, Hubert Henderson, H.J. Spratt, J.M. Keynes, Charles Tennyson and Arthur Bowley. Keynes and Henderson represented Cambridge, Bowley and Beveridge the LSE, Tennyson the Federation of British Industries and Spratt the Central Council of Economic Information. The purpose of the Service was to gather and study material relating to economic conditions not only in the United Kingdom, but also in Germany, France, Belgium and Italy. The Harvard Economic Service generously underwrote the first year's expenses of the new Service with a grant of $5,000: thereafter subscriptions and funding from the Royal Economic Society covered expenses.

While there was no shortage of relevant material in the United Kingdom, the data were in general poorly presented, the various government departments issuing separate and unco-ordinated reports: there was no Central Statistical Office. The Statistical Department of the LSE was to obtain the figures released monthly by the various departments, to prepare comparable series right back to 1913, to take note of changes in content or definition, and to present the data to the Executive Committee before its publication on the 23rd of each month (thus data for June, say, would be published in the Bulletin of 23rd July).

Initially Bowley was editor of the Service's Bulletin, though this function was taken over by an editorial committee in 1927. Bowley served on the Executive Committee from 1923 to 1951, and on the Editorial Committee from 1927 to 1946. Each month the Bulletin gave figures for finance, prices and wages, trade and output, transport, employment and foreign exchanges. Some of the data were given as index numbers: initially with 1913 as base year, which was switched to 1924 in 1928.

The Service enjoyed a close connexion with the Royal Economic

Society (largely because of Keynes), and its office moved to Cambridge in October 1939 and back to London in October 1945. The financial position became worse, and in the mid-1960s publication was taken over by *The Times* of London.

12.5 The Dock Labour Inquiry

In 1920 an Inquiry was held into the wages and conditions of employment of dock labourers. Under the chairmanship of Baron Shaw of Dunfermline (a Scottish lawyer, Lord of Appeal and a Privy Councillor) the Court examined witnesses (one of whom was Arthur Bowley) with the aim not of reaching a decision that would be binding on the parties concerned, but rather of informing the 'Higher Court of Public Opinion'. Ten proposals were expected to be considered by the Court (e.g. the establishment of minimum wages, overtime), but in fact only the proposal that the minimum wage should be 16*s*. per day or 88*s*. for $5\frac{1}{2}$ days of 8 hours was examined.

The dockers were represented by the trade unionist and politician Ernest Bevin[11] (1881-1951), at whose urging an Inquiry rather than a Strike had been agreed to by the Transport and General Workers' Union. Measures of the cost of living, the latter being defined as

> the expense that a family would be put to in providing itself with accommodation, sustenance and all the other elements of common life [Bowley, 1920c, p. 273]

were variously estimated: from Sir Leo Money's £5 3*s*. a week, to Bevin's £6 and Sir Lynden Macassey's (representing the employers) £3 17*s*. for London.

Bevin viewed Bowley's proposed optimal food budget for a family of five of about 40*s* per week with some measure of scepticism. He rather snidely asked Bowley whether he lived on such a budget— 'a strict though not parsimonious economy[12]'— and was no doubt considerably deflated when Bowley, pointing out that his family was slightly larger than that under consideration, said that over the last three weeks his expenditure on food alone was 47s, additional garden produce would bring it up to 52s, reduce the size of the family; 46s, the extra 6s being occasioned by a larger amount of milk. His family had not used butter for the previous three months[13].

Bowley went on to say that his observations had led him to conclude that the working-class could be adequately fed on the kind of food he was advocating, and further that he did not think it a serious hardship that two vegetables should not be served every day of the week. This was common in middle-class families, which he defined as the small professional or clerical families, having £400 to £500 a year (see Laybourn [1999, pp. 99-100]). Patricia Strauss [1970, p. 214] writes that Bevin not only demolished Bowley's figures 'with bull-like ferocity' (he presented ten plates with minute portions of food, provided on Bowley's budget, to the Court), but also spoke for eleven hours! He suggested that the middle-class was better educated in nutritional matters than the working-class, to which Bowley somewhat ambivalently replied 'The education of the middle class woman does not teach her to cook[14]' [Maunder, 1972, p. 16].

As a result of the Inquiry the Court proposed a minimum wage for the dockers at the greater ports of 16s. a day with dockers at the smaller ports getting 15s. (no arguments for the choice of these numbers was given in the Report of the Court). Employers and workers on the National Joint Council for Dock Labour subsequently agreed to a reduction[15], and by July 1923 the wages were 10s. and 9s, resulting in an unofficial strike in London and eleven other ports.

12.6 International Housing Statistics

In the Bowley Archives in the LSE there is a document (Coll. Misc. 0570) by Dorothy S. Thomas entitled 'A Summary and Comparison of Housing Statistics of Certain Countries as brought forward by members of Dr. Bowley's Seminar, 1923-4'. This paper is an introduction to a collection of thirteen other papers (none of them by

Bowley himself) reviewing housing statistics in England and Wales, Scotland, Ireland, South Africa, the United States of America, France, Belgium, Berlin and Cologne, Essen Hamburg and Königshütte, Denmark and Sweden.

Thomas points out the inadequacy of available housing statistics, and the lack of standardisation that makes international comparison almost impossible. For a complete picture of the situation in even one country one would need to know things like (a) the total population of the country, (b) the total number of families and (c) the number of occupied and unoccupied houses. Factors such as overcrowding, rent and rates would need consideration, as well as further classification into urban and rural regions. For international comparison specific definitions would be required: e.g. What is a house? Is a scullery a room? What does overcrowding mean?

Thomas notes that some comparative statistics for England and Wales are available from census returns, but Scotland had adopted different definitions of things like a 'room' or a 'house', and Ireland too had her own definitions. Statistics of the Colonies were meagre, and those of the United States 'are generally inferior to those of Great Britain'. A similar, albeit in some cases brief, discussion of statistics from Europe is given.

The somewhat depressing conclusion is reached that clear-cut comparisons between the countries considered by the members of the seminar cannot be drawn. Further, there is a wide range of variability within any one country (affected by factors like industrial and agricultural development, distribution of wealth and geographical diversification).

Notes

Chapter 1

1. See the preface to Barham [1910]. Dr Peter Pangloss was a character in George Colman the younger's 1797 play *The Heir at Law*.
2. Totteridge is part of the north London Borough of Barnet, some nine miles north-west of Charing Cross.
3. This son, born on the 27th April 1857, was baptised James Lyon on the 23rd May at St Barnabas's Church.
   In 1995 Starr, in a paper on Aleister Crowley's connexions with freemasonry, notes that when Crowley petitioned Anglo-Saxon Lodge No. 343 in 1904, the Reverend James Lyon Bowley signed the petition as the lodge's secretary. According to Crowley Bowley was the chaplain to the British embassy in Paris. Further, initiated in the Apollo University Lodge No. 357 in Oxford in 1889, James was Provincial Grand Organist of the Oxfordshire Provincial Grand Lodge in 1892 but resigned in 1899. He later became a member of the Anglo-Saxon Lodge in Paris. (Aleister, *b.* Edward Alexander, Crowley (1875-1947) was a magus or occultist who is charged with having established satanic temples in London and Italy.)

   The British Census of 1881 shows that at that time Maria (aged 49) had her sons Robert T. (16), Francis B. (13) with a servant Alice Elizabeth Sidney (19) in her household at 11 Freemantle Square in Bristol (Arthur was enumerated at Christ's Hospital). In the 1891 census Maria and Robert were recorded as living on their own means, while Ellen (24) was described as a governess, Arthur a student of mathematics and Francis a solicitor. (Francis Buhner Lyon, to give him his full Christian names, started practising in Hong Kong in

1893. He shared his family's general concern with social matters by publishing a series of articles in a Hong Kong newspaper in 1911 on the British Children's Act of 1908.) The family was now lodging at (or perhaps merely visiting?) 2 Exeter Road, Dawlish, in Devon, the home of the 75-year-old William Libby, pensioner.

4. We are indebted to the present vicar, Canon Malcolm Widdecombe, for providing us with the details of this memorial.
5. The ragged schools were nineteenth century charity schools in the United Kingdom that provided not only education but also, in many cases, food, clothing and accommodation for destitute children.
6. In his book *The Nature and Purpose of the Measurement of Social Phenomena* of 1915 Arthur Bowley describes this phrase as 'middle-class': H.W.Fowler, in *A Dictionary of Modern English Usage*, terms it a 'genteelism'.
7. In Victorian times Christ's School formed one boundary of a neighbourhood of old and run-down houses, known as Little Britain, and thus named because of its having once been the residence of the dukes of Britanny. See 'Little Britain' in Washington Irving's *The Sketch-book of Geoffrey Crayon, Bart*, first published in seven paperbound numbers by C.S. van Winkle, New York, 1819-1820.
8. By the word *hospital* at that time was meant a charitable institution to provide for and maintain the infirm, the needy, the destitute and the aged.
9. Newgate was a gate in the wall of the old city of London in Roman times. A prison was built on the site in the twelfth century and remained in use for felons and debtors for more than 700 years. The Central Criminal Court (the Old Bailey) now stands on the site.
10. Stow [1912, pp. 68, 286] records that, although the *School* was founded in 1553, children, almost four hundred in number, were first taken into Christ's *Hospital* in November 1552.
11. Leigh Hunt (1784-1859) records in his autobiography that the small-clothes, or knee-breeches, were of Russia duck in his time. *Duck* is a strong linen fabric particularly used in earlier times for sails and sailors' outer garments.
12. The river Lea (or 'Lee') rises in Luton, Bedfordshire, and runs for a distance of some eighty-three kilometres to the Thames, through Hertfordshire, Essex and Greater London.

13. Porter was apparently seen in the nineteenth century as a tonic for ill-health; for in *The Lord of Thoulouse* in Barham's *The Ingoldsby Legends* (first published collectively in 1840) we read

    In vain does the family Doctor exhort her
    To take with her chop one poor half-pint of porter.
    [Barham, 1910, p. 355]

14. Hans Holbein the Younger (c. 1497-1543) was a German artist who spent some years in England. He painted many portraits of the court of Henry VIII. The Italian painter Antonio Verrio (1639-1707) was employed in England in decorating Windsor Castle, and he worked under both Stuart and Hanovarian monarchs.
15. William Farr (1807-1883), the father of British vital statistics [Fitz-Patrick, 1956], was a dedicated medical epidemiologist and chief statistician to the General Register Office in England for more than four decades.
16. Benjamin Seebohm Rowntree (1871-1954) was a sociologist and businessman. Being a Quaker he was not able to study at either Oxford or Cambridge, and so he spent some time studying chemistry at Manchester University, though he did not take a degree there. He was, however, awarded an honorary LL.D. by that institution in 1942. His deep concern for the welfare of the poor had an effect on the formulation of Liberal Party policies in the early 1900s.
17. Comparable figures for other British cities, also from the 1891 census (with population parenthesised), are London (4,211,056) 19%, Glasgow (564,981) 59.0%, Liverpool (517,951) 10%, Manchester (505,343) 8%, Birmingham (429,171) 14.0%, Bradford (216,361) 20.0%, Nottingham (211,984) 3.0%, Newcastle-on-Tyne (186,345) 35.0%, Plymouth (84,179) 26.0%, York (72,880) 6.4% and St Helens (71,288) 15.0%. Note that Bradford was roughly the same size as Bristol.
18. These eponymous lectures on the growth of suburban population in England were given across the road from the London School of Economics's premises (at 9 John Street, Adelphi) in the Lecture Hall of the Royal Society of Arts, on the 24th June and 1st July 1921 at 8 p.m. They were established by the Chadwick Trust, which was set up in 1895 in terms of the will of the social reformer and civil

servant Sir Edwin Chadwick (1800-1890), to promote the study of public health.

19. The dates of formation of these three societies are as follows: Manchester, 3rd September 1833; London, 15th March 1834; and Glasgow 1836 (FitzPatrick [1957] and Willcox [1934]). The Statistical Society of London (the precursor of the Royal Statistical Society, which was formed in 1887) was established for 'the collection and classification of all facts illustrative of the present condition and prospects of society, especially as it exists in the British Dominions'.

    Cullen [1975] notes that Manchester was the most important and best-known of the provincial statistical societies. Glasgow had in fact two statistical societies: the Statistical Society of Glasgow, founded on the 22nd of February 1836, and the Glasgow and Clydesdale Statistical Society founded in April of the same year. For a history of these and other statistical societies see Cullen [1975].

20. Samuel Pepys (1633-1703), Member of Parliament and Secretary to the Admiralty, was President of the Royal Society of London from 1684 to 1686.

21. That is, the Trustees, charged with the administration of the foundation.

22. Moore [2005] comments on the remarkable strain placed upon Mathematical Tripos students at Cambridge, particularly on those who aspired to high placement as Wranglers.

23. Bournemouth, in Dorset, England, was a favourite watering-place and regarded as eminently suitable for recuperation after an illness. From a population of a few hundred in the mid-nineteenth century, it grew rapidly with the advent of the railway in the 1870s.

24. The term 'wrangler' dates from the early days of Cambridge University, when the degree examination took the form of a syllogistic disputation, during which the parties to the dispute (the candidate, the examiner, and the Dean as buffer) sat on three-legged stools—hence the term 'Tripos'. In later times essentially everyone who gained a first class in the Mathematical Tripos was designated a Wrangler. See Roth [1971].

25. The trade slump in Britain in the late 1870s, following the explosion in trade union activity in the '60s, led to a decline in Socialism in

that country. However, this trend was reversed under the influence of Karl Marx (1818-1883), with the formation of the Social Democratic Federation and the Socialist League. The trade union movement grew aggressively particularly among semi-skilled workers, leading to the great dock strike of 1889.

26. Lionel Charles Robbins (1898-1984), appointed to the Chair in Political Economy at the London School of Economics in 1929, resigned to take up the chairmanship of the *Financial Times* in 1961. The *Robbins Report*, commissioned by the British Government in the 1960s, led to the considerable expansion of British universities.
27. Fabian Socialists followed the Roman general Quintus Fabius Maximus Verrucosus Cunctator (less impressively, the Warty Delayer) (c.280-203 B.C.), believing rather in a gradual 'wearing down' of the enemy than in an 'all guns blazing' approach (i.e. an evolutionary rather than a revolutionary socialism). See, for example, Michalos & Poff [2002, p. xiii].
28. A copy may be found in the Bowley papers in the Archives of the London School of Economics.
29. A similar opinion of mathematicians was expressed by Bowley in the discussion on a paper by John Hilton, director of statistics in the Ministry of Labour in England in 1924: 'Mathematicians being moderately cheap, I would suggest that the employment of more of them in Government offices might be a considerable saving to the nation' [Hilton, 1924, p. 565].
30. Edward Bellamy (1850-1898) was the great-grandson of Joseph Bellamy (1719-1790), who published *The Millenium, or the Thousand Years of Prosperity* with Jonathan Edwards in 1794. Joseph's vision was Christian Socialism. (There was no payment of *wages* in Edward Bellamy's scheme of things.)
31. Alfred Marshall (1842-1924) had himself a connexion with Bristol. Immediately after the release of his Cambridge Mathematical Tripos results (Second Wrangler) in 1865 Marshall was appointed temporary mathematics master at Clifton College, Bristol. In the autumn of that year he took up a fellowship at St John's College, Cambridge, and in 1868 he became lecturer in Moral Sciences. One of his female

students, Mary Paley (1850-1944), became his wife in 1877. The marriage of course meaning the end of his college fellowship, Marshall became principal of the recently established Bristol University College, an institution that became Bristol University in 1909. Here Mary also lectured, and when Alfred returned to Cambridge in 1884 as Professor of Political Economy, she became Lecturer in Economics at Newnham Hall.

32. In honour of Richard Cobden (1804-1865), a leading figure in the free-trade movement, the Cobden Club established in 1876 a triennial prize of £60 for an essay on a given topic in economics.

33. Adam Smith (1723(?)-1790) was a distinguished moral philosopher and influential political economist. His *An Inquiry into the Nature and Causes of the Wealth of Nations* (1776) is still regarded as a classic.

    Robert Smith (1689-1768) was successor to his cousin Roger Cotes (1682-1716), a well-known mathematician, in the Plumian Professorship at Cambridge and Master of Trinity College. On his death he left a bequest for annual awards to the two students who showed the most progress in mathematics and natural philosophy. See Barrow-Green [1999].

34. The river Mole rises near Gatwick Airport in West Sussex and flows towards Leatherhead. At Molesey it meets the Ember and the conjoined waters run to the Thames. It has been seriously suggested that Jane Austen modelled the town of Highbury in her novel *Emma* on Leatherhead.

35. St John's School has a mathematics prize named for Rutty.

36. Agatha Bowley states elsewhere, however, that Julia took up a 'lectureship in Art and Craft' [A.H. Bowley, 1972, p. vii].

37. Bowley was not unique in this two-fold occupation. Dahrendorf [1995, p. 24] notes that all the lecturers when the School was founded also held other positions elsewhere.

38. On leaving the University of Oxford with a degree in chemistry Gosset (1876-1937) joined a firm of brewers in Dublin, Ireland. His work required an investigation of the relationship between the raw materials used in the production of the beer and the end product, and

Gosset consulted Karl Pearson for advice on the appropriate statistics required. In his article 'The probable error of a mean', published under the pseudonym 'Student' in *Biometrika* in 1908, Gosset found the sampling distributions for the sample variance and standard deviation for samples of independent and identically distributed observations from a Normal distribution (see Zabell [2008]).

39. Agatha [1972, p. 85] has a copy of a letter written by Julia to *The Herts Advertiser* in January 1935, in which a woman's view of the unsatisfactory condition of some cottages in the district is exposed.

40. One can perhaps get a glimpse of Bowley's opinion from something he wrote in his review [1908h] of some reports by the Special Committee on Unskilled Labour. He noted that there were (1) men who were permanently unemployable, (2) men in need of training before becoming employable, (3) casual labourers who were permanently averse to regular work and (4) men unemployed by seasonal fluctuation. He then wrote

    > the only way to help men who belong to none of these four classes is not by relief works [as provided by the authorities] at all, but by decreasing the demand through the ordinary channels of employment, and taking on the best men who offer. [1908h, p. 710]

41. Felt & Sinclair [1991] consider a number of employment indices, e.g., (1) whether a person was employed or unemployed, (2) if employed, whether the employment was full-time or part-time, (3) whether the season was influential in determining employment, and (4) the classification of occupations as derived from census categories. See also Routh [1959, p. 309], Philips [1958, p. 291] and Lipsey [1960, pp. 5, 6]. More will be said on index-numbers in a later chapter.

42. In the early nineteenth century statistical offices and national statistical societies were established in many countries (for example, in Boston in 1839 (the American Statistical Association) and Dublin in 1847). The dynamic Belgian astronomer and mathematician Adolphe Quetelet (1796-1874) attended the Great Exhibition in London in

1851, and as a result of his urging it was decided to hold an international statistical congress. The first of these took place in Brussels in September 1853. Further congresses followed, and led, in 1885, to the foundation of the still influential International Statistical Institute (see Cox [1961] and Nixon [1960]). The I.S.I. has held sessions biennially, except during the war years, since its start.

43. In his discussion of Bowley [1919b] Sir Leo Chiozza Money M.P. said that 'he was always delighted with the way in which humour peeped out in Dr. Bowley's Papers' [p. 369]. Similarly, in supporting the vote of thanks to Bowley on the occasion of the latter's delivery of his inaugural address to the Royal Statistical Society, Lord Stamp said 'in the Presidential Chair he [i.e. Bowley] would be, as always, provocative and whimsical' [Bowley, 1939b, p. 19]. And Dahrendorf [1995, p. 206] refers to Bowley's 'perceptive and often amusing reminiscences of events at the early School [i.e. the LSE]'.
44. Although Cannan lectured at the London School of Economics for thirty-one years, he never moved from his home-base Oxford. His London lectures had to end at 6.57 p.m. sharp to allow his catching the 7.30 p.m. train back home. The only time this routine was upset was during the General Strike of 1926; then Cannan cycled from Oxford to London, and spent the night with William Beveridge, Director of the School from 1919 to 1937.
45. The Bodleian Library is the main research library of the University of Oxford.
46. Bowley was sometimes severely critical of published work with which the Webbs were associated: see for instance his reviews [1922h] and [1922i].
47. See Hayek [1946, p. 3]. Beatrice Webb recorded in her diary [vol. II, p. 56] that, on the 26th July 1894, 'The poor old man [i.e. Hutchinson] blew his brains out, finding his infirmities grow upon him'.
48. Hayek [1946, p. 27] noted that during the second World War, and in accordance with the government's evacuation policy, the London School of Economics was kindly accommodated by Peterhouse College, Cambridge. The Governing Body of the latter had in fact decided as early as the 25th July 1939 to house the School at a cost of three guineas per week for board residence [Dahrendorf, 1995, p. 341].

Part-time classes, however, were continued in London, and the School was reunited in 1945.

The stay in Cambridge seems to have been a pleasant one. For example, Anne Bohm, a staff-member of the London School of Economics from 1942 to 1985, admitted later on that 'I always say we had a very happy war' [Dahrendorf, 1995, p. 349].

49. Dahrendorf [1995, p. 21] remarks that when the School was founded the idea was that all lectures and most classes would be given only in the evenings, between 6 p.m. and 9 p.m. In the early twentieth century the School became a College of the University of London and day classes were instituted.

50. For a statistician, retirement in one's sixty-seventh year by no means implies that one's life is over. Kotz & Johnson [1999] examined the lifelengths of 128 people who had had a considerable impact on statistics, be it in the utilisation or be it in the development thereof. The minimum age at death for those in their sample was 26.9 years (Frank Plumpton Ramsey), the maximum 103.8 (Walter Francis Willcox), while the median was 75.57.

51. Allen [1968] suggests that Bowley was called on far too seldom by the government to advise on official statistics (he was eclipsed in this respect by Keynes).

    For at least ten years Bowley and Keynes had scientific collaboration almost on a daily basis. Their relationship was cordial but not close, and sometimes verged on the coldly polite. Keynes on occasion questioned the validity of Bowley's arguments and results (see, for instance, Keynes [XXII, p. 70]), but as a rule Bowley seems to have been right. Of course, it is hard to compare the colourful and brilliant Keynesian style with Bowley's somewhat prosaic but very clear and lucid descriptions. The difference in background between these two scholars—Bowley a humble son of a preacher in a modest church in Bristol and Keynes the scion of the British academic elite; Bowley with a very happy family life enjoyed for over 40 years and Keynes a prickly and perhaps unhappy marriage to the Russian ballerina Lydia Lopokova; Keynes's legendary national and international activities related to economic problems compared to the 'highlights' of visits to India and Southern Africa achieved by Bowley—heavily influenced their contributions to economics, statistics and probability and their exposition.

52. Clara Collet (1860-1948), a teacher from 1878 to 1885 at Wyggeston Girls' School in Leicester, turned her attention to political economy, and graduated with an M.A. degree from University College, London, in 1885. She participated in Booth's survey that resulted in *Life and Labour of the People in London*, and her published research in general showed her interest in women's occupations and wages.

53. A foreshadowing of a connexion between 'Bowley' and 'social research' appeared in C.M. Smith [1853], *Curiosities of London Life*. In the chapter 'The charitable chums' benefit club' the landlord of the 'Mother Bunch' public-house is one Peter Bowley. He 'visits his tap-room guests, and informs them of a plan which is in operation to improve the condition of the labouring classes'. Unfortunately the scheme (medical aid and assurance) was not a great success.

54. Established on the lines of the Harvard Economic Service, the London and Cambridge Economic Service was directed by an Executive Committee consisting of William Beveridge and Bowley for the LSE and Keynes and Hubert Henderson for Cambridge. On Bowley's editorial skills see Allen [1968, p. 136].

55. Kiaer (sometimes Kiär), head of both the Statistical Division of the Ministry of the Interior and the Central Bureau of Statistics in Norway, not only carried out statistical research into demographic matters, but also undertook investigations in economic statistics. His path-breaking work in the design and use of sample surveys was presented for the first time to the International Statistical Institute at the Berne Session in 1895, and, although his methods were coldly received at first (except by Bowley and Chuprov), they gradually gained acceptance. Kiaer's proposal of the representative method was endorsed by the International Statistical Institute in 1903, the year in which Bowley was elected to membership in that august body (Darnell [1981, p. 163] and Thomsen [2001]).

56. In his review of Volumes III and IV of the *New Survey* Austin Bradford Hill expresses approval of the fact that Bowley was in charge of this part of the Survey. While Charles Booth's standards had also been used here, Hill finds it disturbing that the authors are led to conclude that 'nearly one-half of the persons found to be in

poverty in the week of investigation owed their condition at the time to unemployment and under-employment, while low rates of wages, so important in Charles Booth's day, are no longer a major cause' [1933, p. 109].

57. As a young man Wood (1874-1945) introduced himself to Bowley at a meeting of the British Association in Bristol in 1898. This started a correspondence and eventually resulted in a series of jointly written papers on the statistics of wages in the United Kingdom for various occupations: the papers on the cotton industry were by Wood alone, and resulted in his being awarded the Guy Medal in Silver by the Royal Statistical Society. Wood wrote a number of papers for the *Statistical Journal* (including one on cricket, his main hobby), and frequently participated in the discussions of papers read before the Royal Statistical Society. The major part of his working career was spent in the wool textile industry.
58. The *Elements of Statistics* went through six editions, and the *Elementary Manual of Statistics* seven.
59. Keynes (1883-1946), having completed the Mathematical Tripos at Cambridge (twelfth Wrangler), spent a few years in the India Office before returning to Cambridge as a lecturer in economics. An interest in finance soon developed, and Keynes joined the Treasury. Fundamental writings on money, employment and economics followed (particularly after the end of the Great War), but Keynes is perhaps best known to statisticians for *A Treatise on Probability*, first published in 1912.
60. William Ernest Johnson (1858-1931) was a Cambridge logician who, in 1913, published an influential article on demand theory and utility curves. In 1932 his article 'Probability: the deductive and inductive problems, with an appendix' (ed. R.B. Braithwaite) was published in *Mind* vol. 41. Here an important development of Laplace's Rule of Succession was presented. The article referred to in the quotation was 'The pure theory of utility curves', *The Economic Journal* (1913), 23, No. 92, 483-513.
61. Moore [2005, p. 95] ascribes these views of Marshall's partly to the fact that the applied tradition of the Mathematical Tripos at Cambridge entailed a preference for mathematics that did not intrude on the intuitive approach to a problem.

62. Keynes wrote

> Marshall's mathematical and diagrammatic exercises in Economic Theory were of such a character in their grasp, comprehensiveness, and scientific accuracy, and went so far beyond the 'bright ideas' of his predecessors, that we may justly claim him as the founder of modern diagrammatic economics. [1924, p. 332]

63. Allen (1906-1983) was Bowley's successor in the Chair of Statistics at the London School of Economics, being appointed to that position (vacant since Bowley's retirement) in 1944. He was knighted in 1966. For further details see Grebenik [1984].

64. Herbertson records that nearly four hundred members of the Association, ranging from astronomers and magneticians to school masters, went to Southern Africa for these meetings. 'Never,' he wrote, 'has there been a more impressive demonstration of the advance of the material sciences!' [1905, p. 632]. Among the travellers was the Dutch astronomer Jacobus Cornelius Kapteyn (1851-1922), who read a now famous paper on star-streams. Kapteyn also published a monograph *Skew Frequency Curves in Biology and Statistics* (1903), in which he included a popular discussion of the Normal distribution and its skew extension.

65. Drinkwater changed his name to Drinkwater Bethune in 1836. Hill [1984, p. 139] notes that there are 'minor inaccuracies' in [Anon., 1935].

66. V.D. Barnett, having consulted various Latin scholars, notes that *Aliis Exterendum* may be translated into English as 'to be threshed *by* others' or 'to be threshed *for* others', the translation depending on whether the ablative or the dative respectively is meant. The scholars consulted on the whole went for the latter. See Barnett [1994].

    The cover of the society's *Journal* from the first issue of 1838 carried the seal in which the wheatsheaf was bound by a ribbon bearing the motto: the latter was dropped in 1858. With the current importance and reliance upon computers Ivor Hill [1984, p. 133] suggested that a more appropriate motto might be *machina calculatoria exterendum.*

67. The sampling unit recommended to the Indian Government was the village. In the discussion of Neyman's paper of 1934 Bowley, no doubt somewhat disillusioned, mentioned that the recommendations made 'had no effect'.
68. According to Allen [1957] Bowley attended the following sessions: the 12th (Paris, 1909), 13th (The Hague, 1911), 15th (Brussels, 1923), 16th (Rome, 1925), 18th (Warsaw, 1929), 19th (Tokyo, 1930), 20th (Madrid, 1931), 21st (Mexico, 1933), 22nd (London, 1934), 23rd (Athens, 1936), 24th (Prague, 1938) (a session that started on the 11th of September and was suspended at midnight on the 13th because of the uncertain political situation) and the 26th (Berne, 1949). It appears that Bowley also attended the 17th Session in Cairo in 1927-8 [Richardson, 1930, p. 408].
69. Nixon [1960, p. 37] records that

    > It was a tradition of the Institute that the Treasurer should be a British subject, usually the treasurer or an officer of the Royal Statistical Society. This tradition was maintained for 70 years when an American member (Cox U.S.A.) was appointed treasurer.

    After obtaining a Master's degree in Statistics at Iowa State College, Gertrude Mary Cox (1900-1978) began work for a doctorate in psychology at the University of California, Berkeley. In 1933 she was lured back to Iowa State to work in the newly-established Statistical Laboratory. In 1940 she became head of North Carolina State University's Department of Experimental Statistics. While Cox's early accomplishments in experimental design are recognised, her main contribution to statistics was her ability to organise and administer programmes and workshops and to acquire funding. She was appointed treasurer of the I.S.I. in 1955.
70. In 1890 the British Economic Association was formed, and by Royal Charter it became the Royal Economic Society in December 1902.
71. An unpublished history of the Service is in the Bowley Collection in the London School of Economics.
72. The Cowles Commission for Research in Economics was established in Colorado Springs by Alfred Cowles in 1932. Designed to further

the application and development of mathematical, statistical and logical analytic methods, it was initially under the control of an Advisory Committee. The duties of this committee waned as the Commission waxed, and the need for such a body eventually vanished. In 1939 the Commission moved to Chicago, and in 1955 it moved to Yale, becoming the Cowles Foundation.

73. William Newmarch (1820-1882), economic statistician, was not only a prominent banker and actuary, but also made considerable and noteworthy contributions to the Statistical Society of London. On Newmarch's death, the Society subscribed £1420 14*s.* toward the establishment of the Newmarch Professorship of Economic Science and Statistics at University College, London. Edgeworth delivered six Newmarch Lectures in May and June 1892.
74. When William Pember Reeves (1857-1932), Director of the London School of Economics from 1908 to 1919 indicated his desire to retire, a short list of three names was drawn up of possible replacements: Sir Theodore Morison (principle of Armstrong College, Newcastle-upon-Tyne, and later vice-chancellor of Durham University), Bowley and Sir William Beveridge. The last of these was eventually nominated and appointed.
75. The Guy Medals are named in honour of William Augustus Guy (1810-1885), physician and statistician, and himself an Old Blue. The medal in Bronze is awarded for a paper read at a meeting or conference of the Society or one of its sections or local groups, with preference to those under 35 years of age; Silver is awarded for a paper of special merit with general contributions to statistics; and that in Gold is awarded for an innovative and ground-breaking paper.
76. Agatha Bowley published some fifteen books between 1942 and 1975. As instances we mention *The Natural Development of the Child (a guide for parents, teachers, students, and others)* (E. & S. Livingstone, Edinburgh, 1942), *Seasonal Poems* (Regency Press London, 1975) and, with L. Gardner, *The Handicapped Child: educational and psychological guidance for the organically handicapped* (Churchill Livingstone, Edinburgh, 1969).
77. In addition to a number of scholarly papers (e.g. in *The Review of Economic Studies*) Marian published a number of books, for example

Nassau Senior and Classical Economics (Allen & Unwin, London, 1937) and *Studies in the History of Economic Theory before 1870* (Macmillan, London, 1973).

78. The authors would be grateful to receive any information about Ruth.

Chapter 2

1. In a later work Bowley stated that his subject was best described by the 'barbarous title' Modern Statistical Sociology [1915b, p. 4].
2. E.P. Hennock [1987] emphasises the importance of the Board of Trade investigations into poverty conducted between Rowntree's and Bowley's work.
3. A generous grant to the London School of Economics by Sir Ratan Tata (1871-1918), art connoisseur and noted philanthropist, led to the establishing of the Ratan Tata Foundation, its purpose being 'to promote the study of and further the knowledge of methods of preventing and relieving poverty and destitution' [Bowley & Burnett-Hurst, 1915, p. 5].
4. We have discussed the sample survey aspects of the research elsewhere in this book, and shall accordingly say nothing more on the subject here.
5. It is noted that the figures in Stanley were too small to be expressed as percentages.
6. Details of Rowntree's estimates of the earnings of all wage-earners in York, not published in his *Poverty*, were communicated to the Royal Statistical Society by Bowley [1902e].
7. The tabulation of the data for this paper was done with the help of Burnett-Hurst.
8. This sort of question, examining the worker's needs, was important in matters of wage adjudication.
9. Nowadays a more careful definition would be needed. For example, there is a religious black supremacist group called the Nation of Islam.
10. Drawing on data from the *New Survey of London Life and Labour* Bowley found that some 1,100 sons were in a lower grade of occupation than their fathers, 2,400 were in the same grade and 1,500 were in a higher grade. (Here the grades are unskilled, semi-skilled,

shop assistants, skilled and own account, the latter including owners of small businesses, hawkers, etc.) The division between skilled and unskilled workers based on wages *and* training was essential.

Among the sons nearly 3/5 of the unskilled remained unskilled, while 2/5 were in a higher class. Most surprisingly, nearly 3/10 of the skilled sons dropped down to the unskilled class. The general conclusion may be drawn that it was not easy, in the late 1920s and early 1930s, for the children of working-class fathers to rise to a higher class, and virtually impossible for them to reach the middle class.

11. Later in his book Bowley suggests that Pareto's Law applies better to earned and unearned incomes separately than when they are merged.

    The Pareto distribution is ideally suited to describe income distribution (see Kleiber and Kotz [2003]). For an earlier study with reference to income in India—and of criticisms of Pareto's derivation—see Shirras [1935b]. In 1906 Bowley presented to a Select Committee an illustration of the use of this law in the possible graduating of the income tax, and in his [1914a] he considered whether any new light could be shed on the matter of super-tax in the light of the evidence that had been given to that committee. See also Stamp [1914b].

    Incidentally, Shirras concluded his paper with the words 'There is indeed no Pareto law. It is time that it should be entirely discarded in studies on the distribution of income' [1935b, p. 680].

12. Bowley does not specify *which* skew curves should be used.
13. Greenwood refers to this bad fit as a 'minor point'.
14. In 1937 Ronald F. George presented a new calculation of the poverty line, stating at the outset that 'under no circumstances can the "Poverty Line" be regarded as a desirable level' [1937, p. 74]. He concluded that the 'minimum needs' requirement was in fact much higher than that generally accepted, and as a consequence that the extent of absolute poverty was greater than estimated before. In 1987 A.B. Atkinson published a study of the choice of poverty line, the choice of poverty measure and the question of whether poverty and inequality were sometimes confused. Interestingly, he concluded that Bowley was not apparently greatly concerned with any of these matters. See also Rowntree [1902].

    Marie Stopes, in her discussion of Stevenson [1925], asserted vigorously that the *pregnancy* rate had been found to matter more than the actual *birth* rate in determining the vitality of the stock.

On the 21st of January 1938 a letter, signed by R.F. Harrod, Arthur L. Bowley, James Bonar, D.H. MacGregor, A.C. Pigou, F.A. Hayek, P. Sargant Florence, G.C. Allen, Lindley M. Fraser, J. Marschak, J.M. Keynes, G.D.H. Cole, P. Barrett Whale, R.F. Bretherton, R.L. Hall, Redvers Opie and E.H. Phelps Brown, was published in *The Times*. It was devoted to the Population (Statistics) Bill, discussed in the House of Commons in December 1937. The authors proposed a 'positive proposal', namely, that the Registrar-General be authorised to obtain data pertaining to 'rank or profession' using the same headings that were used in the Census, thus allowing comparison with the latter. This would permit the ascertaining of the reproductive rates not only of occupations but also of social classes. The letter was later cited in full in 'Notes of the Quarter', *Eugenics Review* 30 (April 1938), pp. 3-4.

15. In his [1914c] Bowley presented a long analytical study of the changes in the rural population in England and Wales. Among his findings were (1) the population of rural districts seemed to increase from non-agricultural growth, (2) the number of people employed on farms fell dramatically from 1861 to about 1901, but seemed to recover slightly over the next decade and (3) a large number of people seemed to have started their working life on farms and then to have moved either to other occupations in the country, towns or abroad.
16. Bolton had not been included in *Livelihood and Poverty*.
17. In his 1926 review of *Has Poverty Diminished?* Cannan suggested that the unwillingness of the working-classes to spend the same proportion of their income on rent after the War as they had before (and hence to improve the housing position) was in part because of these Acts, which had encouraged people to think that rents 'ought' not to rise like everything else when money depreciates.
18. In a discussion with Joshua Stamp on the B.B.C. on the 19th February 1930 (reported in *The Listener* on the 26th February) John Maynard Keynes said that Bowley was 'the only man in this country outside Whitehall who really understands the unemployment figures' [Keynes, 1981, p. 319].
19. As used by Rowntree.

20. In his study of the history of sampling procedures Stephan notes that 'the problems of observing and recording data were almost always far more serious than the problems of sampling' [1948, p. 21].
21. In the third volume of 1932 of Llewellyn Smith's *New Survey of London Life and Labour* 'coal range' was replaced by 'coal or other cooking range' [Llewellyn Smith, III, p. 415].
22. In 1887 Booth read a paper before the Royal Statistical Society setting out the first results of his inquiry and sketching the proposed future investigations. The results of the Survey were published during the late 1880s and '90s with various small changes in title.
23. References to the *New Survey* will be given thus in this chapter, the volume being indicated.
24. In 1921 Margaret Hogg published a paper in which she examined the question of women's wages, their relation to those of men and the matter of dependants. She concluded that the percentage of women earners supporting others in whole or in part was underestimated by Rowntree in his *The Human Needs of Labour* and overestimated by the Fabian Women's Group in their 1915 pamphlet *Wage Earning Women and their Dependants.*
25. Even a term like 'genteel poverty' may vary in meaning from place to place. In her short story *The Listerdale Mystery* Agatha Christie suggests that this might describe a white-washed cottage, good but shabby chintzes and a Crown Derby tea service washed by the hostess herself in the country, but frowsy landladies and breakfast haddock that is not quite-quite in London.
26. The Street Surveys were conducted, or at any rate supervised, by Llewellyn Smith: Bowley was responsible for the House Sample Analysis. In 1662 John Graunt estimated the number of families in London from records of births, burials and the number of houses [Hald, 1990, §7.8].
27. In the Survey "M" stands for middle-class. Streets in which more than 50% of the inhabitants were middle-class were designated "red".
28. In his contribution to the discussion of Llewellyn Smith [1929] Bowley commented on the attention that had to be paid to the definition of the sampling factor in a city the size of London, where the difficulty lay in defining the items in the universe and selecting the sample as the universe increased in size.

29. In a review in 1920 Bowley remarked that comments similar to those made here were the result of the 'enthusiastic personalities of the investigators', citing as an example 'Green, spotty face and what seemed like a green, spotty soul' [Bowley, 1920d, p. 106].
30. Volume IX, *London Life and Leisure* of 1935, had two chapters—II, 'Amusements and Entertainments' and III, 'Sports, Games and Hobbies'—by Ruth Bowley. Chapter IV, 'Holidays and Outings', was by Ruth and Llewellyn Smith.
31. In *Poverty: A Study of Town Life*, his first [1901] social survey of York, Rowntree set a minimum daily standard of 3,500 calories of energy value and 125 grams of protein per man. By the time of *The Human Needs of Labour* the figures for unskilled workers and their families were revised to 3,500 calories of energy and 115 grams of protein for men, the corresponding figures for women and children being 2,800 calories, 92 grams and 1,750 calories, $57\frac{1}{2}$ grams.
32. Hagenaars and Van Praag, in an attempt to avoid the categorisation of poverty as either 'absolute' (independent of the general style of living in the society concerned) or 'relative' (dependent on the general style of societal living), examined a new definition of the poverty line. This definition, which turns out to be a generalisation of a number of others, depends on the perception of poverty in society, and seems to be midway on the 'absolute' to 'relative' scale.
33. For instance Carpenter [1932] for a review of Vols I and II, Carr-Saunders [1931] Vol. I, Chaddock [1932] Vol. I, Dearle [1932], [1933], [1934a], [1934b], [1935a], [1935b] Vols II-IX, Lederer [1931] Vol. I, McKenzie [1931] Vol. I, Thomas [1932] Vol. II, Ware [1932] Vol. I. For summaries of the Survey see Bowley [1933d] and Marsh [1930].
34. The day after the signing of the Armistice on the 11th November 1918 Lloyd George, Prime Minister of the United Kingdom, made a speech in which he promised 'habitations fit for the heroes who have won the war'. This led to an Act of 1919 providing for the development of subsidised council houses. These new houses were intended for the artisan elite, thus freeing older houses for others.
35. The figure was open to interpretation: twenty-four, if one included Iceland as being independent of Denmark, and more if Ukrainia, White Russia, Georgia, Armenia and Azerbaijan as independent of Soviet Russia and the Vatican as independent of Italy.

Chapter 3

1. The fiscal controversy of the early twentieth century in Britain was concerned with arguments about the imposition of tariffs, the imposition of preferential duties according to origin, and the introduction of a progressive income tax.
2. Robert Harry Inglis Palgrave (1827-1919) was a prominent banker and economist. President of Section F of the British Association in 1883, he launched the idea of an economic association, resulting in the eventual formation of the Royal Economic Society. Modelled on the *Dictionary of National Biography* and Grove's *Dictionary of Music*, Palgrave's *Dictionary of Political Economy*, a work that took almost a score of years to complete, is a reference work of considerable importance (the name of Alfred Marshall is conspicuously absent from the contributors).
3. Bowley in fact, somewhat optimistically, suggests an *optative* tense interpretation.
4. In his book *The Effect of the War on the External Trade of the United Kingdom* Bowley notes that, when it comes to imports, 'The very important trade in diamonds from South Africa, and to and from the continent, is very imperfectly recorded' [1915a, p. 7].
5. Bowley expressed even harsher criticisms of official statistics in his [1906b] and [1939h].
6. Mandello's paper, read before the 1905 meeting of the International Statistical Institute, was concerned with a number of questions arising in consideration of the probable future of statistics. He noted the progressive increase that was occurring in the number of statistical publications from year to year—indeed, 'statistics have already reached a state where the danger of chaos is impending' [Mandello, 1905, p. 729]—and suggested the cautious restraint in such publication. See also Bowley [1906b, p. 542].
7. The idea of such an office had been suggested by Sir William Petty in the seventeenth century (see Hacking [1975, p. 109]).
8. Money, in his comments, described the post of a newspaper editor as 'onerous, responsible, and arduous' [Bowley, 1908b, p. 484].
9. We shall see in the next section that a similar suggestion had been made by Bowley in his [1904a].

10. *The Mikado*, Act II; less charitably, and from the same source, 'Here's a pretty mess!'

    Bowley notes here that the governments of Australia, Canada, New Zealand and South Africa had recognised their responsibility in the publication of information.
11. In his 1899 review of *The Federal Census* Bowley gloomily noted that the American Census, like the English, suffered owing to the absence of a permanent Census office.
12. In his [1908f] Bowley reviewed a a detailed report of the wages, hours of labour, etc., of some 260,000 railway workers, and found the statistics obtained to be both valuable and detailed.
13. Reviewing *International Comparisons of Cost of Living* in 1935 Bowley noted the extreme difficulty of comparing the cost of living between two countries.
14. *Official Statistics* was designed not for experts in statistics and economics but for those interested in public questions.
15. In 1896 Bowley published a short review of the second issue of the *Abstract of Labour Statistics, 1894-5.* Among the salient features he cited as necessary for the understanding of labour statistics were the following: definiteness; knowledge of the method of collection and compilation of the data; and the omission of figures whose precise significance could not be given. These Bowley found to be satisfactorily met by the *Abstract*, and while indicating lacunæ in published labour statistics he took care to point out praiseworthy aspects.
16. Keynes [1983, p. 62] notes that differences between expenditure (and therefore between cost of living) in different places is due partly to differences between climate, race, religion etc. and also to the fact that some things are cheaper in one place than another. The Board of Trades comparison of England and France simply indicates that on English standards the cost of living is cheaper in England: using the French standard it is cheaper in France.
17. A list of various sources of labour statistics in the United Kingdom from the start of such collections at the end of the nineteenth century to 1950 was published by Ainsworth in 1950.
18. Bowley's reviews of the relevant works by others were [1910e], [1913b], [1930b], [1933b], [1934c], [1937c], [1938d], [1939d], [1939e] and [1947b]. For other relevant work on the national income see Flux [1929a], [1929b] and Bowley [1923e].

19. Dalton, member of the Fabian Society and at one time Chancellor of the Exchequer, was Reader in Economics at the LSE.
20. For a study of the introduction and definition of the term 'occupied class' over the nineteenth century see Topalov [2001]. The definition came to depend on a single criterion: the marketability of one's labour.
21. The southern Irish counties became the Irish Free State in January 1922.
22. See also Cannan et al. [1910].
23. In a memorandum issued by the Executive Committee of the London and Cambridge Economic Service in May 1947 Bowley compared data on wages and prices in 1939-46 with those for 1914-1920. He found among other things that women's wages had increased considerably from 1906 to 1935, although the proportion of women's to men's wages had risen only moderately from 44% to 48%.

Chapter 4

1. In 1954 Schumpeter wrote 'The best reference on wage and unemployment indices is to the outstanding work of A.L. Bowley' [p. 1092].
2. The change in purchasing power was corrected for by using Sauerbeck's index numbers.
3. Is this an example of Bowley's humour—the conjunction of 'marched together' and the wars?
4. The effect of the observer on the result of an experiment in general is well known.
5. Acworth (1850-1925) was a railway economist who served two terms (1911-1915 and 1918) as vice-president of the Royal Statistical Society. He lectured on railway economics at the LSE in the early twentieth century, and published his lectures in a text-book. His expertise was called upon by Canada, India, Rhodesia and the United States of America among a number of countries. He was knighted in 1921.
6. By 'tops' here is presumably meant a bundle of wool that has been combed and is ready to be spun; 'noils' are short pieces or combed-out knots of wool.

7. The series was continued in a number of studies authored by Wood alone. Feinstein noted that these excellent studies ‘remain the indispensable foundation for all modern research on wage movements’ [1990, p. 596]. Wood’s studies on the cotton industry were published in book form in 1910.
8. In a footnote on p. 154 of Part XIV Bowley generously notes that, while the parts on engineering and shipbuilding had been published under the names of both authors, the major part of the preparation of the tables, collating trade union evidence and calculating the index-numbers had been carried out by Wood.
9. Bowley’s Table II contains index-numbers of average rates in engineering and shipbuilding in nineteen districts.
10. In the autumn of 1914 the Division of Economics and History of the Carnegie Endowment for International Peace proposed the publication of a series of studies ‘to attempt to measure the economic cost of the War and the displacement which it was causing in the processes of civilisation’ [Bowley, 1921h, p. v]. One of the chosen authors for this series, all ‘men of judicial temper and adequate training’, was Bowley. For obvious reasons the ambitious programme could not be carried out until after the War.
11. In his analysis of responses to a survey published in 1886 Giffen [1886, p. 61] noted that when labourers said that they had meals ‘with meat’ it meant an occasional bit of bacon.

    In 1909 the *Report of an Inquiry by the Board of Trade into working class rents, housing, and retail prices, together with the rates of wages in certain occupations in the principal industrial towns of France* (‘a stupendous inquiry’) recorded that many Parisians preferred horseflesh to other meat (see Edgar J. Harper [1909, p. 115]).
12. Considerable difficulty was found in gathering data on women’s wages, the difficulty resulting from the facts that conditions had changed considerably during the nineteenth century, the increase in the labour pool, the development of better-paid jobs, and the paucity of available records. An astonishingly detailed investigation was published by Dorothea Barton in 1919.
13. These factors included things like the following: (a) the changes from time- or piece-rates to earnings or *vice versa* were seldom proportional and (b) minimum time-rates were often supplemented by

various systems of bonuses on production. Bowley discussed a method to achieve the separation of these factors in the second of the six appendixes to this work.

14. For further reviews see Caradog Jones [1938] and King [1938].
15. Topalov [2001] discusses the emergence of the concept of 'occupied population' in France, Great Britain and the United States of America during the nineteenth century.
16. The reference is presumably to Edward Bellamy's *Looking Backward: from 2000-1887*.
17. Josiah Charles Stamp (1880-1941) was President of the Royal Statistical Society from 1930 to 1932; Greenwood was President from 1934 to 1936, and (Edward) Hilton Young (1879-1960), created first Baron Kennet and known as Lord Kennet of the Dene, held the same office from 1936 to 1938.

    Greenwood spoke approvingly of Francis Cornford's lecture on ancient views of scientific thought [Cornford, 1938], being reminded, by Bowley's remarks on the use of leisure, of Cornford's opinion that the great increase in happiness experienced by Europeans since the time of Pericles was due to the changed aim of science.

Chapter 5

1. In September 1903 a letter that had appeared in *The Times* on the 15th of August of that year was re-published in *The Economic Journal*. Expressing some thoughts on the fiscal proposals for preferential tariffs under discussion in the United Kingdom at that time, the letter was signed by fourteen prominent economists and statisticians, including Bowley, Edgeworth and Marshall. (Other equally prominent scholars, including Venn and Yule, wrote to the Press giving their reasons for abstaining from signing.)
2. Bowley writes in the Introduction to the second edition that the first was published in 1894.
3. Compare Wainer [2009].
4. The words 'England', 'Great Britain' and 'United Kingdom' seem to be used interchangeably in this book.
5. This procedure was also adopted in other European countries that had colonies.

6. The Corn Laws, introduced in Britain under George III and repealed, after a number of changes, in 1846 in Victoria's reign, were important tariffs designed to protect British corn prices against cheaper foreign imports. ('Corn' here meant any grain, particularly wheat.)
7. The fourteenth edition of the *Encyclopædia Britannica* describes the *Poor Law* in British usage as 'a peculiar system giving public relief to the destitute'.

   William Jevons was somewhat scathing of the consequences of the Poor Law, saying that 'the wise precautions of the present poor law are to a great extent counteracted by the mistaken humanity of charitable people' [1870, p. 311]. The evil done by the aid extended by the upper to the lower classes, he asserted, probably outweighed the good, and small charities, often established by 'mistaken' testators, were, he further suggested, even worse.
8. We are unclear as to the exact meaning to be attached to 'privation' here: synonyms like 'destitution', 'want', 'hardship', 'absence of necessaries' and 'deprivation of what is necessary for comfort' all seem to express different degrees of 'poverty'.
9. In his review of this book Dearle describes this conclusion as 'wise and courageous' [1923, p. 393]
10. It was decreed that in the third of the Special Periods—from April to October 1922—emergency benefits were to be provided in three groups of five weeks, separated by gaps of five weeks each.
11. In addition to the reviews mentioned here see those by Dearle [1923], Feldman [1923] and Lescohier [1923].
12. Bowley cites the 'rather alarmist figures' mentioned by Keynes in an article written in 1922. Owing to a misunderstanding of some of John Brownlee's work, Keynes had incorrectly calculated that the number of males between the ages of 20 and 60 [incorrectly given by Bowley as 69], despite war casualties, was 1,300,000 more in 1922 than it was in 1911. Bowley's corrected figure was 800,000.
13. Herman Feldman was lavish in his praise, describing the work as 'the best on the subject that has appeared in England for some time' [1925, p. 317].
14. For surveys of the evolution of statistics in India, in one form or another, see Chattopadhyay [1987], Ghosh et al. [1999], Lindsay [1934], Meek [1937], Oshima [1951], Rao [1944] and Ray and Sinha [1941].

15. Somewhat later Prasanta Mahalanobis, in discussing the organisation of statistics in India, wrote 'I have learnt by bitter experience the difficulty of securing reliable primary investigators' [1944, p. 73].
16. Referring to some work by Walter Willcox, Shirras noted that India was then the most populous country in the world, with 352,837,778 people as opposed to China's 342,000,000 [1933, p. 58]. Yet despite the high population density he stated that it would be wrong to conclude that the country was becoming overpopulated.
17. The Ad Dharmi are those who believe in Guru Ravidas as their founding prophet. The term means 'Primal Spiritual Path'.
18. A *lakh* is 100,000 rupees. Shirras in his [1929] records that at that time one lakh of rupees was equal to £7,500: at the time of our writing there were roughly 50 Rupees to the U.S. dollar.
19. Shirras [1935a, p. 447] notes that there were 696,831 villages in India in all.
20. In 1932 there were some 13,333 operatives in factories in British India. Not only were these workers of special importance when the question of export arose, but they were also of importance in bringing the factory and cottage industries into statistical relation.
21. A *sirdar* is a person of high rank, a leader or even head-man, in India.
22. One of the present authors (S.K.) had occasion some forty years ago to take part in a similar investigation in a much smaller country, and experienced frustrating episodes before eventually being able to extract the required data from specific households.

Chapter 6

1. According to Kendall & Doig [1968] an English *translation* was printed in the *Bulletin de l'Institut International de Statistique* 18(I), pp. 552-555, and an English *version* as Bowley [1909a].
2. Stigler [2002a, p. 568] points out that, while the beginning student in statistics may well suspect that the analysis used is determined by the form in which the data are presented, the converse is more usually correct.
3. Howie [2002, p. 187] suggests that since statistics was supposed to be concerned with *large* samples, inference was not needed, and hence inverse probability was seldom used in practice.

4. These lectures also formed the substance of Bowley's [1900c], which in a sense was an introduction to the series of papers by Bowley and Wood of wage figures in various industries. See Chapter 1 for details of the Newmarch Lectures.
5. This matter had been noted before. For instance, Augustus de Morgan [1838, p. 147] discussed what should be done when there is a fixed error in an instrument, and even earlier Thomas Bayes (1702?-1761) commented on the suggestion by Thomas Simpson that there was a distinct advantage in taking the mean of a number of astronomical observations rather than using only a single, but carefully made, observation. Bayes noted the importance of having an accurate instrument if one were to rely on an abundance of measurements. See Dale [2003, §§10.2, 10.3] for details.

   Augustus de Morgan (1806-1871), born in Madras, India, studied at Trinity College, Cambridge, and in 1828 he was chosen to be the first professor of mathematics at the newly-founded London University (later University College, London). He made important contributions to logic and symbolic algebra, and his work complemented that of George Boole (1815-1864). Thomas Simpson (1710-1761) was for some time Second Mathematical Master at the Royal Military College, Woolwich. He wrote a number of textbooks and books of essays on fluxions, the laws of chance, actuarial science and statistical error theory among other topics.
6. In formula (6.1) the 'equality' $r\rho/(1+r) = r\rho$ comes from writing the left-hand side as $r\rho\,(1 - r + r^2 - \cdots)$, multiplying out and ignoring terms $r^n\rho$ with n greater than 1. Something similar occurs in the next displayed expression.
7. See, for example, Feller [1966, §7.3] and, for deeper investigations, Gut [2002], Heyde [1963a] & [1963b], Leipnik [1981] and Stoyanov [2000]). The assertion is indeed true for the Normal distribution (see Kendall & Stuart [1977, vol. 1, §4.23]).
8. Yule & Kendall [1950] introduce a *criterion of independence* between two *attributes* A and B by saying that the proportion of As among the Bs should be the same as among the non-Bs.
9. Bowley notes that the approximating conditions he has used here are more carefully examined by Edgeworth—see, for instance, the

latter's [1905, Part I]. Edgeworth uses generating functions.

10. This paper was a revised version of the second appendix to Keynes's Adam Smith Prize-winning essay 'The method of index numbers with special reference to the measurement of general exchange value' of 1909, and published, for the first time, as Chapter 2 of Keynes [1983]. This essay won Keynes a prize of £60, the same amount Bowley had been awarded in 1894.
11. Keynes notes that this median law of error was first given by Pierre-Simon Laplace in 1774 in his 'Mémoire sur la probabilité des causes par les événements'. For a study of the *logical* rather than the *mathematical* nature and use of an average see John Venn [1891].
12. See Stigler [2002a] for a discussion of the early history of contingency tables.
13. For further discussion see Fisher [1922], Pearson [1911b], [1922], [1923b] and Yule [1922].
14. It is interesting to note that Bowley and Connor advocate that all expected frequencies should exceed 10, in contrast to the value 5 usually suggested today.
15. There appears to be a misprint in the original paper here, the χ^2 in this sentence being given as P.
16. On reasons for the choice of the simplest law in statistics see Jeffreys [1961, §1.0].
17. Fisher's reference is to the fourth (1920) edition of Bowley's *Elements of Statistics*.
18. Herbert Hall Turner (1861-1930) was a seismologist and Savilian Professor of Astronomy at Oxford.
19. For a good discussion of the work by Edgeworth and others on this method see Farebrother [1999, §9.7 & Chap. 15].
20. In a paper read before the Twelfth Congress of the International Statistical Institute in Paris in 1909 Bowley [1909a] suggested the use of the median to effect an international comparison of wages.
21. In the third edition of 1888 of *The Logic of Chance* John Venn viewed the problem of determining the 'correct', or most appropriate, average as an *inverse* one, writing

> we are supposed to have a moderate number of 'errors' before us and we are to undertake to say whereabouts is the centre from which they diverge. This resembles the determination of a cause from the observation of an effect. [1888, p. 468]

22. Bowley uses 'r' for both the sample and the population correlation coefficients.
23. We shall examine this second approximation in our next chapter.
24. Bowley often avoids the term 'variance', writing rather of the standard deviation.
25. This is discussed in the chapter on the law of great numbers in Bowley's *Elements of Statistics*: see our next chapter.
26. The word 'passengers' is used here as meaning those who travel in and out of the country in general, and not simply with reference to emigration and immigration. The error due to internal passenger movement, Bowley deduces, may be sensible in Scotland and Northern Ireland but small in England and Wales.
27. See Bowley & Burnett-Hurst [1915] for further details.

Chapter 7

1. Ingraham was right, in his 1929 review of this book, in lamenting that Bowley did not add more to what was essentially an epitome of Edgeworth's work. Stigler said that Bowley's discussion of Edgeworth's work on the law of error was 'extensive (if impenetrable)' [1978, p. 299].
2. Bowley does not specify what these 'fundamental conceptions' are.
3. For further discussion of Bowley's views on the use of probability see Wishart [1939, p. 561]. See also Armatte [2008] for a discussion of the views of Lucien March, who was adamant that probability was unnecessary for statistics.
4. Once again Bowley gives no further details. John Aldrich, in his Internet discussion [Aldrich, 2005/2007] of six reviews of the first edition of Fisher's *Statistical Methods for Research Workers*, declares that there was a strong Bayesian vein in textbooks published around that time (i.e. the 1920s). He mentions Bowley's *Elements of Statistics* and Yule's *Introduction to the Theory of Statistics* in particular. See also Aldrich [2005, p. 304].

5. In 1936 Walter Willcox published a list of over 100 definitions of 'statistics': such a list today would no doubt be considerably longer.
6. Edgeworth [1884] provides the following definition:

> Political economy is an arbitrarily selected fragment of the Calculus of Hedonics; an ill-defined tract of speculation irregularly grouped about a central spot, the theory of exchange, which is distinguished from the general phenomena of human life by the same attribute as the statistically measured belief, namely a certain quasi-mathematical precision. [pp. 225-226]

The *Oxford English Dictionary*, however, provides a tighter definition of this subject as 'the branch of economics dealing with the economic problems of government'.

7. See also Bowley [1926, pp. 7, 9].
8. Robert Giffen, in his [1913], sets out rules for the construction of tables; e.g., tables (a) should be self-explanatory, (b) should be accompanied by full explanatory notes, (c) should be simple, (d) should not be too accurate (for instance, should not give unnecessary decimal figures). (See Bibby [1986, §12].)

   Giffen(1837-1910), economist and statistician, founded the successful journal *The Statist* in 1878. Known too for his espousal and defence of Free Trade, Giffen was knighted in 1895.
9. The translation of this phrase by Edgeworth [1893, p. 670] as 'a Mean Man' perhaps smacks too much of Scrooge! In his review of 1901 of the first edition of the *Elements* Charles Percy Sanger points out that Bowley should have mentioned that the correlation of the magnitudes of various body parts in fact brings Quetelet's results on the average man into question. Edgeworth's views (op. cit.) on this point are worth noting.
10. In his [1923] Irving Fisher expressed some scorn for Bowley's preference for the median, noting that 'with the advent of calculating machines, the one boast of the Median, quickness of calculation, has disappeared' [1923, p. 247]. In 1920 Karl Pearson noted that Galton too used the median rather than the mean, and, again like Bowley, he

preferred probable errors and quartiles to standard deviations. Edgeworth was also one who preferred the median to the mean (at least in certain cases); see, for instance, his [1887a], [1887b] and [1918], and also Keynes [1908, p. 216].

11. See Flux [1921] and Edgeworth's article on index numbers in Higgs [1925-6]. The choice of the geometric mean as most appropriate is generally attributed to Jevons (1835-1882) in his pamphlet entitled 'A serious fall in the price of gold ascertained, and its social effects set forth' published at his own expense in 1863. On the other hand, in a letter to Bowley of the 27th February 1906 Alfred Marshall wrote of 'Jevons's one great analytical mistake, his eulogy of the Geometric mean in general' [Marshall, 1961, p. 776]. A discussion of reasons for the preferring of the geometric to either the arithmetic or the harmonic mean in economics was given by Coggeshall [1886].

    Discussion of possible definitions and desirable properties of means continues (see for instance Dodd [1940], Marichal [2000] and Ostasiewicz and Ostasiewicz [2000]). While it may perhaps be generally agreed that a mean should provide a summary or representation of data by a single value, there may even be argument as to whether the mean $M(x_1, x_2, \ldots, x_n)$ of a set $\{x_i\}_1^n$ of data should satisfy

$$\min\{x_1, \ldots, x_n\} \leq M(x_1, \ldots, x_n) \leq \max\{x_1, \ldots, x_n\}.$$

12. For studies of Gini's mean difference see David [1968] and [1998], David & Nagaraja [2003], Nair [1936] and Snow [1913].

13. Kotz and Seier [2007] present a detailed and useful study of various measures of skewness and kurtosis, many defined in terms of quantiles, for a number of specific distributions. For example, if F denotes a distribution function, and $0 < \alpha < 0.5$, the generalisation of Bowley's skewness is $[Q(1-\alpha) + Q(\alpha) - 2M]/[Q(1-\alpha) - Q(\alpha)]$, where $Q(\alpha) = F^{-1}(\alpha)$.

14. The average times for the four quickest trains between Leatherhead and London Bridge, Victoria and Waterloo were 41, $46\frac{1}{2}$ and $42\frac{1}{2}$ minutes respectively. The median times, for all trains, were 65, 77 and 48 minutes: the respective averages, again for all trains, were 63, 73 and 48 minutes.

15. Writing in *The Lancet* in 1937 Austin Bradford Hill said ‘A colleague of a famous contemporary statesman has been quoted as complaining that he used figures as though they were adjectives’.

    Hill (1897-1991) was a leading medical statistician and epidemiologist. He is often remembered for his advocation of randomised trials in medical research and his controversy with R.A. Fisher on cigarette smoking and lung cancer. Fisher was a strong supporter of the view that *correlation is not causation*, while Doll and Hill’s studies led to the conclusion that cigarette smoking was an important factor in the production of carcinoma of the lung (see, for instance, Doll and Hill [1950], [1954] and [1964]).

16. In the first edition the word ‘relative’ is missing here.

17. Keynes was not taken with Bowley’s definition of index numbers: see his [1983, p. 52]. He also queried whether Bowley’s verbally expressed rule was the sort of thing on which a young statistician should be brought up.

18. Keynes [1983, p. 94] finds that a fairly reliable guide to price movements is provided by the unweighted mean.

19. As references here Bowley mentions H. Secrist’s *An Introduction to Statistical Methods* and the October 1915 issue, Vol. 181, of the *Bulletin of the United States Bureau of Labor Statistics.*

20. The interpolation methods used here by Bowley are essentially given in terms of finite differences (see Jordan [1939/1965] for details): we sketch them briefly here.

    Newton’s formula runs as follows: let $y_0, y_1, \ldots, y_n$ be the values of y corresponding to $x_0, x_0+h, \ldots, x_0+nh$. Further, let $\Delta_0^1 = y_1 - y_0$, $\Delta_1^1 = y_2 - y_1$, $\Delta_0^2 = \Delta_1^1 - \Delta_0^1$, and so on, and consider the equation

$$y = y_0 + \frac{x - x_0}{h}\,\Delta_0^1 + \frac{x - x_0}{h}\,\frac{x - x_0 - h}{2h}\,\Delta_0^2 + \cdots$$

    to $(n+1)$ terms. Then $y = y_0$ for $x = x_0$, $y = y_0 + \Delta_0^1 = y_1$ for $x = x_0 + h$, etc.

    To illustrate Horner’s interpolation formula consider the polynomial

$$f(x) = p_0 x^n + p_1 x^{n-1} + \cdots + p_{n-1} x + p_n.$$

Let $x = y + h$ and suppose that $f(x)$ then becomes

$$q_0 y^n + q_1 y^{n-1} + \cdots + q_{n-1} y + q_n.$$

Substitution of $x - h$ for y then results in

$$q_0(x-h)^n + q_1(x-h)^{n-1} + \cdots + q_{n-1}(x-h) + q_n,$$

and hence q_n is the remainder found on dividing $f(x)$ by $x - h$. On such division the quotient becomes

$$q_0(x-h)^{n-1} + q_1(x-h)^{n-2} + \cdots + q_{n-1},$$

and on division by $x - h$ the remainder q_{n-1} is found. And so on.

Lagrange's Method gives a polynomial passing though (x_0, y_0), $(x_1, y_1), \ldots, (x_n, y_n)$ as

$$y = \sum_{j=0}^{n} y_j \prod_{\substack{i=0 \\ i \neq j}}^{n} \frac{(x - x_i)}{(x_j - x_i)}.$$

As an example of the central difference method Bowley cites Everett's method, one that is of particular use when only tables of differences of even order are available (see, for instance, Everett [1901] or Milne-Thomson [1933, §3.5]).

21. Bowley's reference is apparently to the first edition of 1906: the second appeared in 1927.
22. The work of this single paragraph is spread over several pages in the first edition.
23. The result also holds for non-integral, but positive, x, and may be written in terms of the Gamma function as

$$\Gamma(x) \sim \sqrt{2\pi}\, x^{x-(1/2)}\, e^{-x}.$$

For a proof of the result for $n \in \mathbb{N}$ see Feller [1950/1970], and for the Gamma function version see Diaconis and Freedman [1986]. For a discussion of the work of Stirling and de Moivre see Hald [1990] and Tweddle [1988].

24. Poisson's law of small numbers, published in his book of 1837, was 'lifted' to a great degree from an earlier publication of his of 1830 (see Dale [1989]). For a discussion of the law see Whitaker [1914].

25. Edgeworth [1913, p. 174] distinguishes clearly between the Normal frequency law and 'the more comprehensive "law of great numbers" '. In his [1911a] Bowley, in writing of the Law of Error, writes of 'the first approximation of the Law, which is the Normal curve' [p. 78].

26. Elderton and Johnson [1969] give a compact form of Edgeworth's approximation in terms of the cumulants κ_n as

$$\exp\left[-\frac{\kappa_1}{3!}D^3 + \frac{\kappa_2}{4!}D^4 - \cdots + (-1)^n \frac{\kappa_n}{(n+2)!}D^{n+2}\right] \varphi_0(x),$$

where $D = \frac{d}{dx}$, $\varphi_0(x) = \big(N/(\sigma\sqrt{2\pi})\big)\exp\big(-x^2/(2\sigma^2)\big)$, σ is a standard deviation and N is the total area under the frequency curve. See Edgeworth [1894], [1907], [1917b], Särndal [1971], Cramér [1972], Hald [2000], Elderton [1906] and Kendall and Stuart [1977, vol. 1].

27. Leon Isserlis (1881-1966) was the descendant of a long line of prominent rabbis. Initially head of the Mathematics Department at the West Ham Municipal Technical Institute, he later became Statistician to the British Chamber of Shipping. Isserlis was awarded the Guy Medal in Silver by the Royal Statistical Society in 1939.

28. The fourteenth edition of *The Encyclopædia Britannica* described the Dinka as a Hamitic people of the Anglo-Egyptian Sudan. Figures given here for the average stature (about 1.78 metres) and average cephalic index (about 72) coincide with those given by Bowley. Reference is made in both works to research by the anthropologist and ethnologist Charles Gabriel Seligman (1873-1940).

29. No year is given here for this report, but it is identified in Edgeworth & Bowley [1902].

30. Nixon's work was suggested by Edgeworth: see the latter's paper of 1913. On the distribution of the *first* digit in logarithmic tables see Lee [1989, §3.8] and Fewster [2009].

31. Recall that the inflexion points of the standard Normal density are at ± 0.674.

32. Edgeworth [1913, p. 178] makes the astute observation that while a carefully designed sampling scheme may very well be regarded as yielding a proportion (say) that indeed represents the true proportion that would be found from a census, one must be careful in considering the proportion yielded by a census as representative of proportions at different times and in different places.
33. The Oaks is a race for three-year old thoroughbred fillies over roughly 1 mile, 4 furlongs (the distance varies slightly from one reference to another) or 2,423 metres run annually in Epsom, Surrey, England in June. (A furlong is an eighth of a mile.)
34. For a general discussion of Pearson's system see Ord [1972]. Jacobus Cornelius Kapteyn (1851-1922), an astronomer of considerable note, published several papers on a new theory of skew frequency curves (e.g. Kapteyn [1903] and Kapteyn & van Uven [1916]). One of his investigations was concerned with finding the density of a function of a random variable X where X has a Normal distribution.
35. See Shirras [1935b].
36. Makeham gave his formula, one of particular use in actuarial work, in 1860, with further discussion in 1890. The first form was $\mu_x = A+BC^x$, and this was extended to $\mu_x = A+Hx+BC^x$. Trachtenberg [1924] gave three new formulae for μ_x (perhaps more reminiscent of that given by Bowley), viz. $\exp(\alpha+\beta x+\gamma x^2)$, $(1+\delta x)\exp(\alpha+\beta x+\gamma x^2)$ and $\exp(\alpha+\beta x+\gamma x^2+\delta x^3+\epsilon x^4)$.
37. For instance, $E(X+Y)^2$ may well contain a non-zero covariance term.
38. Kendall & Stuart [1973] show that $Q = (2\omega)/(1+\omega^2)$.
39. See Fisher [1922], Hald [1998, §27.4], Pearson [1923b] and Stigler [1999, Chap. 19] for a discussion of the degrees of freedom to be used in the analysis of contingency tables and in goodness-of-fit tests.
40. Bowley also instances work on smoothing by Persons [1919], and the correlation of differences between successive observations rather than between the observations themselves by Hooker [1905] and Cave-Browne-Cave [1904-5]. (Frances Cave worked for a time with Karl Pearson, and became Director of Studies at Girton College, Cambridge.)

41. Beatrice Mabel Cave-Browne-Cave, (1874-1947), was an applied mathematician who taught mathematics at Clapham High School for eleven years. In 1913 she started working at the Galton Laboratory, University College, under Karl Pearson. Both Beatrice and Frances tended to use simply 'Cave' professionally.
42. Yule sets up the normal equations. He calls the partial correlation coefficient the *net* coefficient of correlation.
43. Keynes [1973, pp. 465-6] did not view Bowley's use of inverse probability in the correlation question with favour. See also Bowley's [1922f] for a review of parts of Keynes's *A Treatise on Probability.*
44. Bowley admits to following Todhunter [1865] here. For a comprehensive discussion of the approximations to the beta integral by Bayes and Price see Dale [2003, §7.3].
45. Bowley cites Edgeworth [1908a, p. 387] as a reference here. Edgeworth noted that 'very generally we are justified in assuming an equal distribution of *a priori* probabilities over that tract of the measurable with which we are concerned' [loc. cit.]: see also Edgeworth [1884] and [1922]. The prior can in fact be made so strong that it completely dominates the data; for example, take the prior $\Pr[\theta = \theta_0] = 1$ (O'Hagan [1994, §3.27]). Slight changes in the prior may well result in considerable changes in the decision: see Berger [1985, p. 111]. What is important is that the likelihood function be concentrated in the 'body' of the prior (Berger, op. cit., p. 233), or, that the likelihood should dominate the prior (Lindley [1965, vol. II, §5.2]).
46. Bowley's method differs from Pearson's in being inductive. (Note that several of the '$\exp(\chi^2/2)$' terms in Pearson's paper should have negative exponents.)
47. Some difficulty is shown to arise in the application of this result in the case of the Normal distribution. For instance, Bowley takes the Normal distribution to be obtained from the Binomial distribution, with $\sigma = \sqrt{pqn}$. For the correct application of the formula it is necessary that both $1/\sqrt{n}$ and the region of integration be finite.
48. Bowley's reference is to Sheppard [1899]. See also Sheppard [1897], and Pearson [1904] (the latter was published as an Editorial, the

journal being 'Edited in consultation with Francis Galton by W.F.R. Weldon, Karl Pearson and C.B. Davenport').

49. This is the expression $K \exp(-\chi^2/2)$ obtained in the derivation of the χ^2 goodness-of-fit test.
50. In a private letter to Bowley on the 7th February 1901 Herbert Foxwell said that in his judgment the *Elements* would be <u>the</u> text-book of statistics for the next generation.
51. Worthington Chauncey Ford (1858-1941) was an American historian. From 1885 to 1889 he was head of the Bureau of Statistics in the United States of America's Department of State, and from 1893 to 1898 he held a similar position in the Treasury Department.
52. Charles Percy Sanger (1871-1930), although a long-standing Fellow of the Royal Statistical Society, was a barrister by profession. Second Wrangler at Cambridge in 1893, he was regarded by Marshall as one of his best pupils, and for some time he lectured in London University.
53. The Preface to the publication of these lectures as Bowley [1903a] was written by 'W.H.', who fortunately seems easier to identify than the 'onlie begetter' of Shakespeare's sonnets. For one William Hughes (1839-1912) was president of the Institute of Actuaries from 1902 to 1904.
54. Some of the terms mentioned in this paragraph may perhaps be unfamiliar to the modern reader: they will be discussed later.
55. Although 'histogram' is listed in the Index to the fifth edition of Bowley's *Elements of Statistics* there is no definition on the cited page.
56. In 1885 Edgeworth called the (sample) variance, the denominator being $n-1$ or n, the *fluctuation*.
57. John Venn described the *median* or *probable error* as 'a technical and decidedly misleading term' [1888, p. 446], while Francis Galton wrote 'The term Probable Error, in its plain English interpretation of the *most* Probable Error, is quite misleading, for it is *not* that' [1889, pp. 58-59]. In the first edition of his *Elements of Statistics* [1901a, p. 282] Bowley suggested that a better term would be *Quartile Deviation*.
58. For a study of de Moivre's derivation of the Normal law see Daw and Pearson [1972].

59. This form of the function with $N/(c\sqrt{\pi})$ is given by several authors (for example, Snedecor [1946] and Yule & Kendall [1950, §8.22]), who prefer the area under the curve to be N rather than 1.
60. See Pearson [1902a] and [1902b].
61. The definition of the correlation coefficient in this way is generally attributed to the French astronomer and physicist Auguste Bravais (1811-1863) in 1846. Various conceptions of correlation, introduced by the English (connected with error theory), the Italian (the theory of connection and concordance) and the Russian (the theory of *á priori* probability) are discussed by Weida [1927-1928]. For a history of the concept of correlation, and the contributions of Bravais, Edgeworth and Galton, see Pearson [1920].

    The 'index of co-relation', or 'Galton function' was re-named the 'coefficient of correlation' by Edgeworth [1892a]. Pearson [1920, p. 33] notes that the modern symbol r is is taken from 'reversion' rather than 'regression'.
62. See Yule [1897] and [1909].
63. Something similar is sometimes mentioned in connexion with experiments in extra-sensory perception: if a subject is asked to identify cards drawn by the experimenter (under appropriate conditions set up to ensure that the subject is not aware in any common way of the actual card drawn), must he, in order to confirm the presence of extra-sensory perception, identify the actual card drawn or can he say 'card N' in draw $M+1$ when card N was in fact drawn in draw M?
64. The reference to the paper by Pearson in the footnote on page 89 should be to Volume 187, not 175, of the *Philosophical Transactions of the Royal Society of London*.
65. The value 0.67 used here is the (approximate) value of z for which $\Phi(z) = 3/4$, where Φ denotes the cumulative distribution function of a standard Normal variate.
66. In June 1919 the Royal Statistical Society set up a committee, with Bowley as chairman, to suggest proposals to improve the 1921 Census [Anon., 1920]. The committee noted that, while many of the suggestions made by the Society in 1908 to improve the 1911 Census had

been adopted, the Census Act had not been made perpetual nor had the suggestion of a quinquennial Census been acted upon. Further, there was no uniformity between the Censuses taken in England and Wales, in Ireland and in Scotland. This lack of uniformity persisted: in 1950 Marian Bowley bemoaned the fact that different methods for the collection and tabulation of data were used in Scotland and in England and Wales, and as recently as 2001 Topalov noted that, in comparison with England and Wales, the Censuses in Scotland and Ireland 'have long been distinct' [op. cit., p. 81].

One of the recommendations of this committee was that some sort of Imperial Census should be taken, and indeed in 1919 a petition was addressed to the British Government urging the establishment of an Imperial Bureau of Census and Statistics (for comments on the reception this petition received from officials see Bowley [1921e]). In 1920 Knibbs published a paper suggesting how such a bureau should be organised, and the matter received considerable and enthusiastic support at a conference of statisticians held under the auspices of the United Kingdom Board of Trade in 1920 [Coats, 1921]. Unfortunately the enthusiasm initially shown by the Dominions for this bureau faded, and nothing came of the idea [Beaud & Prévost, 2005].

67. South Ireland separated from the United Kingdom in 1923.
68. Bowley noted that, at that time, it was important to distinguish the Continental United States from the total, the latter then including Alaska, the Philippines and Cuba.
69. See also Sargant [1866].
70. In the first edition this chapter contained Quetelet's data on the chest measurements of Scottish soldiers. Stigler [2002a] has examined the original data used by Quetelet, from the *Edinburgh Medical and Surgical Journal* of 1817, and finds that three-way contingency tables were actually presented.
71. A quartern-loaf was a four-pound loaf, so called because a quarter of a stone of flour was used in making it. The stone itself was a variable measure, meaning 24lb. (another source says 14lb.) for wool, 22lb. for hay, 16lb. for cheese, 8lb. for fish and meat, etc.
72. The word 'excise' was defined in 1755 by Samuel Johnson as 'a hateful tax levied upon commodities, and adjudged not by the common judges of property, but wretches hired by those to whom excise is

paid'. Today it is less aggressively used for a tax on home goods, either during their manufacture or before they are sold.

73. As a matter of interest, in 1927 the manufacturers of *Lucky Strike* cigarettes aimed an advertising campaign at women, suggesting that they 'reach for a Lucky instead of a sweet'.
74. This review was signed only with Flux's initials. Sir Alfred William Flux, 1867-1942, often commented rather negatively on Bowley's work. One wonders whether there was not some professional rivalry between Bowley and Flux: both delivered Newmarch Lectures at University College, London, both had honorary degrees from Manchester (an LL.D. in Flux's case), they were both knighted, both Presidents of the Royal Statistical Society and both recipients of the Guy Medal in both silver and gold.
75. The review was signed simply 'J.A.F.'.
76. Young (1876-1929), after a varied career including spells at Stanford, Cornell and Harvard, accepted the position of Professor of Economics at the University of London in 1927. A sudden attack of virulent pneumonia led to his early death.

Chapter 8

1. See, for instance, Theil [1960] and [1973].
2. Maunder's bibliography was an updated version of Allen & Buckland [1956].
3. For a study of the early history see Kendall [1969].
4. Kendall [1969] described this monograph as 'well known'!
5. Edgeworth [1896, p. 135] comments that 'it is a well-known proposition that a difference in the system of weights will not make much difference, provided that the number of independent observations is sufficiently great'. Keynes [1983, p. 72 *et seq.*] considers various problems to do with weighting. He finds [p. 78] that there is no one system of weights that is applicable in all cases.
6. Oulton [2006] gives algebraically equivalent expressions, more suitable for calculation, to those given here: $P^L_{rs} = \sum w_{rs}(p_{is}/p_{ir})$, for instance.

7. John Andrew Hamilton (1859-1934), barrister and law lord, was created Baron Sumner in 1913 and became a viscount in 1927. As a young man he was a liberal and supporter of Irish home rule, but in later years he became conservative and vehemently attacked the treaty by which the Irish Free State came into being. As a representative on the reparations committee at the Paris peace talks in 1919 he demanded that Germany pay severe financial penalties.
8. In his Adam Smith Prize-winning essay of 1909 Keynes noted that whereas in Europe price is usually measured 'by naming the number of units of the standard of value which will buy a unit of the commodity' [Keynes, 1983, p. 56], in India it is done by stating how many units of the commodity can be bought by a unit of currency. Thus an arithmetic mean in Europe corresponds to a harmonic mean in India.
9. Keynes proposed a rather more general form of index number in 1909:

> An index number is in itself no more than the measure of the magnitude of an object at one time or in one place in terms of the magnitude of the same or a similar object at another time or in another place. If we are dealing with a series of quantities $f(t_1)$, $f(t_2)$, each of which has the same relation to different moments of time or parts of space or classes of objects, and which are all numerically measurable in terms of a common unit, then the ratio $100f(t_2)/f(t_1)$ is defined as the index number of f for the time or place or class t_2 referred to its value for t_1 as base. [Keynes, 1983, pp. 52-53]

10. Apparently a Type I distribution was found 'unworkable'. Pearson's Type I (the beta) distribution has density function

$$f(x) = [1/B(p,q)]^{-1}\, x^{p-1}(1-x)^{q-1}, \quad x \in [0,1]$$

while the Type III (or gamma) distribution has density

$$f(x) = [1/\Gamma(\lambda)]^{-1}\, x^{\lambda-1}\, e^{-x}, \quad x > 0.$$

11. Bowley had mentioned the first of these in his *Elements of Statistics* as early as 1901. In this same work he also discussed the 'statistical

approach' [Diewert, 1993, p. 37], in which increases in the supply of money result in proportionate increases in prices, ignoring random fluctuations.

12. Letting $A = \sum y_i/n$ Bowley writes $y_i = A(1 + d_s)$. Then

$$G = A\exp(-\mu_2/2 + \mu_3/3 + \cdots) \approx A(1 - \mu_2/2),$$

where μ_2 is the mean square of the d_i's.

13. I_1 and I_2 are the Laspeyres and Paasche indices mentioned earlier.
14. Frisch [1936, §6] finds Bowley's approximation to be unsatisfactory because of Bowley's not having carried the Taylor approximation far enough. In his [1938a] Bowley discussed the relationship between his series and that given by Frisch, with particular attention to the bias involved.
15. In a footnote Bowley states that he now regards the evaluation of the index in his [1919b] as incorrect.
16. To illustrate the effect even *one* erroneous measurement may have on an index number Keynes [1983, p. 95] notes that the index published by the Commercial Intelligence Department of the Indian Government had for many years, and for no obvious reason, included spelter (an impure form of zinc). Spelter, according to Keynes, was of no importance in India and its price had risen considerably over the years in which index numbers had been calculated. He notes further that the index would be depressed by 4% were spelter to be excluded.
17. Sauerbeck used the index (in our notation) $\sum p_i Q_i / \sum P_i Q_i$ for years earlier than 1866-77 and the index $\sum p_i q_i / \sum P_i q_i$ after that period.
18. On the rôle of probability in index numbers see Edgeworth [1925c].
19. Edgeworth [1925b, p. 195], writing of the determination of the change in the money value of articles consumed by a population says that the most refined form of this standard compares amounts of money required to produce the same *satisfaction* at different times.
20. Edgeworth [1925b, p. 329] writes of the 'principle of ponderation'.
21. See also Marshall [1887].

Chapter 9

1. This passage was drawn to the attention of the statistical community by Missiakoulis [2006].
2. Wright [2001] has provided a survey of the development of sampling from its use by Pierre-Simon Laplace to estimate the population of France on the 22nd September 1802 (following on from a plan he published in 1786) to the 1983 National Health and Nutrition Examination Survey on the levels of lead in human blood in the United States of America. See also Laplace [1820] and Edgeworth [1918, p. 187].
3. For a discussion of early uses of sample surveys see Stephan [1948].
4. In the article on sampling in the *International Encyclopedia of Statistics* Alan Stuart points out quite firmly that there is no such thing as "representative," "unbiased," or "fair" sampling: these and similar adjectives are strictly applicable to the sampling *process* and not to the sample itself [Kruskal and Tanur, 1978, p. 886].

   Earlier G.P. Watkins had expressed similar unhappiness in his review of Harald Westergaard's *Scope and Method of Statistics*:

   > The reference to sample statistics as "representative statistics" seems to me infelicitous. The adjective should refer to function and quality rather than to a situation and a numerical relation in which the quality is chiefly assumed, the only reason for alleging that samples are representative being that their selection is presumed to be unbiased. [1916, pp. 289-290]

   The work by Gini and Galvani in 1929, described by Koop as 'commendably frank' [1979, p. 253], showed that proper randomisation was needed to avoid selective bias.
5. For a study of Kiaer's work on sampling see Bellhouse [1988]. On the development of the ideas and methods of representative sampling in Russia, and the possible anticipation there of Kiaer's ideas, see Žarkovič [1956] and [1962], Kruskal & Mosteller [1980, p. 174] and Seneta [1985].

6. For a clear summary of Bowley's work on sampling, in particular on this memorandum, see Aldrich [2005], [2008] and [2009], Bellhouse [1988], Chang [1976], Hansen [1987], Hansen et al. [1985], Jensen [1928], Kruskal & Mosteller [1980] and Smith [1976].

   In his address to the International Congress of Mathematicians at Toronto in 1924 Bowley said:

   > I distinguish the representative method from the method of pure sampling in that the rule of equal chance of inclusion for every person or thing in the universe is not obeyed, but whole sections or classes are chosen which separately or grouped together are held by some criterion to be representative of the whole. [1928e, p. 919]

7. It was in this paper that Neyman (originally Splawa-Neyman) expounded his theory of confidence intervals. Neyman began this paper by paying tribute to the work of the International Statistical Institute and especially 'the personal achievements of Professor A.L. Bowley', and it was thus in a 'somewhat ungrateful' vein, as Bowley himself said, that in the discussion of the paper he had to criticise the theory of probability expounded by Bowley and related to that adopted by Fisher. Further, Bowley said 'I am not at all sure that the "confidence" is not a "confidence trick" ' [Neyman, 1934, p. 609]. Kruskal and Mosteller [1980, p. 187] suggest that the idea of a confidence interval was very nearly reached in Bowley's 1926 memorandum. 'The trouble with confidence interval statements is that they are consistent for any form of random sampling' [Smith, 1976, p. 185].

   In 1935 Sukhatme investigated two methods of selecting a random sample from a stratified population. He concluded that Neyman's method yielded a more precise estimate than the method of proportional sampling.

8. The conclusions of the report delivered by the commission are given in Yates [1946].

9. The reader must be warned that there are several misprints in the original paper.

10. This Bowley finds 'almost co-extensive' with the type of mathematical probability used in Whitworth's *Choice and Chance*.

11. This system Bowley finds to correspond to Poisson's scheme for sampling balls from a number of urns containing balls of different colours and in different proportions.

Chapter 10

1. Philip Mirowski found it curious 'that many of the leading lights of marginalism were also instrumental in the development of probability theory and statistics: Jevons, Edgeworth, Bowley, Keynes, Slutsky and Wald, only to name the most illustrious' [1989, p. 222].
2. As recently as March 2009 Andrew Reamer bemoaned the visible deterioration of the economic statistics system, attributing it in the main to inadequate funding.
3. Ménard suggested 'that the analogy with classical mechanics was mainly responsible for the ignorance of the possibilities offered by probability in economic theory' [1987, p. 144].
4. In 1938 Ronald Fisher noted, with dismay, the wide separation between official and academic statistics in England. Fisher also repeated, from memory, 'witty comments' that Bowley had drafted for an ideal footnote in criticism of official data.
5. Howie [2002, p. 186] comments that *The Mathematical Groundwork of Economics* clearly marked Bowley as one of the founders of mathematical economics.
6. Both Edgeworth [1924, p. 432] and Wicksell [Lindahl, 1958, p. 210] find this an unhappy choice of nomenclature; it might, for example, be confused with a person's supply curve.
7. Figure 10.1 shows what has become known as the *Edgeworth-Bowley Box*, though Pareto's name—and possibly the names of others—should more correctly be associated with it. Its introduction is usually attributed to Edgeworth's *Mathematical Psychics* [1881, pp. 28, 114], though Tarascio [1972] notes that while the idea is perhaps *suggested* there, Edgeworth's graphs cannot be transformed by geometric manipulation into the diagram we know today. For a detailed discussion of the early history of the box diagram see Humphrey [1996] and Weatherby [1976].
8. In his review of the *Groundwork* in 1925 Wicksell suggested that Bowley's curve ${}_1V(x, y) = 0$ should rather be ${}_1V(x, y) = {}_1V(0, 0)$,

since the former might be taken as implying that A had nothing at the beginning of the exchange (see Darnell [1982, p. 165]). Darnell finds that this difference of opinion is caused by the different approaches Bowley and Wicksell had to consumer theory. 'Bowley scaled A's utility index so that the endowed bundle yielded zero utility' [Darnell, 1982, p. 168].

9. Darnell [1982, p. 169] notes that this passage should be understood as saying that A and B exchange quantities between themselves and the other $n - 2$ consumers, and *not* simply as between themselves.
10. See Bowley's [1937b] on the elasticity of substitution.
11. 'Disutility' is much the same as 'utility', but with a negative sign.
12. Inspired by Wicksell's review of 1925, Bowley published a short note on bilateral monopoly in 1928. See also Fountain [1980].
13. The matter of *consumer's* surplus was considered in Bowley [1924c].
14. Darnell [1982, pp. 172-3] finds Bowley's solution to be seriously flawed.
15. How new this was is doubtful: see Cajori [1993, vol. II, §§577, 613].
16. Wicksell's paper is available in English in Lindahl [1958]. The original Swedish version contained a list of errors that is not given in Lindahl: it may however be found in Darnell [1982].
17. For other reviews see Crum [1925], Hankins [1924], Persons [1925a], Sheppard [1925] and Tappan [1925].

Chapter 11

1. Charles Samuel Jackson, better known as 'Slide-rule Jackson' (1867-1916), was Eighth Wrangler at Cambridge (Trinity College) in 1889, and took a First Class in the Law Tripos in 1890. Jackson was a private pupil of the Cambridge mathematical 'coach' Robert Webb (1850-1936), who also coached Bertrand Russell.
2. George Greenhill (1847-1927) spent most of his working life as a professor at the Royal Artillery College in Woolwich.
3. The issue of how much, and what, mathematics the student of statistics should be taught is a thorny one. Many lecturers would agree on the importance of probability theory; for instance, Hotelling wrote

> Without probability theory, statistical methods are of minor value, for although they may put data into forms from which intuitive inferences are easy, such inferences are very likely to be incorrect. [1949, p. 25]

In his discussion of Bartlett [1940] Bowley asserted that the author seemed to limit the term 'statistics' to the mathematics of sampling, and he warned against thinking that this meant that logical and analytic aspects of probability had to be mastered before mathematics could be used by statisticians.

4. Bowley suggests that the phrase 'probable error' be dropped from descriptive statistics and used only, if at all, when error theory is concerned. 'Quartile deviation' is preferable.
5. See, for instance, Yule [1903].
6. Bowley notes the importance of choosing a sufficiently large sample if the attribute under consideration is rare.
7. The usual expressions for $\sum_{k=1}^{n} k^r$, $r \in \{1, 2, 3\}$ are given. For a comprehensive study of such series in general see Edwards [1986].
8. Boyle's Law, first published by Robert Boyle (1627-1691) in 1662, states that the volume of a gas is proportional to its temperature, the pressure remaining constant. His coevals to whom the law is sometimes attributed (at least in part) include Robert Hooke, Edmé Mariotte, Henry Power and Richard Towneley (see West [1999]). Boyle himself actually referred to 'Mr. Towneley's hypothesis'.
9. Bowley uses the notation $a \sim b$, attributed by Cajori [1993] to John Wallis, *Operum mathematicorum pars prima* (1657), for our modern $|a - b|$.
10. The notation '$\lim\limits_{n \to 0}$' was first used by G.H. Hardy in print in 1908 (see Cajori [1993]).
11. It is well known that the multiplication of two convergent infinite series does not necessarily result in a convergent series, though $S = S_1 \times S_2$ if all three series are convergent (see Knopp [1951, §§17, 45]).
12. A non-negative, continuous function $f \not\equiv 0$ on some appropriate structure S (e.g. a Borel subset of $\Re$) that satisfies the Cauchy functional equation $f(x + y) = f(x)f(y)$ for all $x, y \in S$, is said to be

an *exponential function.* For a discussion of the uniqueness of the solution of such an equation see, for instance, Ramachandran & Lau [1991].

13. Quantities that can be expressed as m or m/n where m and n are integral are said to be *commensurable.*
14. Lamb [1956], who also introduces e^x via $E(x)$, notes too that the former is undefined for x irrational, and proposes to define it for any x as the sum of the series $1 + x + x^2/2! + x^3/3! + \cdots$.
15. Does this perhaps not suggest infinitesimals and non-standard analysis?
16. There is a clear case here for using de Morgan's word *comminuent* (see de Morgan [1842, p. 66]) instead of Bowley's circumlocution 'when an increment of x is diminished to zero, the corresponding increment of y also diminishes to zero' [1913a, p. 202].
17. According to Green [1976] Abraham de Moivre (1667-1754) stated and used this result, in various forms, in his writings from 1707 to 1722.
18. This book was listed in the 'New Publications' in the *Bulletin of the American Mathematical Society* (1913), 20, No. 3 under the heading 'Elementary Mathematics'. Although published in 1913 the book has a list of corrigenda dated May 1914.
19. The reference is no doubt to E.W. Hobson's *The Theory of Functions of a Real Variable and the Theory of Fourier's Series* of 1907.
20. Edmund Taylor Whittaker (1873-1956) attended Trinity College, Cambridge, where he was bracketed second Wrangler in 1895. He occupied the chair of astronomy at the University of Dublin (at which time he was also Royal Astronomer of Ireland), and later became professor of mathematics at Edinburgh. His magisterial text *A Course of Modern Analysis* (the first edition was by Whittaker alone: in later editions G.N. Watson was co-author) was first published in 1902. It remained in print throughout the twentieth century, and is still useful today.
21. Godfrey Harold Hardy was fourth Wrangler in the Mathematical Tripos at Cambridge in 1898. After a spell at Oxford in the Savilian Chair of Geometry, Hardy returned to Cambridge and the Sadleirian

Chair. He is remembered for his work in number theory and the theory of functions, and particularly for his discovery of, and collaboration with, the Indian mathematician Srinivasa Ramanujan (1887-1920).

22. The word 'apology' here appears as a justification, vindication or, better, *defence* of mathematics—or even of Hardy's own distinguished career: there is no suggestion of any sense of 'excuse'.

Chapter 12

1. Following hard on the heels of Bowley's obituary of Cannan was one by Lionel Robbins, who had been a student of Cannan's. This is still worth reading today, perhaps particularly for the description of the way in which Cannan lectured and held classes: he was not a 'good' lecturer, and would have been viewed with grim displeasure in modern universities where 'teaching' is viewed as all important. Yet the *attitude* he managed to convey had a profound influence on his students.
2. Bowley's obituary of Wood was followed by a more personal tribute by Edmund Rhodes [1945].
3. See also Bowley [1899e].
4. For Fisher's reply, and Bowley's subsequent response, see Fisher [1923].
5. It is only fair to Mallock to note that Lord Stamp wrote a less harsh review of the work in 1914.
6. Yule's book was the predecessor of the well-known volumes later published by Yule and Kendall, and still later by Kendall and other prominent statisticians, and popularly known today—if somewhat inaccurately—as *Kendall's Advanced Theory of Statistics*.
7. Taken from Logan [1950, p. 132].
8. Quotations from this lecture are from the two-page typed version in the Bowley Archives in the LSE (COLL MISC 0772).
9. In his discussion of a paper by Ernest Snow, William Shaw put in 'a good word for the old method of the graph' [Snow, 1923, p. 381], finding its only demerit perhaps to be that it made more suggestions than it ought.

10. Bowley mentioned specifically work by Raymond Pearl (e.g. Pearl and Reed [1920]) and Udny Yule [1925].
11. Bevin was at one time General Secretary of the Transport and General Workers' Union. During the second World War he was Labour Minister in the Churchill government, and subsequently, in the post-war Labour government under Clement Attlee, he held the position of Foreign Secretary.
12. Richard Barham, *The Ingoldsby Legends*, 'Singular passage in the life of the late Henry Harris, Doctor in Divinity'.
13. Apparently during the proceedings the *Daily News* carried a photograph of Bowley's ten-year-old daughter (probably Agatha), and noted that, even on that weekly budget, she could afford to keep guinea pigs (see Atkinson [1987, p. 751]).
14. Beatrice Webb, whose father had genuinely believed that women were superior to men and had accordingly given his daughters (nine of them) a sound intellectual training, complained during the second world war about the shortage of domestic staff, saying 'If only I had been brought up to know how to cook and clean' [Webb, Vol. 4, p. 479].
15. In his *Prices and Wages in the United Kingdom, 1914-1920* Bowley noted that in this inquiry 'the workers' representatives were prepared to sacrifice both local and occupational differences of pay in order to obtain a universal satisfactory minimum, and it appeared that many of these differences were the results of custom rather than of economic forces' [Bowley, 1921h, p. 108].

Bibliography

While we have been fairly comprehensive in our listing of Arthur Bowley's works the reader is referred for further details to the Select Bibliography in Allen and George [1957] and the holdings in the Archives of the London School of Economics and Political Science.

Ainsworth, R.B. 1950. 'The sources and nature of statistical information in special fields of statistics. United Kingdom labour statistics'. *Journal of the Royal Statistical Society* A, 113, No. 1, 37-49.

Aldrich, J. 2005. 'The statistical education of Harold Jeffreys'. *International Statistical Review* 73, No. 2, 289-307.

Aldrich, J. 2005/2007. Six reviews of the 1st edition of R.A. Fisher's *Statistical Methods for Research Workers.* http://www.economics.soton.ac.uk

Aldrich, J. 2008. 'Professor A.L. Bowley's theory of the representative method'. Discussion Papers in Economics and Econometrics, No. 0801, University of Southampton.

Aldrich, J. 2009. 'Sample surveys and statistical theory: the work of A.L. Bowley'. Preprint.

Allen, R.G.D. 1956. *Mathematical Economics.* London: Macmillan and Co.

Allen, R.G.D. 1957. 'Arthur Lyon Bowley, 1869-1957'. *Review of the International Statistical Institute* 25, No. 1/3, 181-182.

Allen, R.G.D. 1968. 'Bowley, Arthur Lyon'. *International Encyclopedia of the Social Sciences*, vol. 2, pp. 134-137. D.L. Sills (Ed.). New York: Macmillan. Reprinted in 1971 in the *International Encyclopedia of Statistics.* W. Kruskal & J.M. Tanur (Eds). New York: Free Press.

Allen, R.G.D. 1971. 'Bowley, Sir Arthur Lyon'. *Dictionary of National Biography, 1951-1960.* Oxford University Press.

Allen, R.G.D. 1972. Review of A. H. Bowley [1972]. *Journal of the Royal Statistical Society* A 135, No. 4, 628-629.

Allen, R.G.D. & Bowley, A.L. 1935. *Family Expenditure: a study of its variation.* London: P.S. King & Son.

Allen, R.G.D. & George, R.F. 1957. 'Professor Sir Arthur Lyon Bowley: November 6th, 1869–January 21st, 1957'. *Journal of the Royal Statistical Society* A 120, No. 2, 236-241.

Anon. 1838. Introduction. *Journal of the Statistical Society of London* 1, No. 1, 1-5.

Anon. 1913. Review of Bowley [1913a]. *The Mathematical Gazette* 7, No. 108, 214-215.

Anon. 1920. 'Report on the census'. *Journal of the Royal Statistical Society* 83, No. 1, 134-139.

Anon. 1921. Review of the 4th (1920) edition of Bowley [1901a]. *Journal of the Royal Statistical Society* 84, No. 1, 125-126.

Anon. 1929. Review of the 4th (1928) edition of Bowley [1910a]. *Journal of the Royal Statistical Society* 92, No. 1, 136.

Anon. 1934. 'The centenary of the Royal Statistical Society'. *Journal of the Royal Statistical Society* 97, No. 3, 467-472.

Anon. 1935. 'The Royal Statistical Society'. *Journal of the Royal Statistical Society* 98, No. 1, 140-151.

Anstey, V. 1977. *The Economic Development of India.* New York: Arno Press.

Armatte, M. 2008. 'Lucien March (1859-1933): une statistique mathématique sans probabilité?' *Courrier des Statistiques* 123, 5-12.

Astor, J.J., Bowley, A.L., Clay, H., Grant, R., Layton, W.T., Pybus, P.J., Seebohm Rowntree, B., Schuster, G. & Stuart, F.D. 1923. *The Third Winter of Unemployment: The Report of an Enquiry undertaken in the Autumn of 1922.* London: P.S. King and Son, Ltd.

Astor, J.J., Bowley, A.L., Clay, H., Grant, R., Layton, W.T., Pybus, P.J., Seebohm Rowntree, B., Schuster, G. & Stuart, F.D. 1924. *Is Unemployment Inevitable? An Analysis and a Forecast. A Continuation of the Investigations embodied in* The Third Winter of Unemployment. London: Macmillan.

Atkinson, A.B. 1987. 'On the measurement of poverty'. *Econometrica* 55, No. 4, 749-764.

Atkinson, A.C. & Fienberg, S.E. 1985. *A Celebration of Statistics: The ISI Centenary Volume*. New York: Springer-Verlag.

Barham, R.H. 1910. *The Ingoldsby Legends*. 2nd edition. London: Oxford University Press.

Barnett, V.D. 1994. 'Aliis Exterendum and beyond!'. *Royal Statistical Society News* 21, No. 9, 1-2.

Barrow-Green, J. 1999. 'A corrective to the spirit of too exclusively pure mathematics': Robert Smith (1689-1768) and his prizes at Cambridge University'. *Annals of Science* 56, 271-316.

Bartholomew, D.J. 1995. 'What is statistics?'. *Journal of the Royal Statistical Society* A, 158, No. 1, 1-20.

Bartlett, M. 1940. 'The present position of mathematical statistics'. *Journal of the Royal Statistical Society* 103, No. 1, 1-29.

Barton, D.M. 1919. 'The course of women's wages'. *Journal of the Royal Statistical Society* 82, No. 4, 508-553.

Bastable, C.F. et al. 1903e. Letter in *The Economic Journal* 13, No. 51, 446-448.

Beaud, J.-P. & Prévost, J.-G. 2005. 'Statistics of the science of government: the stillborn British Empire statistical bureau, 1918-20'. *The Journal of Imperial and Commonwealth History* 33, No. 3, 369-391.

Bellhouse, D.R. 1988. 'A brief history of random sampling methods', pp. 1-14 in Krishnaiah & Rao [1988].

Berger, J.O. 1985. *Statistical Decision Theory and Bayesian Analysis*. 2nd edition. New York: Springer-Verlag.

Bibby, J. 1986. '1786-1986: Two centuries of teaching statistics'. Paper read at the Second International Conference on Teaching Statistics (ICOTS 2).

Booth, C. 1887. 'The inhabitants of Tower Hamlets (School Board Division), their condition and occupations'. *Journal of the Royal Statistical Society* 50, No. 2, 326-401.

Bowden, W. 1939. Review of Bowley [1937f]. *The American Historical Review* 44, No. 3, 620-621.

Bowley, A.H. 1972. *A Memoir of Professor Sir Arthur Bowley (1869-1957) and his Family*. Printed privately.

Bowley, A.L. 1894. Review of H. de B. Gibbins's *British Colonies and Commerce*. *The Economic Journal* 4, No. 13, 75-76.

Bowley, A.L. 1895a. 'Changes in average wages (nominal and real) in the United Kingdom between 1860 and 1891'. *Journal of the Royal Statistical Society* 58, No. 2, 223-285.

Bowley, A.L. 1895b. 'Comparison of the rates of increase of wages in the United States and in Great Britain, 1860-1891'. *The Economic Journal* 5, No. 19, 369-383.

Bowley, A.L. 1896a. Comments on the 'Abstract of Labour Statistics, 1894-5'. *The Economic Journal* 6, 23, 465-470.

Bowley, A.L. 1896b. Review of J. Bertillon's *Cours élémentaire de Statistique administrative – élaboration des statistiques – organisation des bureaux de statistique – éléments de Démographie*. *The Economic Journal* 6, No. 21, 102-103.

Bowley, A.L. 1897a. 'Import and export index-numbers'. *The Economic Journal* 7, No. 26, 274-278.

Bowley, A.L. 1897b. 'Changes of prices of imports and exports since 1881'. *Journal of the Royal Statistical Society* 60: 437-439.

Bowley, A.L. 1897c. 'Relations between the accuracy of an average and that of its constituent parts'. *Journal of the Royal Statistical Society* 60, No. 4, 855-866. [For errata see the *Journal of the Royal Statistical Society* 63, No. 2, 315.]

Bowley, A.L. 1898a. Review of M.G. Mulhall's *Industries and Wealth of Nations*. *The Economic Journal* 8, No. 29, 94-95.

Bowley, A.L. 1898b. 'Comparison of the changes in wages in France, the United States, and the United Kingdom, from 1840 to 1891'. *The Economic Journal* 8, No. 32, 474-489.

Bowley, A.L. 1898c. 'The statistics of wages in the United Kingdom during the last hundred years. (Part I.) Agricultural Wages'. *Journal of the Royal Statistical Society* 61, No. 4, 702-722.

Bowley, A.L. 1899a. 'The statistics of wages in the United Kingdom during the last hundred years. (Part I.) Agricultural Wages. Contd. Scotland'. *Journal of the Royal Statistical Society* 62, No. 1, 140-150.

Bowley, A.L. 1899b. 'The statistics of wages in the United Kingdom during the last hundred years. (Part III.) Agricultural Wages. Contd. Ireland.' *Journal of the Royal Statistical Society* 62, No. 2, 395-404.

Bowley, A.L. 1899c. 'The statistics of wages in the United Kingdom during the last hundred years. (Part IV.) Agricultural Wages. Concluded. Earnings and General Averages'. *Journal of the Royal Statistical Society* 62, No. 3, 555-570.

Bowley, A.L. 1899d. Review of C.D. Wright's *Outline of Practical Sociology. The Economic Journal* 9, No. 36, 555-557.

Bowley, A.L. 1899e. 'Wages in the United States and Europe'. *The Economic Journal* 9, No. 33, 136-140. [Review of the *Bulletin of the Department of Labor*, Washington, No. 18, September 1898.]

Bowley, A.L. 1899f. Review of *The Federal Census. Critical Essays by Members of the American Economic Association.* Collected and edited by a Special Committee. *The Economic Journal* 9, No. 35, 428-432.

Bowley, A.L. 1899g. 'Wages, nominal and real'. *Dictionary of Political Economy*, vol. 3, ed. R.H. Inglis Palgrave. 640-651.

Bowley, A.L. 1900a. 'The statistics of wages in the United Kingdom during the last hundred years. (Part VI.) Wages in the Building Trades.— English Towns'. *Journal of the Royal Statistical Society* 63, No. 2, 297-315.

Bowley, A.L. 1900b. 'The statistics of wages in the United Kingdom during the last hundred years. (Part VII.) Contd. Scotland and Ireland'. *Journal of the Royal Statistical Society* 63, No. 3, 485-497.

Bowley, A.L. 1900c. *Wages in the United Kingdom in the Nineteenth Century. Notes for the use of students of social and economic questions.* London: Cambridge University Press.

Bowley, A.L. 1901a. *Elements of Statistics.* London: P.S. King & Son, Ltd.

Bowley, A.L. 1901b. Review of R. Benini's *Principii di Demografia. The Economic Journal* 11, No. 42, 216-217.

Bowley, A.L. 1901c. 'The statistics of wages in the United Kingdom during the last hundred years. (Part VIII.) Wages in the Building Trades— Concluded'. *Journal of the Royal Statistical Society* 64, No. 1, 102-112. [This Part contains, on p. 106, a list of errata in Parts VI and VII.]

Bowley, A.L. 1902a. Review of F.H. Giddings's *Inductive Sociology; a Syllabus of Methods, Analyses and Classifications, and provisionally formulated Laws. The Economic Journal* 12, No. 47, 376-377.

Bowley, A.L. 1902b. 'The statistics of wages in the United Kingdom during the last hundred years. (Part IX.) Wages in the Worsted and Woollen Manufactures of the West Riding of Yorkshire'. *Journal of the Royal Statistical Society* 65, No. 1, 102-126. [This is also listed as 'Report on the West Riding of Yorkshire', *British Association for the Advancement of Science* 1902, pp. 5-9.]

Bowley, A.L. 1902c. Review of H. Westergaard [1901]. *The Economic Journal* 12, No. 45, 92-94.

Bowley, A.L. 1902d. Review of J.P. Norton's *Statistical Studies in the New York Money Market, preceded by a Brief Analysis under the Theory of Money and Credit, with Statistical Tables, Diagrams, and Folding Chart. The Economic Journal* 12, No. 48, 516-518.

Bowley, A.L. 1902e. 'Wages in York in 1899'. From Mr. B.S. Rowntree's investigations. Communicated by A.L. Bowley, M.A. *Journal of the Royal Statistical Society* 65, No. 2, 359-361.

Bowley, A.L. 1903a. *The Measurement of Groups and Series: A Course of Lectures.* London: Charles and Edwin Layton.

Bowley, A.L. 1903b. 'The prices of imports and exports of the United Kingdom and Germany'. *The Economic Journal* 13, No. 52, 628-632.

Bowley, A.L. 1903c. 'Statistical methods and the fiscal controversy'. *The Economic Journal* 13, No. 51, 303-312.

Bowley, A.L. 1903d. 'Notes on Mr. Wilson Fox's paper'. *Journal of the Royal Statistical Society* 66: 598-601.

Bowley, A.L. 1903e. Letter by Bastable, Bowley et al. *The Economic Journal* 13, No. 51, 446-448.

Bowley, A.L. 1903f. Review of W. Lexis's *Abhandlungen zur Theorie der Bevölkerungs- und Moralstatistik. The Economic Journal* 31, No. 50, 233-235.

Bowley, A.L. 1904a. *Statistical Studies relating to National Progress in Wealth and Trade since 1882; a plea for further inquiry.* London: P.S. King & Son, Ltd.

Bowley, A.L. 1904b. 'Tests of national progress'. *The Economic Journal* 14, No. 55, 457-465.

Bowley, A.L. 1904c. Review of S.J. Chapman's *Work and Wages. Part I. Foreign Competition. The Economic Journal* 14, No. 56, 612-616.

Bowley, A.L. 1905a. Review of A. Liesse's *La Statistique: ses difficultés, ses procédés, ses résultats. The Economic Journal* 15, No. 59, 412-413.

Bowley, A.L. 1905b. Review of Edgeworth [1905]. *Journal of the Royal Statistical Society* 68, No. 4, 747-750.

Bowley, A.L. 1905c. 'Note on the statistics of the woollen industries'. *The Economic Journal* 15, No. 60, 584-590.

Bowley, A.L. 1905d. *A Short Account of England's Foreign Trade in the Nineteenth Century, its Economic and Social Results.* London: George Allen & Unwin, Ltd.

Bowley, A.L. 1905e. 'Economics in South Africa'. *The Economic Journal* 15, No. 60,590-595.

Bowley, A.L. 1906a. Review of M. Igier's *Des Retenues sur les Salaires opérées pour le Compte du Patron. Journal of the Royal Statistical Society* 69, No. 2, 455-457.

Bowley, A.L. 1906b. 'Address to the Economic Science and Statistics Section of the British Association for the Advancement of Science'. *Journal of the Royal Statistical Society* 69, No. 3, 540-558.

Bowley, A.L. 1906c. Review of E. von Halle's *Die Weltwirtschaft. Ein Jahr- und Lesebuch. I. Teil, Internationale Ubersichten; II. Teil, Deutschland. The Economic Journal* 16, No. 64, 605.

Bowley, A.L. 1906d. Review of H.Hauser's *Ouvriers du Temps Passé (*15^e–16^e *siècles). Journal of the Royal Statistical Society* 69, No. 3, 603-604.

Bowley, A.L. 1906e. Review of E. Blaschke's *Vorlesungen über Mathematische Statistik (Die Lehre von den Statistichen Masszahlen). Journal of the Royal Statistical Society* 69, No. 4, 757.

Bowley, A.L. 1906f. Review of Ch. Henry's *Instituts Solvay. Travaux de l'Institut de Sociologie. Notes et Mémoires. Fascicule 6. Mesure des Capacités Intellectuelle et Energique. Journal of the Royal Statistical Society* 69, No. 4, 758-759.

Bowley, A.L. 1906g. Review of J.C. Smith's *Inter-Temporary Values, or the Distribution of the Produce in Time. Journal of the Royal Statistical Society* 69, No. 4, 762-763.

Bowley, A.L. 1906h. Review of *Imports and Exports at Prices of 1900. Journal of the Royal Statistical Society* 69, No. 4, 772-773.

Bowley, A.L. 1906i. Review of W.H. Dawson's *The German Workman: a Study in National Efficiency. Journal of the Royal Statistical Society* 69, No. 3, 601-603.

Bowley, A.L. 1907a. Review of P.N. Pinkus's *Das Problem des Normalen in der Nationalökonomie. Beitrag zur Erforschung der Störungen im Wirtschaftsleben. The Economic Journal* 17, No. 65, 113.

Bowley, A.L. 1907b. Review of E. von Halle's *Die Weltwirtschaft. Ein Jahr- und Lesebuch. I. Jahrgang, 1906. III. Teil, Das Ausland. The Economic Journal* 17, No. 66, 267.

Bowley, A.L. 1907c. Review of *Album graphique de la Statistique générale de la France. Journal of the Royal Statistical Society* 70, No. 3, 489-490.

Bowley, A.L. 1907d. Review of E. Lauterbach's *Die Staats- und Kommunal-Besteuerung in Deutschland, England, den Vereinigten Staaten von Nordamerika und den englischen Kolonien. Journal of the Royal Statistical Society* 70, No. 2, 358-359.

Bowley, A.L. 1907e. Review of the Gainsborough Commission's *Life and Labour in Germany. Journal of the Royal Statistical Society* 70, No. 2, 347-349.

Bowley, A.L. 1907f. Review of H.R. Seager's *Introduction to Economics. Journal of the Royal Statistical Society* 70, No. 1, 143-144.

Bowley, A.L. 1907g. Review of M.G. Spencer & H.J. Falk's *Employment Pictures from the Census. Journal of the Royal Statistical Society* 70, No. 1, 134-135.

Bowley, A.L. 1907h. Review of W. Palin Elderton's *Frequency-Curves and Correlation. Journal of the Royal Statistical Society* 70, No. 1, 132-134.

Bowley, A.L. 1907i. Review of F. Simiand's *Le Salaire des Ouvriers des Mines de Charbon en France. Journal of the Royal Statistical Society* 70, No. 3, 508-509.

Bowley, A.L. 1908a. Review of S.J. Chapman's *Work and Wages. II. Wages and Employment. The Economic Journal* 18, No. 70, 288-290.

Bowley, A.L. 1908b. 'The improvement of official statistics'. *Journal of the Royal Statistical Society* 71, No. 3, 459-495.

Bowley, A.L. 1908c. Review of E.G.F. Gryzanovski's *On Collective Phenomena and the Scientific Value of Statistical Data. The Economic Journal* 18, No. 71, 460.

Bowley, A.L. 1908d. Review of R. Calwer's *Das Wirtschaftsjahr, 1905. Jahresberichte über den Wirtschaftsund Arbeitsmarkt. The Economic Journal* 18, No. 71, 459.

Bowley, A.L. 1908e. ' Wages and cost of living in Germany: the Board of Trade and Mr. Ellis Barker'. *The Economic Journal* 18, No. 72, 657-662.

Bowley, A.L. 1908f. Review of 'Amalgamated Society of Railway Servants. Report giving Statement of the Results of the Census of Wages, Hours of Labour, &c., of Men employed on the Railways of the United Kingdom.' *Journal of the Royal Statistical Society* 71, No. 1, 213-214.

Bowley, A.L. 1908g. Review of M. Meyer's *Statistik der Streiks und Aussperrungen im In- und Auslande. Journal of the Royal Statistical Society* 71, No. 4, 711-712.

Bowley, A.L. 1908h. Review of *The Central (Unemployed) Body for London. 1. Preliminary Report. 2. Second Report. 3. Special Committee on Unskilled Labour. Report and Minutes of Evidence. Journal of the Royal Statistical Society* 71, No. 4, 709-711.

Bowley, A.L. 1908i. 'Change of kinetic energy due to mutual action of two particles, and loss of kinetic energy by collision'. *The Mathematical Gazette* 4, No. 74, 327.

Bowley, A.L. 1909a. 'A suggestion for the international comparison of wages by the use of the median'. *Journal of the Royal Statistical Society* 72, No. 4, 718-721.

Bowley, A.L. 1909b. Review of E. von Halle's *Die Weltwirtschaft: Ein Jahr- und Lesebuch.* (III. Jahrgang, 1908. I. and II. Teil.) *The Economic Journal* 19, No. 75, 449.

Bowley A.L. 1909c. 'Comparaison internationale des salaires a l'aide de la mediane'. *Bulletin de l'Institut International de Statistique* 18(I), 278-282.

Bowley, A.L. 1909d. Review of M. Lazard's *Le chômage et la profession. Contribution à l'étude statistique du chômage et de son coefficient professional. Journal of the Royal Statistical Society* 72, No. 2, 412-414.

Bowley, A.L. 1909e. Review of D. Zolla's (preface by A. Lery-Beaulieu) *La grève, les salaires et le contrat de travail. Journal of the Royal Statistical Society* 72, No. 1, 135-136.

Bowley, A.L. 1909f. Review of F. Žižek's *Die Statistichen Mittelwerte. Eine methodologische Untersuchung. Journal of the Royal Statistical Society* 72, No. 1, 110-112.

Bowley, A.L. 1909g. Review of C. Cornelissen's *Théorie du salaire et du travail salarié. Journal of the Royal Statistical Society* 72, No. 1, 112-113.

Bowley, A.L. 1910a. *An Elementary Manual of Statistics.* London: MacDonald and Evans.

Bowley, A.L. 1910b. 'The amount and distribution of income (other than wages) below the income tax exemption limit in the United Kingdom'. Report of the committee of the British Association, consisting of Professors E. Cannan (Chairman), A.L. Bowley (Secretary), F.Y. Edgeworth, and H.B. Lees Smith, and Dr. W.R. Scott. *Journal of the Royal Statistical Society* 74, No. 1, 37-66.

Bowley, A.L. 1910c. Review of Ireson [1910]. *Journal of the Royal Statistical Society* 73, No. 4, 442-445.

Bowley, A.L. 1910d. Review of G. von Mayr's *Statistik und Gesellschaftslehre* Dritter Band. *Journal of the Royal Statistical Society* 73, No. 4, 437-439.

Bowley, A.L. 1910e. Review of F.W. Raffety & W.H. Cartwright Sharp's *The Nation's Income. An outline of national and local taxation. Journal of the Royal Statistical Society* 73, No. 2, 172.

Bowley, A.L. 1910f. 'The insufficiency of official statistics'. *XIXth Century* 67, 931-945.

Bowley, A.L. 1911a. 'The measurement of the accuracy of an average'. *Journal of the Royal Statistical Society* 75, No. 1, 77-88.

Bowley, A.L. 1911b. Review of G. Udny Yule's *An Introduction to the Theory of Statistics. The Economic Journal* 21, No. 82, 263-264.

Bowley, A.L. 1912a. 'Wages and the mobility of labour'. *The Economic Journal* 22, No. 85, 46-52.

Bowley, A.L. 1912b. 'The measurement of employment: an experiment'. *Journal of the Royal Statistical Society* 75, No. 8, 791-829.

Bowley, A.L. 1912c. Review of W.I. King's *Elements of Statistical Method. Journal of the Royal Statistical Society* 76, No. 1, 111-112.

Bowley, A.L. 1913a. *A General Course of Pure Mathematics from Indices to Solid Analytical Geometry.* Oxford: Clarendon Press.

Bowley, A.L. 1913b. Review of F.H. Streightoff's *The Distribution of Incomes in the United States*. *The Economic Journal* 23, No. 91, 425-427.

Bowley, A.L. 1913c. Review of Kaufmann [1913]. *The Economic Journal* 23, No. 92, 608-609.

Bowley, A.L. 1913d. 'The relation between the changes in wholesale and retail prices of food'. *The Economic Journal* 23, No. 92, 514-523.

Bowley, A.L. 1913e. Review of F. Žižek's *Soziologie und Statistik*. *Journal of the Royal Statistical Society* 76, No. 3, 325-326.

Bowley, A.L. 1913f. 'The census of production and the national dividend'. *The Economic Journal* 25, No. 89, 53-61.

Bowley, A.L. 1913g. Review of *"Official" Finance in Government Departments. A series of critical remarks upon the business and financial methods of Government Departments*. *Journal of the Royal Statistical Society* 77, No. 1, 96-97.

Bowley, A.L. 1913h. Review of B. Raynaud's *Vers le salaire minimum. Etude d'économie et de législation industrielles*. *Journal of the Royal Statistical Society* 76, No. 7, 718-719.

Bowley, A.L. 1913i. Review of M. Halbwachs's *La classe ouvrière et les niveaux de vie. Recherches sur al hiérarchie des besoins dans les sociétiés industrielles contemporaires*. *Journal of the Royal Statistical Society* 76, No. 7, 719-720.

Bowley, A.L. 1913j. Review of R. von Kuczynski's *Arbeitslohn und Arbeitszeit in Europa und Amerika, 1870-1909*. *Journal of the Royal Statistical Society* 76, No. 7, 720.

Bowley, A.L. 1913k. 'Working-class households in Reading'. *Journal of the Royal Statistical Society* 76, No. 7, 672-701.

Bowley, A.L. 1914a. 'The British super-tax and the distribution of income'. *The Quarterly Journal of Economics* 28, No. 2, 255-268.

Bowley, A.L. 1914b. Review of Mallock [1914]. *The Economic Journal* 24, No. 95, 438-441.

Bowley, A.L. 1914c. 'Rural population in England and Wales: a study of the changes of density, occupations, and ages'. *Journal of the Royal Statistical Society* 77, No. 6, 597-652.

Bowley, A.L. 1914d. Review of C.K. Hobson's *The Export of Capital*. *The Economic Journal* 24, No. 96, 563-565.

Bowley, A.L. 1914e. Review of R. Giffen's *Statistics. The Economic Journal* 24, No. 93, 92-94.

Bowley, A.L. 1914f. Review of R. Lennard's *Economic Notes on English Agricultural Wages. Journal of the Royal Statistical Society* 77, No. 8, 875-876.

Bowley, A.L. 1915a. *The Effect of the War on the External Trade of the United Kingdom; an analysis of the monthly statistics, 1906-1914.* London: Cambridge University Press.

Bowley, A.L. 1915b. *The Nature and Purpose of the Measurement of Social Phenomena.* 2nd ed. 1923. London: P.S. King & Son, Ltd.

Bowley, A.L. 1917. Review of A. W. Kirkaldy's (Ed.) *Labour, Finance and the War (being the results of inquiries, arranged by the Section of Economic Science and Statistics of the British Association for the Advancement of Science, during the years 1915 and 1916). Journal of the Royal Statistical Society* 80, No. 1, 99-101.

Bowley, A.L. 1918. Review of B. Seebohm Rowntree's *The Human Needs of Labour. The Economic Journal* 28, No. 112, 418-421.

Bowley, A.L. 1919a. *The Division of the Product of Industry; an analysis of national income before the war.* Oxford: Clarendon Press.

Bowley, A.L. 1919b. 'The measurement of changes in the cost of living'. *Journal of the Royal Statistical Society* 82, No. 3, 343-372.

Bowley, A.L. 1919c. Review of S. Secerov's *Economic Phenomena Before and After the War. A Statistical Theory of Modern Wars. The Economic Journal* 29, No. 115, 340-341.

Bowley, A.L. 1919d. Review of K. Morimoto's *The Standard of Living in Japan. Journal of the Royal Statistical Society* 82, No. 2, 245-246.

Bowley, A.L. 1919e. 'Note on the biology of a life-table'. *Journal of the Royal Statistical Society* 82, No. 1, 34-77.

Bowley, A.L. 1920a. 'The theoretical effects of rationing on prices'. *The Economic Journal* 30, No. 119, 340-347.

Bowley, A.L. 1920b. Reviews of (i) E.E. Wood's *The Housing of the Unskilled Wage-earner*, (ii) H.P. Douglass's *The Little Town* and (iii) W.R. Hughes's *New Town. The Economic Journal* 30, No. 117, 104-106.

Bowley, A.L. 1920c. 'Conditions of employment of dock labour'. *The Economic Journal* 30, No. 118, 272-275.

Bowley, A.L. 1920d. Review of *The Equipment of the Workers*, by the St. Philip's (Y.M.C.A.) Settlement Education and Economics Research Society of Sheffield. *The Economic Journal* 30, No. 117, 106-107.

Bowley, A.L. 1920e. 'Cost of living and wage determination'. *The Economic Journal* 30, No. 117, 114-117.

Bowley, A.L. 1920f. *The Change in the Distribution of the National Income, 1880-1913.* Oxford: Clarendon Press.

Bowley, A.L. 1920g. *Official Statistics of Retail Prices in Foreign Countries.* London: printed for the League of Nations, Harrison & Sons, Ltd.

Bowley, A.L. 1920h. 'Density and death-rate: Farr's Law'. *Journal of the Royal Statistical Society* 83, No. 2, 280-283.

Bowley, A.L. 1921a. 'Recent official statistical publications'. *Economica* 3, 302-306.

Bowley, A.L. 1921b. *Official Statistics: what they contain and how to use them.* London: Oxford University Press.

Bowley, A.L. 1921c. Review of D. Friday's *Profits, Wages and Prices. The Economic Journal* 31, No. 122, 219-220.

Bowley, A.L. 1921d. 'The survival of small firms'. *Economica* 2, 113-115.

Bowley, A.L. 1921e. 'An index of the physical volume of production'. *The Economic Journal* 31, No. 122, 196-205.

Bowley, A.L. 1921f. 'The labour report on the cost of living'. *The Economic Journal* 31, No. 123, 406-411.

Bowley, A.L. 1921g. 'Earners and dependants in English towns in 1911'. *Economica* 2, 101-112.

Bowley, A.L. 1921h. *Prices and Wages in the United Kingdom, 1914-1920.* Oxford: Clarendon Press.

Bowley, A.L. 1921i. Two Chadwick Public Lectures on 'The Growth of Suburban Population in England'. London School of Economics, COLL. MISC. 07772, Box V.

Bowley, A.L. 1922a. 'The definition of national income'. *The Economic Journal* 32, No. 125, 1-11.

Bowley, A.L. 1922b. 'An index of British economic conditions: 1919-22'. *The Review of Economic Statistics* 4, Supplement 2, 145-156.

Bowley, A.L. 1922c. Review of R.A. Lehfeldt's *The National Resources of South Africa. The Economic Journal* 32, No. 128, 549-551.

Bowley, A.L. 1922d. Review of B. Seebohm Rowntree's *The Human Factor in Business. The Economic Journal* 32, No. 126, 228-229.

Bowley, A.L. 1922e. 'The relation between wholesale and retail prices since the war'. *Economica* 6, 195-207.

Bowley, A.L. 1922f. 'Studies in Probability. II. The logic of probability and statistics'. *Economica* 4, 97-100.

Bowley, A.L. 1922g. Review of F. Stuart Chapin's *Field Work and Social Research. Journal of the Royal Statistical Society* 85, No. 3, 507-508.

Bowley, A.L. 1922h. Review of *Wages, Prices, and Profits: A Report Prepared by the Labour Research Department*, with a preface by Sidney Webb. *Journal of the Royal Statistical Society* 85, No. 3, 500-502.

Bowley, A.L. 1922i. Review of S. & B. Webb's *The Consumer's Co-operative Movement. Journal of the Royal Statistical Society* 85, No. 3, 502-505.

Bowley, A.L. 1923a. 'Death-rates, density, population, and housing'. *Journal of the Royal Statistical Society* 86, No. 4, 516-546.

Bowley, A.L. 1923b. 'The precision of measurements estimated from samples'. *Metron* 2, 494-500.

Bowley, A.L. 1923c. Review of I. Fisher [1922]. *The Economic Journal* 33, No. 129, 90-94.

Bowley, A.L. 1923d. Review of G. Findlay Shirras's *Report on an Enquiry into working Class Budgets in Bombay. The Economic Journal* 33, No. 131, 401-402.

Bowley, A.L. 1923e. 'Income in the United States'. *The Quarterly Journal of Economics* 37, No. 3, 510-517. (Review of King et al. [1922])

Bowley, A.L. 1923f. Review of T.L. Kelley's *Statistical Method. Economica* 9, 256-258.

Bowley, A.L. 1923g. Review of G. Findlay Shirras's *Report on an Enquiry into the Wages and Hours of Labour in the Cotton Mill Industry. Journal of the Royal Statistical Society* 86, No. 3, 436-437.

Bowley, A.L. 1923h. Reviews of (i) *Guide to Current Official Statistics* and (ii) *Prices and Price Indexes, 1918-1922. Economica* 9, 258-259.

Bowley, A.L. 1923i. Review of E.G. Peake's *An Academic Study of Some Money Markets and Other Statistics. The Economic Journal* 33, No. 131, 377-378.

Bowley, A.L. 1924a. 'Births and population in Great Britain'. *The Economic Journal* 34, No. 134, 188-192.

Bowley, A.L. 1924b. Review of A. Andréadès's *La Population Anglaise; avant, pendant et après la grande guerre. The Economic Journal* 34, No. 134, 257-259.

Bowley, A.L. 1924c. 'Does mathematical analysis explain? A note on consumer's surplus'. *Economica* 11, 135-139.

Bowley, A.L. 1924d. *The Mathematical Groundwork of Economics. An Introductory Treatise.* Oxford: Clarendon Press.

Bowley, A.L. 1925a. Review of C. Gini's *Quelques considérations au sujet de la construction des nombres indices des prix et des questions analogues. The Economic Journal* 35, No. 139, 451-453.

Bowley, A.L. 1925b. Review of H. Secrist's *Expenses, Profits and Losses in Retail Meat Stores: How Much and Why. The Economic Journal* 35, No. 137, 138-140.

Bowley, A.L. 1925c. Review of *The Problem of Business Forecasting*, edited by W.M. Persons, W.T. Foster and A.J. Hettinger, Jr. [1924]. *The Economic Journal* 35, No. 137, 96-98.

Bowley, A.L. 1926a. 'Measurement of the precision attained in sampling'. *Bulletin of the International Statistical Institute* 22, Supplement to Book I, 1-62.

Bowley, A.L. 1926b. Review of H. Secrist's *An Introduction to Statistical Methods. The Economic Journal* 36, No. 141, 115-116.

Bowley, A.L. 1926c. Review of W.L. Crum & A.C. Patton's *An Introduction to the Methods of Economic Statistics. The Economic Journal* 36, No. 142, 272-274.

Bowley, A.L. 1926d. 'The influence on the precision of index-numbers of correlation between the prices of commodities'. *Journal of the Royal Statistical Society* 89, No. 2, 300-319.

Bowley, A.L. 1927a. Review of S.S. Kuznets's *Cyclical Fluctuations. Retail and Wholesale Trade, United States. Journal of the Royal Statistical Society* 90, No. 2, 374-376.

Bowley, A.L. 1927b. Review of E. Rogowski's *Das Deutsche Volkseinkommen. The Economic Journal* 37, No. 145, 92-94.

Bowley, A.L. 1928a. 'Bilateral monopoly'. *The Economic Journal* 38, No. 152, 651-659.

Bowley, A.L. 1928b. 'Notes on index numbers'. *The Economic Journal* 38, No. 150, 216-237.

Bowley, A.L. 1928c. 'Some tests of the trustworthiness of public statistics'. *Economica* 24, 253-276.

Bowley, A.L. 1928d. 'The standard deviation of the correlation coefficient'. *Journal of the American Statistical Association* 23, No. 161, 31-34.

Bowley, A.L. 1928e. 'Use of mathematics in economic, social and public statistics'. *International Congress of Mathematicians, Toronto 1924*. Vol. 2, 907-920.

Bowley, A.L. 1928f. 'Economic and social survey of London'. *Journal of the American Statistical Association* 23, No. 164, 440.

Bowley, A.L. 1928g. *F.Y. Edgeworth's Contributions to Mathematical Statistics.* London: Royal Statistical Society. Reprinted 1972 by Augustus M. Kelley, Clifton, NJ.

Bowley, A.L. 1929a. Review of H. Schultz's *Statistical Laws of Demand and Supply, with special application to Sugar. The Economic Journal* 39, No. 154, 245-248.

Bowley, A.L. 1929b. 'The low birth-rate and unemployment'. *The Banker* 10, 140-148.

Bowley, A.L. 1929c. Review of *Cours de Statistique, professé en 1927-8 à la Faculté de Droit. Journal of the Royal Statistical Society* 92, No. 2, 268-269.

Bowley, A.L. 1930a. Review of I. Ferenczi's *International Migrations.* Volume I: Statistics. *Economica* 28, 109-110.

Bowley, A.L. 1930b. Review of W.I. King's *The National Income and its Purchasing Power. The Economic Journal* 40, No. 160, 657-660.

Bowley, A.L. 1930c. 'The relation between wholesale and retail prices'. *Lloyds Bank Monthly Review*, New Series, 1, 104-110.

Bowley, A.L. 1930d. *Some Economic Consequences of the Great War.* London: T. Butterworth Ltd.

Bowley, A.L. 1931a. 'The International Institute of Statistics'. *Journal of the Royal Statistical Society* 94, No. 1, 77-82.

Bowley, A.L. 1931b. Review of O. Weinberger's *Mathematische Volkswirthschlaftslehre. The Mathematical Gazette* 15, No. 214, 444.

Bowley, A.L. 1932a. Review of G.C. Evans's *Mathematical Introduction to Economics. The Economic Journal* 42, No. 165, 93-94.

Bowley, A.L. 1932b. Review of R. Frisch's *New Methods of Measuring Marginal Utility. The Economic Journal* 42, No. 166, 252-256.

Bowley, A.L. 1932c. 'The International Institute of Statistics, Madrid, 1931'. *Journal of the Royal Statistical Society* 95, No. 1, 105-107.

Bowley, A.L. 1932d. 'Economic aspects of the tendency of population in Great Britain'. pp. 47-51 in Pitt-Rivers [1932].

Bowley, A.L. 1933a. 'The action of economic forces in producing frequency distributions of income, prices, and other phenomena: a suggestion for study'. *Econometrica* 1, 358-372.

Bowley, A.L. 1933b. Review of C. Clark's *The National Income, 1924-1931. The American Economic Review* 23, No. 2, 371-372.

Bowley, A.L. 1933c. 'The national income of the United Kingdom in 1924'. *Economica* 40, 138-142.

Bowley, A.L. 1933d. 'Methods of investigating the economic and social conditions of a great city'. *Review of the International Statistical Institute* 1, No. 2, 7-18.

Bowley, A.L. 1934a. 'Francis Ysidro Edgeworth'. *Econometrica* 2, No. 2, 113-124.

Bowley, A.L. 1934b. Review of F. Vinci's *Manuele di Statistica. Introduzione allo Studio Quantitativo dei Fatti Sociale. Economica*, New Series, 1, No. 3, 372-373.

Bowley, A.L. 1934c. Review of *Wages, Cost of Living and National Income in Sweden, 1860-1930*, by the Staff of the Institute for Social Sciences, University of Stockholm. Vol. I, *The Cost of Living in Sweden, 1830-1930*, by G.Myrdal. Vol. II, *Wages in Sweden, 1830-1930*, by G. Bagge, E. Lundberg & I. Svennilson. *Economica*, New Series, 1, No. 3, 373-375.

Bowley, A.L. 1934d. 'The twenty-second session of the International Institute of Statistics'. *Journal of the Royal Statistical Society* 97, No. 3, 473-477.

Bowley, A.L. 1934e. *An Elementary Manual of Statistics.* London: MacDonald and Evans. (5th edition of Bowley [1910a])

Bowley, A.L. 1935a. Review of E.W. Gilboy's *Wages in Eighteenth-Century England. Journal of the Royal Statistical Society* 98, No. 1, 160-161.

Bowley, A.L. 1935b. 'Number of children in working-class families in London, 1929-30'. *Journal of the Royal Statistical Society* 98, No. 2, 363-375.

Bowley, A.L. 1935c. Review of *International Comparisons of Cost of Living. The Economic Journal* 45, No. 178, 301-303.

Bowley, A.L. 1935d. 'The occupations of fathers and of their children'. *Economica*, New Series, 2, No. 8, 400-407.

Bowley, A.L. 1935e. Obituary of Edwin Cannan. *The Economic Journal* 45, No. 178, 385-392.

Bowley, A.L. 1935f. Review of L. Lenti's *Analisi de Statistica Economica. Economica*, New Series, 2, No. 7, 356-357.

Bowley, A.L. 1935g. Review of C.F. Roos's *Dynamic Economics: Theoretical and Statistical Studies of Demand, Production, and Prices. Journal of the Royal Statistical Society* 98, No. 4, 726-728.

Bowley, A.L. 1936a. 'The application of sampling to economic and sociological problems'. *Journal of the American Statistical Association* 31, No. 195, 474-480.

Bowley, A.L. 1936b. 'The New Survey of London Life and Labour. Effect of modifying the poverty line'. *Journal of the Royal Statistical Society* 99, No. 2, 364-366.

Bowley, A.L. 1936c. Review of G.W. Daniels & H. Campion's *The Distribution of National Capital. The Economic Journal* 46, No. 184, 714-716.

Bowley, A.L. 1936d. Review of G. Tintner's *Prices in the Trade Cycle. Journal of the Royal Statistical Society* 99, No. 4, 790-793.

Bowley, A.L. 1937a. 'The twenty-third session of the International Institute of Statistics, 1936'. *Journal of the Royal Statistical Society* 100, No. 1, 100-102.

Bowley, A.L. 1937b. 'Numerical computation of the elasticity of substitution'. *The Review of Economic Studies* 5, No. 1, 600-65.

Bowley, A.L. 1937c. Review of C. Clark's *National Income and Outlay. Economica*, New Series, 4, No. 15, 350-353.

Bowley, A.L. 1937d. Reviews of (i) D. Tenderini's *Analisi sui prezzi in Italia dal 1901 al 1932* and (ii) G. Veronese's *Contributo allo studio della vischiosità dei prezzi delle mercie. The Economic Journal* 47, No. 188, 726-727.

Bowley, A.L. 1937e. Review of C.J. Bullock's *Economic Essays. Economica*, New Series, 4, No. 16, 462-463.

Bowley, A.L. 1937f. *Wages and Income in the United Kingdom since 1860.* Cambridge: University Press.

Bowley, A.L. 1937g. Review of Kuczynski [1937]. *Journal of the Royal Statistical Society* 100, No. 2, 309-310.

Bowley, A.L. 1938a. 'Note on Professor Frisch's "The problem of index numbers" '. *Econometrica* 6, No. 1, 83-84.

Bowley, A.L. 1938b. Review of A.M. Carr-Saunders & D. Caradog Jones's *A Survey of the Social Structure of England and Wales, as illustrated by Statistics*. *Economica*, New Seies, 5, No. 17, 103-104.

Bowley, A.L. 1938c. Review of J.R. Hicks's *Théorie Mathématique de la Valeur en régime de Libre Concurrence*. *The Economic Journal* 48, No. 191, 513-515.

Bowley, A.L. 1938d. Review of (i) *Wages, Cost of Living and National Income in Sweden, 1860-1930*, by the Staff of the Institute for Social Sciences, University of Stockholm, and (ii) *Vol. III, National Income of Sweden, 1860-1930*, by E. Lindahl, E. Dahlgren & K. Kock. *Economica*, New Series, 5, No. 18, 235-237.

Bowley, A.L. 1938e. *Three Studies on the National Income; being The division of the product of industry, by A.L. Bowley; The change in the distribution of the national income, 1880-1913, by A.L. Bowley; The National Income, 1924, by A.L. Bowley & Sir Josiah Stamp*. Reprinted by the London School of Economics and Political Science.

Bowley, A.L. 1939a. Review of H. Schultz's *The Theory and Measurement of Demand*. *Economica*, New Series, 6, No. 22, 213-216.

Bowley, A.L. 1939b. 'Production and efficiency'. *Journal of the Royal Statistical Society* 102, No. 1, 1-20.

Bowley, A.L. 1939c. Review of H. Campion's *Public and Private Property in Great Britain*. *The Economic Journal* 49, No. 194, 325-326.

Bowley, A.L. 1939d. Review of S. Kuznets's *National Income and Capital Formation, 1919-1935: A Preliminary Report*. *Economica*, New Series, 6, No. 21, 90-92.

Bowley, A.L. 1939e. Review of *Studies in Income and Wealth*. Vol. I. By the Conference on Research in National Income and Wealth. *Economica*, New Series, 6, No. 21, 90-92.

Bowley, A.L. 1939f. 'The International Institute of Statistics'. *Journal of the Royal Statistical Society* 102, No. 1, 83-85.

Bowley, A.L. 1939g. Obituary of Godfrey Isaac Howard Lloyd. *Journal of the Royal Statistical Society* 102, No. 2, 333.

Bowley, A.L. 1939h. 'Current social statistics: poverty and population'. *The Political Quarterly* 10, No. 1, 114-119.

Bowley, A.L. 1940a. Review of W. Beveridge's *Prices and Wages in England from the Twelfth to the Nineteenth Century: Vol. I, Price Tables: Mercantile Era*. *Economica*, New Series, 7, No. 27, 327-328.

Bowley, A.L. 1940b. 'Some constituents of the national income'. *Journal of the Royal Statistical Society* 103, No. 4, 491-518.

Bowley, A.L. 1940c. 'Working-class expenditure'. *The Economic Journal* 50, No. 200, 517-524.

Bowley, A.L. 1940d. 'The measurement of real income'. *The Economic Journal* 50, No. 198/199, 340-32.

Bowley, A.L. 1941a. Obituary of Lord Stamp. *Journal of the Royal Statistical Society* 104, No. 2, 193-196.

Bowley, A.L. 1941b. 'Earnings and prices, 1904, 1914, 1937-8'. *The Review of Economic Studies* 8, No. 3, 129-142.

Bowley, A.L. 1941c. Review of Rowntree [1941]. *The Economic Journal* 51, No. 204, 473-476.

Bowley, A.L. 1942a. 'National income in America and the United Kingdom'. *Economica*, New Series, 9, No. 35, 227-236.

Bowley, A.L. 1942b. Review of O. Lange, F. McIntyre & T.O. Yntema's (Eds) *Studies in Mathematical Economics and Econometrics. In Memory of Henry Schultz. The Economic Journal* 52, No. 208, 354-356.

Bowley, A.L. (Ed.) 1942c. *Studies in the National Income, 1924-1938.* Cambridge: Cambridge University Press.

Bowley, A.L. 1943a. 'Note on Mr. Barna's valuation of stocks'. *Economica*, New Series, 10, No. 37, 64-65.

Bowley, A.L. 1943b. Review of H.A. Shannon & E. Grebenik's *The Population of Bristol. Economica*, New Series, 10, No. 39, 259-260.

Bowley, A.L. 1943c. 'The volume of industrial production'. *Journal of the Royal Statistical Society* 106, No. 2, 167-170.

Bowley, A.L. 1944. Review of R.M. Titmuss's *Birth, Poverty and Wealth. Economica*, New Series, 11, No. 43, 143-144.

Bowley, A.L. 1945a. Obituary of Sir Hubert Llewellyn Smith, G.C.B. *Journal of the Royal Statistical Society* 108, No. 3/4, 480-481.

Bowley, A.L. 1945b. Obituary of George Henry Wood. *Journal of the Royal Statistical Society* 108, No. 3/4, 485-486.

Bowley, A.L. 1946a. 'Rural population in England and Wales, 1911 and 1931'. *Economica*, New Series, 13, No. 50, 97-118.

Bowley, A.L. 1946b. 'Communications sur la méthodologie, la législation. l'organisation et l'administration statistiques'. *Review of the International Statistical Institute* 14, No. 1/4, 52-72.

Bowley, A.L. 1947a. *Wages, Earnings and Hours of Work, 1914-1947.* Executive Committee of the London & Cambridge Economic Service.

Bowley, A.L. 1947b. Review of S. Kuznets's *National Income. A Summary of Findings. Economica*, New Series, 14, No. 53, 63-64.

Bowley, A.L. 1950. 'Clara E. Collet (September 10, 1860-August 3, 1948)'. *The Economic Journal* 60, No. 238, 408-410.

Bowley, A.L. 1952a. 'Index-numbers of wage-rates and cost of living'. *Journal of the Royal Statistical Society* A, 115, No. 4, 500-506.

Bowley, A.L. 1952b. Obituary of M.J. Elsas. *The Economic Journal* 62, No. 247, 677-678.

Bowley, A.L. 1952c. Obituary of Langford Lovell Price, 1862-1950. *Review of the International Statistical Institute* 20, No. 1, 91.

Bowley, A.L. 1953a. Obituary of Augustus D. Webb, 1880-1953. *Journal of the Royal Statistical Society* A, 116, No. 1, 104.

Bowley, A.L. 1953b. Review of A.L. Chapman's *Wages and Salaries in the United Kingdom, 1920-1938. Journal of the Royal Statistical Society* A, 116, No. 2, 210-213.

Bowley, A.L. 1955a. Obituary of B. Seebohm Rowntree, C.H. 1871-1954. *Journal of the Royal Statistical Society* A, 118, No. 1, 125-126.

Bowley, A.L. 1955b. Reviews of (i) R. Stone's *The Measurement of Consumers' Expenditure and Behaviour in the United Kingdom, 1920-1938*, vol. I, and (ii) A.R. Prest's *Consumers' Expenditure in the United Kingdom, 1900-1919. Journal of the Royal Statistical Society* A, 118, No. 1, 114-116.

Bowley, A.L. 1956. Obituary of H.W. Methorst—1868-1955. *Journal of the Royal Statistical Society* A, 119, No. 1, 102.

Bowley, A.L. & Burnett-Hurst, A.R. 1915. *Livelihood and Poverty; a study in the economic conditions of working-class households in Northampton, Warrington, Stanley and Reading.* London: G. Bell and Sons.

Bowley, A.L. & Connor, L.R. 1923. 'Tests of correspondence between statistical grouping and formulae'. *Economica* 7, 1-9.

Bowley, A.L. & Hogg, M.H. 1923. 'Wages and production in a Durham colliery'. *Economica* 9, 229-235.

Bowley, A.L. & Hogg, M.H. 1925. *Has Poverty Diminished? A Sequel to Livelihood and Poverty*. London: P.S. King.

Bowley, A.L. & Robertson, D.H. 1934. *A Scheme for an Economic Census of India, with special reference to a census of production and reorganisation of statistics*. New Delhi: Government of India Press.

Bowley, A.L., Schwartz, G.L. & Smith, K.C. 1931. 'A new index of prices of securities'. London and Cambridge Economic Service special memorandum 33.

Bowley, A.L. & Stamp, J.C. 1927. *The National Income, 1924; a comparative study of the income of the United Kingdom in 1911 and 1924*. Oxford: Clarendon Press.

Bowley, A.L. & Wood, G.H. 1899. 'The statistics of wages in the United Kingdom during the last hundred years. (Part V.) Printers'. *Journal of the Royal Statistical Society* 62, No. 4, 708-715.

Bowley, A.L. & Wood, G.H. 1905a. 'The statistics of wages in the United Kingdom during the last hundred years. (Part X.) Engineering and Shipbuilding. A. Trade Union Standard Rates'. *Journal of the Royal Statistical Society* 68, No. 1, 104-137.

Bowley, A.L. & Wood, G.H. 1905b. 'The statistics of wages in the United Kingdom during the last hundred years. (Part XI.) Engineering and Shipbuilding. B. Statements of Wages from Non-Trade Union Sources in General Engineering'. *Journal of the Royal Statistical Society* 68, No. 2, 373-391.

Bowley, A.L. & Wood, G.H. 1905c. 'The statistics of wages in the United Kingdom during the last hundred years. (Part XII.) Engineering and Shipbuilding. C. Statements of Wages from Non-Tade Union Sources in Shipbuilding and Engineering at Shipbuilding Centres'. *Journal of the Royal Statistical Society* 68, No. 3, 563-614.

Bowley, A.L. & Wood, G.H. 1905d. 'The statistics of wages in the United Kingdom during the last hundred years. (Part XIII.) Engineering and Shipbuilding. D. Dockyards and Railway Centres'. *Journal of the Royal Statistical Society* 68, No. 4, 704-715.

Bowley, A.L. & Wood, G.H. 1906. 'The statistics of wages in the United Kingdom during the last hundred years. (Part XIV.) Engineering and Shipbuilding. E. Averages, Index Numbers, and General Results'. *Journal of the Royal Statistical Society* 69, No. 1, 148-196.

Bowley, M. 1950. 'The sources and nature of statistical information in special fields of statistics: the housing statistics of Great Britain'. *Journal of the Royal Statistical Society* A, 113, No. 3, 396-411.

Bowley, M. 1972. Review of Lord Robbins's *Autobiography of an Economist. The Economic Journal* 82, No. 326, 808-810.

Bowley, R. 1934. 'The cost of living of girls professionally employed in the County of London'. *The Economic Journal* 44, No. 174, 328-334.

Box, J.F. 1978. *R.A. Fisher: the Life of a Scientist.* New York: John Wiley & Sons.

Bradford Hill, A. 1931. Review of Llewellyn Smith, Vol. I [1930]. *Journal of the Royal Statistical Society* 94, No. 3, 445-447.

Bradford Hill, A. 1932. Review of Llewellyn Smith, Vol. II [1931]. *Journal of the Royal Statistical Society* 95, No. 2, 326-328.

Bradford Hill, A. 1933. Review of Llewellyn Smith, Vols III & IV [1932]. *Journal of the Royal Statistical Society* 96, No. 1, 108-109.

Bradford Hill, A. 1934a. Review of Llewellyn Smith, Vol. V [1933]. *Journal of the Royal Statistical Society* 97, No. 1, 163-165.

Bradford Hill, A. 1934b. Review of Llewellyn Smith, Vols VI & VII [1934]. *Journal of the Royal Statistical Society* 97, No. 3, 490-492.

Bradford Hill, A. 1935a. Review of Llewellyn Smith, Vol. VIII [1934]. *Journal of the Royal Statistical Society* 98, No. 1, 152-154.

Bradford Hill, A. 1935b. Review of Llewellyn Smith, Vol. IX [1935]. *Journal of the Royal Statistical Society* 98, No. 4, 724-726.

Bradford Hill, A. 1937. 'Mathematics and medicine". *The Lancet* 229, Issue 5914, 31.

Bravais, A. 1846. 'Analyse mathématique sur les probabilités des erreurs de situation d'un point'. *Mémoires présentés par divers Savants à l'Académie royale des Sciences de l'Institut de France*, *II*^{e}*Serie*, IX, 255-332.

Brown, F. 1939. Review of Bowley [1937f] and Kuczynski [1937]. *The Economic History Review* 9, No. 2, 215-217.

Brownlee, J. 1915. 'Studies in the meaning and relationships of birth and death rates. II. Density of population and death rate (Farr's Law)'. *The Journal of Hygiene* 15, No. 1, 11-16.

Buckland, W.R. 1953. Review of the 7th (1952) edition of Bowley [1910a]. *Journal of the Royal Statistical Society* A, 116, No. 1, 89-90.

Burns, E.M. 1923. Review of Astor et al. [1923]. *Economica* 9, 247-249.

Cajori, F. 1993. *A History of Mathematical Notations.* Two volumes bound as one (originally published in 1928 and 1929). New York: Dover.

Cannan, E. et al. 1910. 'The amount and distribution of income (other than wages) below the income tax exemption limit in the United Kingdom. Report of the committee of the British Association, consisting of Professors E. Cannan (Chairman), A.L. Bowley (Secretary), F.Y. Edgeworth, and H.B. Lees Smith, and Dr. W.R. Scott'. *Journal of the Royal Statistical Society* 74, No. 1, 37-66.

Cannan, E. 1919. Review of Bowley [1919a]. *The Economic Journal* 29, No. 114, 207-213.

Cannan, E. 1926. Review of Bowley & Hogg [1925]. *Economica* 17, 221-222.

Caradog Jones, D. 1938. Review of Bowley [1937f]. *The Economic Journal* 48, No. 190, 291-294.

Carpenter. N. 1932. Review of Llewellyn Smith, Vol. I [1930]. *The Quarterly Journal of Economics* 47, No. 1, 150-159.

Carr-Saunders, A.M. 1931. Review of Llewellyn Smith, Vol. I [1930]. *The Economic Journal* 41, No. 161, 80-84.

Cave, B.M. & Pearson, K. 1914. 'Numerical illustrations of the variate difference correlation method'. *Biometrika* 10, No. 2/3, 340-355.

Cave-Browne-Cave, F.E. 1904-5. 'On the influence of the time factor on the correlation between the barometric heights at stations more than 1000 miles apart'. *Proceedings of the Royal Society of London* 74, 403-411.

Chaddock, R.E. 1916. Review of Bowley [1915b]. *Political Science Quarterly* 31, No. 2, 334-335.

Chaddock, R.E. 1932. Review of Llewellyn Smith, Vol. I [1930]. *The Journal of Political Economy* 40, No. 4, 544-547.

Chang, W.-I. 1976. 'Sampling theories and sampling practice'. pp. 299-315 in Owen [1976].

Chattopadhyay, R. 1987. ‘An early British Government initiative in the genesis of Indian planning’. *Economic and Political Weekly* 22, No. 5, PE19-PE29.

Coats, R.H. 1921. ‘Improvements in British Empire trade information—the Imperial Statistical Conference’. *Annals of the American Academy of Political and Social Science* 94, 153-156.

Coggeshall, F. 1886. ‘The arithmetic, geometric, and harmonic means’. *The Quarterly Journal of Economics* 1, No. 1, 83-86.

Collet, C.E. 1945. ‘Charles Booth, The Denison Club and H. Llewellyn Smith’. *Journal of the Royal Statistical Society* 108, No. 3/4, 482-485.

Cornford, F.M. 1938. ‘Greek Natural Philosophy and Modern Science’. pp. 3-22 in Needham & Pagel [1938].

Cox, G.M. 1961. Review of Nixon [1960]. *Journal of the American Statistical Association* 56, No. 294, 416-417.

Cramér, H. 1972. ‘Studies in the history of probability and statistics. XXVIII. On the history of certain expansions used in mathematical statistics’. *Biometrika* 59, No. 1, 205-207.

Crum, W.L. 1925. Review of Bowley [1924d]. *The Quarterly Journal of Economics* 39, No. 2, 313-315.

Cullen, M.J. 1975. *The Statistical Movement in Early Victorian Britain: The Foundations of Empirical Social Research.* New York: Harvester Press.

Dahrendorf, R. 1995. *LSE: A History of the London School of Economics and Political Science 1895-1995.* Oxford: Oxford University Press.

Dale, A.I. 1989. ‘An early occurrence of the Poisson distribution’. *Statistics and Probability Letters* 7, 21-22.

Dale, A.I. 2001. ‘Arthur Lyon Bowley’. pp. 278-282 in Heyde & Seneta [2001].

Dale, A.I. 2003. *Most Honourable Remembrance: the Life and Work of Thomas Bayes.* New York: Springer-Verlag.

Darnell, A. 1981. ‘A.L. Bowley, 1869-1957’. pp. 140-174 in O’Brien & Presley [1981].

Darnell, A.C. 1982. ‘Bowley, Wicksell and the development of mathematical economics’. *Scottish Journal of Political Economy* 29, No. 2, 156-180.

David, H.A. 1968. 'Gini's mean difference rediscovered'. *Biometrika* 55, No. 3, 573-575.

David, H.A. 1998. 'Early sample methods of variability'. *Statistical Science* 13, No. 4, 368-377.

David, H.A. & Nagaraja, H.N. 2003. *Order Statistics.* 3rd edition. Hoboken, NJ: Wiley.

Daw, R.H. & Pearson, E.S. 1972. 'Studies in the History of Probability and Statistics. XXX. Abraham de Moivre's 1733 derivation of the normal curve: a bibliographic note'. *Biometrika* 59, No. 3, 677-680.

Dearle, N.B. 1923. Review of Astor et al. [1923]. *The Economic Journal* 33, No. 131, 391-394.

Dearle, N.B. 1924. Review of Astor et al. [1924]. *The Economic Journal* 34, No. 136, 602-606.

Dearle, N.B. 1932. Review of Llewellyn Smith, Vol. II [1931]. *The Economic Journal* 42, No. 166, 281-285.

Dearle, N.B. 1933. Review of Llewellyn Smith, Vols III & IV [1932]. *The Economic Journal* 43, No. 170, 315-319.

Dearle, N.B. 1934a. Review of Llewellyn Smith, Vol. V [1933]. *The Economic Journal* 44, No. 173, 112-116.

Dearle, N.B. 1934b. Review of Llewellyn Smith, Vol. VI & VII [1934]. *The Economic Journal* 44, No. 174, 301-305.

Dearle, N.B. 1935a. Review of Llewellyn Smith, Vol. VIII [1934]. *The Economic Journal* 45, No. 178, 326-327.

Dearle, N.B. 1935b. Review Of Llewellyn Smith, Vol. IX [1935]. *The Economic Journal* 45, No. 180, 747-749.

de Gans, H.A. 2002. 'Law or Speculation? A debate on the method of forecasting population size in the 1920s'. *Population (English Edition)* 57, No. 1, 83-108.

de Morgan, A. 1838. *An Essay on Probabilities and on their Application to Life Contingencies and Insurance Offices.* London: Longman, Orme, Brown, Green, & Longmans.

de Morgan, A. 1842. *The Differential and Integral Calculus.* London: Baldwin and Cradock.

Diaconis, P. & Freedman, D. 1986. 'An elementary proof of Stirling's formula'. *The American Mathematical Monthly* 93, No. 2, 123-125.

Dodd, E.L. 1940. 'The substitutive mean and certain subclasses of this general mean'. *The Annals of Mathematical Statistics* 11, No. 2, 163-176.

Doll, R. & Hill, A. Bradford. 1950. 'Smoking and carcinoma of the lung'. *British Medical Journal* 2(4682), 739-748.

Doll, R. & Hill, A. Bradford. 1954. 'The mortality of doctors in relation to their smoking habits'. *British Medical Journal* 1(4877), 1451-1455.

Doll, R. & Hill, A. Bradford. 1964. 'Mortality in relation to smoking: ten years' observations of British doctors'. *British Medical Journal* 1(5396), 1460-1467.

Edgeworth, F.Y. 1881. *Mathematical Psychics. An Essay on the Application of Mathematics to the Moral Sciences.* London: C. Kegan Paul & Co.

Edgeworth, F.Y. 1884. '*A priori* probabilities'. *London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science* 18, 204-210.

Edgeworth, F.Y. 1885. 'Methods of statistics'. *Jubilee Volume of the Journal of the Royal Statistical Society*, 181-217.

Edgeworth, F.Y. 1887a. 'Observations and statistics: an essay on the theory of errors of observation and the first principles of statistics'. *Transactions of the Cambridge Philosophical Society* 14, Part II, 138-169.

Edgeworth, F.Y. 1887b. 'The choice of means'. *London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, Fifth Series, 24, No. 148, 268-271.

Edgeworth, F.Y. 1888a. 'On observations relating to several quantities'. *Hermathena* 13, 279-285.

Edgeworth, F.Y. 1888b. 'On a new method of reducing observations relating to several quantities'. *London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, Fifth Series, 25, No. 154, 184-191.

Edgeworth, F.Y. 1892a. 'Correlated averages'. *London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, Fifth Series, XXXIV (CCVII), 190-204.

Edgeworth, F.Y. 1892b. 'The law of error and correlated averages'. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, Fifth Series, XXXIV (CCX), 429-438; XXXIV (CCXI), 518-526.

Edgeworth, F.Y. 1893. 'Statistical correlation between social phenomena'. *Journal of the Royal Statistical Society* 56, 670-675.

Edgeworth, F.Y. 1894. 'The asymmetrical probability curve'. *Proceedings of the Royal Society of London* LVI (337), 271-272.

Edgeworth, F.Y. 1904. Review of Bowley [1904a] *The Economic Journal* 14, No. 54, 268-271.

Edgeworth, F.Y. 1905. 'The law of error'. *Transactions of the Cambridge Philosophical Society* XX, Part I, 36-65; Part II, 113-141.

Edgeworth, F.Y. 1907. 'On the representation of statistical frequency by a series'. *Journal of the Royal Statistical Society* 70, No. 1, 102-106.

Edgeworth, F.Y. 1908a. 'On the probable errors of frequency-constants'. *Journal of the Royal Statistical Society* 71, No. 2, 381-397.

Edgeworth, F.Y. 1908b. 'On the probable errors of frequency-constants (contd)'. *Journal of the Royal Statistical Society* 71, No. 3, 499-512.

Edgeworth, F.Y. 1913. 'On the use of the theory of probabilities in statistics relating to society'. *Journal of the Royal Statistical Society* 76, No. 2, 165-193.

Edgeworth, F.Y. 1917a. 'On the mathematical representation of statistical data. Section IV.—Surfaces'. *Journal of the Royal Statistical Society* 80, No. 2, 266-288.

Edgeworth, F.Y. 1917b. Review of H. Westergaard's *Scope and Method of Statistics*. *Journal of the Royal Statistical Society* 80, 546-551.

Edgeworth, F.Y. 1918. 'The doctrine of index-numbers according to Professor Wesley Mitchell'. *The Economic Journal* 28, No. 110, 176-197.

Edgeworth, F.Y. 1920. Review of Bowley [1920f]. *Journal of the Royal Statistical Society* 83, No. 3, 482-484.

Edgeworth, F.Y. 1922. 'The philosophy of chance'. *Mind* (New Series), XXXI (123), 257-283.

Edgeworth, F.Y. 1923. 'On the use of medians for reducing observations relating to several quantities'. *London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, Sixth Series, 46, 1074-1088.

Edgeworth, F.Y. 1924. Review of Bowley [1924d]. *The Economic Journal* 34, No. 135, 430-434.

Edgeworth, F.Y. & Bowley, A.L. 1902. 'Methods of representing statistics of wages and other groups not fulfilling the normal law of error'. *Journal of the Royal Statistical Society* 65, No. 2, 325-354. [Part I: Theoretical considerations, by Edgeworth; pp. 325-331. Part II: Applications to wage statistics and other groups, by Bowley; pp. 331-354.]

Edwards, A.W.F. 1963. 'The measure of association in a 2×2 table'. *Journal of the Royal Statistical Society* A, 126, No. 1, 109-114.

Edwards, A.W.F. 1986. 'A quick route to sums of powers'. *The American Mathematical Monthly* 93, No. 6, 451-455.

Elderton, W.P. 1906. Review of two works by F.Y. Edgeworth and four by C.V.L. Charlier. *Biometrika* 5, No. 1/2, 206-210.

Elderton, W.P. 1953. *Frequency Curves and Correlation.* 4th edition (1st edition 1906). Washington, D.C.: Harren Press.

Elderton, W.P. & Johnson, N.L. 1969. *Systems of Frequency Curves.* Cambridge: Cambridge University Press.

Everett, J.D. 1901. 'On a new interpolation formula'. *Journal of the Institute of Actuaries* 35, 452-458.

Fabricant, S. 1943. Review of Bowley [1942c]. *The American Economic Review* 33, No. 3, 636-640.

Farebrother, R.W. 1999. *Fitting Linear Relationships. A History of the Calculus of Observations 1750-1900.* New York: Springer-Verlag.

Farr, W. 1859. 'On the construction of life-tables, illustrated by a new life-table of the healthy districts of England'. *Philosophical Transactions of the Royal Society of London* 149, 837-878.

Feinstein, C. 1990. 'New estimates of average earnings in the United Kingdom, 1880-1913'. *The Economic History Review* 43, No. 4, 595-632.

Feldman, H. 1923. Review of Astor et al. [1923]. *Political Science Quarterly* 38, No. 3, 521-525.

Feldman, H. 1925. Review of Astor et al. [1924]. *Political Science Quarterly* 40, No. 2, 317-319.

Feller, W. 1950. *An Introduction to Probability Theory and Its Applications.* Vol. I. 3rd edition, revised printing 1970. New York: John Wiley & Sons.

Feller, W. 1966. *An Introduction to Probability Theory and Its Applications.* Vol. II. New York: John Wiley & Sons.

Felt, L.F. & Sinclair, P.R. 1991. 'Home sweet home!: Dimensions and determinants of life satisfaction in an underdeveloped region'. *Canadian Journal of Sociology* 16, No. 1, 1-21.

Fewster, R.M. 2009. 'A simple explanation of Benford's Law'. *The American Statistician* 63, No. 1, 26-32.

Field, J.A. 1910. Review of Bowley [1910a]. *The Journal of Political Economy* 18, No. 7, 563-564.

Field, J.A. 1916. Review of Bowley [1915b]. *The Journal of Political Economy* 24, No. 4, 408-409.

Fisher, I. 1923. 'Professor Bowley on index-numbers'. *The Economic Journal* 33, No. 130, 246-252.

Fisher, R.A. 1910. Review of Bowley [1910a]. *The Economic Journal* 20, No. 78, 268-271.

Fisher, R.A. 1922. 'On the interpretation of χ^2 from contingency tables, and the calculation of P'. *Journal of the Royal Statistical Society* 85, No. 1, 87-94.

Fisher, R.A. 1923. 'Statistical tests of agreement between observation and hypothesis'. *Economica* 8, 139-147.

Fisher, R.A. 1925. *Statistical Methods for Research Workers*. Edinburgh: Oliver and Boyd Ltd.

Fisher, R.A. 1935. 'The logic of inductive inference'. *Journal of the Royal Statistical Society* 98, No. 1, 39-82.

Fisher, R.A. 1938. 'Presidential Address: First Indian Statistical Conference, 1938'. *Sankhyā: The Indian Journal of Statistics* 4, 14-17.

FitzPatrick, P.J. 1956. 'Leading British statisticians of the nineteenth century'. *Journal of the American Statistical Association* 55, No. 289, 38-70.

FitzPatrick, P.J. 1957. 'Statistical societies in the United States in the nineteenth century'. *The American Statistician* 11, No. 5, 15-21.

Flux, A.W. 1910a. Review of Bowley [1910a]. *Journal of the Royal Statistical Society* 73, No. 4, 435-437.

Flux, A.W. 1910b. Review of Bowley [1910a]. *The Economic Journal* 20, No. 78, 268-271.

Flux, A.W. 1921. 'The measurement of price changes'. *Journal of the Royal Statistical Society* 84, No. 2, 167-215.

Flux, A.W. 1927. Review of Bowley and Stamp [1927]. *The Economic Journal* 37, No. 146, 255-257.

Flux, A.W. 1929a. 'The National Income'. *Journal of the Royal Statistical Society* 92, No. 1, 1-33.

Flux, A.W. 1929b. 'Discussion on the National Income'. *Journal of the Royal Statistical Society* 92, No. 2, 163-182.

Ford, W.C. 1901. Review of Bowley [1901a]. *The Journal of Political Economy* 9, No. 3, 443-450.

Fountain, J. 1980. 'Bowley's analysis of bilateral monopoly and Sen's liberal paradox in collective choice theory: a note'. *The Quarterly Journal of Economics* 94, No. 4, 809-812.

Furniss, H.S. 1915. Review of Bowley [1915b] *The Economic Journal* 25, No. 99, 430-431.

Galton, F. 1889. *Natural Inheritance.* London: Macmillan.

Gavett, G.I. 1927. Review of the 5th (1926) edition of Bowley [1901a]. *Journal of the American Statistical Association* 22, No. 160, 523-527.

George, R.F. 1937. 'A new calculation of the poverty line'. *Journal of the Royal Statistical Society* 100, No. 1, 74-95.

Ghosh, J.K., Maiti, P., Rao, T.J. & Sinha, B.K. 1999. 'Evolution of statistics in India'. *International Statistical Review* 67, No. 1, 13-34.

Giffen, R. 1883. 'The progress of the working classes in the last half century'. *Journal of the Statistical Society of London* 46, No. 4, 593-622.

Giffen, R. 1886. 'Further notes on the progress of the working classes in the last half century'. *Journal of the Statistical Society of London* 49, No. 1, 28-100.

Giffen, R. 1913. *Statistics...Written about the years 1898-1900.* Edited with an introduction by Henry Higgs, C.B., with the assistance of George Udny Yule. London: Macmillan and Co.

Gilbert, M. 1943. Review of Bowley [1942c]. *Journal of the American Statistical Association* 38, No. 224, 473-475.

Gini, C. 1912. *Variabilità e Mutabilità. Contributo allo studio delle distribuzioni e delle relazioni statistche.* Bologna.

Gini, C. & Galvani, L. 1929. 'Di una applicazione del metoda rappresentative all' ultimo censimento italiano della populazione'. *Annali di Statistica* 4, 1-107.

Graunt, J. 1662. *Natural and Political Observations made upon the Bills of Mortality.* London: Martyn.

Grebenik, E. 1984. 'Obituary Roy George Douglas Allen 1906-1983'. *Journal of the Royal Statistical Society* A 147, No. 5, 706-707.

Green, D.R. 1976. 'The historical development of complex numbers'. *The Mathematical Gazette* 60, No. 412, 99-107.

Greenwood, M. 1915. Review of Bowley [1915b]. *Journal of the Royal Statistical Society* 78, No. 4, 616-617.

Greenwood, M. & Yule, G.U. 1915. 'The statistics of anti-typhoid and anti-cholera inoculations, and the interpretation of such statistics in general'. *Proceedings of the Royal Society of Medicine*, Section of Epidemiology and State Medicine 8, 113-190.

Gut, A. 2002. 'On the moment problem'. *Bernoulli* 8, No. 3, 407-421.

Hacking, I. 1975. *The Emergence of Probability.* London: Cambridge University Press.

Hagenaars, A.J.M. & Van Praag, B.M.S. 1985. 'A synthesis of poverty line definitions'. *Review of Income and Wealth* 31, No. 2, 139-154.

Hald, A. 1990. *A History of Probability and Statistics and Their Applications before 1750.* New York: John Wiley & Sons.

Hald, A. 1998. *A History of Mathematical Statistics from 1750 to 1930.* New York: John Wiley & Sons.

Hald, A. 2000. 'The early history of the cumulants and the Gram-Charlier series'. *International Statistical Review* 68, No. 2, 137-153.

Hankins, F.H. 1924. Review of Bowley [1924d]. *Journal of Social Forces* 3, No. 1, 185.

Hansen, M.H. 1987. 'Some history and reminiscences on survey sampling'. *Statistical Science* 2, No. 2, 180-190.

Hansen, M.H., Dalenius, T. & Tepping, B.J. 1985. 'The development of sample surveys of finite populations'. Chapter 13 in Atkinson & Fienberg [1985].

Hardy, G.H. 1908. *A Course of Pure Mathematics.* (2nd edition 1914). Cambridge: University Press.

Hardy, G.H. 1914. Review of Bowley [1913a]. *Cambridge Review* XXXV, No. 879, 394-395.

Hardy, G.H. 1915. 'The definition of a complex number'. *The Mathematical Gazette* 8, No. 116, 48-49.

Harper, E.J. 1909. Review of *Report of an Inquiry by the Board of Trade into working class rents, housing, and retail prices, together with the rates of wages in certain occupations in the principal industrial towns of France. Journal of the Royal Statistical Society* 72, No. 1, 114-118.

Harrod, R.F. et al. 1938. Letter to *The Times.* (21st January, p. 13.)

Harrod, R.F. 1951. *The Life of John Maynard Keynes.* London: Macmillan & Co.

Hayek, F.A. 1946. 'The London School of Economics 1895-1945'. *Economica*, New Series, 13, No. 49, 1-31.

Hazlitt, W. 1908. *Table Talk.* Everyman edition. London: J.M. Dent & Sons.

Hennock, E.P. 1987. 'The measurement of urban poverty: from the metropolis to the nation, 1880-1920'. *Economic History Review* 2nd Series, 40, No. 2, 208-227.

Herbertson, A.J. 1905. 'The visit of the British Association to South Africa'. *The Geographical Journal* 26, No. 6, 632-641.

Heyde, C.C. 1963a. 'Some remarks on the moment problem (I)'. *The Quarterly Journal of Mathematics* 14, 91-96.

Heyde, C.C. 1963b. 'Some remarks on the moment problem (II)'. *The Quarterly Journal of Mathematics* 14, 97-105.

Heyde, C.C. & Seneta, E. 2001. *Statisticians of the Centuries.* New York: Springer-Verlag.

Higgs, H. (Ed.) 1925-6. *Palgrave's Dictionary of Political Economy.* 1963 reprint: New York: Augustus M. Kelley.

Hill, I.D. 1984. 'Statistical Society of London—Royal Statistical Society: the first 100 years: 1834-1934'. *Journal of the Royal Statistical Society* A 147, No. 2, 130-139.

Hilton, J. 1924. 'Enquiry by sample: an experiment and its results'. *Journal of the Royal Statistical Society* 87, No. 4, 544-570.

Hilts, V.L. 1978. '*Aliis exterendum*, or, the origins of the Statistical Society of London'. *Isis* 69, No. 1, 21-43.

Hogg, M.H. 1921. 'Dependants on women wage-earners'. *Economica* 1, 69-86.

Hooker, R.H. 1901. 'Correlation of the marriage-rate with trade'. *Journal of the Royal Statistical Society* 64, No. 3, 485-492.

Hooker, R.H. 1905. 'On the correlation of successive observations'. *Journal of the Royal Statistical Society* 68, No. 4, 696-703.

Hotelling, H. 1949. 'The place of statistics in the university'. *Proceedings of the Berkeley Symposium on Mathematical Statistics and Probability*, 21-40. (Ed. J. Neyman) Berkeley: University of California Press.

Howarth, E.G. & Wilson, M. 1907. *West Ham: A Study in Social and Industrial Problems.* London: J.M. Dent.

Howie, D. 2002. *Interpreting Probability: Controversies and Developments in the Early Twentieth Century.* Cambridge: Cambridge University Press.

Humphrey, T.M. 1996. 'The early history of the box diagram'. Federal Reserve Bank of Richmond *Economic Quarterly* 82, No. 1, 37-75.

Hunt, L. 1906. *Autobiography.* London: Smith, Elder, & Co. (First printed in 1850.)

Ingraham, M.H. 1929. Review of Bowley [1928g]. *The American Economic Review* 19, No. 4, 738-739.

Ireson, F. 1910. *The People's Progress. A Study of the Facts of National Wealth, with some Answers to Socialists.* London: John Murray.

Isserlis, L. 1918. 'On the value of a mean as calculated from a sample'. *Journal of the Royal Statistical Society* 81, No. 1, 75-81.

Jeffreys, H. 1961. *Theory of Probability.* 3rd ed. Oxford: Clarendon Press.

Jensen, A. 1928. 'Purposive selection'. *Journal of the Royal Statistical Society* 91, No. 4, 541-547.

Jevons, W.S. 1870. Opening Address of the President of Section F of the British Association for the Advancement of Science. *Journal of the Statistical Society of London* 33, No. 3, 309-326.

Jordan, C. 1939/1965. *Calculus of Finite Differences.* New York: Chelsea Publishing Company.

Kapteyn, J.C. 1903. *Skew Frequency Curves in Biology and Statistics.* Groningen: Astronomical Laboratory.

Kapteyn, J.C. & van Uven, M.J. 1916. *Skew Frequency Curves in Biology and Statistics.* (Second paper). Groningen: Astronomical Laboratory.

Kaufmann, Al. A. 1913. *Theorie und Methoden der Statistik. Ein Lehr- und Lesebuch für Studierende und Praktiker.* Tübingen: J.C.B. Mohr.

Kendall, M.G. 1938. Review of the 6th (1937) edition of Bowley [1901a]. *Journal of the Royal Statistical Society* 101, No. 2, 458-459.

Kendall, M.G. 1939. Review of Thomas and Sundararama Sastry [1939]. *Journal of the Royal Statistical Society* 103, No. 3, 407-409.

Kendall, M.G. 1969. 'The early history of index numbers'. *Review of the International Statistical Institute* 37, No. 1, 1-12. Reprinted in Kendall & Plackett [1977].

Kendall, M.G. & Doig, A.G. 1968. *Bibliography of Statistical Literature. Pre-1940.* Edinburgh and London: Oliver and Boyd.

Kendall, M.G. & Plackett, R.L. 1977. *Studies in the History of Statistics and Probability.* Vol. II. London: Charles Griffin & Company.

Kendall, M.G. & Stuart, A. 1973. *The Advanced Theory of Statistics.* Vol. II, 3rd ed. London: Charles Griffin & Company.

Kendall, M.G. & Stuart, A. 1977. *The Advanced Theory of Statistics.* Vol. I, 4th ed. London: Charles Griffin & Company.

Keynes, J.M. 1908. Review of Howarth & Wilson [1907]. *Journal of the Royal Statistical Society* 71, No. 1, 215-217.

Keynes, J.M. 1909. 'The method of index numbers with special reference to the measurement of general exchange value'. Printed as Chapter 2 of Keynes [1983].

Keynes, J.M. 1911. 'The principal averages and the laws of error which lead to them'. *Journal of the Royal Statistical Society* 74, No. 3, 322-331.

Keynes, J.M. 1922. 'An economist's view of population'. *Manchester Guardian Commercial* Sixth Supplement, 11th August 1922. Reprinted in Keynes [1977, pp. 440-44].

Keynes, J.M. 1924. 'Alfred Marshall, 1842-1924'. *The Economic Journal* 34, No. 135, 311-372. Reprinted in Keynes [1972].

Keynes, J.M. 1972. *Essays in Biography.* Vol. X of *The Collected Works of John Maynard Keynes.* London: published for the Royal Economic Society by the Macmillan Press.

Keynes, J.M. 1973. *A Treatise on Probability.* (1st edition 1921.) Vol. VIII of *The Collected Writings of John Maynard Keynes.* London: published for the Royal Economic Society by the Macmillan Press.

Keynes, J.M. 1977. *Activities 1920-1922: treaty revision and reconstruction.* Vol. XVII of *The Collected Writings of John Maynard Keynes.* London: published for the Royal Economic Society by the Macmillan Press.

Keynes, J.M. 1981. *Activities 1929-1931. Rethinking Unemployment and Unemployment Policies.* Vol. XX of *The Collected Writings of John Maynard Keynes.* London: published for the Royal Economic Society by the Macmillan Press.

Keynes, J.M. 1983. *Economic Articles and Correspondence: Academic.* Vol. XI of *The Collected Works of John Maynard Keynes.* London: published for the Royal Economic Society by the Macmillan Press.

Kiaer, A.N. 1895-6. 'Observations et expériences concernant des dénombrements représentatifs'. *Bulletin de l'Institut International de Statistique* 9(II), 176-183. [Berne, 1895].

Kiaer, A.N. 1899. 'Sur les méthodes representatives ou typologiques appliquées à la statistique'. *Bulletin de l'Institut International de Statistique* 11(I), 180-185. [St Petersburg, 1897].

Kiaer, A.N. 1903a. 'Sur les méthodes representatives ou typologiques'. *Bulletin de l'Institut International de Statistique* 13(I), 66-70. [Budapest, 1901].

Kiaer, A.N. 1903b. Address on the representative method. *Bulletin de l'Institut International de Statistique* 14(I), 119-126. [Berlin, 1903].

King, W.I. 1919. Review of Bowley [1919a]. *The American Economic Review* 9, No. 3, 617-620.

King, W.I. 1925. Review of Astor et al. [1924]. *The American Economic Review* 15, No. 1, 126-128.

King, W.I. 1938. Review of Bowley [1937f]. *The American Economic Review* 28, No. 3, 605-607.

King, W.I. 1943. Review of Bowley [1942c]. *American Sociological Review* 8, No. 5, 612-613.

King, W.I., Knauth, O.W. & Macaulay, R.R. 1922. *Income in the United States. Its Amount and Distribution 1909-1919.* (Ed. W.C. Mitchell.) New York: National Bureau of Economic Research.

Kleiber, C. & Kotz, S. 2003. *Statistical Size Distributions in Economics and Actuarial Sciences.* New York: John Wiley & Sons.

Knibbs, G.H. 1920. 'The organisation of imperial statistics'. *Journal of the Royal Statistical Society* 83, No. 2, 201-224.

Knopp, K. 1951. *Theory and Application of Infinite Series.* 2nd English edition. London: Blackie. (Reprinted Dover Publications, New York, 1990.)

Koop, J.C. 1979. 'On statistical inference in sample surveys and the underlying role of randomisation'. *Annals of the Institute of Statistical Mathematics* 31, Part A, 253-269.

Kotz, S. & Johnson, N.L. 1992. *Breakthroughs in Statistics. Volume II. Methodology and Distribution.* New York: Springer-Verlag.

Kotz, S. & Johnson, N.L. 1999. 'Lifelengths of "leading personalities" in statistics'. *Chance* 12, No. 4, 31-35.

Kotz, S. & Seier, E. 2007. 'An analysis of quantile measures of kurtosis: center and tails'. *Statistical Papers* 49, No. 1. (Published online October 2007; March 2008.)

Krishnaiah, P.R. & Rao, C.R. 1988. *Handbook of Statistics. Vol. 6. Sampling.* Amsterdam: Elsevier Science Publishers.

Krüger, L., Gigerenzer, G. & Morgan, M.S. 1987. *The Probabilistic Revolution.* Vol. 2. Ideas in the Sciences. Camb., Mass.: MIT Press.

Kruskal, W. & Mosteller, F. 1980. 'Representative sampling, IV: the history of the concept in statistics, 1895-1939'. *International Statistical Review* 48, No. 2, 169-195.

Kruskal, W. & Tanur, J.M. (Eds) 1978. *International Encyclopedia of Statistics*, Vol. 2. New York: The Free Press.

Kuczynski, J. 1937. *Labour Conditions in Western Europe 1820 to 1935.* London: Lawrence & Wishart.

Kuznets, S. 1938. Review of Bowley [1937f]. *Journal of the American Statistical Association* 33, No. 202, 456-458.

Lamb, C. 1885. 'Recollections of Christ's Hospital'. Reprinted in *The Works of Charles Lamb*, 1885. London: George Bell & Sons.

Lamb, H. 1956. *An Elementary Course of Infinitesimal Calculus.* Cambridge: University Press.

Laplace, P.-S. 1786. 'Sur les naissances, les mariages et les morts à Paris, depuis 1771 jusqu'en 1784, et dans toute l'étendue de la France, pendant les années 1781 et 1782'. *Mémoires de l'Académie royale des Sciences de Paris* (1783), 693-702. Reprinted in *Œuvres Complètes de Laplace* XI, 35-46.

Laplace, P.-S. 1820. *Théorie analytique des probabilités.* 3rd edition. Reissued in two volumes in 1995, Paris: Jacques Gabay.

Laybourn, K. (Ed.) 1999. *Modern Britain Since 1906.* London: I.B. Tauris.

Lederer, E. 1931. Review of Llewellyn Smith, Vol. 1 [1930]. *Economica* 33, 342-349.

Lee, P.M. 1989. *Bayesian Statistics: an introduction.* London: Edward Arnold.

Leipnik, R. 1981. 'The lognormal distribution and strong non-uniqueness of the moment problem'. *Theory of Probability and Its Applications* 26, No. 4, 850-852.

Lescohier, D.D. 1923. Review of Astor et al. [1923]. *Journal of the American Statistical Association* 18, No. 144, 1069-1071.

Lindahl, E. (Ed.) 1958. *Knut Wicksell. Selected Papers on Economic Theory.* London: George Allen & Unwin. (Reprinted Augustus M. Kelley, 1969.)

Lindley, D.V. 1965. *Introduction to Probability and Statistics from a Bayesian Viewpoint.* 2 vols. Cambridge: Cambridge University Press.

Lindsay, H.A.F. 1934. 'India's trade and industrial statistics, past, present and future'. *Journal of the Royal Statistical Society* 97, No. 3, 399-422.

Linsley, C.A. & Linsley, C.L. 1993. 'Booth, Rowntree, and Llewellyn Smith: a reassessment of interwar poverty'. *Economic History Review* 46, No. 1, 88-104.

Lipsey, R.G. 1960. 'The relation between unemployment and the rate of change of money wage rates in the United Kingdom, 1862-1957: a further analysis'. *Economica*, New Series, 27, No. 105, 1-31.

Llewellyn Smith, H. 1929. 'The New Survey of London Life and Labour'. *Journal of the Royal Statistical Society* 92, No. 4, 530-558.

Llewellyn Smith, H. 1930-1935. *New Survey of London Life and Labour.* 9 vols. London: P.S. King & Son.

Logan, W.P.D. 1950. 'Mortality in England and Wales from 1848 to 1947. A survey of the changing causes of death during the past hundred years'. *Population Studies* 4, No. 2, 132-178.

MacGregor, D.C. 1945. Review of Bowley [1942c]. *The Canadian Journal of Economics and Political Science* 11, No. 2, 270-280.

McKenzie, R.D. 1931. Review of Llewellyn Smith, Vol. I [1930]. *The American Economic Review* 21, No. 4, 712-713.

Macrosty, H.W. 1927. Review of Bowley and Stamp [1927]. *Journal of the Royal Statistical Society* 90, No. 2, 376-378.

Macrosty, H.W. 1938. Review of Bowley [1937f]. *Journal of the Royal Statistical Society* 101, No. 2, 461-463.

Mahalanobis, P.C. 1944. 'Organisation of statistics in the post-war period'. *Proceedings of the National Institute of Sciences of India* 10, No. 1, 69-78.

Makeham, W.M. 1860. 'On the law of mortality and the construction of annuity tables'. *Journal of the Institute of Actuaries* 18, 301-310.

Makeham, W.M. 1890. 'On the further development of Gompertz's law'. *Journal of the Institute of Actuaries* 28, 152-159; 185-192; 316-332.

Mallery, O.T. 1923. Review of Astor et al. [1923]. *Annals of the American Academy of Political and Social Science* 110, 224-225.

Mallock, W.H. 1914. *Social Reform, as related to Realities and Delusions: An Examination of the Increase and Distribution of Wealth from 1801 to 1910.* London: John Murray.

Mandello, J.G. 1905. 'The future of statistics'. *Journal of the Royal Statistical Society* 68, No. 4, 725-732.

Marichal, J.-L. 2000. 'On an axiomatisation of the quasi-arithmetic mean values without the symmetry axiom'. *Aequationes Mathematicae* 59, Nos 1-2, 74-83.

Marsh, L.C. 1930. 'New Survey of London Life and Labor'. *Monthly Labor Review* 31, 35-41.

Marshall, A. 1961. *Principles of Economics.* Ninth (variorum) edition, with annotations by C.W. Guillebaud. Vol. II. London: Macmillan and Co., for the Royal Economic Society.

Martin, R.M. 1838. *The History, Antiquities, Topography, and Statistics of Eastern India.* London: W.H. Allen & Co.

Maunder, W.F. 1972. *Sir Arthur Lyon Bowley (1869-1957).* Exeter: University of Exeter. Reprinted in Kendall & Plackett [1977].

Meek, D.B. 1937. 'Some measures of economic activity in India'. *Journal of the Royal Statistical Society* 100, No. 3, 363-395.

Meller, H.E. 1976. *Leisure and the Changing City, 1870-1914.* London: Routledge & Kegan Paul.

Ménard, C. 1987. 'Why was there no probabilistic revolution in economic thought?' pp. 139-146 in Krüger et al. [1987].

Michalos, A.C. & Poff, D.C. (Eds.) 2002. *Bernard Shaw and the Webbs.* Toronto: University of Toronto Press.

Milne-Thomson, L.M. 1933. *The Calculus of Finite Differences.* London: Macmillan Press.

Mirowski, P. 1989. 'The probabilistic counter-revolution, or how stochastic concepts came to neoclassical economic theory'. *Oxford Economic Papers*, New Series, vol. 41, 1, History and Methodology of Econometrics. pp. 217-235.

Mirowski, P. (Ed.) 1994. *Edgeworth on Chance, Economic Hazard, and Statistics.* Lanham, Maryland: Rowman & Littlefield Publishers, Inc.

Missiakoulis, S. 2006. 'Plutarch and Lucullus' applied sampling'. *The American Statistician* 60, No. 3, 269-270.

Moore, G.C.G. 2005. '*Masters of Theory* and its relevance to the history of economic thought'. *History of Economics Review* 42, 77-99.

Naik, Y.S. 1963. 'Development of industrial statistics in India'. *Sankhyā: The Indian Journal of Statistics* Series B, 25, No. 3/4, 283-322.

Nair, U.S. 1936. 'The standard error of Gini's mean difference'. *Biometrika* 28, No. 3/4, 428-436.

Needham, J. & Pagel, W. 1938. *Background to Modern Science.* Cambridge: University Press.

Neyman, J. 1934. 'On the two different aspects of the representative method: the method of stratified sampling and the method of purposive selection'. *Journal of the Royal Statistical Society* 97, No. 4, 558-625.

Nichols, J.G. (Ed.) 1852. *Chronicle of the Grey Friars of London.* London: printed for the Camden Society, Old Series, vol. 53.

Nixon, J.W. 1913. 'An experimental test of the Normal law of error'. *Journal of the Royal Statistical Society* 76, No. 7, 702-706.

Nixon, J.W. 1960. *A History of the International Statistical Institute, 1885-1960.* The Hague: International Statistical Institute.

O'Brien, D.P. & Presley, J.R. (Eds) 1981. *Pioneers of Modern Economics in Britain.* London and Basingstoke: Macmillan Press.

O'Hagan, A. 1994. *Kendall's Advanced Theory of Statistics. Volume 2B. Bayesian Inference.* London: Edward Arnold.

Ord, J.K. 1972. *Families of Frequency Distributions.* London: Charles Griffin and Company.

Oshima, H. 1951. 'The price system and national income and product'. *The Review of Economics and Statistics* 33, No. 3, 248-254.

Ostasiewicz, S. & Ostasiewicz, W. 2000. 'Means and their applications'. *Annals of Operations Research* 97, 337-355.

Owen, D.B. (Ed.) 1976. *On the History of Statistics and Probability.* New York: Marcel Dekker.

Palmer, G.W. 1916. 'The definition of a complex number'. *The Mathematical Gazette* 8, No. 124, 305.

Pareto, V. 1906. *Manuale di Economie Politica con una Introduzione alla Scienza Sociale.* Milan: Societa editrice libraria.

Pearl, R. & Reed, L.J. 1920. 'On the growth rate of the population of the United States since 1790 and its mathematical representation'. *Proceedings of the National Academy of Sciences* 6, No. 6, 275-288.

Pearson, K. 1895. 'Contributions to the mathematical theory of evolution. II. Skew variation in homogeneous material'. *Philosophical Transactions of the Royal Society of London* A, 186, 343-414.

Pearson, K. 1900a. 'Mathematical contributions to the theory of evolution VII. On the correlation of characters not quantitatively measurable'. *Philosophical Transactions of the Royal Society of London* A, 195, 1-47+405.

Pearson, K. 1900b. 'On the criterion that a given system of deviations from the probable in the case of a correlated system of variables is such that it can be reasonably supposed to have arisen from random sampling'. *London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, Fifth Series, 50, 157-175. Reprinted in Kotz & Johnson [1992].

Pearson, K. 1902a. 'On the systematic fitting of curves to observations and measurements'. *Biometrika* 1, No. 3, 265-303.

Pearson, K. 1902b. 'On the systematic fitting of curves to observations and measurements'. *Biometrika* 2, No. 1, 1-23.

Pearson, K. 1904. 'On an elementary proof of Sheppard's formulae for correcting raw moments and on other allied points'. *Biometrika* 3, No. 2/3, 308-312.

Pearson, K. 1905. 'On the general theory of skew correlation and non-linear regression'. *Drapers' Company Research Memoirs, Biometric Series II.*

Pearson, K. 1911a. 'On a correction to be made to the correlation ratio η'. *Biometrika* 8, No. 1/2, 254-256.

Pearson, K. 1911b. ‘On the probability that two independent distributions of frequency are really samples from the same population’. *Biometrika* 8, No. 1/2, 250-254.

Pearson, K. 1920. ‘Notes on the history of correlation’. *Biometrika* 13, No. 1, 25-45.

Pearson, K. 1922. ‘On the χ^2 test of goodness of fit’. *Biometrika* 14, No. 1/2, 186-191.

Pearson, K. 1923a. ‘On the correction necessary for the correlation ratio η’. *Biometrika* 14, No. 3/4, 412-417.

Pearson, K. 1923b. ‘Further note on the χ^2 test of goodness of fit’. *Biometrika* 14, No. 3/4, 418.

Persons, W.M. 1919. ‘An index of general business conditions’. *The Review of Economic Statistics* 1, No. 2. In four parts. I. The index: a statement of results, 111-117; II. The method used, 117-139; III. Application of the method to the data, (A) the individual series, 139-181; IV. Application of the method to the data, (B) the groups of series, 182-205.

Persons, W.M. 1925a. Review of Bowley [1924d]. *Bulletin of the American Mathematical Society* 3, 469.

Persons, W.M. 1925b. ‘Statistics and economic theory’. *The Review of Economics and Statistics* 7, No. 3, 179-197.

Philips, A.W. 1958. ‘The relation between unemployment and the rate of change of money wage rates in the United Kingdom, 1861-1957’. *Economica*, New Series, 25, No. 100, 283-299.

Pitt-Rivers, G.H.L.F. (Ed.) 1932. *Problems of Population*, being the Report of the Proceedings of the Second General Assembly of the International Union for the Scientific Investigation of Population Problems. London: George Allen & Unwin.

Poisson, S.-D. 1830. ‘Mémoire sur la proportion des naissances des filles et des garçons’. *Mem. Acad. Sci. Paris* 9, 239-308.

Poisson, S.-D. 1837. *Recherches sur la probabilité des jugements en matière criminelle et en matière civile, précéedes des régles générales du calcul des probabilités.* Paris: Bachelier.

Quetelet, L.A.J. 1835. *Sur l’Homme et le Développement de ses Facultés ou Essai de physique sociale.* 2 vols. Paris: Bachelier. Translated as Quetelet [1842].

Quetelet, L.A.J. 1842. *A Treatise on Man and the Development of his Faculties.* Edinburgh: William and Robert Chambers. Reprinted in 1969; Gainesville, Fla.: Scholars' Facsimiles & Reprints.

Ramachandran, B. & Lau, K.S. 1991. *Functional Equations in Probability Theory.* San Diego: Academic Press.

Rao, V.K.R.V. 1944. 'National income of India'. *Annals of the American Academy of Political and Social Science* 233, 99-105.

Ray, P.R. & Sinha, H.C. 1941. 'Index of business activity in India'. *Sankhyā: The Indian Journal of Statistics* 5, No. 2, 179-190.

Reamer, A. 2009. 'In dire straits: the urgent need to improve economic statistics'. *Amstat News*, March issue, 33-35.

Rew, R. H. 1909. 'The International Statistical Institute at Paris'. *Journal of the Royal Statistical Society* 72, No. 3, 594-603.

Rhodes, E.C. 1945. Note on G.H. Wood. *Journal of the Royal Statistical Society* 108, No. 3/4, 486-487.

Richardson, J.H. 1930. 'International comparisons of real wages'. *Journal of the Royal Statistical Society* 93, No. 3, 398-441.

Robbins, L.C. 1935. 'A student's recollections of Edwin Cannan'. *The Economic Journal* 45, No. 178, 393-398.

Robbins, L.C. 1971. *Autobiography of an Economist.* London: Macmillan.

Roth, L. 1971. 'Old Cambridge days'. *The American Mathematical Monthly* 78, No. 3, 223-236.

Rothbarth, E. 1943. Review of Bowley [1942c]. *The Economic Journal* 53, No. 209, 55-59.

Routh, G. 1959. 'The relation between unemployment and the rate of change of money wage rates: a comment'. *Economica*, New Series, 26, No. 104, 299-315.

Rowntree, B. S. 1901. *Poverty: a Study of Town Life.* 3rd edn. (1902). London: Macmillan and Co.

Rowntree, B.S. 1915. Review of Bowley and Burnett-Hurst [1915]. *The Economic Journal* 25, No. 99, 427-430.

Rowntree, B.S. 1919/1937. *Human Needs of Labour*. London: T. Nelson & Sons. (Published in 1937 as *The Human Needs of Labour.* London: Longmans.)

Rowntree, B.S. 1941. *Poverty and Progress: A Second Social Survey of York.* London: Longmans, Green and Co.

Sanger, C.P. 1901. Review of Bowley [1901a]. *The Economic Journal* 11, No. 42, 193-197.

Sargant, W.L. 1866. 'On the vital statistics of Birmingham and seven other large towns'. *Journal of the Statistical Society of London* 29, No. 1, 92-111.

Särndal, C.-E. 1971. 'Studies in the history of probability and statistics. XXVII. The hypothesis of elementary errors and the Scandinavian school in statistical theory'. *Biometrika* 58, No. 2, 375-391.

Sauerbeck, A. 1896. 'Prices of commodities in 1895'. *Journal of the Royal Statistical Society* 59, No. 1, 186-201.

Schumpeter, J.A. 1954. *History of Economic Analysis.* London: George Allen and Unwin.

Seligman, C.G. 1930. *Races of Africa.* London: Thornton Butterworth Ltd.

Seneta, E. 1985. 'A sketch of the history of survey sampling in Russia'. *Journal of the Royal Statistical Society* A 148, No. 2, 118-125.

Seng, Y.P. 1951. 'Historical survey of the development of sampling theories and practice'. *Journal of the Royal Statistical Society* A 114, No. 2, 214-231.

Sheppard, W.F. 1897. 'On the calculation of the most probable values of frequency-constants, for data arranged according to equidistant division of a scale'. *Proceedings of the London Mathematical Society* XXIV, 353-380.

Sheppard, W.F. 1899. 'On the application of the theory of error to cases of Normal distribution and Normal correlation'. *Philosophical Transactions of the Royal Society of London* A, 192, 101-167+531.

Sheppard, W.F. 1925. Review of Bowley [1924d]. *The Mathematical Gazette* 12, No. 174, 292.

Shirras, G.F. 1929. 'Gold and British capital in India'. *The Economic Journal* 39, No. 156, 629-636.

Shirras, G.F. 1933. 'The population problem in India'. *The Economic Journal* 43, No. 169, 56-73.

Shirras, G.F. 1935a. 'The census of India, 1931'. *Geographical Review* 25, No. 3, 434-448.

Shirras, G.F. 1935b. 'The Pareto Law and the distribution of income'. *The Economic Journal* 45, No. 180, 663-681.

Smith, C.M. 1853. *Curiosities of London Life; or phases, physiological and social, of the great metropolis.* London: A.W. Bennett.

Smith, T.M.F. 1976. 'The foundations of survey sampling: a review'. *Journal of the Royal Statistical Society* A 139, No. 2, 183-204.

Smith, T.M.F. 1997. 'Social surveys and social science'. *The Canadian Journal of Statistics* 25, No. 1, 23-44.

Snedecor, G.W. 1946. *Statistical Methods Applied to Experiments in Agriculture and Biology.* 4th ed. Ames, Iowa: Iowa State College Press.

Snow, E.C. 1913. Review of Gini [1912]. *Journal of the Royal Statistical Society* 76, No. 3, 326-327.

Snow, E.C. 1923. 'Trade forecasting and prices'. *Journal of the Royal Statistical Society* 86, No. 3, 332-398.

Stamp, J.C. 1914a. Review of Mallock [1914]. *Journal of the Royal Statistical Society* 77, No. 6, 675-678.

Stamp, J.C. 1914b. 'A new illustration of Pareto's Law'. *Journal of the Royal Statistical Society* 77, No. 2, 200-204.

Stamp, J.C. 1937. *The Science of Social Adjustment.* London: Macmillan and Co.

Starr, M.P. 1995. 'Aleister Crowley: freemason!'. *Ars Quatuor Coronatorum. Transactions of Quatuor Corona Lodge, No. 2076*, 108, 150-161.

Stephan, F.F. 1948. 'History of the uses of modern sampling procedures'. *Journal of the American Statistical Association* 43, No. 241, 12-39.

Stevenson, T.H.C. 1925. 'The laws governing population'. *Journal of the Royal Statistical Society* 88, No. 1, 63-90.

Stigler, S.M. 1978. 'Francis Ysidro Edgeworth, statistician'. *Journal of the Royal Statistical Society* A, 141, No. 3, 287-322. Reprinted in Stigler [1999].

Stigler, S.M. 1999. *Statistics on the Table: The History of Statistical Concepts and Methods.* Cambridge, Mass.: Harvard University Press.

Stigler, S.M. 2002a. 'The missing early history of contingency tables'. *Annales de la faculté des sciences de Toulouse* 6^{e} série 11, No. 4, 563-573.

Stigler, S.M. 2002b. 'Statisticians and the history of economics'. *Journal of the History of Economic Thought* 24, No. 2, 155-164.

Stone, R. 1943. Review of Bowley [1942c]. *Economica* 10, No. 40, 312-315.

Stow, K. 1603/1912. *A Survay of London. Conteyning the Originall, Antiquity, Increase, Moderne estate, and description of that City, written in the yeare 1598, by John Stow Citizen of London. Since by the same Author increased, with diuers rare notes of Antiquity, and published in the yeare, 1603. Also an Apologie (or defence) against the opinion of some men, concerning that Citie, the greatnesse thereof. With an Appendix, containing in Latine* Libellum de situ *&* nobilitate Londinii*: Written by William Fitzstephen, in the raigne of Henry the second.* Reprinted in 1912; London: J.M. Dent & Sons.

Stoyanov, J. 2000. 'Krein condition in probabilistic moment problems'. *Bernoulli* 6, No. 5, 939-949.

Strauss, P. 1970. *Bevin and Co: The Leaders of British Labour.* Manchester: Ayer Company Publishing.

Sukhatme, P.V. 1935. 'Contribution to the theory of the representative method'. *Supplement to the Journal of the Royal Statistical Society* 2, No. 2, 253-268.

Szreter, S. & Hardy, A. 2000. 'Urban fertility and mortality patterns'. Chapter 20 in *The Cambridge Urban History of Britain, vol. III 1840-1950.* Ed. M. Daunton. Cambridge: Cambridge University Press.

Szreter, S. & Smith, R. 1996. *Fertility, Class and Gender in Britain, 1860-1940.* Cambridge: Cambridge University Press.

Tappan, M. 1925. Review of Bowley [1924d]. *Economica* 15, 334-338.

Tarascio, V.J. 1972. 'A correction: on the genealogy of the so-called Edgeworth-Bowley box'. *Western Economic Journal* 10, 193-197.

Theil, H. 1958. Review of Allen [1956]. *Econometrica* 26, No. 2, 336-337.

Thomas, B. 1932. Review of Llewellyn Smith, Vol. II [1931]. *Economica* 35, 128-130.

Thomas, P.J. 1937. 'The census as an agency for economic plannning'. *Sankhyā: The Indian Journal of Statistics* 5, No. 3, 247-248.

Thomas, P.J. & Sundararama Sastry, N. 1939. *Indian Agricultural Statistics. An Introductory Study.* University of Madras.

Thomsen, I. 2001. 'Anders Nicolai Kiaer'. pp. 208-211 in Heyde & Seneta [2001].

Thornbury, G.W. 1881. *Old and New London*, vol. 2. Published in six volumes 1879-1885. London: Cassell, Petter, Galpin & Co.

Todhunter, I. 1865. *A History of the Mathematical Theory of Probability: from the time of Pascal to that of Laplace*. Cambridge.

Topalov, C. 2001. 'A revolution in representations of work: the emergence over the 19th century of the statistical category "occupied population" in France, Great Britain, and the United States'. *Revue Française de Sociologie* 42, Supplement, 79-106.

Trachtenberg, H.L. 1924. 'The wider application of the Gompertz law of mortality'. *Journal of the Royal Statistical Society* 87, No. 2, 278-290.

Turner, H.H. 1888. 'On Mr. Edgeworth's method of reducing observations relating to several quantities'. *London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, Fifth Series, 24, 466-470.

Tweddle, I. 1988. *James Stirling 'This about series and such things'*. Edinburgh: Scottish Academic Press.

Uncle Jonathan. 1895. *Walks In and Around London*. 3rd edition. (1st ed. 1884.) London: T. Woolmer.

Venn, J. 1866. *The Logic of Chance*. London: Macmillan. 3rd edition of 1888 reprinted as 4th edition, 1962, New York: Chelsea.

Venn, J. 1891. 'On the nature and use of averages'. *Journal of the Royal Statistical Society* 54, No. 3, 429-456.

Wainer, H. 2009. 'A good table can beat a bad graph: it matters who plays Mozart'. *Chance* 22, No. 4, 55-57.

Ward, A. 1891. *Artemus Ward, His Book*. London: George Routledge and Sons.

Ware, C.F. 1932. Review of Llewellyn Smith, Vol. I [1930]. *Political Science Quarterly* 47, No. 1, 141-143.

Watkins, G.P. 1916. 'Comment on Westergaard's "Scope and Method of Statistics" '. *Publications of the American Statistical Association* 15, No. 115, 288-291.

Watkins, G.P. 1927. Review of Bowley and Stamp [1927]. *Journal of the American Statistical Association* 22, No. 159, 404-405.

Weatherby, J.L. 1976. 'Why was it called an Edgeworth-Bowley box? A possible explanation'. *Economic Inquiry* 14, 294-296.

Webb, A.D. 1921. Review of Bowley [1921h]. *The Economic Journal* 31, No. 124, 498-501.

Webb, B. 1982-1985. *The Diary of Beatrice Webb.* (Ed. N. & J. MacKenzie) 4 vols. London: Virago Press, in association with the London School of Economics and Political Science.

Weida, F.M. 1927-1928. 'On various conceptions of correlation'. *The Annals of Mathematics*, 2nd Series, 29, No. 1/4, 276-312.

West, J.B. 1999. 'The original presentation of Boyle's law'. *Journal of Applied Physiology* 87, 1543-1545.

Westergaard, H. 1901. *Die Lehre von der Mortalität und Morbilität.* Jena: Gustav Fischer.

Whitaker, L. 1914. 'On the Poisson law of small numbers'. *Biometrika* X, No. 1, 36-71.

Whittaker, E.T. & Watson, G.N. 1915. *A Course of Modern Analysis.* 2nd edition. Cambridge: Cambridge University Press.

Whitworth, W.A. 1867. *Choice and Chance, Two Chapters of Arithmetic; with an appendix containing the algebraical treatment of permutations and combinations newly set forth.* Cambridge: Deighton, Bell and Co.

Whitworth, W.A. 1901. *Choice and Chance, with 1000 exercises.* 5th edition. Reprinted 1942. Cambridge: Deighton, Bell and Co.

Wicksell, K. 1925. 'Matamatisk nationalekonomi'. *Eknonomisk Tidskrif* 103-125. (English translation in Lindahl [1958].)

Willcox, W.F. 1934. 'Note on the chronology of statistical societies'. *Journal of the American Statistical Society* 29, No. 188, 418-420.

Willcox, W.F. 1936. 'Definitions of statistics'. *Review of the International Statistical Institute* 3, No. 4, 388-399.

Wishart, J. 1939. 'Some aspects of the teaching of statistics'. *Journal of the Royal Statistical Society* 102, No. 4, 532-564.

Wright, T. 2001. 'Selected moments in the development of probability sampling: theory and practice'. *Newsletter of the Section on Survey Research Methods of the American Statistical Association* Issue 13 (July), 1-6.

Yates, F. 1946. 'A review of recent statistical developments in sampling and sampling surveys'. *Journal of the Royal Statistical Society* 109, No. 1, 12-43.

Young, A.A. 1910. Review of Bowley [1910a]. *Publications of the American Statistical Society* 12, No. 92, 385-386.

Young, A.A. 1925. Review of Bowley [1924d]. *Journal of the American Statistical Association* 20, No. 149, 133-135.

Yule, G.U. 1897. 'On the theory of correlation'. *Journal of the Royal Statistical Society* 60, No. 4, 812-854.

Yule, G.U. 1903. 'Notes on the theory of association of attributes in statistics'. *Biometrika* 2, No. 2, 121-134.

Yule, G.U. 1909. 'The applications of the method of correlation to social and economic statistics'. *Journal of the Royal Statistical Society* 72, No. 4, 721-730.

Yule, G.U. 1921. Review of the 4th (1920) edition of Bowley [1901a]. *The Economic Journal* 31, No. 122, 220-224.

Yule, G.U. 1922. 'On the application of the χ^2 method to association and contingency-tables, with experimental illustrations'. *Journal of the Royal Statistical Society* 85, No. 1, 95-104.

Yule, G.U. 1925. 'The growth of population and the factors which control it'. *Journal of the Royal Statistical Society* 88, No. 1, 1-58.

Yule, G.U. & Kendall, M. 1950. *An Introduction to the Theory of Statistics*. 14th edition. London: Charles Griffin & Company Ltd.

Zabell, S.L. 2008. 'On Student's 1908 article "The probable error of a mean" '. *Journal of the American Statistical Association* 103, No. 481, 1-20.

Žarkovič, S.S. 1956. 'Notes on the history of sampling methods in Russia'. *Journal of the Royal Statistical Society* A 119, No. 3, 336-338. (Reprinted in Kendall & Plackett [1977].)

Žarkovič, S.S. 1962. 'A supplement to "Note on the history of sampling methods in Russia" '. *Journal of the Royal Statistical Society* A 125, No. 4, 580-582. (Reprinted in Kendall & Plackett [1977].)

This page is intentionally left blank

Index

ARTHUR L BOWLEY

A Pioneer in Modern Statistics and Economics

Arthur Lyon Bowley, the founding father of modern statistics, was an important and colorful figure and a leader in cementing the foundations of statistical methodology, including survey methodology, and of the applications of statistics to economical and social issues during the late 19th and early 20th centuries. In many respects, he was ahead of his time.

The giants in this field around that time were largely concentrated in the British Isles and Scandinavian countries; among these contributors, Arthur Bowley was one of the most active in revolutionizing statistical methodology and its economic applications. However, Bowley has been vastly undervalued by subsequent commentators — while hundreds of articles and books have been written on Karl Pearson, those on Arthur Bowley amount to a dozen or fewer. This book seeks to remedy this and fill in an important omission in the monographical literature on the history of statistics. In particular, the recent resurgence of interest in poverty research has led to a renewed interest in Bowley's legacy.

World Scientific
www.worldscientific.com
7049 hc

ISBN-13 978-981-283-550-5
ISBN-10 981-283-550-4