ORGANIC COMPOSITION OF CAPITAL AND AVERAGE PERIOD OF PRODUCTION

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1. — Introduction

Böhm-Bawerk and Marx are considered to be the protagonists of two different and quarreling schools of thought in capital theory. Their method of analysis seems to differ widely, the purpose of their investigation seems to be exactly opposed, and their only similarity seems to be their liking for polemics and controversy (1). In the following I want to show that the analytical concepts which they use and the strategies of simplification which they choose have a striking similarity. Moreover, even from the point of view of present day economics these simplifications have some relevance as approximations. This fact casts some light on the recent controversies in capital theory.

The mathematical repertoire which was available to Marx and Böhm-Bawerk was considerably inferior to the one available to modern economics. In presenting Marx and Böhm-Bawerk's theories in a consistent way, it is therefore necessary to provide some mathematical polishing. They use certain basic concepts, such as the organic composition of capital or the average period of production. In presenting their theories I want to preserve the role which these central concepts play, but, within that constraint, I want to come as close as possible to an exposition of the theories which is convincing even to present day readers. This obviously is not completely

(1) Cf. for example Marx' discussion of Nassau Senior's theory of abstinence (Das Kapital, vol. I, pp. 626-628), a forerunner of Böhm-Bawerk's own theories and on the other hand Böhm-Bawerk's critique of Marx: E. v. Böhm-Bawerk, Zum Abschluß des Marxschen Systems, in O. Häring (ed.), *Staatswissenschaftliche Arbeiten, Festgaben für Karl Knies*, Berlin 1896. Here and in the following page, quotations from Das Kapital are from Karl Marx, Das Kapital, vol. I, II, III, Berlin, Dietz-Verlag, 1959.

possible. There remains a gap between what I consider a consistent presentation of Marx' and Böhm-Bawerk's theories and what we would consider a theory up to modern standards of consistency. But, I believe, we can interpret their theories as sound approximations of « correct » theories, just as sound as the approximation, which was used so much in empirical work : the macroeconomic production function, i. e. not sufficient for all theoretical purposes, but good enough for certain practical applications or for simple heuristic explanations.

2. — Marx' law of the tendency of a falling rate of profit (2)

It is well-known that the tendency of a falling rate of profit is one of the cornerstones of Marxist analysis of capitalism. Not only Marx himself but also his followers use the law to demonstrate the increasing contradiction between the productive forces developed through capitalism and the production relations prevailing under capitalism.

In the fourth chapter of volume III of Das Kapital, Marx provides the following formula for the rate of profit (3)

$$\text{rate of profit} = \frac{\text{rate of exploitation times variable capital times speed of turnover of variable capital}}{\text{variable capital} + \text{constant capital}}.$$

This is a formula applicable to the economy at large. Its numerator is an expression of the sum of surplus value generated within a certain period of time, say, a year. Its denominator is the amount of capital invested and owned by the capitalist class. The falling profit rate is explained by Marx' proposition of a rising ratio of constant capital to variable capital, what he calls a rising organic or value composition of capital. From the modern point of view, we could criticize the formula, as has been discussed by Morishima (4), because the right hand side is in value terms rather than price terms whereas the rate of profit should be in price terms. But I want to ignore this criticism : a correct solution of this problem would have

(2) For this section see Das Kapital, vol. III, section III, chapter 13-15.
(3) Cf. Das Kapital, vol. III, pp. 94-95.
(4) M. Morishima, *Marx' Economics*, Cambridge, 1973, in particular chapter 6.

surpassed Marx' mathematical capacity, since the difference between the value and production price of constant and variable capital depends itself on the disaggregated structure of the economy and the profit rate. I then want to use the formula, as if it were correct. We may think of it as the first approximation of the correct solution. To explain the falling rate of profit by the rising organic composition of capital is only valid ceteris paribus. Now Marx is aware of the fact that the rate of exploitation may rise. He thus makes the even stronger statement that the organic composition has a tendency to rise faster than the rate of exploitation. Let us therefore assume that the rate of exploitation does not interfere with Marx' law a falling profit rate. There remains the speed of turnover of variable capital. Only those phenomena explain the falling profit rate which cause the value composition to rise but which do not at the same time cause the speed of turnover of variable capital to rise in equal proportion. As we shall see, this will give us a hint for a more consistent interpretation of what the organic composition of capital really is.

What is the speed of turnover of variable capital ? For a given firm this is a clear concept : the variable capital which the owner of the firm has to keep in his business is the capital tied down in wages paid to his workers before their product is sold on the market. Constant capital is the amount of capital tied down in inputs purchased from other firms before the products made with these inputs are sold on the market. To a certain extent it is possible to separate out specific pieces of capital which only (or almost only) contain constant capital : inventories of raw materials not yet transformed in the production process of the firm are purely constant capital, newly bought equipment is also constant capital only. But inventories of intermediate products already represent a mixture of constant and variable capital. They contain labour expended in the process of the production within the firm. Finished products, before they are sold, contain a maximum proportion of variable capital. It is thus a mistake to say that constant capital corresponds to the value of the physical items in the factory, so to speak that the constant capital is what you can touch with your fingers. For, unless the firm itself represents capital and value independent from these physical items (organisational capital), there would be nothing but constant capital.

A tendency to rise for the organic composition of capital thus

means that the value proportion of inputs bought from other firms to inputs bought from workers is rising through time. The « value added » of the production process in the firm is declining as a proportion of the value of sold finished products of the firm. If we aggregate over all firms in the economy this means that for any given value added of the economy total sales of all firms have a tendency to rise. Now, orthodox economists are well aware of the fact that the ratio of total sales to total value added is a good measure of the average degree of vertical nonintegration in the economy. Indeed, any merger of the vertical integration type will reduce total sales in the economy without by itself reducing value added.

But did Marx really say that there is a tendency towards vertical deconcentration as capitalism advances ? On the contrary, throughout his work he emphasizes the tendency towards concentration of capital in fewer and fewer hands, and I am not aware of any passage in his work, where he excludes vertical integration from this general tendency towards concentration. Whatever Marx said, empirical facts do not indicate a tendency towards vertical deconcentration. Whereas I believe that my interpretation of variable and constant capital is the only reasonable interpretation of what Marx meant by these terms, I am quite certain that he would have revised his definitions, had be seen the consequences just drawn from them. The process which he wants to describe is the process of an increasing amount of value represented in the machinery and working capital used by a given work force. Constant capital represents the increasing value of the means of production combined with a given work-force, whereas variable capital represents the size of the work-force itself. The careful reader of chapter 13, in volume III of Das Kapital, will agree with me. But obviously variable capital can represent the size of the work-force only for a given speed of turnover of variable capital. If this speed of turnover is cut in half, we need twice the former variable capital to represent the same work-force. Hence, what Marx really was after, is the ratio of constant capital (a stock) to the product of variable capital and the speed of turnover of variable capital (a flow). And indeed, if you want to transform the rate of exploitation (a variable without dimension) into the rate of profit (which has the dimension « one over time ») you need a coefficient which itself is of the dimension « one over time ». And this coefficient, or its inverse, the capital/wage bill ratio is something which is not directly affected by mergers of firms or other forms of vertical integration. It and not, as Marx thought, the dimensionless constant/variable capital ratio represents the value of capital which is combined with a given work-force.

Of course, we could save the original interpretation of the organic composition by assuming that the speed of turnover of variable capital and the definition of variable capital itself, always refer to some technologically defined stage of production, perhaps the smallest stage of production, in the sense that it would be infeasible or at least clearly inefficient to split it up and let different parts of it take place at different locations or in different firms. But then the falling profit rate would presuppose that the time length of these indivisible stages of production does not change, as the productive forces are further developed. Should this be the case (which is implausible), we would without loss of generality define this time length of minimal production stages as our unit period so that the speed of turnover of variable capital will be unity, by construction. Indeed this is, what is implicitely done, whenever production is modelled in discrete time units, which in my opinion has the disadvantage that stocks and flows are easily confused. But, let us see what the organic composition of capital signifies in such a model of production stages of unit length. Let the different elementary production processes in the whole economy be numbered through from 1 to N, where N is the number of different such processes. Let s_i be a number indicating the average number of stages through which the constant capital went, before it entered stage i. Let $\bar{s}_i$ be the average number of stages through which the products of stage i went before reaching stage i plus stage i itself. If stage i did not use any labour, then, of course, $\bar{s}_i$ were $s_i + 1$. But it is less, because $\bar{s}_i$ is a weighted average of $s_i + 1$ (for the constant capital) and unity for the labour inputs, i. e. the value added of stage i. These labour inputs are $(1 + e)\, v_i$, where e is the rate of exploitation and v_i is variable capital of stage i. Let constant capital of stage i be c_i and thus the organic composition of this stage c_i/v_i. We then obtain the formula

$$\bar{s}_i = \frac{(1+e)\, v_i + c_i(s_i + 1)}{(1+e)\, v_i + c_i}.$$

Let us now try to develop a corresponding formula for the economy at large. Let s be the appropriate average of the s_i and let $\bar{s}$ be the corresponding average of the $\bar{s}_i$. Let v and c be variable and constant

capital of the economy respectively. We define (summation sign Σ indicates summation over all $i = 1, 2, ..., \mathrm{N}$):

$$s = \frac{\Sigma c_i s_i}{\Sigma c_i} = \frac{\Sigma c_i s_i}{c}$$

$$\bar{s} = \frac{\Sigma((1+e)\, v_i + c_i)\, \bar{s}_i}{\Sigma(1+e)\, v_i + c_i} = \frac{\Sigma((1+e)\, v_i + c_i)\, \bar{s}_i}{(1+e)\, v + c}.$$

We then obtain

$$\bar{s} = \frac{\Sigma[(1+e)\, v_i + c_i(s_i+1)]}{(1+e)\, v + c}$$

$$= \frac{(1+e)\, v}{(1+e)\, v + c} + \frac{c}{(1+e)\, v + c}\, \frac{\Sigma c_i(s_i+1)}{c}$$

$$= \frac{(1+e)\, v}{(1+e)\, v + c} + \frac{c}{(1+e)\, v + c}(s+1) = 1 + \frac{c}{(1+e)\, v + c}\, s.$$

If we now assume a stationary system, the value of consumption goods delivered to workers and capitalists, y, equals $(1 + e)\, v$, the value added. What is the average $\bar{s}$ of consumption goods, which we call s_y? Total output consists of the consumption goods and the constant capital employed in the next period. Since the system is in stationary equilibrium, the s of the constant capital of next period is the same as the s of this period. We thus obtain the equation

$$\bar{s} = \frac{y}{(1+e)\, v + c}\, s_y + \frac{c}{(1+e)\, v + c}\, s =$$

$$= \frac{(1+e)\, v}{(1+e)\, v + c}\, s_y + \frac{c}{(1+e)\, v + c}\, s.$$

Thus, we now have two equations involving $\bar{s}$: one explaining $\bar{s}$ from the inputs of the production process and the other relating $\bar{s}$ to the use of the product for final and, as Marx calls it, productive consumption. These two equations combined imply after elimination of $\bar{s}$

$$1 + \frac{c}{(1+e)\, v + c}\, s = \frac{(1+e)\, v}{(1+e)\, v + c}\, s_y + \frac{c}{(1+e)\, v + c}\, s$$

which obviously yields

$$1 = \frac{(1+e)\, v}{(1+e)\, v + c}\, s_y$$

or

$$s_y = \frac{c}{(1+e)\,v} + 1 = \frac{1}{1+e}\,\frac{c}{v} + 1\,.$$

c/v is the organic composition of capital, which therefore is an indicator of the average number of standardised stages which are involved in the production of the final output, the consumption goods. Given that these stages are of equal time length, s_y can be interpreted as the average time distance of labour inputs and consumption good output. Indeed, nothing would have changed in the formal argument, had we called $\bar{s}_i$ the average time distance of original labour inputs leading up to the products of production stage i.

Whichever route we take, the organic composition of capital, if interpreted in such a way as to really explain the falling rate of profit, turns out to have the dimension « time » and, indeed, is closely related to the Böhm-Bawerkian average period of production.

3. — The transformation problem I : Ricardo's influence

There is little doubt that Marx was aware of the discrepancy between labour values and prices from the time that he seriously studied Political Economy. For it is all in Ricardo's Principles (5), chapter 1, section 4 and 5. Ricardo, and indeed Adam Smith, knew the principle of the uniform rate of profit, which is a consequence of the intersectoral mobility of capital. This mobility of capital itself is a necessary assumption for the classical theory of natural price, as opposed to the daily market price.

But not only did Marx read in Ricardo, that a uniform profit rate is normally incompatible with prices equal to labour values. In his work he also gave Ricardo's reason for this discrepancy. Section 4 in chapter 1 of Ricardo's Principles bears the title : « The principle that the quantity of labour bestowed on the production of commodities regulates their relative value, considerably modified by the employment of machinery and other fixed and durable capital. » He then explains why with increasing wages and falling profits the

(5) D. Ricardo, *On the Principles of Political Economy and Taxation*, ed. by P. Sraffa, Cambridge 1951, reprinted 1953.

price of those commodities, which are produced with much fixed capital, falls relative to those commodities which require little fixed capital. What does he really mean by fixed capital ? « According as capital is rapidly perishable, and requires to be frequently reproduced, or is of slow consumption, it is classed under the heads of circulating, or of fixed capital », to which he adds in a footnote : « A division not essential and on which the line of demarcation cannot be accurately drawn (6). » The fixity of capital is thus a matter of degree for Ricardo, and it would be translated in modern terms as capital intensity. And thus Ricardo provides the reason for the discrepancy between labour values and prices which we would give today : prices are not equal to labour values because different industries have different capital intensity.

As I said, Marx adopts Ricardo's explanation by explaining the price-value discrepancy by differences in the organic composition of capital. Now, of course, Marx thinks that his concept of organic composition is quite different from Ricardo's concept of fixity of capital. Contrary to Ricardo's own intentions, he interprets the difference between fixed and circulating capital in Ricardo as one of quality rather than degree and he explicitely rejects the identification of this distinction with his distinction of constant and variable capital. But then his own concept of organic composition of capital is only able to be related to the rate of profit if it has the dimension « time » and thus is an indicator of the time structure of production. On the other hand, Ricardo was quite clear and explicit that his concept of fixity of capital is just a special case of the general concept of time difference between inputs and outputs. For he writes : « On account then of the different degrees of durability of their capital, or, which is the same thing, on account of the time which must elapse before one set of commodities can be brought to market, they will be valuable, not exactly in proportion to the quantity of labour bestowed on them (7) », or again : « ... the superior price of one commodity is owing to the greater length of time which must elapse before it can be brought to market (8) ».

Before I continue, I want to discuss a little further the distinction between fixed and circulating capital, which basically for Ricardo

(6) Ricardo, *op. cit.*, p. 31.
(7) Ricardo, *op. cit.*, p. 34.
(8) Ricardo, *op. cit.*, p. 37.

was one of degree reflecting different time distances of inputs and outputs. In modern mathematical economics it has become more fundamental. Activity analysis as it developed in recent decades tended to use the dichotomy : single output activity models versus joint output activity models. If the time dimension was explicitely used, it became convenient to subsume the phenomenon of fixed capital as a special case under the heading of joint production. The von Neumann growth model in particular made this approach popular (9). In discussing certain problems of capital theory, it was mathematically convenient to use single output models, and using the Leontief Input-Output model in a discrete time setting, models were developed which incorporated the phenomenon of capital without having to bother about joint production. They were pure circulating capital models.

A large part of Morishima's discussion of Marx and many other publications (including some of my own) made use of this convenient device. Thus, our intuitive understanding of the difference between fixed capital (« machines », « buildings ») and circulating capital (« things used up as they enter the production process ») were analytically reenforced by the fundamental distinction between joint product and single product activities. But I believe that this distinction is, as Ricardo already said, nothing more than one of degree. This becomes clear when one starts using a continuous time medel. Then, the only reasonable distinction is between production processes producing a homogeneous flow of outputs and those producing a vector flow of outputs. Whenever capital is involved, it will not be a flow, but a stock, and it is possible for a stock to disappear in an instant only on a set of points of measure zero. Thus, for the economist neither the stocks of fixed nor the stocks of circulating capital disappear in a moment of time. They can all be treated alike ; if you wish, they are all fixed capital. From a point of view, which is not related to the single product-joint product distinction, there may be fundamental differences between fixed and circulating capital. Fixed capital can perhaps be defined as capital consisting of goods which are involved in the proper production processes and

(9) P. Sraffa, *Production of Commodities by Means of Commodities*, Cambridge 1960, and Morshima, *op. cit.* build their analysis on this approach. For Morishima the « von Neumann » revolution, as he calls it, is the basic reason for rejecting Marx' labour theory of value.

circulating capital are goods which are buffer stocks of one kind or another. But this is quite a different story, which is usually not of interest in the kind of problems discussed in capital theory.

There may be mathematical and other disadvantages for continuous time models in capital theory. But there is no doubt that they are superior for the purpose of maintaining the difference between stocks and flows. In the following analysis, I shall therefore use a continuous time model.

4. — The Transformation Problem : Mathematical analysis (10)

The following mathematical model tries to follow Marx's analysis of the transformation problem as closely as possible. It can be considered as a generalisation of the numerical example developed by Marx. Using this mathematical model, we try to obtain an approximate solution of the transformation problem which keeps the close connection between the organic composition of capital and the production price-value discrepancy of any industry.

For the purposes of simpler notation and computation all the following variables are defined with reference to a unit value output of a firm (or industry). Thus, we could imagine that the firm is producing one unit of output in labour value terms per year. Let c be the stock of constant capital in this firm (or industry : it does not matter here). Let c' be constant capital being used up per year ; it is a flow. Let v be the stock of variable capital, and let v' be the flow of variable capital used up per year. In other words, v' is the annual wage bill of the firm. Let $u = c'/c$ be the speed of turnover of constant capital and let $n = v'/v$ be the speed of turnover of variable capital. Let e be the rate of exploitation. Let m' be the flow of surplus value accruing in the firm. Since total value production in the firm is unity per year, we obtain

$$m' + v' + c' = 1$$

and, remembering the definition of e,

$$(1 + e)\, v' + c' = 1 \qquad \text{or} \qquad v' + c' = 1 - ev' .$$

(10) For this section see Das Kapital, vol. III, section II : chapters 8-12, in particular chapter 9.

Using now Marx's approach, which first disregards price-value differences on the input side, the cost price k is equal to

$$k = v' + c'$$

and the production price p is

$$p = v' + c' + r(v + c)$$

where r is the rate of profit. The rate of profit must be derived from the rate of exploitation by using the relevant formula for the national economy. The letters with a bar have the analogous meaning for the economy which their counterparts without bars have for the firm under consideration. Thus the profit rate is

$$r = e\frac{\bar{v}'}{\bar{v} + \bar{c}} = e\frac{\overline{vn}}{\bar{v} + \bar{c}}$$

or

$$e = r\frac{\bar{c} + \bar{v}}{\overline{vn}}.$$

This implies for p

$$p = v' + c' + r(c + v) = 1 - ev' + r(c + v) =$$

$$= 1 - r\frac{\bar{c} + \bar{v}}{\overline{vn}}v' + r\frac{c + v}{vn}v' = 1 + rv'\left[\frac{c + v}{nv} - \frac{\bar{c} + \bar{v}}{\overline{nv}}\right].$$

The first Marxian approximation to the transformation problem yields production prices as linear functions of expressions which are very closely related to the organic composition of capital. Indeed, let me call

$$z = \frac{c + v}{nv}$$

the modified organic composition. It has the dimension « time », being the quotient of a stock and a flow. Using the symbol z, we have the formula

$$p = 1 + rv'(z - \bar{z}).$$

The correct formula for p would be

$$p = 1 - ev' + (p_{v'} - 1)\,v' + (p_1' - 1)\,c' + r(c + v) + r(p_1 - 1)\,(c + v)$$

where $p_{v'}$ is the price index of the means of subsistence of the workers, p_1' is the price index of the constant capital flow c', and p_1 is the price index of the stock of capital $c + v$. Now, of course, the correct solution for all prices in the economy involves the solution of a simultaneous equation system, which was beyond Marx' mathematical capacity.

I now shall try an approximation to the correct solution which is in the spirit of Marx, because it maintains the explanatory power of the organic composition of capital. For this purpose, we try a second approximation, by replacing p_1' in the correct formula by the formula for its first Marxian approximation and by ignoring the terms

$$(p_{v'} - 1)\,v' \quad \text{and} \quad r(p_1 - 1)\,(c + v)$$

of the correct formula. Why do we choose this as the second approximation ? It remains linear in the rate of profit (r is only multiplied with expressions not depending on r), and, economically speaking, it leads back to the first Marxian approximation, if we consider the firm as vertically integrated with those parts of the rest of the economy which deliver the flows of constant capital.

The second approximation thus reads

$$p = 1 + rv'(z - \bar{z}) + (p_1' - 1)\,c' = 1 + rv'(z - \bar{z}) + rc'\,v_1'(z_1 - \bar{z})$$

where v_1' is the average quantity of wages paid per unit of output value in the industries delivering commodities to our firm. The weights in the average are in proportion to the delivery flows (in value terms) to our firm. Similarly z_1 is the average « modified » organic composition in the delivering industries. I now want to show that the second approximation is very close to Marx' first approximation, because it would be the first approximation, would those parts of the delivering industries working for our firm be owned by our firm. The output sold on the market by this combined firm, would still be the same as the old firm, since the additions to the old firm sold their output only to the old firm, and after the amalgamation, these are no longer sales on the market. What are the relevant variables of this larger hypothetical firm. We denote them by c^1, c'^1, v^1, v'^1, etc. The wage bill v'^1 is of course the sum of the two wage bills v' and $c'\,v'_1$, hence

$$v'^1 = v' + c'\,v'_1\,.$$

The stock of capital $c^1 + v^1$ is also the corresponding sum

$$c^1 + v^1 = c + v + (c_1 + v_1)\,c'\,.$$

The modified organic composition thus is

$$z^1 = \frac{c^1 + v^1}{v'^1} = \frac{c + v + (c_1 + v_1)\,c'}{v' + v'_1\,c'} = \frac{v'}{v'^1}\,z + \frac{v'_1\,c'}{v'^1}\,z_1\,.$$

It is a weighted average of the modified organic compositions of the component firms. These formulae imply for our second approximation

$$p = 1 + r[v'(z - \bar{z}) + c'\,v'_1(z_1 - \bar{z})] =$$

$$= 1 + rv'^1\left[\frac{v'}{v'^1}\,z + \frac{c'\,v'_1}{v'^1}\,z_1\right] - rv'^1\bar{z} = 1 + rv'^1(z^1 - \bar{z})\,.$$

The second approximation then is the first Marxian approximation for the amalgamated firm. We therefore denote this second approximation by p^1.

Now we may proceed to the third approximation, resulting from the formula

$$p = 1 + rv'(z - \bar{z}) + (p_1 - 1)\,c'$$

by using the second approximation for p_1, which again means using the first approximation for p_2, the average price in the industries delivering to the industries delivering to our firm. It is now straightforward to show that the third approximation consists of the first approximation for a firm amalgamated from our amalgamated firm I and the parts of the industries which deliver to it. On the other hand,

$$p^2 = 1 + rv'(z - \bar{z}) + rc'\,v'_1(z_1 - \bar{z}) + rc'\,c'_1\,v'_2(z_2 - \bar{z})\,.$$

In a similar fashion, we arrive at fourth, fifth, sixth..., approximations which always can be interpreted as a first approximation of an

amalgamated firm. This sequence of approximation converges to a price $\hat{p}$ which is given by

$$\hat{p} = 1 + r[v'(z - \bar{z}) + c'\,v'_1(z_1 - \bar{z}) + c'\,c'_1\,v'_2(z_2 - \bar{z}) + \\ + c'\,c'_1\,c'_2\,v'_3(z_3 - \bar{z}) + \cdots]$$

(convergence is ensured, whenever labour values in the Marxian sense are defined).

And this price $\hat{p}$ can be interpreted as the first Marxian approximation of the production price of a firm selling on the market the product which the original firm sells, but being completely integrated in the sense that it buys nothing but labour from outside.

Indeed, the modified organic composition of capital of the completely integrated firm can be computed in the following way. Its stock of capital is the sum of the capital stocks of its components, i. e. it is

$$\hat{c} + \hat{v} = c + v + c'(c_1 + v_1) + c'\,c'_1(c_2 + v_2) + c'\,c'_1\,c'_2(c_3 + v_3) + \cdots.$$

Its total flow of wages is the sum of its component firms i. e. it is

$$\hat{v}' = v' + c'\,v'_1 + c'\,c'_1\,v'_2 + c'\,c'_1\,c'_2\,v'_3 + \cdots.$$

The quotient of these two expressions is

$$\hat{z} = \frac{\hat{c} + \hat{v}}{\hat{v}'} = \frac{v'}{\hat{v}'}\,\frac{c + v}{v'} + \frac{c'\,v'_1}{\hat{v}'}\,\frac{(c_1 + v_1)}{v'_1} + \frac{c'\,c'_1\,v'_2}{\hat{v}'}\,\frac{c_2 + v_2}{v'_2} + \cdots$$

$$= \frac{v'}{\hat{v}'}\,z + \frac{c'\,v'_1}{\hat{v}'}\,z_1 + \frac{c'\,c'_1\,v'_2}{\hat{v}'}\,z_2 + \cdots$$

$\hat{z}$ is a weighted average of the values of z in the component elements of the completely integrated firm, where the weights are given by the share of the components in the total labour input.

Using the last formula and the formula for $\hat{v}'$, our price equation yields

$$\hat{p} = 1 + r\hat{v}'\,\hat{z} - r\bar{z}[v' + c'\,v'_1 + c'\,c'_1\,v'_2 + \cdots]$$

$$= 1 + r\hat{v}'[\hat{z} - \bar{z}]\,.$$

This is what we wanted to show: the price of the product is now explained by the discrepancy between the organic composition in the completely integrated firm and the organic composition in the economy at large in just the same way as in Marx' own approximation.

It remains an approximation, to be sure, but a better one, as we shall see later, and it keeps the explanatory reason for the price-value discrepancy, which Marx gave.

5. — Böhm-Bawerk's average period of production (11)

The Austrian School conceives of the production process as a sequence of production stages such that output of one stage is — together with labour — the input of the next stage, until consumption goods emanate from the production process as its final result. Even if this social production process is distributed over several firms its essential characteristics do not change. Böhm-Bawerk tells clearly that the location of the borderlines between firms is inessential from the social point of view, a point which probably Marx would also have admitted, even though as we have shown, his own understanding of his own concepts is somewhat confused in this regard. Böhm-Bawerk often speaks of the production process as if all stages of production were in the hands of one firm.

The crucial assumption, which Böhm-Bawerk makes, is the identification of the capital stock with the sum of wages paid in the past for labour inputs whose final outputs (consumption goods) still are to be expected in the future. Hence his capital stock can be interpreted as a subsistence fund for the payment of wages. The capital required to organize an ongoing production process is equal to the annual wage bill multiplied by the average period of production, i. e., the average time distance between labour input and consumption good output (12). This heuristically appealing approach is of course a simplification and the simplification turns out to be equivalent to the assumption of simple interest instead of compound interest payment. For simple interest payment can be interpreted as an arrangement so that interest on borrowed capital does not have to be paid periodically but only at the end of the loan contract together with the repayment of the capital. No interest has to be paid on the deferred interest payments themselves, as

(11) For this section see E. von Böhm-Bawerk, Positive Theorie des Kapitales, vol. I and II, 4th ed., Jena 1921. A useful modern discussion of Böhm-Bawerk is E. Wolfstetter, Die Kapitaltheorie bei E. von Böhm-Bawerk, Diplomarbeit, Dept. of Economics, University of Heidelberg, 1970.

(12) Cf. Böhm-Bawerk, *op. cit.*, pp. 443-464.

would be the case in a compound interest arrangement. The only interest bearing capital is then the capital originally borrowed. A completely vertically integrated firm only borrows capital to pay wages and thus only the wages-subsistence fund bears interest in a simple interest arrangement. In the following we shall always assume completely integrated firms.

It should be noted that this identification of capital and wage subsistence fund from the social point of view can only be described in the way Böhm-Bawerk does, if one accepts his story of the completely integrated firm. If the production process were distributed over several firms, then Böhm-Bawerk's story would break down. Böhm-Bawerk does not seem to be aware of this fact. He clearly seems to believe that his story of the completely integrated firm is a legitimate expository device which is not intrinsically necessary for his theory. It appears that both, Marx and Böhm-Bawerk got somewhat confused by the fact that the social production process is taking place in different firms.

Let $l(t)$ be the flow of labour inputs which lead up to the production of the consumption good, where t is the time distance between the labour input and the availability of output. The cash payment for this input $l(t)$ is $wl(t)$, where w is the wage rate. If the firm borrows the money from the bank to be repaid, after the output becomes available, then the simple interest arrangement implies interest payments for this loan of $rwl(t)\ t$ where r is the rate of interest per annum. The cost of this input for the firm is cash payment of wages plus interest payment $wl(t)\,(1 + rt)$. Total cost of the production process is

$$w \int_0^n l(t)\,(1 + rt)\,dt$$

where n is the total time length of the production process. Under certain conditions ensuring convergence there exists no problem in principle to assume n to be infinite. (Böhm-Bawerk did not assume this, and his theory was criticised by Knight and others because under realistic conditions there is no way around the assumption $n = \infty$, which seemed ridiculous, because it implies that the production process was already in existence forever. But modern methodology teaches us that model assumptions can be interpreted as assumptions about objects which behave as if they would have the property which is assumed. It is irrelevant whether they really do have this property. Looked at in this light the assumption of an infinite duration of the production process is not less defensible and not more ridiculous than the assumption that « the firm » maximizes profits. All these assumptions, if taken literally are mystifications, but to the extent that theory can deduce observable consequences from these assumptions and to the extent that our understanding of reality is enhanced by making these assumptions, I don't see any problem in making them, even though, if taken literally they are meaningless or patently unrealistic.)

Let us denote the output flow by x. The quantity of labour expended in the production process is denoted by L, i. e.,

$$\int_0^n l(t)\, dt = L\,.$$

We now introduce the concept of average period of production. It is the average time distance of labour inputs and the consumption good output. If we denote it by T we obtain

$$T = \int_0^n l(t)\, t\, dt \Big/ \int_0^n l(t)\, dt = \frac{1}{L}\int_0^n l(t)\, t\, dt\,.$$

By choosing units in the appropriate way we can arrange $x = L$. Under equilibrium conditions the price must be equal to total production cost divided by x, or

$$px = w\int_0^n l(t)\,(1 + rt)\, dt = Lw(1 + rT)$$

or

$$p = w(1 + rT)\,.$$

The price is a linear function of the rate of interest, given T, but also a linear function of the period of production, given r.

Let us now look at the Böhm-Bawerkian economy at large. It consists of many completely integrated firms of the type considered which produce many different consumption goods. There is no loss of generality, if we assume that each firm produces a different output : some of these outputs could be complete substitutes, which means they are really the same commodity. If each firm gets an index $i = 1, ..., m$, then the value of total output is

$$y = \sum_{i=1}^{m} p_i\, x_i = w\sum_{i=1}^{m} L_i(1 + rT_i) = w\overline{L}(1 + r\overline{T})$$

where

$$\overline{L} = \sum_{i=1}^{m} L_i \quad \text{and} \quad \overline{T} = \sum_{i=1}^{m} \frac{L_i}{L}\, T_i\,.$$

The latter expression is the average period of production of the economy at large. It is the average of the average periods of production of each firm. $\overline{L}$ is the total labour content of the flow of consumption goods produced today. In a stationary economy $\overline{L}$ also represents the present flow of labour inputs. Let us now normalize prices in such a way that $y = \overline{L}$, i. e. that the flow of outputs is of equal magnitude as its labour content. This means

$$1 = w(1 + r\overline{T})\,.$$

Using this equation we can go back to our individual price equation and write

$$p = w(1 + rT) = 1 - w(1 + r\overline{T}) + w(1 + rT) = 1 + rw(T - \overline{T})\,.$$

The price of a commodity with a labour content of unity is explained by the difference of its specific average period of production and the average period of production of the whole economy.

Comparing this equation with the approximation in the spirit of Marx

$$p = 1 + rv(\hat{z} - \overline{z})$$

we realize that the two are almost identical. For the variable capital v in the Marx formula corresponds to the wage rate w in the Böhm-Bawerk formula. The expression for the modified organic composition of capital $\hat{z}$ of an integrated firm is just equal to Böhm-Bawerk's period of production. Consider a stationary economy. Then the quantity of (labour) values tied up in the capital corresponding to a nnit flow of labour input is equal to the average time distance of the labour inputs and their outputs. Since, in the completely integrated firm all capital is variable capital (i. e. wages paid with their return for the capitalist still in the future), the ratio of capital to the flow of wages is the same as the ratio of labour values contained in the means of production to the flow of labour inputs. Thus the modified organic composition of capital and the average period of production of the completely integrated firm are identical. And both are the central elements in the extension of the Marxian and the Böhm-Bawerkian theory of relative prices and their deviations from relative labour values.

Before we discuss the « correctness » of these approximations let us look at Böhm-Bawerk's law of increased productivity of greater roundaboutness of production. As he points out in the later editions of his « Positive Theorie des Kapitalzinses » the law does not say that any mode of production with increased roundaboutness will provide more output per input than any mode of production with a lower degree of roundaboutness. Only an appropriately chosen method of production with a greater average period of production will be more productive than any method of production with a lower period of production. And an appropriate choice is made by profit maximizing firms. Böhm-Bawerk's law is used for an explanation of the rate of interest. For, a positive rate of interest or rate of return on capital is necessary to induce capitalist entrepreneurs not to seek a method of production with maximum roundaboutness and productivity. But this is socially necessary, if there exists not enough capital to sustain every degree of roundaboutness.

Given the wage rate, the capitalist entrepreneur determines the return maximizing period of production in the following straightforward manner: the number of workers who can be employed is proportional to the inverse of the period of production (this follows from Böhm-Bawerk's identification of capital and wage subsistence fund). The profit per worker is the difference between the productivity of the worker and the wage rate.

Taking the price of the product as given and defining $x(\mathrm{T})$ as output for worker, the profit π on the capital k is

$$\pi = (px(\mathrm{T}) - w)\,\mathrm{L}$$

where

$$\mathrm{L} = \frac{k}{w\mathrm{T}}.$$

Hence

$$\pi = (px(\mathrm{T}) - w)\frac{k}{w\mathrm{T}}.$$

Differentiating this with respect to T and putting the derivative zero yields

$$\frac{d\pi}{d\mathrm{T}} = k\frac{w\mathrm{T}px'(\mathrm{T}) - w(px(\mathrm{T}) - w)}{(w\mathrm{T})^2} = 0$$

$$\mathrm{T}px'(\mathrm{T}) = px(\mathrm{T}) - w.$$

Böhm-Bawerk assumes $x''(\mathrm{T}) < 0$ which ensures that the profit is maximized, if the last equation is fulfilled. By differentiating this equation with respect to w we obtain

$$p[\mathrm{T}x''(\mathrm{T}) + x'(\mathrm{T})]\frac{d\mathrm{T}}{dw} = px'(\mathrm{T})\frac{d\mathrm{T}}{dw} - 1$$

$$p\mathrm{T}x''(\mathrm{T})\frac{d\mathrm{T}}{dw} = -1$$

which means

$$\frac{d\mathrm{T}}{dw} > 0.$$

The rate of profit π/k is

$$r = \frac{\pi}{k} = \frac{px(\mathrm{T}) - w}{w\mathrm{T}}.$$

Using the profit maximum condition, we obtain

$$r = \frac{\pi}{k} = \frac{p\mathrm{T}x'(\mathrm{T})}{\mathrm{T}w} = \frac{px'(\mathrm{T})}{w}.$$

The rate of profit then is an indicator of the marginal productivity of roundaboutness $x'(\mathrm{T})$. Böhm-Bawerk's notion of capital allows him to deduce how the market mechanism determines the method of production appropriate for the given amount of capital available in the economy. Since the demand for labour is inversely related to the period of production and since the choice of the period of production is determined by the wage rate, demand for labour is related to the wage rate: the higher the wage rate, the lower is demand for labour. Given a certain supply of labour, there exists an equilibrium in which wage rate, period of production, and rate of profit are determined. As the capital stock rises, the wage rate rises, and the rate of profit falls.

There is a passage in volume III of Das Kapital, which is not widely quoted, but which allows us to relate Böhm-Bawerk's law of roundaboutness to Marx's law of the falling rate of profit (13), Marx does not only say that increasing development of productive forces implies a rising organic composition of capital, he does not only

(13) Das Kapital, vol. III, pp. 289-292, see also vol. I, p. 411.

observe that this implies a contradiction to the capitalistic mode of production, but he also states that capitalists deliberately choose not to implement the most advanced methods of production, whenever their greater productivity is overcompensated by their greater capital requirements, so that they actually would imply a lower rate of profit than the prevailing one. Even though the purpose of the argument is just in the opposite direction of Böhm-Bawerk's, the formal analogy of the argument is very close : entrepreneur-capitalists have a choice of techniques of production characterized by different periods of production, or different organic compositions of capital. They do not choose that one which is the most productive but that one which maximizes the rate of profit. Unless the maximum rate of profit is not greater than zero, the rate of profit is maximized with a method of production which implies a lower period of production (or organic composition) than the one which maximizes productivity. The analogous function of the two concepts period of production and organic composition of capital supports our argument that a reasonable and consistent understanding of them makes them identical concepts. Moreover, in the passage referred to here, Marx anticipates a law of increased productivity of greater roundaboutness which is the central core of Böhm-Bawerk's theory.

6. — Ricardo's, Marx's and Böhm-Bawerk's theories as identical approximations of the « true » theory

Is it worthwhile to compare two theories and to try to establish that they are surprisingly close together, if both these theories are not correct ? Does it make sense to « improve » the theories for that purpose (as I have done with the Marxian theory), if even the improved version remains faulty ? Should we not simply replace these old, faulty theories by our better ones ? The answer to all these questions could easily be yes, if these old theories were totally unrelated to what we today consider to be the correct theory. But this is not the case. To find out how they are related, let us look at the correct formula for the Marxian transformation problem and the correct solution of Böhm-Bawerk's model. Let us start with Marx and the transformation problem. The correct price equation is, as was discussed already on page 209

$$p = 1 - ev' + (p'_v - 1)\, v' + (p'_1 - 1)\, c' + r(c + v) + r(p_1 - 1)\,(c + v)\,.$$

Let us now apply this formula to our hypothetical completely integrated firm. This firm does not buy other commodities than abour and hence, for it c and c' are zero. The equation then reads

$$p = 1 - e\hat{v}' + (p'_v - l)\,\hat{v}' + r\hat{v} + r(p_1 - 1)\,\hat{v}$$

$p_1 \hat{v}$ is now simply to be understood as the true market value of the capital stock employed (we remember that our numeraire is the labour value unit). It is therefore appropriate to change the notation from p_1 to $\hat{p}_v$. The expression $\hat{p}_v - 1$ indicates the difference between the market price and the labour value of a representative unit of capital employed in the integrated firm. In addition, since c' is now zero, we have $e\hat{v}' + \hat{v}' = 1$ and therefore the equation is

$$p = \hat{v}' + (p'_v - 1)\,\hat{v}' + r\hat{v} + r(\hat{p}_v - 1)\,\hat{v}\,.$$

Using our notation of the modified organic composition of capital, we have $\hat{z} = \hat{v}/\hat{v}'$ and thus we can write

$$p = \hat{v}'\,[p'_v + r\hat{z}\hat{p}_v]\,.$$

Now, there exists, of course, a relationship between the rate of profit r and the degree of exploitation. This relationship is a macroeconomic one. We simply have to amalgamate the whole economy into one hypothetical firm, which now is also a completely integrated one (ignoring foreign trade). Hence *mutatis mutandis*, the last equation also has to hold for the economy at large. But the price of the total product is equal to its value and hence the macroeconomic equation reads

$$1 = v'[p'_v + r\overline{z}\overline{p}_v]\,.$$

This is the correct equation for the rate of profit, given v' or the rate of exploitation. But it by itself does not enable one to compute r, since p'_v and $\overline{p}_v$ are not constant. They themselves depend on the rate of profit (or the degree of exploitation). We would have to know more about the technology to know precisely, how p'_v and $\overline{p}_v$ depend on r. But even $\overline{z}$ depends on r, since r will influence the composition of the industries.

So far therefore, we have not solved anything, and it is not our purpose to give a correct solution of the transformation problem, which — for special technologies — has been provided by Bortkiewicz, and later by Samuelson, Morishima, and many others. Let us look instead at a situation with a low rate of exploitation, or a low rate of profit, which can be approximated by a first degree Taylor expansion around $r = 0$. The prices $p_v'(r)$, $\bar{p}_v(r)$, $\widehat{p}_v(r)$ are of course equal to unity at $r = 0$. With no profits, commodities are exchanged at prices equal to their labour values. Let us now differentiate our equation with respect to r. Differentiation of the macroeconomic equation provides us with

$$0 = \frac{dv'}{dr}[p_v' + r\bar{z}p_v] + v'\left[\frac{dp_v'}{dr} + \bar{z}p_v + rp_v\frac{d\bar{z}}{dr} + r\bar{z}\frac{dp_v}{dr}\right].$$

Evaluated at $r = 0$ and hence $v' = 1$ it boils down to

$$0 = \frac{dv'}{dr} + \frac{dp_v'}{dr} + \bar{z}$$

which means that the derivative of the wage rate (in price terms) is equal to $-\bar{z}$. Actually, we could show, but don't need to that

$$\frac{dp_v'}{dr} = 0 \qquad \text{at} \qquad r = 0.$$

Let us now compute the derivative of the price equation for the individual integrated firm

$$p = v'[p_v' + r\widehat{z}\widehat{p}_v].$$

We obtain

$$\frac{dp}{dr} = \frac{dv'}{dr}[p_v' + r\widehat{z}\widehat{p}_v] + v'\left[\frac{dp_v'}{dr} + \widehat{z}\widehat{p}_v + r\widehat{z}\frac{d\widehat{p}_v}{dr}\right]$$

which evaluated at $r = 0$ provides us with

$$\left.\frac{dp}{dr}\right|_{r=0} = \frac{dv'}{dr} + \frac{dp_{v'}}{dr} + \widehat{z}.$$

This together with the corresponding macroeconomic equation yields

$$\left.\frac{dp}{dr}\right|_{r=0} = \widehat{z} - \bar{z}.$$

The correct derivative of the Marxian production price with respect to the rate of profit at $r = 0$ is equal to the difference between the relevant modified organic composition of capital and the average modified organic composition of the economy at large.

Let us now go back to the Marxian approximation applied to our integrated firm. It is

$$\widehat{p} = 1 + rv'[\widehat{z} - \bar{z}].$$

Its derivative with respect to r is

$$\frac{d\widehat{p}}{dr} = v'[\widehat{z} - \bar{z}] + r\frac{dv'}{dr}[\widehat{z} - \bar{z}] + rv'\frac{d[\widehat{z} - \bar{z}]}{dr}.$$

Evaluated at $r = 0$ this is

$$\left.\frac{d\widehat{p}}{dr}\right|_{r=0} = [\widehat{z} - \bar{z}].$$

It is equal to the corresponding derivative of the true price equation. In addition $p = \widehat{p} = 1$ at $r = 0$. Thus the extended Marxian approximation is close to the true price for small values of r. It indicates the direction in which the price structure moves as the rate of profit becomes positive. If labour values can be considered to be a zeroth approximation of exchange ratios in a capitalist economy, the organic composition of capital (in this modified form) provides the explanatory basis for a first approximation beyond the crude labour theory of value.

Let us now discuss Böhm-Bawerk's theory. We shall use the mathematical setup which we introduced for the presentation of his theory, but try to avoid his mistakes or simplifications. The flow of labour inputs of the integrated firm $l(t)$ remains the basis of its cost accounting. But now we introduce the correct (from the point of view of present day theory) method of compound interest. Thus the costs of the present flow of outputs of the firm are

$$w\int_0^n l(t)\,e^{rt}\,dt.$$

This must be equal to the market value of the output and therefore we have

$$px = w\int_0^n l(t)\,e^{rt}\,dt.$$

Or, if we define units such that $x = L$.

$$p = w\frac{\int_0^n l(t)\,e^{rt}\,dt}{L}.$$

Again, at the macroeconomic level we obtain by summation over all firms

$$y = \sum_i p_i x_i = w \sum_i \int_0^{n_i} l_i(t)\, e^{rt}\, dt .$$

Let us normalize this in such a way that $y = \overline{L}$ so that

$$w = \frac{\overline{L}}{\sum_i \int_0^{n_i} l_i(t)\, e^{rt}\, dt} .$$

Our price equation for the output of a specific integrated firm then reads

$$p = \frac{\overline{L}}{L} \frac{\int_0^{n} l(t)\, e^{rt}\, dt}{\sum_i \int_0^{n_i} l_i(t)\, e^{rt}\, dt} .$$

Let us now differentiate this expression with respect to r

$$\frac{dp}{dr} = \frac{L \sum_i \int_0^{n_i} l_i(t)\, e^{rt}\, dt . \overline{L} \int_0^{n} l(t)\, e^{rt}\, t\, dt}{\left[L \sum_i \int_0^{n_i} l_i(t)\, e^{rt}\, dt\right]^2} - \frac{\overline{L} \int_0^{n} l(t)\, e^{rt}\, dt . L \sum_i \int_0^{n_i} l_i(t)\, e^{rt}\, t\, dt}{\left[L \sum_i \int_0^{n_i} l_i(t)\, e^{rt}\, dt\right]^2}$$

$$= p \frac{\int_0^{n} l(t)\, e^{rt}\, t\, dt}{\int_0^{n} l(t)\, e^{rt}\, dt} - p \frac{\sum_i \int_0^{n_i} l_i(t)\, e^{rt}\, t\, dt}{\sum_i \int_0^{n_i} l_i(t)\, e^{rt}\, dt} .$$

Let us define

$$\theta = \frac{\int_0^{n} l(t)\, e^{rt}\, t\, dt}{\int_0^{n} l(t)\, e^{rt}\, dt}, \qquad \overline{\theta} = \frac{\sum_i \int_0^{n_i} l_i(t)\, e^{rt}\, t\, dt}{\sum_i \int_0^{n_i} l_i(t)\, e^{rt}\, dt} .$$

We then have

$$\frac{dp}{dr} = p[\theta - \overline{\theta}] .$$

It is obvious that the value of θ and $\overline{\theta}$ at $r = 0$ is equal to the Böhm-Bawerkian periods of production for the firm, T, and for the economy at large $\overline{T}$. Moreover, p is equal to unity at $r = 0$. Hence

$$\left.\frac{dp}{dr}\right|_{r=0} = T - \overline{T} .$$

Let us now look at the Böhm-Bawerkian approximation

$$\hat{p} = 1 + rw[T - \overline{T}] .$$

Differentiating this with respect to r yields

$$\frac{d\hat{p}}{dr} = w(T - \overline{T}) + r \frac{dw}{dr} [T - \overline{T}]$$

which at $r = 0$ means

$$\left.\frac{d\hat{p}}{dr}\right|_{r=0} = T - \overline{T} .$$

Again the price formula derived from Böhm-Bawerk's theory has the same value and first derivative at $r = 0$ as the correct price formula.

Let us now look at Böhm-Bawerk's theory of roundaboutness of production. In a stationary system output per unit of labour input is equal to x/L with x being the output of the time consuming production process and L being the total labour input during the lifetime of this process.

Let α be a variable taking on real numbers (so that we will be able to differentiate with respect to α). The variable α is an index of the production process : the entrepreneur can choose α, i. e., he can choose one of the available production processes. Let us standardize production processes in such a way that the total labour requirement of a unit process is unity. Let us assume that the time flow $\{ l(t) \}$ of labour inputs is a differentiable function of α, so that we have

$$l = l(t, \alpha) , \quad \frac{\partial l}{\partial \alpha} \quad \text{exists and} \quad \int_0^{n} \frac{\partial l(t, \alpha)}{\partial \alpha}\, dt = 0 .$$

The last equation is due to the standardization condition

$$\int_0^n l(t, \alpha)\, dt = 1 \qquad \text{for all } \alpha .$$

We also assume that the output x of the standard process is a differentiable function of α. An equilibrium condition is one in which entrepreneurs choose the profit maximizing α and one in which at this α profit (beyond the rate of return r on capital) is zero. Hence we have the price equation

$$px(\alpha) = w \int_0^n l(t, \alpha)\, e^{rt}\, dt$$

and, because of profit maximization,

$$px'(\alpha) = w \int_0^n \frac{\partial l(t, \alpha)}{\partial \alpha} e^{rt}\, dt .$$

We now introduce again the period of production

$$T = \int_0^n l(t, \alpha)\, t\, dt \Big/ \int_0^n l(t, \alpha)\, dt = \int_0^n l(t, \alpha)\, t\, dt .$$

We differentiate T with respect to α

$$\frac{dT}{d\alpha} = \int_0^n \frac{\partial l(t, \alpha)}{\partial \alpha}\, t\, dt .$$

Let us now use the Taylor expansion of e^{rt}

$$e^{rt} = 1 + rt + \frac{r^2 t^2}{2} + \frac{r^3 t^3}{3} + \cdots$$

to write the profit maximizing condition

$$\begin{aligned} px'(\alpha) &= w \int_0^n \frac{\partial l(t, \alpha)}{\partial \alpha} \left(1 + rt + \frac{r^2 t^2}{2} + \frac{r^3 t^3}{3\,!} + \cdots\right) dt \\ &= w \int_0^n \frac{\partial l(t, \alpha)}{\partial \alpha} \left(rt + \frac{r^2 t^2}{2} + \frac{r^3 t^3}{3\,!} + \cdots\right) dt \\ &= wr \frac{dT}{d\alpha} + w \int_0^n \frac{\partial l(t, \alpha)}{\partial \alpha} \left(\frac{r^2 t^2}{2} + \frac{r^3 t^3}{3\,!} + \cdots\right) dt \end{aligned}$$

$$\frac{px'(\alpha)}{w} = r \frac{dT}{d\alpha} + O(r)$$

where $\frac{O(r)}{r}$ goes to zero as r approaches zero. This we may write

$$\frac{p \frac{dx}{dT}}{w} = r + O(r) .$$

Except for $O(r)$ this is precisely the formula derived from Böhm-Bawerk's theory in which the rate of return on capital is an indicator of the social rate of return for lengthening the period of production. Thus Böhm-Bawerk's theory of roundaboutness is a good approximation of the true theory for small values of r.

What about Marx's law of the tendency of the profit rate to fall ? Marx does not give a mathematical or quasi-mathematical analysis of his law that the organic composition has a tendency to rise. For him it is a manifestation of the development of the productive forces under capitalism which is connected with the evolvement of the social character of work and production. Viewed from the individual producer this increasing social character is reflected in the fact that more and more work already had to be done, before his labour could be used productively. I do not have enough space to discuss the « truth » or plausibility of Marx' law. But, this being a paper on the similarities between Marx and Böhm-Bawerk, I want to point out that Böhm-Bawerk strongly believed in a tendency for the average period of production to rise through time. In his book he defends his view at some length against people like Lexis, who held the opposite view. Considering the similarity, or if well understood, identity of the organic composition of capital and the average period of production, we find another point of agreement between Marx and Böhm-Bawerk.

7. — *The Aggregate Production Function.*

The aggregate production function has been the subject of much controversy. Those, who have used it, have never claimed anything else for it, but to be a crude approximation. Let us relate it to the Böhm-Bawerk and Marx approximations which we have discussed. It is well known that the derivative of output with respect to capital (if prices are kept constant at the point where the derivative is taken) is equal to the rate of return on capital under rather general conditions (14). But this is not the interesting question here. The macroeconomic production function relates the current value of capital per worker to the current value of output, where the rate of interest is interpreted to be the derivative of the latter with respect to the former. Now, it is also well-known (15) that on the Golden Rule path this theorem is true. Indeed, it is an almost obvious corollary of the Golden Rule itself. For, if y is real output (in terms of a basket of consumption goods) and if v is the value of means of production employed per worker, then on a steady state path with growth rate g and consumption per head c we have

$$y = c + gv$$

i. e. output is consumption plus investment. Let us now differentiate his expression with respect to v by moving from one steady state to another one. We obtain

$$\frac{dy}{dv} = \frac{dc}{dv} + g .$$

On the Golden Rule Path two things hold : the rate of interest r equals g and $dc/dv = 0$, we thus obtain there

$$\left.\frac{dy}{dv}\right|_{r=g} = 0 + g = r .$$

In a neighborhood of the Golden Rule path the macroeconomic production function is a good approximation.

In a similar way the quantity of capital can be used for an approximation of relative prices. Let p_i be the price of the product i. Let us look at the production system which similarly as above can be interpreted as a completely integrated industry with only labour as an input. We are in a steady state system where all industries grow at the same rate g. Let λ_i be current labour input per unit of current

(14) Cf. for example C. Bliss, *Capital Theory and the Distribution of Income*, Amsterdam-Oxford 1975, chapter 5, or C. C. von Weizsäcker, *Steady State Capital Theory*, Heidelberg-New York 1971, chapter 9 of part II.

(15) This was first pointed out in C. C. von Weizsäcker, *Bemerkungen zu einem Symposium über Wachstumstheorie und Produktionsfunktionen*, Kyklos 1973. This paper has been quoted in the literature on several occasions, but apparently has been misunderstood to contain only a one commodity model. In a one commodity model the theorem does not make sense, because there we *are* in a world of an aggregate production function. These are the dangers of publishing in an obscure language.

final output in industry i. Note that λ_i is not the Marxian labour value. It is what Samuelson and I called synchronised labour costs (16) of good i. Let v_i be the value of capital per worker. We get a price equation from the consideration, that payments received must be equal to payments made. Payments received per unit of output are p_i from the sale of the commodity and $gv_i \lambda_i$ from new investment funds. Payments made are $\lambda_i w$ for wages and $rv_i \lambda_i$ for interest. We thus obtain

$$p_i + gv_i \lambda_i = \lambda_i w + r\lambda_i v_i$$

or

$$p_i = \lambda_i[w + (r - g) v_i] .$$

Upon differentiation with respect to r we get

$$\frac{dp_i}{dr} = \frac{d\lambda_i}{dr} [w + (r - g) v_i] + \lambda_i \left[\frac{dw}{dr} + v_i + (r - g) \frac{dv_i}{dr}\right] .$$

Let us now evaluate this expression at the Golden Rule path, where $r = g$. Because of the Golden Rule we know that λ_i attains a minimum at $r = g$, hence

$$\left.\frac{d\lambda_i}{dr}\right|_{r=g} = 0 .$$

Moreover, it is known (17) that

$$\left.\frac{dw}{dr}\right|_{r=g} = - v$$

where v is the average capital intensity in the economy. Of course, the term $(r - g)\, dv_i/dr$ is zero at $r = g$. So we have

$$\left.\frac{dp_i}{dr}\right|_{r=g} = \lambda_i[v_i - v] .$$

(16) C. C. von Weizsäcker and P. A. Samuelson, *A New Labor Theory of Value for Rational Planning through Use of the Bourgeois Profit Rate*, *Proceedings of the National Academy of Sciences*, Jnne 1971, reprinted in : P. A. Samuelson, *The Collected Scientific Papers*, vol. III, Cambridge, Mass. 1972.

(17) Cf. C. C. von Weizsäcker, *Steady State Capital Theory*, pp. 42-43.

In addition, for $r = g$

$$p_i = \lambda_i w .$$

Thus, in a neighborhood of the Golden Rule path the relative prices are well approximated by the minimum synchronised labour requirements and the relative capital intensities. This approximation is quite similar in spirit to the Marxian and Böhm-Bawerkian approximations which we have discussed above. Indeed, for $g = 0$ all three approximations are identical.

8. — *Opposing points of view of the same phenomenon.*

It would be ridiculous to claim that Marx and Böhm-Bawerk had the same theories of capital. What I wanted to show is that the basic analytical concepts which they used are much more alike than usually assumed. Moreover, as I believe, these concepts, if interpreted correctly, are basically sound, even from the modern point of view. But on the interpretative level the two authors differed widely, and the phenomena at hand lent themselves to opposing modes of interpretation. The basic fact is that in a capitalist economy workers combine with produced means of production to produce economic goods or commodities. The question arises : What is the product of the worker, and hence, in a way, due to the worker. According to Marx, the product of the worker is the value added to the ongoing production process, the result of which is « due » to the owner of the means of production and the worker in proportion to the value of their respective contribution of the production process. The « capitalist » would then just get the replacement value of the depreciation of his means of production, and the worker would get the rest. The particular commodity in question may not be of any use value to the worker, so he would exchange it, value unit for value unit, for commodities with use value for him. But in fact, the worker does not get his full product. The capitalist, who provides the means of production — consisting of products of past labour — is in a position of power which forces the worker to accept a deal in which the division of the product is less favourable for the worker. The basis of this power of the capitalist is the fact that the worker can only be productive, if he combines his work with the products of past labour. The social character of production (in this particular instance : the intertemporal character of production) and the institutions of capitalism force the worker to accept a position of exploitation. Let us note that the emphasis is on the combination of labour with products of *past* labour.

But, and this is Böhm-Bawerk's view, if the process of production is intertemporal in its very nature, then present labour normally is only productive, if it is combined with *future* labour, or, which comes to the same thing, the social product of labour only comes to fruition some time, perhaps a substantial time after the labour input occurs. The worker must be interested in the present consumption goods rather than the products, which he himself produces, which lie in the future. Thus, he exchanges these future products for present products at the going market rates. Exploitation is not involved. Here the emphasis is on the forward looking time aspect, i. e. on looking into the future rather than the past. The relative prices of present and future goods are determined in a social process, which involves the entrepreneurial allocation decision about the period of production, i. e. about the use of present resources for future goods. It is the relative shortage of present vis a vis future goods, which give present goods a value premium and put the owners of these goods (owners of capital) in an advantage compared to the producers of future goods (the workers). This premium, moreover, induces the market society to choose techniques of production which are in line with the prevailing relative shortage of present goods.

Both points of view have their faults and their merits, and in a sense, rather than contradicting each other, they can be considered complementary. Moreover, they seem to be perennial : much of the modern controversies in capital theory seem to be closely connected to these two points of view. The orthodox school emphasizes the relative scarcity and the allocational point of view : the rate of return on capital is also the social rate of return on investment. The closeness to Böhm-Bawerk's view is obvious, even though the particular theory of roundaboutness of production no longer is accepted. The antiorthodox schools emphasize the class and power relations in society which to them are more relevant for the explanation of distribution than allocational considerations. Again, I would say that these points of view are complementary.

Much of the controversy in capital theory in recent years was about issues of secondary importance. An example is the debate about the double switching hypothesis. Let me try to use an analogy. I am on a hike and I want to catch a train to bring me back home. There are two railway stations which the train will pass. One station is two miles from where I am now, the other four miles. This I know from my map. In all likelihood, I walk to the nearer station, unless special information influences me otherwise. Of course there is no law of nature or logical necessity, why I should have the greater chance to catch the train by walking to the nearest station. In a similar way, there is no logical necessity why a higher organic composition of capital (or period of production) in the production of commodity 1 compared to commodity 2 should imply that the price labour content ratio of the first commodity should be greater than for the second at a profit rate of, say, 20 %. But, unless I have further specific information, I consider it much more likely that at this profit rate the first commodity will have a higher price labour content ratio. In other words, the information used in the Marx-Böhm-Bawerk approximation can serve as a useful guideline in the absence of further information. Now, of course, they thought that their laws were not approximations but scientific laws. They were analogous to people who propose as a law of nature that the shortest distance always requires a minimum of time to walk. This law is false and it is useful for didactical purposes to construct counter examples. After we have constructed the counter-example, what shall we do ? Discard the law altogether and organise a crusade against its use in all walks of life ? Or should we maintain it as a useful heuristic principle, providing some guidance in a state of ignorance ? I would opt for the latter, which for many practical purposes means that I would opt for the use of the macroeconomic production function. There is, I believe, no ideological implication in such a proposal. If such different people as Marx and Böhm-Bawerk used similar devices, if, as I have pointed out, Marx had some notion of higher productivity of higher organic composition of capital, I do not see any apologetic or ideological purpose served by the use of this simplifying device.

But let me draw the analogy further. In asking, which is the quickest route to *walk*, I may ask the wrong question. Perhaps other means of transportation are available, which would bring me faster to one of the stations. My map may indicate that half a mile from where I am, there is a road probably with substantial car traffic. I may be able to hitchhike. Similarly, we may ask ourselves, whether the models, which we use to discuss all these problems, are good models. Could it not be that uncertainty or the social relations in the factory or the economic value of going concerns (as opposed to the value of its assets) are much more important for capital theory than the discrepancies between the approximations discussed in this paper and the « true » solutions of the system of equations we so far have favoured in capital theory ?

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