The logic of prices as values

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Abstract

There are two major lines of criticism moved at Marx's approach to the transformation of values into prices. The circularity critique holds that constant and variable capital appear in Marx's numerical examples as inputs at their individual values and as outputs at their social, transformed value (or price of production). This critique is rejected as being foreign to Marx's methodology. Rather, the problem, when correctly formulated, is why and how the value incorporated in the constant and variable capital at the moment of their realization as outputs can differ from the value appropriated by them at the moment of their realization as inputs (and vice versa). The infinite regression critique submits that Marx's approach implies following the formation of value step by step backward ad infinitum. This critique too is rejected on logical grounds and it is submitted that the problem, rather, is that of bringing up to the present the value which has been formed in the past. After the transformation problem has been thus reformulated, a solution is provided. Seen from this angle, which I argue is Marx's own, there is no inconsistency in Marx's numerical examples.

Introduction

Marx's transformation of values into prices has been the object of hotly debated controversies since Bohm-Bawerk's attack (1973) on the third volume of Capital. In short, to compute the price of production (P_t) of commodities, Marx adds to the constant capital (c) and variable capital (v) needed for those commodities' production the average rate of profit (Marx, 1967c, p. 164). Two types of critique have emerged as the most influential. The first is the circularity critique. On the one hand, c is said to be an individual value, i.e. a value not yet transformed into a price of production. On the other hand, the same c is itself a product of other production processes and, when sold, must be sold at its price of production. There is, so runs the argument, a logical mistake, a circular reasoning which is why the conditions of equilibrium cannot be respected any longer (Sweezy, 1968). The second type of critique is complementary to the one just mentioned. This is the infinite regression argument, according to which to compute the value of a commodity we must know the value of c, but to know the value of c we must know the value of the c which went into its production in the previous period, and so on in an infinite series of steps backwards in time. In what follows, I intend to analyze Marx's approach and both the circularity and the infinite regression critiques. In the process of answering these two types of critique, I shall put forward my own interpretation and solution of the transformation problem.

1 Marx's 'solution' of the transformation problem

Let us start with a few basic concepts. Marx works with the following ones. The individual value of a commodity is given by the amount of labour actually expended in its production and more generally by any value which deviates from the social value of that commodity. The individual value is a potential social value, a social value before its realization through exchange. The social value constitutes itself at the moment of the commodity's realization and is the value of the commodity produced under average conditions of production, given a certain distribution of society's purchasing power among the various branches. The social value can take several forms according to the scope of the problematic, i.e. according to the comprehensiveness of the analysis. Marx considers three forms. First, the market value which on the one hand is to be viewed as the average value of commodities produced in a single sphere and, on the other, as the individual value of the commodities produced under average conditions of their respective sphere and forming the bulk of the production of that sphere (Marx, 1967c, p. 176). The market value is then the social, i.e. average, value when reference is made to a single branch, taken in (artificial) isolation. Competition within that branch tends both to create different rates of profit (the countercurrent) and to equalize them (the tendency) thus producing the market value. But the different branches compete also with each other since 'capital withdraws from a sphere with a low rate of profit and invades other, which yield a higher profit' (Marx, 1967c, p. 195). The equalization of the rates of profit in this case results in the formation of prices of production. The price of production in a certain branch, then, differs from the market value because the equalization of that branch's rate of profit is now subject to the influence also of other branches. In this case, the social, i.e. average, value is given by the price of production rather than by the market value. As is well known, the formation of the prices of production is due to the fact that under capitalism, profits 'are not distributed in proportion to the surplus value produced in each
special sphere of production, but rather in proportion to the mass of capital employed in each sphere, so that equal masses of capital, whatever their composition, receive equal aliquot shares of the total surplus value produced by the total social capital (Marx, 1967c, p. 194). The price of production is thus computed by adding to constant and variable capital (the cost-price) the average rate of profit computed on the cost-price itself. Finally, the price of production is, in its turn, the centre around which the daily market prices fluctuate and tend to equalize one another within definite periods. Against this background, given three branches of production, the transformation of individual values into prices of production is carried out by Marx as follows (Marx, 1967c, p. 164)

Table 1

<table>
<thead>
<tr>
<th></th>
<th>c</th>
<th>v</th>
<th>s</th>
<th>Value</th>
<th>PrPr</th>
<th>PrPr-Value</th>
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<tbody>
<tr>
<td>I</td>
<td>80</td>
<td>20</td>
<td>20</td>
<td>120</td>
<td>120</td>
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<tr>
<td>II</td>
<td>90</td>
<td>10</td>
<td>10</td>
<td>110</td>
<td>120</td>
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</tr>
<tr>
<td>III</td>
<td>70</td>
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<td>120</td>
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<tr>
<td></td>
<td>240</td>
<td>60</td>
<td>60</td>
<td>360</td>
<td>360</td>
<td>0</td>
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where c = constant capital; v = variable capital; s = surplus value; PrPr = price of production; and where the average rate of profit (\(\bar{\pi} = 60/300 = 20\%\)) is used to compute the surplus value accruing to each capital of 100, i.e. to compute the PrPr. Therefore, the total value produced is equal to the total value distributed on the basis of a PrPr equal to 120. In this case, thus, the equalization of the rate of profit takes place through the equalization of the PrPr in all three branches and it will be around this level that the market price will fluctuate.

The usefulness of this procedure is that it isolates the essence of the problem, i.e. it shows that even when the rate of profit in the three branches is the same, there is inherent in the equalization of the rates of profit through the price mechanism a transfer of value or unequal exchange, from lower composition capitals (in this case branch III) to higher composition capitals (in this case branch II). In other words, the higher organic composition of capital in branch II will cause less production (in percentage terms) of surplus value than, but appropriation of more surplus value through redistribution from, branch III. Unequal exchange is thus inherent in and is the specific feature of the transformation problem. Or, again, the equalization of the rate of profit, the fact that all capitals (branches) must appropriate the same rate of profit (20%) on the total capital invested, takes place through a PrPr that ensures that part of the surplus value produced by the less productive branch (i.e. III) is transferred to, appropriated through distribution by, the more productive branch, i.e. II. The height of the organic composition of capital is here an index of productivity and the transfer of value to the more productive, higher composition, capitals is the way capitalist society rewards the introduction of capital intensive technologies, and thus stimulates a constant growth in the organic composition of capital. It is my contention that this is the purpose of Marx's scheme illustrating the formation of prices of production: to show the unequal exchange inherent in price formation and rewarding the most productive capitals at the expense of the least productive ones; i.e. to show how the price mechanism makes possible the functioning of competitive capitalism.

2 The circularity critique

Perhaps the most well-known line of critique and discussion around the transformation problem, is that originated by Bohm-Bawerk (1973) which, with the reply by R. Hilferding (1973) and the seminal contribution by von Bortkiewicz (1973), has been brought to the attention of a wide readership by the classical work of P. Sweezy (1968). Building upon that discussion, Sweezy points out that, after the transformation has been carried out, (a) the value incorporated into a commodity and the value appropriated through its sale do not coincide any longer, something which 'results in the violation of equilibrium of Simple Reproduction' (1968, p. 114) and (b) in the computation of prices of production, 'the capitalists' outlay on constant and variable capital are left exactly as they were in the value scheme; in other words the constant capital and the variable capital used in production are still expressed in value terms' (1968, p. 115). This second point has been later formulated in more modern terminology as follows: inputs are expressed as values but outputs are expressed as prices of production; this is a logical flaw since the same commodity is bought as an input and sold as an output at the same price. In terms of table 1 above, the constant and variable capital entering the transformation process are said to be individual, embodied, labour. However, inasmuch as they are outputs of their respective branches, they are social, i.e. transformed values, prices of production. Looking again at table 1 above, suppose that I produces means of production, II produces wage goods, and III produces...
luxury goods; then I sells its product at 120 (the price of production) but a portion of that product is bought by I for a value of 80, another portion by II for 90, and yet another by III for 70, i.e. for total of 240. The means of production are sold at prices of production but bought at their individual values. For Sweezy, (b) is the source of the violation mentioned in (a). By far the most influential solution is that offered by von Bortkiewicz (1973).

2a The von Bortkiewicz answer

Von Bortkiewicz' solution, reduced to its essentials, assumes a situation of simple reproduction, given the three above-mentioned sectors, i.e.

\[ c_1 + v_1 + s_1 = V_1 \]
\[ c_2 + v_2 + s_2 = V_2 \]
\[ c_3 + v_3 + s_3 = V_3 \]

where \( c, v \) and \( s \) are respectively constant capital, variable capital and surplus value and where the subscripts refer to the branch producing means of production (1), wage goods (2), and luxury goods (3). If demand equals supply then

\[ c_1 + v_1 + s_1 = V_1 = c_1 + c_2 + c_3 \]
\[ c_2 + v_2 + s_2 = V_2 = v_1 + v_2 + v_3 \]
\[ c_3 + v_3 + s_3 = V_3 = s_1 + s_2 + s_3 \]

The assumption is then made that with the transformation of values into prices of production the price of the means of production becomes \( x \) times greater than their value, that of the workers' articles of consumption becomes \( y \) times greater than their value and that of the capitalists' luxury goods becomes \( z \) times higher than their value. If we call the average rate of profit in price terms \( r \), then the model of simple reproduction transformed in prices of production becomes

\[ c_1 x + v_1 y + r(c_1 x + v_1 y) = (c_1 + c_2 + c_3)x \]
\[ c_2 x + v_2 y + r(c_2 x + v_2 y) = (v_1 + v_2 + v_3)y \]
\[ c_3 x + v_3 y + r(c_3 x + v_3 y) = (s_1 + s_2 + s_3)z \]

Bortkiewicz obtains thus three equations with four unknowns \((x, y, z, \text{ and } r)\). In terms of mathematics, to solve this system we must supply a fourth equation. In terms of economics, this means that we must choose between two equally undesirable solutions. Either we assume that the total of prices equals the total of values, but then the equality between surplus value and profit is not respected any more; or we assume that the total of profit equals the total of surplus value but then the total of prices and of values

will not coincide any more. The two equalities do not hold, in general, at the same time. After Bortkiewicz, many other authors have worked out improved or more complete, equally 'consistent', solutions which, however, share with Bortkiewicz' solution the same characteristic of severing either the equality between prices and values or that between surplus value and profits. The significance of this is almost unanimously played down. Yet, if the former equality does not hold, it makes no sense any more to speak of 'transformation' of values into prices (which is precisely what Marx set out to do and is, as we have seen, fundamental for an understanding of capital reproduction and accumulation); and if the latter equality does not hold, profits do not come necessarily any more from surplus value and the theory of exploitation is dealt a fatal blow. Both conclusions are devastating for the Marxian value theory.\(^5\)

2b Why von Bortkiewicz was off the mark

It should be pointed out right away that there is a basic logical mistake which invalidates the Bortkiewicz-inspired critique and 'solution': the tying of the transformation problem to the reproduction schemes and thus the collapsing of one type of problematic into a different type of problematic. These schemes concern themselves with 'the reconversion of one portion of the value of the product into capital and the passing of another portion into the individual consumption of the capitalist, as well as the working class' (Marx, 1967b, p. 394). In other words, these schemes concern themselves with the redistribution of the social product (in terms of use and exchange value) after that product has been realized through sale, in such a way that the equilibrium conditions of simple (or expanded) reproduction are met. The point that this has nothing to do with Marx' transformation problem is not new. It was already made in 1948 by J. Winternitz (1948). Winternitz' solution, however, even though applicable to expanded reproduction, is not more satisfactory and in line with Marx's approach than Bortkiewicz' solution. In fact, Winternitz, instead of tying the transformation problem to the reproduction schemes, considers it in the light of the input-output framework, in this followed by most commentators on the transformation problem, from Seton, who generalized Winternitz' three-departments model to N commodities,\(^6\) onwards. But to consider the transformation of values into prices as an input-output scheme does not come closer to the problem as posed by Marx than Bortkiewicz and Sweezy did. Both inputs and outputs are commodities whose value has already been produced and realized so that — obviously — a
commodity must be sold (as output) and bought (as input) at the same price (market price). To consider constant and variable capital as inputs in an input-output sense, means to have already left the transformation problem behind, to deal with already realized values; it means to disregard the interplay between the individual and social values of \( c \) and \( v \) both as inputs and as outputs.

But the nature of the problem under consideration is totally different. The problem is neither the analysis of how specific products must be used in the following period as constant and variable capital for equilibrium (under conditions of simple or several departments in the following period as inputs. The transformation problem is the problem of why and how the value incorporated in \( c \) and \( v \) at the moment of their realization as outputs differs from the value appropriated by them at the moment of their realization as inputs (and vice versa). This is the definition of the problem. This definition seems to be the same as the one submitted by Marx's critics, but it is not. The reasons why the two formulations are not the same as well as the reasons for choosing this formulation will become clear after the two basic critiques have been discussed and rejected. As far as the boundaries of the problematic are concerned, my thesis will be that the transformation problem is first of all a problem of logic, it concerns itself with the logical problem of why and how values exist only inasmuch as they at the same time can manifest (realize) themselves; why and how they must manifest themselves in a modified, social, form, i.e. as prices of production; and why and how realized social values can become again individual (or potential social) values. Secondly, I will submit that the transformation problem depicts a real transformation, a real process. This process must be seen both as a chronological sequence of different moments of distribution (realization), thus as a chronological sequence of transfers of value, and as a logical sequence, i.e. within each of these moments individual values precede logically (but not chronologically) the social, realized values. Finally, the solution is summarized at the end of section 3 and for reasons of exposition will be given in three steps, of which two arise from the discussion of the circularity critique and the third one from the discussion of the infinite regression critique. 7

2c Redressing the balance in favour of dialectical logic
I have argued above that the problem is one of dialectics. The proper interpretative scheme has been developed elsewhere. 8 Here I will only mention what is strictly necessary for the purpose of this article. First of all, a distinction must be made between determination in the last instance (which deals with possible conditions of existence or of supersession) and realized determination (which deals with the realization of some of those possibilities). As far as determination in the last instance is concerned, given two instances, \( A \) and \( B \), \( A \) is said to determine in the last instance \( B \) if \( B \) is a potential condition of reproduction (existence) or of supersession of \( A \). \( A \) cannot then be theorized independently from \( B \), as if \( B \) did not exist, not even as a first approximation to a more advanced stage of research in which \( A \) and \( B \) will be considered together. Rather, \( B \) must be considered to be a potential condition of reproduction of supersession of \( A \), and thus inherent in \( A \), from the very beginning of the theorization (but not necessarily from the beginning of the exposition). But determination in the last instance does not explain the realization of one or some of the several possibilities. If \( A \) calls into existence \( B \) as a condition of its own existence (or supersession), \( B \) in its turn reacts upon and modifies \( A \). The theoretical explanation of how this is possible is provided in my above-mentioned works. All that can be said here is that both \( A \) and \( B \) realize themselves in their mutual inter-relation, i.e. they constitute themselves reciprocally in the act of their realization and this realization is at the same time their reciprocal modification. These are some of the concepts dealing with realization in general. This is only a necessary but preliminary stage. The next stage in dealing with realization is to inquire into the specific mechanisms, or processes, through which the specific categories of phenomena realize themselves.

Particularly important is the question of how particular categories of individual phenomena realize themselves as social phenomena. In the context of this article, the question becomes that of the realization of individual values into social values. It could be submitted that prices of production (the social values of commodities in a capitalist economy, if the assumption is made that the prices of production coincide with market prices) are the determined instance, the conditions of existence of values. This interpretation would seem to be supported by Marx when he says that values 'lie beneath the prices of production and . . . determine them in the last instance' (1967c, p. 208). This, however, raises two objections. First, if there were a relation of determination between individual values and prices of production, both instances would realize themselves and they would do so in a modified way. In fact, what is realized is neither, but rather the market prices. Second, even if we assumed an equality between market prices and
prices of production, the objection still remains that it is not values and prices of production which realize themselves, but values as prices of production. We must therefore conclude that individual values realize themselves in a modified form, as social values, i.e. as prices of production.

There is then no relation of determination between values and prices; rather the structure of the prices of production is the concrete, social, form (when it coincides with the market prices) taken by the structure of individual values due to these values’ interrelation with all other social phenomena through the process of, and at the moment of, exchange. Individual values are the potential, not yet realized, social values and the prices of production are the realized social values, the form taken by the individual values through, and at the moment of, exchange. This interpretation is consistent not only with the discussion carried out by Marx; it is also consistent with the concept, to which Marx keeps coming back, that the prices of production are a modified form of value. This holds for all commodities, including those which are inputs for the production of other commodities, i.e. c. In other words, c cannot appear in table 1 as an input at its embodied value and must appear as an output at its price of production. This is the first step in the solution of the transformation problem: the explanation of why and how c as an input cannot appear at its embodied value and, therefore, the indication that its individual value as an input must already be a transformed value. In short, the individual value of c as an input cannot be its embodied value.

But there is also a second mistake inherent in the circularity critique. First of all, it is important to underline that there are two production periods. The former is the period in which c is the output, the latter is the period in which c is the input of a new output, say of a. The circularity critique collapses these two periods into just one. A real, and thus temporal process, is reduced to one in which two production periods are superimposed. But the producer of c realizes its (social) value when c is sold (as output) while the producer of a realizes the social value of c as input when a is sold. That is, the moment of realization of c as output is not the moment of realization of c as input. Thus, what is inconsistent is not Marx’s procedure, but the application of the logic of the input-output scheme (i.e. to consider that c realizes its value both as output and as input at the same moment, i.e. when c is sold by the producer of c and bought by the producer of a) to that procedure. It should thus be clear that while I use the terms ‘input’ and ‘output’, I apply a totally different reasoning than that implicit in the input-output tables.

Since two production periods and thus two moments of realiza-

tion are collapsed into one, the illusion is created that c is exchanged at both its individual and social value. To unravel the skin created by the circularity critique we must clearly see two things. First, the transformation problem depicts a transformation process and thus a chronological sequence of processes of production of value and surplus value and of moments of realization/distribution not only of surplus value but also of the value of c as an input. Secondly, within each of these moments of realization there is chronological contemporaneity (individual values can manifest themselves only as social values) which is at the same time a logical sequence (the individual values precede logically the social ones). This complexity is reduced by the circularity critique to the absurd accusation that there is implicit in Marx’s procedure the notion that c appears (realizes itself) both as an individual and as a social value and that therefore it is bought and sold at different prices (values). In the last analysis, the circularity critique makes the double mistake of considering as a chronological sequence what in fact is a chronologically contemporaneous process (the realization of individual values as social ones) and of considering as a chronologically contemporaneous process what is in fact a chronological sequence (the realization of the social value of c both as an input and as an output). For Marx, on the other hand, the value of c can appear only in its realized, social form and (contrary to the logic inherent in the input-output tables) the moment of c’s realization as an output is chronologically different from the moment of its realization as an input. Thus, the social value which realizes itself at time t, i.e. when c is sold as an output and bought as an input, is its PrPr as output and not as input (since this latter realizes itself only at time t + 1, when a realizes its social value). But since t is the moment at which both the previous production process ends and the new one begins, the social value of c as an output enters the new production process as the individual value of c as an input. But this individual value neither is embodied value (it is rather an old price of production, an already transformed value) nor it is a price different from the price at which c has been sold as an output. This is the second step in the solution of the transformation problem: the explanation of why the individual value of c as an input is its PrPr as the output of the previous period. But, since table 1 refers to time t + 1, the time of a’s realization, the value of c which appears in it cannot be its individual value (the old PrPr) but must be its social value. This will be determined in the next section.

There is thus no mistake in the transformation procedure of which Marx would have been aware but did nothing about. The mistake is the critics’ who do not understand Marx’s dialectical
method and who misinterpret a mathematical example aimed at showing how, under developed capitalism, values must realize, transform themselves into prices of production through the mechanism of the equalization of the rate of profit, i.e. aimed at showing how surplus value must be redistributed at the moment of realization in order for the rate of profit to be equal, as if it were a perverse input-output table depicting a type of transaction, in which the same commodity is bought by someone and sold by someone else at different prices. The mistake is to apply the logic of the input-output tables (which collapses the two moments of realization/distribution into just one but separates realization from distribution) to Marx's numerical examples in which realization and distribution of surplus value are chronologically contemporaneous but in which there are two chronologically different moments of realization/distribution (the first is the realization of c as an output, when c is sold, and the second is the realization of c as an input, when a is sold). The mistake is thus not only to consider Marx's solution of the transformation problem as a mathematical computation. The mistake is to do this without making explicit the methodology which supports it, thus leaving room for the implicit or explicit adoption of a different method which in turn changes the meaning and significance of that computation.

3 The infinite regression critique

But, it can be argued, if c is already expressed as realized, social value, computed as a PrPr, then to compute the value of c (i.e. the value of the means of production going into the value of a certain commodity, say a) we must go back to the previous period and from there to the previous one, thus falling into infinite regression. As J. Robinson puts it:

the constant capital was produced in the past by labour time working with then pre-existing constant capital and so on, ad infinitum backwards. It therefore cannot be reduced simply to a number of labour hours that can be added to the net value of the current year. And there is no advantage in trying to do so.

(1972, p. 202)

This highly sophisticated piece of methodology is wrong at least on two accounts. As far as method goes, the principle which must be adopted, and which is the only one which makes scientific research possible by avoiding infinite regression both in time and in logical causation, is that the choice of the starting point must be both subjective and objective. It is subjective in the sense that it depends upon the purpose of the research so that the starting point need not be analyzed in terms of its antecedents. It is objective because the starting point could be analyzed and must be analyzable by the same laws which govern the phenomena of which that point is the starting one. In other words, we could apply the same principles to the study of the starting point itself if (for reasons of historical analysis or because of the need to widen the logical chain of causations and not because of methodological considerations) we decided to go further back in time or in the chain of causations in the inquiry of the phenomenon under consideration. The value of c is given and need not be determined if the aim is to compute the value of a.

From the point of view of determination, it is necessary to consider the transformation process as a real process and thus as a sequence of real processes. As we know, the value of c has been produced in a certain period and is realized at a certain moment, say t, and is thus expressed as its PrPr at moment t. Therefore, c is sold as output and bought as input at its PrPr. There has been a production of value and of surplus value and a redistribution of that surplus value at the time c is sold as output and bought as input. In short, the individual value of c expresses itself as social value at the time of its realization as a product. Now a new production period starts and c enters in it as an input. The product, a, realizes its social value at time t + 1. The social value of c, now considered as an input, as an element of a at time t + 1, will be the value given by the socially necessary labour time at time t + 1 both to re-produce c and to produce a.

More specifically, if the average conditions of production of c change between t and t + 1, the value going into the value of a will be the one given at time t + 1; and if a certain producer of the commodity a has employed more (or less) c than it is socially necessary to produce a, then the value going into that particular a will be that of the average quantity of c at time t + 1. Thus, c is a social value in the double and interrelated sense that (1) it, as we have seen, as an output of the previous production process, is an individual value that cannot but realize itself as a social value, it counts as the quantity of labour socially necessary (i.e. average labour) to produce it (rather than the amount of labour actually spent to produce it) at the moment of its realization as an output; and (2) as an input in the present production period, it counts only as the quantity and quality of c employed in a's average production process and produced according to c's average production process at the time of a's realization. If it is not the average quantity and quality of c needed for a certain production process, if more (or less) socially necessary labour time is used at t + 1 to
produce c than it was needed at time t, then the transfer of value will apply not only to the surplus value produced but also to the c which deviates from its average value. As Marx puts it in Capital, Volume I:

if the capitalist has a foible for using golden spindles instead of steel ones, the only labour that counts for anything in the value of the yarn remains that which would be required to produce a steel spindle, because no more is necessary under the given social conditions (quoted in Nichols, 1980, p. 52).

This is the third and last step in the solution of the transformation problem: the explanation of why the social value of c as the input in the present production period is the modified quantity of its social value as the output of the previous period. It is this value which appears in Table 1.

To sum up, from the point of view of method, the infinite regression critique makes no sense because the value of c is given and we do not need to determine it, but we could, if we wanted to, by following the procedure outlined above. From the point of view of value determination, the critique makes even less sense because the value of c as an input of a is its value at the time of a’s realization, i.e. the re-production value of the average quantity and quality of c needed to produce a. The question is not that of following the formation of value step by step backwards ad infinitum but that of bringing up to the present the value which has been formed in the past. Or, in other words, to counter the infinite regression movement, we first ‘stop’ it by means of the above mentioned methodological principle and then ‘reverse’ its direction by means of the above mentioned principle of value determination.

It should be now clear why Marx, after having performed the transformation as in Table 1, says that also the c going into a commodity’s cost-price (i.e. c + v) should be computed at its price of production (1967c, p. 208). For Marx the question is not whether the c of a commodity of average composition is expressed as an individual value or as a price of production: if it goes into a commodity of average composition, the surplus value contained in it will be equal to the average profit and its individual value will be equal to its price of production. In other words, Marx considers deviations of individual values from prices of production in c as input in the context of the question as to whether these deviations affect the price of production of a commodity of average composition (of which c is an input). The answer is no, as shown above. This is not the question to which the critics address themselves and according to which c is bought as an input at its (individual) value and is sold as an output at its price (of production). When Marx considers c as an input, he addresses himself to a completely different question, namely whether a deviation of the value of c as an input from its price of production affects the price of production of a; but since this deviation can manifest itself only when a is exchanged, the problem posed by Marx is in fact the one formulated above in section 2b, i.e. why and how the value incorporated in c as an output differs from the value appropriated by it as an input.

We can now summarize the solution to the transformation problem. The value incorporated in c as an output can differ from the value appropriated by it at the moment of its realization, say t, because of the transformation process as depicted in Table 1. If we are concerned with the transformation of values into prices of production at time t + 1, it is methodologically correct to take this price of production as given. But whether or not this price of production coincides with its value at the moment of its realization as an output (t), the possibility arises of a deviation of this price of production at time t from its price of production at time t + 1 (the moment of its realization as an input), if the average conditions of production of c change between t and t + 1. Thus, the value of c as an input is determined by the PrPr of c in the preceding period (a given social value which need not be determined because of specific methodological reasons) as modified by the change in the average conditions of production of c in the present period. Thus c is not an individual value: if it is sold as an output at the end of the previous period and bought as an input at the beginning of the present period (the two moments coincide chronologically) it must be bought and sold at its social value. Or, c is a social (i.e. average) value because it is a realized social phenomenon. This is the answer to the von Bortkiewiczian critique: individual values can manifest themselves on the market only as social values, production and realization are distinct but indissolubly tied moments, a commodity sold (as output) and bought (as input) on the market is valued at its price of production at the time of its realization. But c as an input of the present period will realize its social value only at the end of the present period when the output, a, of which it is an input, will be sold, will realize its social value. If its individual value does not correspond to its social value (if, e.g. more than the average c has been used to produce a certain a), it must count as social value (i.e. realize itself as social value) because, when a is realized, what the market gives the producer of a is not only the average rate of profit, but also the average c needed for the production of a. Thus, what goes into the present period’s product is the PrPr of c as
given in the previous period and as modified in the present period. There is no need to go backwards ad infinitum either.

As long as the dialectic between individual and social values is lost sight of, i.e. as long as c as an input is considered to appear as an individual value and as an output is considered to be an immutable social value, a realized social value which cannot become again an individual value, i.e. as long as c as an input is not seen as a previously produced and realized value the magnitude of which, however, can change if its conditions of production change, the way is open to the two above mentioned critiques. There is a peculiar division of labour between the circularity critique and the infinite regression critique: each specializes in a mistaken interpretation of the relation between potential and realized social values. The circularity critique does not see that c as an output of the previous process and thus as an input of the present process is a transformed social value. The value of c as an input cannot be given by the labour embodied in it: its individual value is already a transformed value (an old PrPr). This value must appear in table 1 as a modified social value, as the modification of that old PrPr, as the social value of c as an output. The infinite regression critique does not see that, when c enters the present production process as the input of a commodity a, its realized social value becomes again a potential social value, an individual value (which, as such, cannot appear in table 1 which depicts the situation at the time of a's realization). This individual value, this previous social value, will become again a social value (the new PrPr) only when the output, a, of which c is an input, will realize its value. In short, the circularity critique mistakes the PrPr of the previous period for labour embodied in the present period while the infinite regression critique mistakes the same PrPr for an immutable magnitude which is not modified by changes in the average conditions of production in the present period.

In the last analysis, neither type of critique sees (1) that c can have at the same time both an individual (but not embodied) value and a social value if we consider that moment, t, which marks the end of the previous production process and the beginning of the present production process; and (2) that this does not mean that c is bought at its individual value and sold at its social value, i.e. at two different prices: when it is sold by the producer of c to the producer of a, it is bought and sold at its PrPr, or social value, at time t. In short, while c must be bought and sold at the same price, this price is at the same time its social value as an output (and it is why the transaction is carried out on the basis of this price) of the previous period and its individual (but not embodied) value as an input of the present period.

4 Summary and conclusions

It can now be seen that the individual values (120 for branch I, 110 for branch II, and 130 for branch III) in table 1 above are the values given by how much surplus value has been added to the average, socially necessary, quantity and quality of c and v, before this surplus value is redistributed through the equalization of the rate of profit (the same reasoning applied to c can now be extended to v too). At the moment of the realization of c and v as outputs of the previous period there is a redistribution of surplus value, i.e. the formation of PrPr's through the equalization of the rates of profit. A certain redistribution of income takes place through the price mechanism. But the sale of c and v as outputs marks also the beginning of a new production period in which c and v are now the inputs. If the conditions of production of c and v do not change, the PrPr does not change either. If they do change, c and v have to adjust their social value at the end of the present production period, when the product, a, is sold. If, say, c has become cheaper, the producer of a will realize less value in proportion to the fall in c's value to the advantage of the other producers with whom our producer exchanges a. In the new, present, production period, the social value of c has to adjust itself to the new condition, it becomes the average cost of re-producing (under the new condition) c. Therefore, the individual values of 110, 120, 130 are already social as far as c and v are concerned, both as outputs of the previous period (since c and v are sold as outputs and bought as inputs), they are valued at their social magnitude, they must be PrPr's, realized social magnitudes) and as inputs of the new production period (since they transfer to the commodity of which they have become inputs only the average social value, i.e. only the average cost of production of the average quantity and quality of c and v needed under the new conditions of production). The values of 110, 120 and 130 are individual only inasmuch as the surplus value component is concerned since redistribution of surplus value has not been carried out yet.

This approach could be, mistakenly I think, criticized as relating solely to the sphere of exchange since the c and v entering the computation of the prices of production are already social values, prices of production. But this is not so. I do establish a link between production and exchange, between the individual and social value of c and v. My approach differs from the usual one in that this link is a logical (in terms of dialectical logic) rather than a mathematical one. I explain why the individual values of c and v cannot appear as such, must appear as social values, and how the individual value of a is transformed into its social value, i.e. how
the value of c and v as inputs as well as the surplus value produced by using these inputs are redistributed in the act of exchange. Production and exchange are two distinct, but indissolubly tied moments of the same process. Both are necessary (even if the former, production, is the determinant one and the latter, exchange, is the determined one) for individual values to be able to express, realize, themselves, i.e. to become social phenomena. If this approach is followed, the way is open to mathematics, to the computation of how capitals with different organic compositions can realize amounts of surplus value different from those produced by them.\textsuperscript{10}

It becomes thus clear how much Marx has been misunderstood on this score. The approach which is usually (but mistakenly) submitted as being Marx’s own rests upon the mathematical relation between two realized forms of value, the individual and the social. But to treat individual values as if they were realized social phenomena is nonsensical in terms of Marx’s problematic. Yet, it is this assumption which is behind the approach which is mistakenly attributed to Marx and upon which the critique is based, according to which commodities as inputs are exchanged at individual values and as outputs are exchanged at social values, i.e. at their prices of production. We can now see the importance of the emphasis placed upon the fact that prices are not determined by values but rather are their concrete form of existence. Far from being a philosophical quibble, this result allows us to stress the importance placed upon the fact that prices are not determined by values but rather are their concrete form of existence. Far from being a philosophical quibble, this result allows us to stress the importance of the emphasis placed upon the fact that prices are not determined by values but rather are their concrete form of existence.

The backwards ad infinitum argument is a measure of Marx’s critics’ proficiency in methodological questions. On this point they score rather low. In fact, they not only fail to look at the methodological nature of their objection but they fail also to see that its application would make science impossible. If there is anything backward here, it is Marx’s critics’ understanding of his method and theory.

How should we then interpret Marx’ computations as exemplified in table 1 above? It is a basic misunderstanding of Marx’s method to consider his mathematical computations as the ‘solution’ to the transformation problem. Those examples only depict the computational specificity of the transformation problem and can acquire their proper significance only when immersed in their proper methodological context. Only when the method and the purpose of the numerical examples have been properly understood, can those examples be seen for what they are, not as examples of a redistribution of already realized value functional for the equilibrium conditions of simple or expanded reproduction, not as an input-output table but — once it has been understood that we deal with that particular moment in which realization and redistribution of value and surplus value coincide chronologically — as examples of how surplus value must be redistributed if the possibility for competitive capitalism to function, i.e. for capital to accumulate, must be accounted for theoretically.

Once the transformation problem is posed in its proper problematic and solved, it can be seen that:
(1) in Marx's numerical examples there is no inconsistency. These examples depict the computational specificity of the transformation problem (not its solution).

(2) these examples are also correct in the sense that, if individual values realize themselves as prices of production, as social values, there must be equality between the total sum of individual and social values of the commodities produced and realized in a certain period, and that total surplus value must be equal to the total sum of profits (assuming, of course, that no surplus value is appropriated by other, unproductive, capitalists, by the state as taxes, etc.).

(3) the objection that the condition of equilibrium (simple or expanded) are violated becomes irrelevant, since the transformation problem has nothing to do with the reproduction schemes and with the conditions of equilibrium they are supposed to explicate.

From von Bortkiewicz on, the transformation problem has been dealt with on the terrain of the ideological opponent, a problem of dialectical logic has been debased to one of mathematical computation implicitly immersed into an alien problematic. Even within the Marxist camp, the validity of Bortkiewicz' objection has been and is still accepted almost unanimously. As B. Rowthorn puts it, 'Marxists still find themselves trapped within a debate whose terms of reference were laid down by vulgar economists such as Böhm-Bawerk, on the one hand, and neo-Ricardians such as Bortkiewicz on the other' (1979, p. 75). Nor is there anything to be gained in choosing the Sraffian path which is based on the consideration that 'since both inputs including labour power, and outputs have to be transformed into price-terms, and hence in all probability the rate of profits will be affected, these have all to be determined simultaneously and interdependently, i.e. by solving a set of simultaneous equations' (Dobb, 1973, pp. 159-160). This approach too rests on the false assumption that the values of both c and a as outputs realize themselves simultaneously. This erroneous conception bars the study of technical progress so that not by chance the neo-Ricardian model of the capitalist economy assumes no technical change.11 This is one of the many features which indicate the width of the gulf which separates Marx from Sraffa (so that the Sraffa-based critique cannot be thought of as immanent to Marx's theory) as well as the lack of realism which characterizes the Sraffian model. All mathematical 'improvements' of the Bortkiewicz-Winternitz-Seton, and further type or of the Sraffa type are mathematical expressions of a logic alien to Marx's. The transformation problem has to be re-defined, in terms of the logic inherent in Marx's theoretical construction, before it can be solved.12 When this is done, the Ricardian transformation problem turns out to be like the character of a play who, after having wandered in a world of papier mâché, finds out that he does not exist.

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Notes
* This is a shorter version of Carchedi, 1983d. Copies of it can be requested to the author, Department of Economics, University of Amsterdam, Jodenbreeestraat 23, Amsterdam. The Netherlands.
1. For a very good discussion of Böhm-Bawerk, see G. Kay, 1975, pp. 46-67.
2. Historically, the formation of prices of production requires a 'higher development of capitalist production' than the formation of market values. (Marx, 1967c, p. 180). In my opinion, whether this is so or not, is irrelevant for the explanation of the mechanism through which values appear as prices. Historical and logical explanations are related but different aspects of the explanatory scheme. The former helps, but does not provide the key for the understanding of the latter. For a different view, see R. Murray, 1977.
3. Marx, 1967c, p. 179. It should be noted that for Marx the transformation problem is the problem of the transformation of individual values into market prices through market values and prices of production and not one of the transformation of values into prices of production (as it is usually assumed). Thus, to deal with the transformation problem means at the same time to deal with the Marxian theory of prices, or at least with a facet of it. (For a different interpretation, see G. Dostaler, 1982). However, in what follows, I will deal only with the transformation of individual values into prices of production, in order to counter the most commonly accepted objections to Marx's procedure. The analysis of the formation of market prices as well as their manifestation in money terms is subsequent to the solution of the transformation of production and is not the specific topic of this paper. In a companion article to this one, I shall deal with the mechanism through which values realize themselves as international market prices through international prices of production. But, for the purposes of the present article, it is the relation between values and prices of production which must be analyzed.
4. Two points deserve to be mentioned here summarily, for lack of space. First, table 1 does not depict the process of the equalization of the rate of profit. It depicts neither the increase in the number of use values produced as a result of the introduction of more efficient techniques (and which, by being sold at a price of production higher than their individual value, allow an appropriation of surplus value through unequal exchange, thus generating a hierarchy of rates of profit) nor the incessant movements of capitals from the less to the more efficient techniques and/or branches (which works towards the equalization of these different rates since capital inflows increase supply, decrease prices of production and thus the surplus value appropriated). The
movement towards a hierarchy of rates of profit is the counter-tendency, the movement towards the equalization of the different rates is the tendency. Table 1 does not depict the interplay of technological innovation and of capital movement, nor the succession of tendency and counter-tendency but gives, and is meant to give, only a static and incomplete view of the real process: it is meant to show the unequal exchange inherent in the formation of the prices of production under tendencial conditions. More specifically, we should distinguish between two levels of analysis in order to assess correctly the theoretical status of table 1. When only production is considered (i.e. when exchange is not allowed to exert any influence), the assumption is made that tendentially the value produced is equal to the value appropriated. Under these assumptions, the only way the rate of profit can be equalized is through an equalization of the organic compositions. The tendency is thus towards an equalization of the rates of profit through an equalization of the organic compositions (due to capital movements) and the counter-tendency is towards a hierarchy of rates of profit through a hierarchy of organic compositions (due to technological innovations). The price mechanism here explains how different capitals get a uniform rate of profit under tendencial conditions (i.e. when the organic compositions are equal) through equal exchange. When exchange is considered, the assumption is made that tendentially the value produced differs from the value appropriated. Under these conditions we have to assume unequal exchange and thus different organic compositions of capital. In other words, there is not only a move towards a hierarchy of organic compositions (clustered around an average) such that each organic composition gets the average rate of profit. The counter-tendency is the upsetting of this condition through the introduction of new (i.e. higher organic composition) techniques which allow the appropriation of a higher than average rate of profit. Capital movements will tend to restore that hierarchy in which all capitals will get the average rate of profit.

On the basis of these considerations we can now understand correctly the nature of table 1. Table 1 is, of course, at the level of abstraction considering both production and exchange. However, it does not depict the succession of tendency (the equalization of the rate of profit on the basis of a hierarchy of organic compositions brought to this tendencial state through capital movements and counter-tendency (the upsetting of this hierarchy due to technological innovations). Better said, this table does not depict the succession of tendency towards a tendencial state (which only by chance realizes itself) through the constant realization of a series of counter-tendencial moments. Table 1 gives only a static picture, that of the tendency, in order to isolate the cause of unequal exchange, the difference in the organic compositions. In other words, table 1 shows the unequal exchange inherent in the formation of the prices of production when the cause of unequal exchange (the difference in organic compositions) is isolated for analysis. Or, table 1 shows how the price mechanism rewards the high composition capitals and penalizes the low composition ones under tendencial conditions, when all capitals get the average rate of profit (through the price mechanism) so that there is no need for capital movements. Secondly, table 1 shows that there must be unequal exchange among capitals with different organic compositions. This does not contradict the basic notion that, to understand the production of surplus value and thus capital accumulation, we must assume that commodities (and thus also labour power) must exchange at their value. In fact, equal exchange must be assumed at the highest level of abstraction in order to explain the production of surplus value (the excess value above the value of labour power); unequal exchange must be assumed at a more concrete level of abstraction to explain the realization of surplus value through exchange. As Dobb (1973) correctly points out, the origin of surplus value must be analyzed before we can explain how that surplus value is realized/distributed due to differences in organic compositions. At the highest levels of abstraction (that of Capital I) exchange is not considered explicitly but is inherent in the theory of production since the equality of the organic compositions is implicitly assumed. At the lower level of abstraction (that of Capital III) exchange is considered explicitly by developing what is inherent in Capital I, i.e. by modifying the assumption of equal organic compositions. There is thus no need to choose between equal and unequal exchange as B. Bradley (1975), for example, does. For a more detailed treatment of these points, see Carchedi, 1983d.

5. For a discussion of the conditions under which both equalities hold, see P. Salama, 1975, p. 159.

6. 1975. Fine and Harris claim that 'Seton's difference from neo-Ricardianism arises because he does transform values into prices of production without reference to the technical relations of production which are so fundamental to neo-Ricardianism. This is simply done by setting up simultaneous equations between the price rate of profit and the ratios of prices of production to values. This involves correcting Marx's failure to transform the original costs of production from values into prices of production.' I disagree. Seton set up his system of simultaneous equations by multiplying cost inputs by prices. Seton's cost inputs are the amount of product of industry j 'reckoned in terms of labour time' (as well as other's) contribution is W. Baumol, 1974, pp. 53-54. See I.I. Rubin, 1977, pp. 236–7. The third correctly stresses that 'the conception of that which-has-to-be-calculated' must come before calculation and develops an ingenious procedure (the iterative one) but fails in its own terms. In fact, that procedure is alien to the Marxian one so that implicitly a dialectic solution is reduced to a numerical one which, moreover, shares with the von Bortkiewiczian solution the disadvantage of having to break the quantitative relation between surplus value and profits. See A. Shaikh, 1977, pp. 106–139. The fourth makes a more definite attempt to rediscover Marx's problematic but it too does not manage to break away from the temptation.
to look for a numerical solution starting from the premise of the validity of von Bortkiewicz' critique. See I. Bernstein, 1976, p. 254. For a more detailed discussion of these authors, see Carchedi, 1983d. After this article had been submitted and accepted for publication, I saw B. Fine, 1983. His note supports some of the arguments to be submitted below and adds the interesting point that Marx's critics fail to distinguish between the organic and the value composition of capital: this failure is the cause of their mistaken formulation of the transformation problem. Fine's note, however, offers no adequate answer to the circularity and infinite regression critiques.


9. For example, according to B. Fine and L. Harris 'this Marx recognizes for he observes that the value of capital advanced may diverge from the price of production of that capital, but he makes no effort to correct this discrepancy' (1979, p. 25). Marx's text reads 'We have seen how a deviation in the price of production of a commodity which deviates from the value of a commodity, entering into the cost price of other commodities as one of its elements, so that the cost price of a commodity may already contain a deviation from value in those means of production consumed by it, quite aside from a deviation of its own which may arise through a difference between the average profit and the surplus value. It is therefore possible that even the cost price of a commodity produced at a price equal to its average composition may differ from the sum of the values of the elements which make up this component of their price of production.' (1967c, pp. 206-7).

10. The relation between mathematical and dialectical logic is a complex one and cannot be dealt with here for lack of space. For a treatment of this problem, see Carchedi, 1983a.

11. J.R. Ernst correctly emphasizes the lack of chronological sequence of different production periods in the neo-Ricardian model and the significance for this of their theorization of an economy in which technical change is absent (1982, pp. 85-94). However, Ernst thinks that this is admissible if the conditions of production do not change from period to period. I, on the other hand, argue that the simultaneous determination of values in the Ricardian scheme is incorrect to depict even the case of an economy with no technical progress. In fact, the determination of the value of c as an output must always be separated chronologically from that of a as an output. In case of joint production, the value of c as an input will remain the same as the value it had (in the previous period) as an output, but this does not justify the theoretical mistake of collapsing the two production periods and moments of realization into just one.

12. In this paper, I have considered the Sraffian critique of the transformation problem only in so far as it is a variation of the circularity critique. According to I. Steedman, Marx assumes that by dividing the surplus value by the constant and variable capitals (453) we get the rate of profit, 'but then derives the result that prices diverge from values, which means precisely, in general, that S/(C+V) is not the rate of profit' (1977, p. 31). This paper disposeds, I believe, of this critique. The same author mentions also two further objections. First, the transformation problem is deemed to be a pseudo-problem and a redundant one. There is, in fact, so runs, no need to derive profits from surplus value because the rate of profit and the prices of production can be computed once the technical conditions of production and the real wages, both specified in physical quantities, are known. Values can be determined if the physical data relating to methods of production are known, but such a determination is redundant. A reply to this objection would show that the Sraffian prices refer to individual, embodied, labour time rather than to socially necessary labour time and that this is the source of inconsistency, circularity, and a simplistic (rather than a simplifying) model of the capitalist economy. Secondly, there is the objection that in case of joint production the calculation of value can give either indeterminate (Sraffa) or negative (Steedman) results. In Sraffa's view, the result is determined from the total price of the products (and thus prices to be determined) than there are processes (and thus equations) to determine them. Negative values result — in Steedman's view — because positive profits can coexist with negative surplus value. The answer to this objection would show both that it is possible to determine the value of the individual components of a joint product within Marx's theory of value and that the Sraffa-based treatment of joint production is one of the weakest spots in the Sraffian system, the one point where the logic of the assumptions from values arises from the fact that joint products are produced.

References

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The logic of prices as values